UGANDA MARTYRS UNIVERSITY, NKOZI

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

BSC II GEN AND BSC II FM END OF SEMESTER 1 EXAMINATIONS

MTC2101 CALCULUS III

YEAR 2013/2014

DATE: 19th **DEC** 2013
TIME: $10:00am - 1:00Pm$

Instructions

(i) *Attempt any five questions*

(ii) *Read through the paper carefully and follow instructions on the answer booklet.*

(iii) *Calculators and mathematical tables may be used.*

(iii) *Neat work is highly recommended.*

(1) (a) Given two vectors A and B, define the following;

(i) Dot product of A and B (2 marks)

(ii) Cross product of A and B. (2 marks)

(b) Prove that $A.(B \times C) = B.(C \times A) = C.(A \times B)$ (6 marks)

(c) If $A = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$,$B = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $C = \mathbf{i} + \mathbf{3j} - \mathbf{2k}$. Find

(i) $|(2A + B).(A - 2B)|$ (4 marks)

(ii) $(A \times B) \times C$ (5 marks)

(d) If $\mathbf{R} = \mathbf{x^2yi - 2y^2zj + xy^2z^2k}$, find $|\frac{\partial^2 \mathbf{R}}{\partial x^2} \times \frac{\partial^2 \mathbf{R}}{\partial y^2}|$at the point (2,1,-2)(6 marks)

(2) (a) Find a polar equation corresponding to the given rectangular equation.

(i) $y^2 - x^2 = 4$ (3 marks)

(ii) $x^2 + y^2 = x$ (3 marks)

(b) Find the area inside the limacon r=$3 + 2cos0$ and outside the circle $r = 2$(14 marks)

(3) (a) State what is meant by curl and divergence of a vector field $\mathbf{F}$. (04 marks)

(b) If ϕ=$xy + yz + zx$ and $A = x^2y\mathbf{i} + y^2z\mathbf{j} + z^2x\mathbf{k}$

(i) $(A.\nabla\phi)$ (05 marks)

(iii) $\nabla.(\phi A)$ (05 marks)

(c) Find a unit normal to the surface $x^2y - 2xz + 2y^2z^4 = 10$ at a point (2,1,-1) (06 marks)

4a(i) (i) State what is meant by a function $f(t)$ being vector valued. (2 marks)

(ii) For avector valued function $f(t)$, define what is meant by the derivative $f'(t_0)$ of $f(t)$ at a point t_0. (2 marks)

(b) Given a vector valued function$\frac{1-t^2}{1+t}\mathbf{i} + e^{-2t}\mathbf{j} + (1-t^3)\mathbf{k}$ find;

(i) $lim_{t \to -1} f(t)$ (4 marks)

(ii) $f'(2)$ (2 marks)

(iii) $\int f(t)dt$ (5 marks)

(c) Show that $r = e^-t(C_1 cos2t + C_2 sin2t)$, where C_1, $C_2$are constant vectors, is a solution of the differential equation $\frac{dr^2}{dt^2} + 2\frac{dr}{dt} + 5\mathbf{r}{=}0$, (5 marks)

5(a) Given the region R in the xy plane bounded by $x+y=6$, $x-y=2$ and $y=0$, Find the area bounded by R (06 marks)

(b) If $\phi = 2xyz^2$, $F = xy\mathbf{i} - z\mathbf{j} + x^2\mathbf{k}$ and C is the curve $x = t^2$, $y = 2t$, $Z = t^3$ from $t = 0$ to $t = 1$, evaluate the line integrals

(i) $\int_C \phi dr$ (05 marks)

(ii) $\int_C F \times dr$ (05 marks)

(c) Show that $\mathbf{F} = (\mathbf{2xy} + \mathbf{z^3})\mathbf{i} + \mathbf{x^2}\mathbf{j} + \mathbf{3xz^2}\mathbf{k}$ is a conservative force field (04 marks)

6a(i) Define arc length of a curve (2 marks)

(ii) A curve is described by a pair of parametric equations $x(t) = 1 - 2cost$, $y(t) = 2 + 3sint$,on the interval $0 \le t \le 2\pi$, find the length of the arc of the curve.(6 marks)

(b) Find the arc length of $r = 2 - 2sin\theta$ (8 marks)

(c) Find all polar coordinate representation for the rectangular point (-3, 1) (4 marks)

7(a) Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}{=}(x-3y)\mathbf{i} + (y-2x)\mathbf{j}$ and C is the closed curve in the xy plane, $x = 2cost$, $y = 3sint$ from $t = 0$ to $t = 2\pi$ (6 marks)

(b) Compute the value of triple integral $\int\int\int_D f(x,y,z)dv$ for :$f(x,y,z) = xysinz$;D is a cube bounded by $0 \leq x \leq \pi$,$0 \leq y \leq \pi$ and $0 \leq z \leq \pi$ (6 marks)

(c) Let $\mathbf{F}$=$2xz\mathbf{i} - x\mathbf{j} + y^2\mathbf{k}$. Evaluate $\int\int\int_V \mathbf{F}dV$ where V is the region bounded by the surfaces $x = 0$, $y = 0$,$y = 6$, $z = x^2$, $z = 4$ (8 marks)