UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

University Examinations 2015/2016

Final Assessment for BSc II FM and BSc II GENERAL

Friday May 6, 2016

MTC 2201 : PARTIAL DIFFERENTIAL EQUATIONS

Time allowed: 3 hours

Instructions

(i) Answer **five** questions.

(ii) Write both sides of the paper but begin a new question on a fresh page.

(iii) Only approved basic scientific calculators may be used in this examination.

(iv) Show all your solutions clearly and neatly.

(v) Where applicable, use $P = Z_x$ and $Q = Z_y$.

1 (a) (i) When is a function said to be piecewise continuous?

(2 marks)

(ii) Consider an infinite series $\sum_{n=1}^{\infty} U_n(x)$. What does it mean to say that this infinite series converges to $f(x)$ in some interval?

(2 marks)

(b) Graph $f(x)$ that has period 6 given that

$$f(x) = \begin{cases} 5 & 0 \leq x < 2 \\ -4 & 2 \leq x < 4 \\ 2 & 4 \leq x < 6 \end{cases}$$

(2 marks)

(c) (i) Define a Fourier series.

(ii) Compute the Fourier series for $f(x)$ with period 8 where

$$f(x) = \begin{cases} 2 - x & 0 < x < 4 \\ x - 6 & 4 < x < 8 \end{cases}$$

(14 marks)

2 (a) (i) Explain the following:

- a solution to the PDE,
- the order of a PDE.

(2 marks)

(ii) Explain any two applications of the Partial Differential Equations in real life situations.

(4 marks)

(b) Explain how a two-dimensional Laplace's equation is derived from a two-dimensional heat flow equation.

(4 marks)

(c) Separate the PDE $t^3 U_{xx} + x^3 U_{tt} = 0$ into two ordinary differential equations in t and x.

(5 marks)

(d) Determine the solution to the PDE:

$$Py + Qx - PQ = 0$$

(5 marks)

3 (a) Prove that $\frac{1}{2}+\cos t+\cos 2t+\cos 3t+\ldots+\cos Mt = \frac{\sin(M+\frac{1}{2})t}{2\sin\frac{1}{2}t}$.

(5 marks)

(b) By the method of separation of variables, show that the heat equation $\frac{\partial u}{\partial t}(x,t) = \beta\frac{\partial^2 u}{\partial x^2}(x,t)$; $0 < x < l$, $t > 0$ can be *reduced* to solving two ordinary differential equations.

(4 marks)

(c) Solve $\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2}$, $0 < x < 5$, $t > 0$, given that $u(0,t) = u(5,t) = 0$, $u(x,0) = 3\sin 5\pi x - 7\sin 4\pi x - 2\sin 3\pi x + 11\sin 8\pi x$, $|u(x,t)| < M$ where the last condition states that u is bounded for $0 < x < 4$, $t > 0$.

(11 marks)

4 (a) Classify each of the following equations as elliptic, hyperbolic or parabolic.

(i) $9U_{xx} = U_{tt} + 6U_t$,

(ii) $k^2U_{xx} = U_{tt}$.

(3 marks)

(b) Solve the following PDEs

(i) $\frac{\partial^2 z}{\partial x^3} - 5\frac{\partial^2 z}{\partial x\partial y} + 6\frac{\partial^2 z}{\partial y^2} = e^{x+y}$,

(4 marks)

(ii) $Z_{tt} - 3Z_t + 2t = 3e^{-t} - 10\cos 3t$.

(6 marks)

(c) (i) State the superposition principle.

(ii) Solve the boundary value problem $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ subject to condition $u(x,0) = 12e^{6x} - 11e^{-5x} + 7e^{2x} - 8e^{-3x}$.

(7 marks)

5 (a) Find a PDE whose solution is the surface of $Z = f(x^2+y^2)$ where f is an arbitrary differential function of x and y.

(4 marks)

(b) Formulate a partial differential equation from the function $ax^2 + by^2 + z^2 = 1$ where a and b are constants, and x, y and z are variables.

(5 marks)

(c) Solve the following wave equation subject to the boundary conditions given.

$$\frac{\partial^2 y}{\partial t^2} = 25\frac{\partial^2 y}{\partial x^2}$$

$$y(0,t) = y(5,t) = 0,\ y(x,0) = 0,\ y_t(x,0) = f(x),$$
$$(0 < x < 5,\ t > 0)$$

where $f(x) = 5\sin 9\pi x - 7\sin 11\pi x - 11\sin 15\pi x$.

(11 marks)

6. (a) (i) With relevent examples, differentiate between the following PDEs:-

- a partial differential equation(PDE) and an Ordinary differential equation,
- linear and non linear PDE,
- second order linear and second order non linear PDE.

(5 marks)

(ii) A steady-state temperature function of $u(x,y)$ for a thin, flat plate satisfies Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Is $u(x,y) = e^{-x}\sin y$ a solution of Laplace's equation above.

(4 marks)

(b) Solve the following PDE.

$$Z_{tt} - 3Z_t + 2Z = 3e^{-t} - 10\cos 3t$$

(6 marks)

(c) (i) Solve the equation $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$

(ii) Find the particular solution for which $z(x,0) = x^2$, $z(1,y) = \cos y$.

(5 marks)

END