UGANDA MARTYRS UNIVERSITY
NKOZI

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ONE FINAL ASESSMENT

CALCULUS
MTH

DATE: 4TH DECEMBER 2014

TIME 2:00 PM - 5: PM

DURATION: 3HRS

Instructions:

1. *Carefully read through ALL the questions before attempting*
2. ***ANSWER FIVE (5) Questions*** *ONLY. (Each question carries equal marks)*
3. *No* ***names*** *should be written anywhere on the examination book.*
4. *Ensure that your* ***ID number*** *is indicated on all pages of the examination answer booklet.*
5. *Ensure your work is* ***clear*** *and* ***readable****. Untidy work shall be penalized*
6. *Any type of examination Malpractice will lead to automatic disqualification*
7. *Do not write anything on the questions paper.*

1. a) Solve for x: $x^2 - x > 0$ **(3 marks)**

   b) i) Give the difference between a relation and a function. **(2 marks)**

   ii) Give an example of a relation that is not a function. **(2 marks)**

   c) Using ray diagrams, show all possible functions from the set A = {1,2,3} to the set B = {a,b}. Hence state the number of possible functions from A to B. **(9 marks)**

   d) Let $f(x) = \frac{x+1}{1-x}$. Find a formula for f^{-1} and hence state $f^{-1}(0)$. **(4 marks)**

2. a) Define the following terms as used in Calculus:

   i. range of a function **(2 marks)**

   ii. an even function **(2 marks)**

   iii. a bijective function **(3 marks)**

   b) Find the domains of the following functions:

   i. $f(x) = x^2 + 3$. **(2 marks)**

   ii. $F(x) = \frac{\sqrt{x+2}}{x^3-3x^2+2x}$. **(5 marks)**

   c) A function $f : \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = 2x - 1$. Find a formula for the composition functions:

   i. $f^{(2)} = f \circ f$ **(2 marks)**

   ii. $f^{(3)} = f \circ f \circ f$ **(2 marks)**

   iii. $f^{(4)} = f \circ f \circ f \circ f$ **(1 marks)**

   Hence state $f^{(k)}$, where k is a positive integer. **(1 marks)**

3. a) i. What is an odd function? **(2 marks)**

   ii. Write true or false for the following statement:

   "a function cannot be both even and odd" **(2 marks)**

   iii. Determine whether $f(x) = x - x^3$ is even or odd or neither. **(3 marks)**

   b) A function $f(x) = 3 - x$ has a domain {1,2,3,4,5} and another function g is defined as $g(x) = x^2 + 1$. Find the domain and range of $g \circ f$. **(6 marks)**

   c) For the functions $g(x) = 3x + 2$ and $h(x) = \frac{x-1}{x+1}$. Find a formula for the following functions:

   i. $g + h$ **(2 marks)**

   ii. $\frac{g}{h}$ **(2 marks)**

   Hence state the domain of $\frac{g}{h}$ and $\left(\frac{g}{h}\right)(3)$. **(3 marks)**

4. a) Define the limit of a process or function? **(2 marks)**

b) A function f is defined as $f(x) = \frac{2-\sqrt{x}}{x-4}$. Find:

i. the left side limit $\lim_{x \to 4^-} f(x)$. **(3 marks)**

ii. the right side limit $\lim_{x \to 4^+} f(x)$. **(3 marks)**

Hence state the limit $\lim_{x \to 4} f(x)$. **(2 marks)**

c) Compute the limits $\lim_{x \to 3} f(x)$ of the following functions:

i. $f(x) = x^3 - 2x^2$ **(3 marks)**

ii. $6x - x^2 - 18 \leq f(x) \leq x^2 - 6x$. **(3 marks)**

iii. $f(x) = \frac{2x-6}{x^3-9x}$. **(4 marks)**

5. a) *The function g(x) = 2x is continuous at the point x = 2.* What is meant by "continuity of a function at a point"? **(2 marks)**

b) Give an example of

i. a continuous function that is not differentiable **(2 marks)**

ii. a differentiable function that is not continuous **(1 marks)**

c) Find the values of *a* and *b* for which the following function *g* is continuous.

$$g(x) = \begin{cases} a - x, & for\ x \leq -2 \\ 2, & for\ -2 < x \leq 2. \\ bx, & for\ x > 2 \end{cases}$$ **(4 marks)**

d) For the function $f(x) = \frac{x+2}{x^2-9x}$. Find

i. the intercepts of *f*. **(2 marks)**

ii. the extreme points of *f*. **(4 marks)**

iii. $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ **(2 marks)**

Hence sketch the graph of *f*. **(3 marks)**

6. a) i. State the limit definition of the derivative of a function f at a point c. **(2 marks)**

ii. Use the definition above to find the derivative f' of $f(x) = x^2 - x$. **(4 marks)**

b) Find the derivatives of the following functions:

i. $f(x) = x\sqrt{1-2x}$ **(4 marks)**

ii. $f(x) = \frac{cosx}{x}$. **(3 marks)**

c) Find the equation of the tangent line to $y = \frac{2}{3}x^3$ at $x = 1$. **(4 marks)**

d) Use linear approximation to estimate $\sqrt{80.9}$. **(3 marks)**

END