UGANDA MARTYRS UNIVERSITY
FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

University Examinations, Semester I 2012/2013

First Year Examination for the Degree of Bachelor of Science
(FM, IT, ECON, GEN)

MTC 1102 Elements of Mathematics

Friday, 14 December 2012 Time: 9:00 - 12:00 noon

Instructions

(i) Answer **Five** questions

(ii) Write on both sides of the paper but begin a new question on a fresh page.

1. (a) State in words and then write in tabular form

   (i) $A = \{x \mid x^2 - 3x - 2 = 0\}$ (2 Marks)

   (ii) $B = \{x \mid x \text{ is positive}, x \text{ is negative}\}$ (2 Marks)

   (b) Write these sets in set - builder form

   (i) $A = \{1, 3, 5, 7, ...\}$ (2 Marks)

   (ii) $B = \{a, b, c, d, e\}$ (2 Marks)

   (c) (i) What does it mean to say that sets A and B are equal? (2 Marks)

   (ii) Show that if A is a subset of B and B is a subset of C, then A is a subset of C. (4 Marks)

   (d) Let A be any set.

   (i) Define the power set of A. (1 Mark)

   (ii) Let $A = \{a, b, c, d\}$. Find the power set of A. (2 Marks)

   (e) (i) Distinguish between a null set and an infinite set. (2 Marks)

   (ii) Let $A = \{x \mid x^2 = 4 \text{ and } x \text{ is odd}\}$. Classify set A as infinite or null set. (1 Mark)

2. (a) Find x and y such that $\begin{bmatrix} 3x & 5 \\ -1 & 4x \end{bmatrix} + \begin{bmatrix} 2y & -3 \\ -6 & -y \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ -7 & 2 \end{bmatrix}$ (3 Marks)

   (b) Find the determinant of each of the following matrices

   (i) $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ (1 Mark)

   (ii) $\mathbf{B} = \begin{bmatrix} 3 & -2 & 1 \\ -1 & 5 & 1 \\ 2 & 3 & 1 \end{bmatrix}$ (3 Marks)

   (c) Using Cramer's rule, solve the following systems of linear equations

   (i)

$$\begin{aligned} 2x - 3y &= 1 \\ -4x - 5y &= 2 \end{aligned}$$

   (3 Marks)

   (ii)

$$\begin{aligned} 2x + y &= 2 \\ x - y + z &= -1 \\ x + y + z &= 2 \end{aligned}$$

   (5 Marks)

   (d) (i) Define the inverse of a square matrix A. (1 Mark)

   (ii) Let $\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 1 & 3 \end{bmatrix}$. By finding the inverse of A solve the linear system

$$\begin{aligned} 3x - 2y &= 2 \\ x + 3y &= -1 \end{aligned}$$

   (4 Marks)

3. (a) (i) Define the terms conjunction, disjunction and negation. (3 Marks)
   (ii) State the fundamental property of a statement. (1 Mark)
   (iii) Consider the statement $2 \times 2 = 9$. State the negation of this statement. (1 Mark)

   (b) Let p be "He is tall" and q be "He is ugly". Give a simple verbal sentence which describes each of the following
   (i) $p \wedge q$
   (ii) $\sim (\sim p \vee q)$
   (iii) $p \vee (\sim p \wedge q)$ (3 Marks)

   (c) Determine the truth value of each of the following statements
   (i) It is not true that $1 + 1 = 3$ or $2 + 1 = 3$. (2 Marks)
   (ii) If $3 + 2 = 7$, then $4 + 4 = 8$. (2 Marks)

   (d) (i) Let $f(p, q) = \sim p \vee (p \longrightarrow q)$ and $g(p, q) = (p \longleftrightarrow \sim q) \wedge q$. Find $f(p, q) \wedge g(p, q)$ and $f(p, q) \longrightarrow g(p, q)$. (4 Marks)
   (ii) Let $f(p, q) = \sim p \wedge (p \longrightarrow q)$ and let p_o be "$2 + 2 = 5$" and q_o be "$1 + 1 = 2$". Give a verbal sentence for $f(p_o, q_o)$ and find its truth value. (4 Marks)

4. (a) Find the truth table of each proposition
   (i) $\sim p \wedge q$ (2 Marks)
   (ii) $(p \wedge q) \longrightarrow (p \vee q)$ (3 Marks)

   (b) (i) Distinguish between a tautology and a contradiction. (2 Marks)
   (ii) Show that $(p \wedge q) \longrightarrow (p \vee q)$ is a tautology and $p \wedge \sim p$ is a contradiction. (4 Marks)

   (c) (i) Explain what it means to say that two propositions $P(p, q, ...)$ and $Q(p, q, ...)$ are logically equivalent. (1 Mark)
   (ii) Verify that $p \vee q \equiv \sim (\sim p \wedge \sim q)$. (3 Marks)

   (d) Let A, B and C denote electrical switches and Let A and A' denote switches with the property that if one is on, then the other is off. Describe the bahaviour of the circuit $(A \wedge B') \vee [(A' \vee C) \wedge B]$. (5 marks)

5. (a) (i) Distinguish between a permutation and a combination. (2 Marks)

(ii) Define the permutation of n objects taken r at a time. (1 Mark)

(ii) State the number of combinations of n objects taken r at a time. (1 Mark)

(b) Find

(i) C_3^8 (2 Marks)

(ii) n if $C_2^n = 15$ (4 Marks)

(c) (i) From a committee of 8 people, in how many ways can we choose a subcommittee of 2 people? (2 Marks)

(ii) Out of a standard 52 card deck, how many 5 card hands will have 3 aces and 2 kings? (3 Marks)

(d) A catering service offers 8 appetizers, 10 main courses and 7 desserts. A banquet chairperson is to select 3 appetizers with no repeats, 4 main courses and 2 desserts. In how many ways can this be done? (5 Marks)

6. (a) Prove that

(i) $(A \cup B) \cap (A \cup B') = A$ (3 Marks)

(ii) $A \cap (A' \cup B) = A \cap B$ (3 Marks)

(b) Define a function $f : \mathbb{R} \longrightarrow \mathbb{R}$ by $f(x) = x^2 - 3x + 2$. Find

(i) $f(-3)$

(ii) $\frac{f(x+h)-f(x)}{h}$ (5 Marks)

(c) Let $f(x) = \frac{x}{2x^2-x+6}$. Find the values of x for f(x) is undefined. (4 Marks)

(d) Let $f(x) = x^2 + 5$. Find the formula for the inverse function f^{-1} and hence compute $f^{-1}(9)$. (5 Marks)