UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

UNIVERSITY EXAMINATIONS
SEMESTER I, 2011/12

SECOND YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE (FM)

MTF 2101: GENERAL TOPOLOGY

DATE: 21ST DECEMBER 2012

TIME: 9:00 – 12:00 NOON

Instructions:

*i) Attempt **(five)** questions.*

Question 1

(a) Given A,B and C any three sets, state

- (i) The Distributive laws of sets. [2 Marks]
- (ii) The Identity laws of sets. [2 Marks]

(b) (i) Simplify $(A \cap B) \cup (A' \cap B) \cup (A \cap B') \cup (A' \cap B')$, Using the laws of sets. [3 Marks]

(ii) Prove that $(A - B) - C = (A - C) - (B-C)$, using the laws of sets. [3 Marks]

(c) Show by shading on a venn diagram the region represented by $[A' \cap B' \cap C] \cup [(B-A) \cap C']$ [3 Marks]

(d) (i) Define a family. [1 Mark]

(ii) When are two sets M and N said to be equal? [1 Mark]

(iii) Prove one of the distributive laws mentioned in a (i) above. [3 Marks]

(iv) If $M = \{2,3,5\}$ list the elements in the power set of M.

Question 2

(a) (i) Define a relation. [1 Mark]

(ii) When is a relation on a set A said to be reflexive? [1 Mark]

(b) Consider the following four relations on the set $A = \{4,5,6\}$.

$R = \{(4,4),(4,5),(4,6),(6,6)\}$

$S = \{(4,4),(4,5),(5,4),(5,5),(6,6)\}$

$T = \{4,4),(4,5),(5,5),(5,6)\}$

$U = \text{The universal relation on } A.$

(i) List the elements of the universal relation on A.

(ii) Determine which of the above relations on A are Symmetric, reflexive and transitive. [3 Marks]

(c) Let R be the relation "***is greater than by 2***" from $A = \{4,6,8\}$ *to* $B = \{2,3,4,6\}$, that is (a, b) ε R if and only if $(a + 2) = b$, for $a \in A$ *and* $b \in B$.

- (i) List the elements of the relation R. [2 Marks]
- (ii) Find the Domain and range of R. [2 Marks]
- (iii) Find R o R^{-1}. [2 Marks]

(d) Given the sets A= {h, i}, B={2,3,5} and C={β, α, θ}.
The relations R and P are defined as
R: is a relation from A to B.
R = {(h , 2), (h,3), (i,5), (i.2)}
P is a relation from B to C such that
P = {(2, β), (2, θ), (3, β), (5, α), (3,θ)}

(i) Find the following relations

(a) R o P (b) P^c (c) R^{-1} o P^{-1} [6 Marks]

(ii) Compute $M = M_R M_P M_R^{-1}$, the product of the Matrices of the relations R, P and R^{-1}. [3 Marks]

Question 3

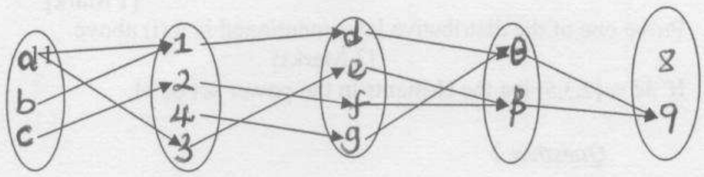
(a) (i) Define a function. [1 Mark]

(ii) When is a function said to be

(i) one to one

(ii) Onto surjective? [2 Marks]

(ii) Consider the functions f: A→B, g: B→C, h: C→D and k: D→E



A ——→ B ——→ C ——→ D ——→ E

Determine with reasons which of the functions f, g, h and k are onto, one to one or both. [5 Marks]

(iii) Determine with reasons which of the following expressions are functions

(i) $f(x) = y = 5\sqrt{(x)} + 4$ (ii) $f(x) = y = 5x^2$, where the domain in both cases is the Real line. [3 Marks]

(b) (i) When is a function f: R→R, said to be continuous at a point $x = x_0$. [1 Mark]

(ii) Let f : A→B and g: B→C, be one to one and onto functions, then show that

$(g\ 0\ f)\ 0\ (g\ 0\ f)^{-1} = 1_C$, where 1_C is the identity function of C. [4 Marks]

(c) Given the functions

$f(x) = x^2 + 5, \quad g(x) = 9x - 5, \quad h(x) = \sqrt{9 - x^2}$,

Find an expression for the function $(f^{-1}\ 0\ g^{-1}\ 0\ h)(x)$.

[4 Marks]

Question 4

(a) (i) The class of subsets
$T = \{a,e,i,o,u\},\{\},\{a\},\{a,i,o\},\{a,e,o,u\},\{a,o\}\}$
forms a topology on a set $A = \{a,e,i,o,u\}$. Give the three axioms that are satisfied by the class T, in order for it to form a topology on A. [3 Marks]

(iii) The class of subsets
$T_1 = \{a,e,i,o,u\},\{\},\{a\},\{a,i,o\},\{i,o\},\{e,i,o\},\{a,e,i\}\}$, does not form a topology on the set A in a(i) above, Give with clear examples the reasons why it does not form a topology on A. [3 Marks]

(iv) Formulate another class T_2 on the set A, above that forms a topology on A. [2 Marks]

(b) (i) Define with clear examples in each case
(a) an open set. [2 Marks]
(b) a closed set [2 Marks]

(ii) Prove that the intersection of any number of closed sets is also closed. [3 Marks]

(c) Given the Point set $A = \left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},......\right\} = \left\{\frac{1}{n}\right\}$

n = 1, 2,3.......

Determine with reasons whether

(i) A is bounded. [2 Marks]
(ii) The Limit points of A. [1 Mark]
(iii) A is closed. [1 Mark]
(iv) A is countable. [2 Marks]

Question 5

(a) When is a Topological space (X,T) said to be
(i) a first Countable space? [2 Marks]

(ii) a second countable space? [2 Mark]

(b) (i) Define a relative Topology on a set A. [2 Marks]

(ii) Consider the following topology on a set $A = \{1,2,3,4,5\}$
$T = \{1,2,3,4,5\},\{\},\{2\},\{3,4\},\{2,3,4\},\{1,3,4,5\}\}$ and the subset $A = \{2,4,5\}$. Determine the relativization of T on A. [3 Marks]

(c) Consider the topology
$T = \{a,b,c,d,e\},\{\},\{a\},\{a,b\},\{a,c,d\},\{a,b,c,d\},\{a,b,e\}\}$ on a set $X = \{a,b,c,d,e\}$.

(i) With reference to the topology T, list the closed subsets of X. [4 Marks]

(ii) Determine the closure of the sets $\{a\},\{b\},\{c,e\},\{b,d\}$. [6 Marks]

(d) Define a Topological structure on a given set A. [1 Mark]

Question 6

(a) State the Heine Borel Theorem. [2 Marks]

(b) (i) Give the conditions that must be satisfied for two subsets A and B to be separated. [2 Marks]

(ii) Consider the following intervals on the real line $A = (0,2)$, $B = (2,4)$, $C = [4,5)$, $D = (5,6]$

Determine with a proper working, whether

(i) A and B are separated. [2 Marks]

(ii) B and C are separated. [3 Marks]

(iii) C and D are not separated. [2 Marks]

(c) Show that if A and B are non empty separated sets, then $A \bigcup B$ is disconnected. [4 Marks]

(d) Define a subspace topology. [1 Mark]

(e) Find x and y if $(4x+3, 8) = (7, 2x+y)$. [3 Marks]

(f) State the Urysohn's Lemma. [2 Marks]

END