UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

UNIVERSITY EXAMINATIONS

Semester II 2013/2014

Second Year Examination for Bachelor of Science

(General and Financial Mathematics)

STA 2201: Advanced Probability Theory

Date: 9^{th} May, 2014

Time: 10:00 - 13:00 Hours

Instructions

1. *Do not write any thing on this question paper.*
2. Attempt any **FIVE** (5) questions.
3. Begin answering each question from a fresh page of the Answer Booklet.

1. a) Given that A, B and C are events.

   i) Prove that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$ [6 marks]

   ii) If it is known that A and B are independent, are A^c and B also independent? [4 marks]

   iii) If B and C are both mutually exclusive and exhaustive, show that $P(C) = P(B^c)$. [4 marks]

   b) A and B are events such that P(A) = 1/3, P(B) = 1/5 and P(A/B) + P(B/A) = 2/3.

   i) Calculate $P(A \cap B)$. [2 marks]

   ii) Find the value of $P(A^c \cap B^c)$. [4 marks]

2. a) State Baye's Rule. [2 marks]

   b) Three florists; X, Y and Z have equal plots in a circular piece of flower land. The boundaries are clearly marked and seen. X has 80 red and 20 white flowers in his field, Y has 30 red and 40 white flowers whereas Z has 10 red and 60 white flowers. Their regular customer, KK wants a flower for an occasion.

   i) Find the probability that KK picks a red flower if she chooses a flower at random from the garden, ignoring the boundaries. [3 marks]

   ii) Calculate the probability that KK picks a red flower if she first chooses a plot at random. [4 marks]

   iii) If KK picks a red flower by the method in (ii) above, obtain the probability that it comes from Y's plot. [3 marks]

   c) i) Given that $X \sim U(0, \frac{3\pi}{4})$, find $P(\frac{\pi}{4} \leq X \leq \frac{\pi}{2})$. [3 marks]

   ii) The random variable X is exponential with parameter μ. Find the cumulative distribution function of X, F(x) and F(2) if $\mu = 0.5$. [5 marks]

3. a) Consider a random variable X from a distribution whose probability density function, pdf is given as $f(x) = \sqrt{(kx)}$; $0 < x < 1$. Find;

   i) the value of the constant k [3 marks]

   ii) E(X) [2 marks]

iii) E(X^2) [3 marks]

iv) Var(X) [2 marks]

b) Given that $X \sim$ bin (n, p). Show that;

i) E(X) = np [4 mark]

ii) Var(X) = npq; where q = 1 - p. [6 marks]

4. a) A random variable X follows a Poisson distribution with parameter λ, i.e. $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ for $x = 0, 1, 2, \ldots$

   i) Derive the probability generating function of X, $G_X(t)$. [6 marks]

   ii) If $\lambda = 3$, find P(X = 3) using your $G_X(t)$. [4 marks]

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample from Bernoulli(p). Use the moment generating function, mgf, technique to determine the distribution of $Y = \sum_{i=1}^{n} X_i$. [4 marks]

   c) Given that the mgf, of a random variable X is given by $M_X(t) = e^{3t+8t^2}$. Find the mgf of a random variable $Z = \frac{1}{4}(X - 3)$ and use it to deduce E(Z) and Var(Z). [6 marks]

5. a) The joint pdf of two random variables X and Y is given as;
   $f(x, y) = ky^2,\ 0 \le x \le 2;\ 0 \le y \le 1$.

   i) Find the value of the constant k. [2 marks]

   ii) Determine the marginal pdfs, $f_X(x)$ and $f_Y(y)$. [4 marks]

   iii) Are the random variables X and Y independent? [4 marks]

   iv) Are events $\{X < 1\}$ and $\{Y \le 1/2\}$ independent? [4 marks]

   b) The joint pdf of random variables X_1 and X_2 is given as;
   $f(x_1, x_2) = e^{-(x_1+x_2)};\ x_1 > 0,\ x_2 > 0$.
   Find the joint pdf of $Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1 + X_2)$. [6 marks]

6. a) The joint probability mass function, pmf of two discrete random variables X and Y is given as shown below.
   $f(x, y) = kxy,\ x = 1, 2, 3;\ y = 1, 2, 3$.
   Calculate:

   i) the value of the constant k. [2 marks]

ii) the Covariance between X and Y, Cov(X, Y). [5 marks]

iii) the correlation ρ(X, Y) between X and Y. [5 marks]

b) Let the random variables X, Y and Z have the following joint pmf.

(x, y, z)	(0,0,0)	(0,0,1)	(0,1,1)	(1,0,1)	(1,1,0)	(1,1,1)
P(X=x, Y=y, Z=z)	1/8	3/8	1/8	1/8	1/8	1/8

Find the pmf of U = X + Y + Z and V = $|\ (Z - Y)\ |$. [8 marks]

END.

$G^{O^{O^D}}$ $L_{U_{C_K}}$