UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

END OF SEMESTER FINAL ASSESSMENT

SEMESTER I 2023/24

FIRST YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE WITH EDUCATION

(BSc Educ 1 MASAKA CAMPUS)

ELEMENTS OF PROBABILITY AND STATISTICS

MTH 1202

DATE : 14/12/2023

TIME : 2:00 - 5:00pm

TIME : 3 Hours

Instructions

1. *Carefully read through ALL the questions before attempting.*
2. *ANSWER FOUR (4)Questions (All questions carry equal marks).*
3. *No* ***names*** *should be written anywhere on the examination booklet.*
4. *Ensure that your* ***Reg. number*** *and* ***Course*** *are indicated on all pages of your work.*
5. *Ensure that your work is* ***clear*** *and* ***readable****. Untidy work will be penalized.*
6. *Any type of examination Malpractice will lead to automatic disqualification.*

QUESTION ONE

(a) Given $P(A) = 0.59$, $P(B) = 0.30$ and $P(A \cap B) = 0.21$, find

(i) $P(A' \cup B')$ [4 Marks]

(ii) $P(A/B')$ [4 Marks]

(b) A, B and C are events in the sample space

(i) What is meant by A and B being independent? [2 Marks]

(ii) What is meant by conditional probability between A and B? [2 Marks]

(iii) Show that $P(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$ and what is the expression in case of independence of events? [4 Marks]

(c) A balanced die is rolled. Let A be the event that an even number appears, B the event that a number not greater than 4 shows up and C the event that one of the numbers 2,3 and 4 appears.

(i) Show that A and B are independent. [5 Marks]

(ii) Are C and A independent? [4 Marks]

QUESTION TWO

(a) (i) A random variable X has a Binomial distribution on parameters n and p. Write down the probability mass function of X. Show that $f(X)$ satisfies the conditions for a probability mass function. [7 Marks]

(ii) A fair coin is tossed until a tail appears. Let X denote the number of trials until a tail appears. Show that the probability mass of X is

$$P(X = x) = \frac{1}{2^x},$$

and hence find the distribution function $F(X)$ of X. [8 Marks]

(b) State the relationship between the distribution function and the probability density function of a random variable. [2 Marks]

The total lifetime (in years) of five-year-old dogs of a certain breed is a random variable whose distribution function is given by

$$F(x) = \begin{cases} 1 - e^{-\frac{1}{12}x} & ; \quad x \geq 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that such a five-year-old dog will live anywhere from 12 to 15 years.

Calculate the expected lifetime of such five-year-old dogs. [8 Marks]

QUESTION THREE

(a) There are five defective items in a lot of 25 items. A sample of 10 items is taken without replacement. Let X denote the number of defective items in the sample.

(i) Write down the probability mass function of X. [2 Marks]

(ii) Find the probability that the sample contains exactly two defective items. [4 Marks]

(iii) Find the probability that the sample contains at most four defective items. [4 Marks]

(b) A random variable X has a Poisson distribution with probability mass function

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,\ 1,\ 2,\ 3,\ \ldots$$

Find $E[X(X-1)]$ and hence determine the variance of X. [7 Marks]

(c) A certain area in Western Uganda is hit on average by a medium strength earthquake 6 times a year. Assuming the frequency of such earthquakes follows a Poisson distribution, find the probability that the area will be hit by

(i) exactly 4 earthquakes. [4 Marks]

(ii) between 5 and 7 earthquakes. [4 Marks]

QUESTION FOUR

(a) (i) Consider $x_1, x_2, x_3, \ldots\ldots x_n$ to be values of sample random variables $X_1, X_2, X_3, \ldots\ldots X_n$ respectively. Given that random variable $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is a sample mean, show that $E(\overline{X}) = \mu$. [5 Marks]

(ii) The weights of packages received by a departmental store are normally distributed with mean of 40 kg and standard deviation of 5 kg. What is the probability that a package received at random and put on the shelf will not exceed the safety limit of the shelf which is 42.5 kg? [8 Marks]

(b) Thirty students in Physics laboratory make determination of the speed of sound. The average of their determinations is 3330 ms^{-1} and sample standard deviation of 61 ms^{-1}. Find a 90% confidence interval for the true speed of sound in the laboratory at that time.

[12 Marks]

QUESTION FIVE

(a) Let$x_1, x_2, x_3, \ldots\ldots x_n$ be a random sample, define the sample variance S^2 and show that S^2 can be written in the form

$$S^2 = \frac{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}{n(n-1)}$$

[8 Marks]

(b) The variable $\overline{X_1} - \overline{X_2}$ has a normal distribution with mean $\mu_1 - \mu_2$ and variance $\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$ where n_1 and n_2 are the sample sizes from which $\overline{X_1}$ and $\overline{X_2}$ are computed respectively. Establish a $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$. [8 Marks]

(c) A car manufacturer has found that on average, it took 80 minutes with a standard deviation of 19 minutes to repair a type of engine after having 60 breakdowns. But with type 2 engine, the average is 90 minutes with a standard deviation of 18 minutes after repairing 70 of them. Find the difference in the true average amount of time it takes to repair these engines with 99% confidence. [9 Marks]

