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FINAL ASSESSMENT 2ND SEMESTER 2007/2008
BSc II & III DIFFERENTIAL EQUATIONS II

Date : APRIL 30th , 2008
Time: 9: 00 AM – 12:00 NOON

INSTRUCTIONS
Read through the paper carefully
Attempt **ALL** questions in Section **A** and **THREE** from Section **B** but answer (a) OR (b) from questions 8, 9 and 10.
Show all your solutions clearly and neatly.

SECTION A:

1. Define the following:
   (a) A partial differential equation (PDE)
   (b) An order of PDE
   (c) A linear PDE
   (d) A general solution of PDE

2. Find a solution of $6\frac{\partial^2 y}{\partial x^2}+\frac{\partial^2 y}{\partial x \partial y}-2\frac{\partial^2 y}{\partial y^2}=0$ which contains two arbitrary functions.

3. Determine whether the given function is even, odd or neither
   (a) $f(x)=x^3 \sin 2x$
   (b) $f(x)=e^{-x}\cos 3x$

4. Prove the following:
   (a) If *f* and *g* are odd functions, then *fg* is an even function.
   (b) If *f* is an even function and *g* is an odd function, the *fg* is an odd function.

5. Compute the Fourier series for f(x) = x(1 – x), -p < x < p

6. Compute the Fourier sine series for f(x) = x^2, 0 < x < π

7. Compute the Fourier cosine series for f(x) = e^x , 0 < x < 1

SECTION B

8. Find a formal solution to the given initial-boundary value problem:
   (a) $\frac{\partial u}{\partial t}=5\frac{\partial^2 u}{\partial x^2}$, 0 < x < p, t > 0
   u(0, t) = u(p - t) = 0, t > 0

u(x, 0) = x(p – x), 0 < x < p

(b) $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$

$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0$, $t > 0$

u(x, 0) = x, $0 < x < \pi$

9. (a) Find a formal solution to the vibrating string problem governed by the given initial-boundary value problem:

$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$

u(0, t) = u(π, t) = 0, $t > 0$

$u(x,0) = x^2(\pi - x)$, $0 < x < \pi$

$\frac{\partial u}{\partial t}(x,0) = 0$, $0 < x < \pi$

(b) Find a solution to the initial value problem

$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$, $-\infty < x < \infty$

$u(x,0) = \cos 2x$, $-\infty < x < \infty$

$\frac{\partial u}{\partial t}(x,0) = 1 - x$, $-\infty < x < \infty$

10. (a) Find a formal solution to the given boundary value problem:

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $0 < x < \pi$, $0 < y < \pi$

$\frac{\partial u}{\partial x}(0,y) = \frac{\partial u}{\partial x}(\pi,y) = 0$, $0 \le y \le \pi$

u(x, 0) = cos x – 2 cos 4x, $0 \le x \le \pi$

u(x, π) = 0, $0 \le x \le \pi$

(b) Find a solution to the Dirichlet boundary value problem for a disk:

$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$, $0 \le r \le 2$, $-\pi \le \theta \le \pi$

$u(2,0) = \cos^2\theta$, $-\pi \le \theta \le \pi$