UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL EXAMS FOR BSc. Economics and Statistics YEAR 1

SEMESTER II, 2021/2022

MTC 1102: FOUNDATIONS OF MATHEMATICS

DATE: 22/7/2022

TIME: 9:30 am – 12:30 pm

Instructions

1. Attempt ANY FOUR questions
2. Ensure that your name and registration number is indicated on the cover page of your work.
3. Where applicable, leave your answer in fractional form or round it to 2dp
4. Only Non-Programmable calculators are allowed

Question 1

(a) With relevant examples, explain the three methods used to represent a set **[3 marks]**

(b) Differentiate between the following terms as used in set theory

(i) Singleton and binary set **[1 mark]**

(ii) Set difference and set symmetric difference **[1 mark]**

(iii) Sub set and proper sub set **[2 marks]**

(c) In a recent survey people were asked if they took a vacation in the summer, winter, or spring in the past year. The results were; 73 took a vacation in the summer, 51 took a vacation in the winter, 27 took a vacation in the spring, and 2 had taken no vacation. Also, 10 had taken vacations at all three times, 33 had taken both a summer and a winter vacation, 18 had taken only a winter vacation, and 5 had taken both a summer and spring vacation only.

(i) Summarize the above information in set language **[4 marks]**

(ii) Represent the summarized information in a Venn diagram **[4 marks]**

(iii) How many people had taken vacations during at most one time of the year? **[2 marks]**

(iv) What percentage had taken vacations during both summer and winter only? **[2 marks]**

(d) A company that makes a certain brand of chairs has fixed costs of $5,000 and variable costs of $30 per chair. The company sells the chairs for $50 each. Determine:-

(i) Cost function **[2 marks]**

(ii) Revenue function **[2 marks]**

(iii) Marginal cost function **[2 marks]**

Question 2

(a) Using examples of your own, describe three ways in which a complex number can be represented **[3 marks]**

(b) Represent the following numbers on the same Argand diagram **[6 marks]**

(i) $-5+2i$

(ii) $3-4i$

(iii) $\frac{2+i}{3-i}$

(c) Given the complex number $z=x+iy$; show that $z\bar{z}=x^2+y^2$ **[3 marks]**

(d) Reduce the following complex numbers in the form $a+bi$

(i) $(4-3i)(3+2i)+(3-4i)(5-6i)$ ..... [4 marks]

(ii) $\frac{3-2i}{9+2i}+\frac{4+8i}{6+2i}$ ..... [4 marks]

(iii) $(1-2i)^6$ ..... [5 marks]

Question 3

(a) Solve the following complex numbers

(i) $z_1+z_2=5$ *and* $3z_1-2iz_2=9+4i$ ..... [3 marks]

(ii) $z_1-2iz_2=6$ *and* $2z_1-5z_2=23i$ ..... [3 marks]

(iii) *Find all the* 4^{th} *roots of* $2-i\sqrt{12}$ ..... [5 marks]

(b)

(i) *Given that* $z=2+i$ is a root of $2z^3+pz^2+22z-15=0$. Determine the value of p and other roots **[6 marks]**

(ii) Verify that $z_1=2+i$ is a root of the equation $z^4-5z^3+3z^2+19z-30=0$. Hence find the other roots of the equation **[8 marks]**

Question 4

(a) Determine the number of ways in which you can mix up the alphabets of the following words

(i) "multiple"

[2 marks]

(ii) 'Lomerikaaciiikit"

[3 marks]

(b) Ramesh wants to invest \$5 million in two projects; A and B without equal allocation in each project. He decided to invest \$3 million in the most promising project and \$2 million in the less promising project. If seven projects for potential investment are shortlisted, determine the number of possible arrangements available for investment decision. **[3 marks]**

(c) There are seven distinct objects and Sam would like to take three of them at a time without considering order. Determine the number of ways in which Sam can arrange them if repetition of objects is allowed.

