UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL ASSESSMENT
SEMESTER II

SECOND YEAR EXAMINATIONS FOR BSc Education & BSc (Gen)

MTC 2201: Partial Differential Equations

DATE: Friday 19^{th} May, 2023
TIME: 9:30am - 12:30pm
DURATION: 3 Hours

Instructions:

1. *Carefully read through ALL the questions before attempting the examination.*
2. ***ANSWER ANY FOUR Questions*** *(Each question carries a total of 25 marks)*
3. *No* ***names*** *should be written anywhere on the examination book.*
4. *Ensure that your* ***Reg number*** *is indicated on all pages of the examination answer booklet.*
5. *Ensure your work is* ***clear*** *and* ***readable****. Untidy work shall be penalized*
6. *Any type of examination Malpractice will lead to automatic disqualification*
7. *Do not write anything on the questions paper.*

1. a. Classify each of the following equations as ordinary or partial differential equations; state the order and degree of each equation; and determine whether the equation under consideration is linear or non-linear.

i. $\dfrac{\partial^2 u}{\partial x^2} = \left(\dfrac{\partial^2 u}{\partial x^2}\right)^3 + y$

ii. $\dfrac{du}{dx} - \left(\dfrac{du}{dx}\right)^2 = x$

[6 marks]

b. Find a solution to the given initial-boundary value problem

$$\begin{cases} u_t = u_{xx} & 0 < x < \pi \\ u(0,t) = 0, u(\pi,t) = 3\pi & t \geq 0 \\ u(x,0) = \pi - x & 0 < x < \pi. \end{cases}$$

[19 marks]

2. a. Obtain the Fourier series expansion of the following functions

i.

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi. \end{cases}$$

ii. $f(x) = |x|, -1 < x < 1$

[15 marks]

b. Classify the following differential equations as parabolic, Hyperbolic or elliptic. Give reasons for your answers.

i. $\dfrac{\partial^2 u}{\partial y^2} - 3\dfrac{\partial^2 u}{\partial x^2} = -2\dfrac{\partial^2 u}{\partial x \partial y}$

ii. $\dfrac{\partial^2 u}{\partial y^2} = -\dfrac{\partial^2 u}{\partial x^2}$

[10 marks]

3. a. Find the Fourier series expansion of $f(x) = x$ on $[0, 1]$. [13 marks]

b. Hence solve the differential equation

$$y'' + 2y = x$$

with boundary conditions $y(0) = y(1) = 0$. [12 marks]

4. a. Find the general solution of

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

where $u = u(x, y)$. **[6 marks]**

b. Let $u = u(x, y)$. By integration, find the general solution to $u_{xx} = 0$. **[6 marks]**

c. i. Classify the function as odd or even

$$g(x) = \begin{cases} -2, & -\pi < x < 0 \\ 2, & 0 < x < \pi. \end{cases}$$

ii. Hence, determine the Fourier series expansion of the function g. **[13 marks]**

5. a. Determine the regions in the xy-plane for which the equation

$$(xy - 1)\frac{\partial^2 u}{\partial x^2} + (x - 2y)\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + x^2 y u = 0$$

is:

i. hyperbolic,
ii. parabolic,
iii. elliptic.

[9 marks]

b. Show that $u(x, y) = \cos(x^2 + y^2)$ satisfies the equation

$$x u_y - y u_x = 0$$

subject to $u(0, y) = \cos y^2$. **[4 marks]**

c. Find a relationship between a and b if $u(x, y) = f(ax + by)$ is a solution to the equation $3u_x - 7u_y = 0$ for any differentiable function f such that $f'(x) \neq 0$ for all x. **[6 marks]**

d. Use the method of separation of variables to solve the differential equation $u_x - u = 0$ where $u = u(x, t)$. **[6 marks]**

END

Use where required: