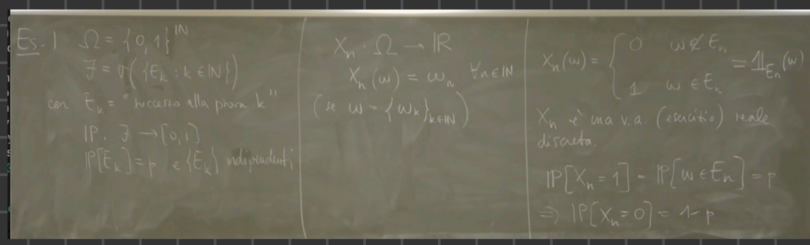
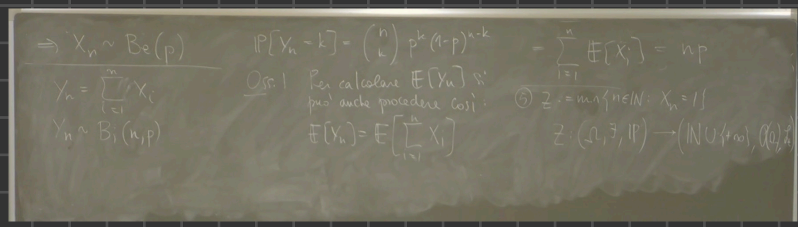
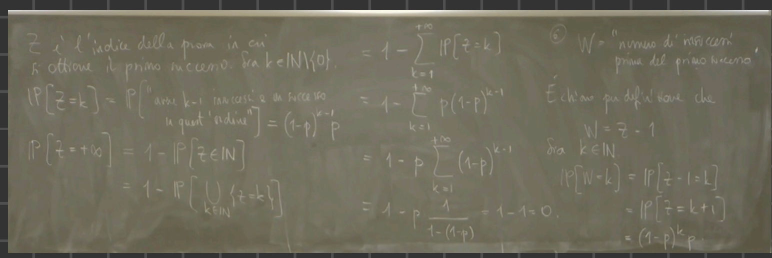
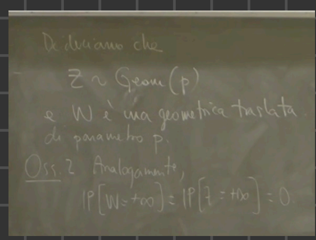
Esercitazione 6

- esercizio 1









finito nello specifico

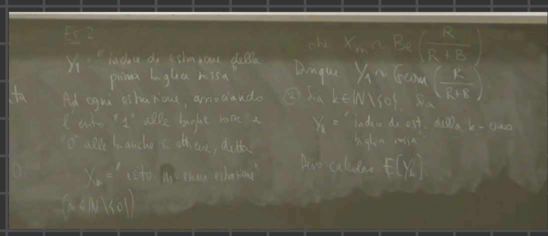
- X_n è una v.A (esempio) reale discreta P

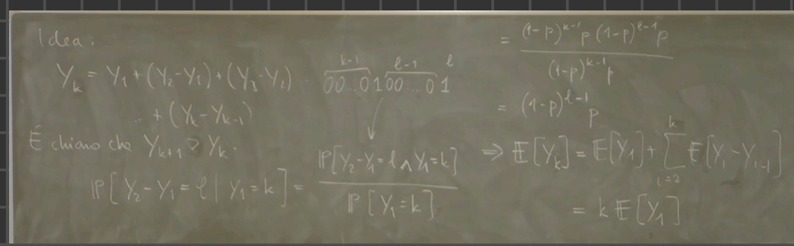
- significato: n-esima prova successo

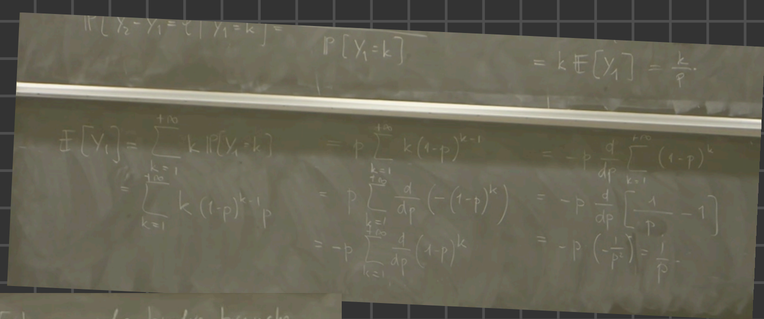
- $\mathbb{P}[X_n = 1] = \mathbb{P}[\omega \in E_n] = p \Rightarrow \mathbb{P}[X_n = 0] = 1-p$