[3 marks]

(d) Given that $f(x)=2x+1$ and $g(x)=7x^2+15$. Determine

$(i)(f\circ g)(x)$ ..... [3 *marks*]

$(ii)(g\circ f)(x)$ ..... [3 *marks*]

(iii) *Value of* x *for which* $(f\circ g)(x)=(g\circ f)(x)$ ..... [4 *marks*]

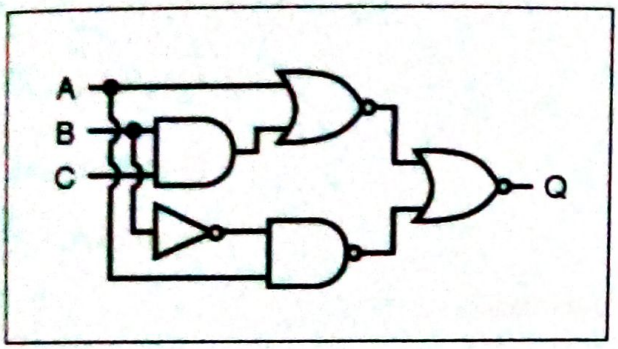
$(iv)\ f^{-1}(x)$ ..... [2 *marks*]

$(v)\ f\bullet g$ ..... [2 *marks*]

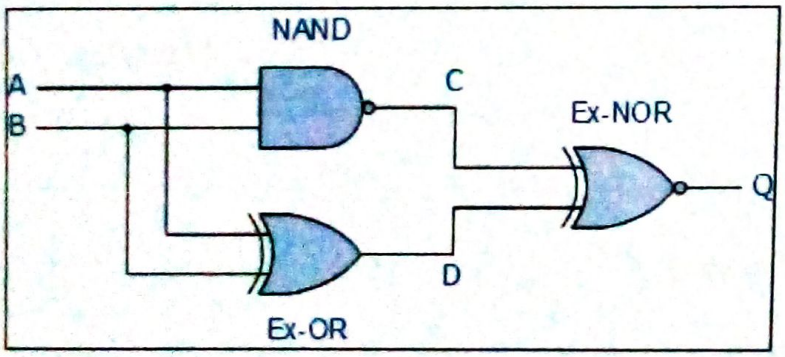
Question 5

(a) For each the following circuits write the Boolean expression for the output Q, simplify the expression and draw an equivalent circuit diagram

(i) **[4 marks]**



(ii) **[5 marks]**



(b) Use De-Morgan's theorem to simplify the following Boolean expressions

(i) $\overline{\overline{A+BC}+\overline{A\overline{B}}}$ .....[3 *marks*]

(ii) $\overline{A\oplus\overline{B}}$ .....[4 *marks*]

(iii) $\overline{AB+\overline{B}A}$ .....[3 *marks*]

(c) Construct circuits that produce the following out puts

(i) $\overline{(A.B+\overline{C})}$.....[3 *marks*]

(ii) $\overline{(A+B)}\overline{(C+\overline{D})}$.....[3 *marks*]

Question 6

(a) Using truth tables, identify whether the following statements are a tautology, a contradiction or a contingency

(i) PL: $((P \wedge Q) \rightarrow \neg R)$ **[4 marks]**

(ii) PL: $\neg [(P \rightarrow Q) \vee (Q \rightarrow P)]$ **[4 marks]**

(iii) PL: $((A \wedge B) \rightarrow C) \leftrightarrow (A \rightarrow (B \rightarrow C))$ **[4 marks]**

(b) Which of the following statements are logically equivalent?

(i) PL: $((P \wedge Q) \rightarrow \neg R)$ and PL: $(P \rightarrow Q) \wedge (Q \rightarrow R)$. **[5 marks]**

(ii) PL: $\neg(P \rightarrow \neg Q)$ and PL: $(P \wedge Q)$ **[4 marks]**

(iii) $\neg (Q \rightarrow R)$ and $Q \wedge \neg R$ **[4 marks]**

Question 7

(a) Given that set $Q = \{3, 4, 5, 6\}$ and that set $P = \{1, 2, 3, 4\}$

(i) Define a binary relation R from set P to set Q

(ii) Determine the number of relations in R

(iii) Determine the sub relations $R_1 = \{(a, b) \mid a < b\}$; $R_2 = \{(a, b) \mid a + b < 7\}$ and $R_3 = \{(a, b) \mid b - a = 2\}$

in R:-

(b) Determine the following combined relations:-

(i) $R_1 \cap R_2$

(ii) $R_2 \cup R_3$

(iii) $R_3 - R_2$

(iv) $R_2 \Delta R_3$

(c) Given a set F = {1, 2, 3, 4}; and that R is a binary relation on set F. Determine whether R is:-

(i) Reflexive

(ii) Transitive

(iii) Symmetric

END