End

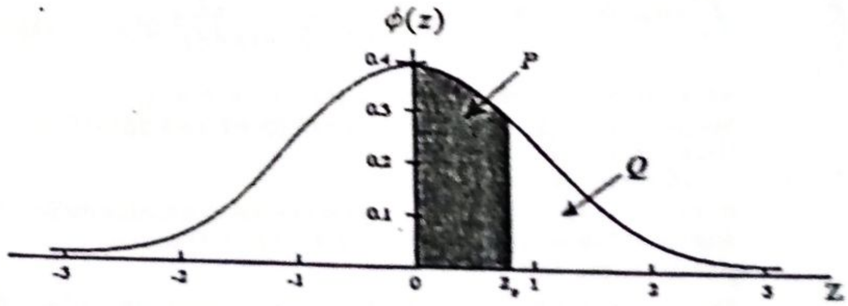
CUMULATIVE NORMAL DISTRIBUTION $P(z)$

z	0	1	2	3	4	5	6	7	8	9	ADD 1	2	3	4	5	6	7	8	9
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
											3	6	9	12	15	19	22	25	28
0.7	0.2580	2611	2642	2673	2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
											3	6	8	11	14	17	20	22	25
0.8	0.2881	2910	2939	2967	2995	3023	3051	3078	3106	3133	3	5	8	11	13	16	19	22	24
											3	5	8	10	13	16	18	21	23
0.9	0.3159	3186	3212	3238	3264	3289	3315	3340	3365	3389	2	5	7	10	12	15	17	20	22
											2	5	7	10	12	14	17	19	22
1.0	0.3413	3438	3461	3485	3508	3531	3554	3577	3599	3621	2	4	7	9	11	13	15	18	20
											2	4	6	8	11	13	15	17	19
1.1	0.3643	3665	3686	3708	3729	3749	3770	3790	3810	3830	2	4	6	8	10	12	14	16	18
											2	4	6	8	10	11	13	15	17
1.2	0.3849	3869	3888	3907	3925	3944	3962	3980	3997	4015	2	4	5	7	9	11	13	14	16
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	3	5	6	8	10	11	13	14
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	1	3	4	6	7	8	10	11	13
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	1	2	4	5	6	7	8	10	11
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	1	2	3	4	5	6	7	8	9
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	1	2	3	3	4	5	6	7	8
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	1	1	2	3	4	4	5	6	6
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	4	5	5
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	0	1	1	2	2	3	3	4	4
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4957	0	1	1	2	2	2	3	3	4
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	0	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952									
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964									
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974									
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981									
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986									
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000									

The table gives $P(z) = \int_0^z \phi(z)dz$

If the random variable Z is distributed as the standard normal distribution N(0,1) then:

1. $P(o < Z < z_p) = P(\text{Shaded Area})$
2. $P(Z > Z_p) = Q = \frac{1}{2} - P$
3. $P(Z > |Z_p|) = 1 - 2P = 2Q$

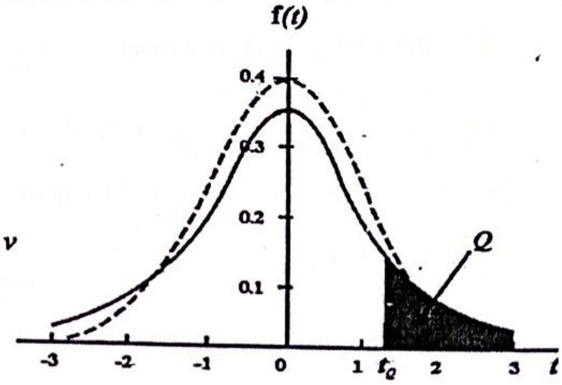


PERCENTAGE POINTS OF STUDENT'S t – DISTRIBUTION t_Q

ν	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.050	0.01 0.02	0.005 0.010	0.0025 0.0050	0.001 0.002	0.0005 0.0010	Q $2Q$
					Probability*					
1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6	
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60	
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92	
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610	
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869	
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959	
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408	
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041	
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781	
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587	
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437	
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318	
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221	
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140	
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073	
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015	
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965	
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922	
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883	
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850	
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819	
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792	
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767	
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745	
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725	
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707	
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690	
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674	$\frac{120}{\nu}$
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659	
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646	4
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551	3
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460	2
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373	1
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291	0

The function tabulated is t_Q defined by

$$\int_{t_Q}^{\infty} f(t)dt = Q; \quad f(t) = \frac{(\tfrac{1}{2}\nu - \tfrac{1}{2})!}{\sqrt{(\nu\pi)}(\tfrac{1}{2}\nu - 1)!} \cdot \frac{1}{(1 + t^2/\nu)^{(\nu+1)/2}}$$



where $f(t)$ is the probability density of the t-distribution.
Interpolation ν-wise should be linear in $120/\nu$ for $\nu > 30$.
Use (i) upper row for one tail-tests
(i) lower row for two tail-tests

If x is a random variable with the t-probability distribution for ν degrees of freedom, the probability that $x > t_Q$ is Q and the probability that $|x| > t_Q$ is 2Q.

The graph shows the form of the distribution for $\nu = 2$. The shaded area represents the probability Q. For large ν the distribution approximates to the normal distribution N(0,1), shown by the dotted line.