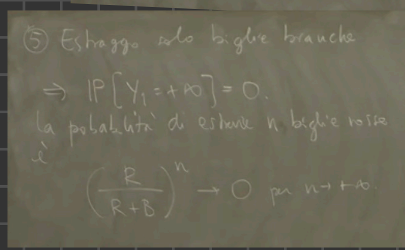
Esercizio 2

- caso con reimissione

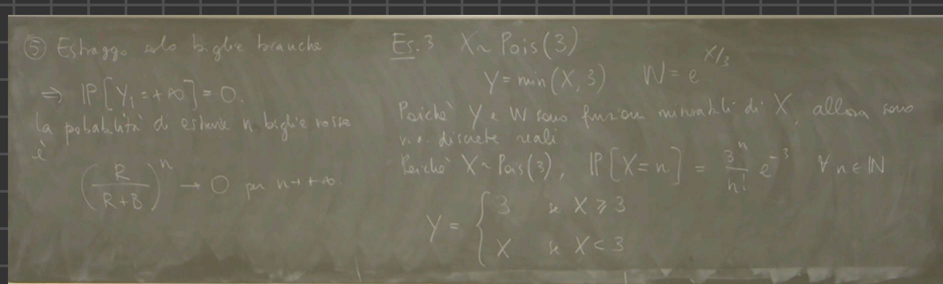


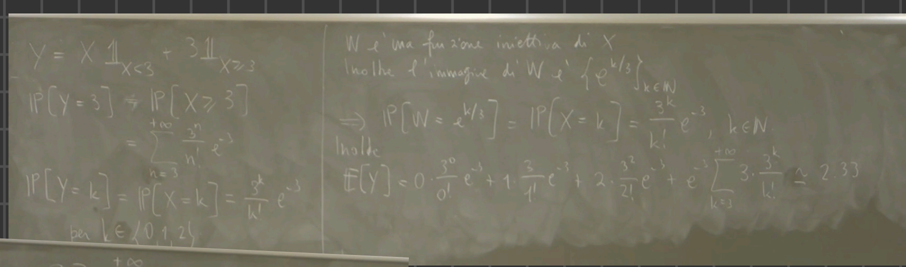


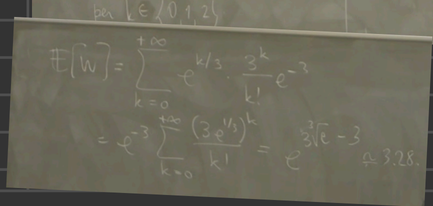




Esercizio 3







$$\mathbb{E}[W] = \sum_{k=0}^{+\infty} e^{k/3} \cdot \frac{3^k}{k!} e^{-3} = e^{-3} \sum_{k=0}^{+\infty} \frac{(3e^{1/3})^k}{k!} = e^{3\sqrt[3]{e}-3} \simeq 3.28.$$

Es. 4

$$\Omega = \{B, RB, RRB, \dots, \underbrace{R\dots R}_{9}B, \underbrace{R\dots R}_{10}\}.$$

Presa come σ-algebra $\mathcal{P}(\Omega)$ calcoliamo $\mathbb{P}$.

$$\mathbb{P}[B] = \frac{1}{2}$$

$$\mathbb{P}[RB] = \frac{1}{2} \cdot \frac{1}{3}$$

$$\mathbb{P}[RRB] = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4}$$

$$\mathbb{P}[\underbrace{RR\dots R}_{k}B] = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{4}} \cdots \frac{k}{k+1} \cdot \frac{1}{k+2} \quad (k \in \{1, \dots, 9\}) = \frac{1}{(k+1)(k+2)}$$

$$\mathbb{P}[\underbrace{RR\dots R}_{10}] = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{4}} \cdots \frac{\cancel{10}}{11} = \frac{1}{11}.$$

Se X è la v.a. che conta le estrazioni

$$\mathbb{P}[X = k+1] = \frac{1}{(k+1)(k+2)} \qquad k = 0, \dots, 8$$

$$\mathbb{P}[X = 10] = \frac{1}{11} + \frac{1}{10 \cdot 11} = \frac{1}{10}$$

$$\mathbb{E}[X] = \sum_{k=0}^{8} \frac{\cancel{k+1}}{\cancel{(k+1)}(k+2)} + 10 \cdot \frac{1}{10} \simeq 2.9\dot{3}$$

Y = "estrazioni effettuate"

$$\mathbb{P}[Y \geq n] = \mathbb{P}[\text{prime } n-1 \text{ biglie rosse}] = \frac{1}{2} \cdot \frac{2}{3} \cdots \frac{n-1}{n} = \frac{1}{n}$$

$$\mathbb{P}[Y = +\infty] = \mathbb{P}\Big[\bigcap_{n \in \mathbb{N}} \{Y \geq n\}\Big] = \lim_{n \to +\infty} \frac{1}{n} = 0.$$

$$\mathbb{E}[Y] = \sum_{k=1}^{+\infty} k\, \mathbb{P}[Y = k]$$

$$= \sum_{k=1}^{+\infty} k\, \mathbb{P}[\underbrace{RR\dots R}_{k-1}B]$$

$$= \sum_{k=1}^{+\infty} \cancel{k} \left[\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} \cdots \frac{k-1}{\cancel{k}} \cdot \frac{1}{k+1}\right]$$

$$= \sum_{k=1}^{+\infty} \frac{1}{k+1} = +\infty.$$