

UGANDA MARTYRS UNIVERSITY, NKOZI

FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS

UNIVERSITY SPECIAL/SUPPLEMENTARY EXAMINATIONS

2014/2015

First Year Bachelor of Science (FM and GEN)

MTC 1202 Ordinary Differential Equations(ODE)

Date : Aug, 2015

Time : 3 Hours (10:00pm - :100 pm)

Instructions

- (i) *Read through the paper carefully and follow instructions on the answer booklet.*
- (ii) *Attempt any **Four** (4) questions.*
- (iii) *Do not write any thing on this question paper:*
- (iv) *Calculators and mathematical tables may be used.*
- (v) *Neat work is highly recommended.*

1. a(i) Define a differential equation.

[2 marks]

(ii) Give two examples of differential equations

[2 marks]

- (b) Solve $\frac{dy}{dx} = \frac{x^2 + 2}{y}$, with $y(0) = 0$ [6 marks]
- (c) A bacteria culture is known to grow at a rate proportional to the amount present. After one hour, 1000 strands of the bacteria are observed in the culture, and after four hours 3000 strands; Find
- an expression for the approximate number of strands of the bacteria present in the culture at any time(t) [7 marks]
 - the approximate number of strands of the bacteria originally in the culture [3 marks]
2. (a) Solve $y'' + y' + y = 0$ with $y(0) = 1$ and $y'(0) = 1$. [7 marks]
- (b) Find a particular solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - x = \sin x$ [7 marks]
- (c) Determine whether $ydx + xdy = 0$ with boundary condition $y(1) = 2$ is exact hence find the solution [6 marks]
3. a(i) Give typical steps of Euler's method for approximating IVP. [5 marks]
- (ii) Use Euler's method with a step size of $h = 0.5$ to approximate the solution of the equation $y' = x^2 + 2$, satisfying $y(0) = 0$ for $0 \leq x \leq 2$. [8 marks]
- (iii) How does your solution in (ii) compare with the exact solution [4 marks]
- (b) State three errors encountered while finding approximations obtained by numerical methods to solve IVP [3 marks]
4. a(i) Define a power series in x . [2 marks]

- (ii) Determine the radius of convergence and interval of convergence for

$$\sum_{n=1}^{\infty} \frac{2^n}{n} (4x - 8)^n \quad [7 \text{ marks}]$$

- (b) Find the first four terms in each portion of the series solution around

$$x_0 = 0 \text{ for}$$

$$(x^2 + 1)y'' - 4xy' + 6y = 0. \quad [11 \text{ marks}]$$

5. (a) Use the method of variation of parameters to solve

$$y''' + y' = \cos x \quad [8 \text{ marks}]$$

- b(i) Show that the three solutions $y_1(x) = x$, $y_2(x) = x \ln x$ and $y_3(x) = x^2$ of the third order equation

$$x^3 y''' - x^2 y'' + xy' - 2y = 0 \text{ are linearly independent on the interval } x > 0 \quad [5 \text{ marks}]$$

- (ii) Find the particular solution that satisfies the initial conditions $y(1) = 3$,

$$y'(1) = 2, \quad y''(0) = 0 \quad [7 \text{ marks}]$$

6. (a) Define the following terms as applied to differential equations;

- (i) solution [2 marks]

- (ii) order [2 marks]

- (b) Solve $(D^3 - 5D + 6)y = e^x$ [5 marks]

- (c) Show that $y = A \sin \omega x$ and $y = B \cos \omega x$ will satisfy $\frac{d^2 y}{dx^2} + \omega^2 y = 0$, where A and B are constants [6 marks]

- (d) A quantity undergoes natural decay with relative growth rate -1. Initially there are 2 units of the quantity. At what time will there remain 1 unit of the quantity? (5 marks)