UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

University Examinations 2013/2014

Supplementary Assessment for BSc II FM and BSc II GENERAL

Monday August 4th, 2014

MTC 3103 COMPLEX VARIABLES

Time allowed: 3 hours

Instructions

(i) Answer **FIVE** questions.

(ii) Write both sides of the paper but begin a new question on a fresh page.

(iii) Only approved basic scientific calculators may be used in this Examination.

1 (a) (i) Let $a_n, a_{n-1}, \ldots, a_1, a_0$ be real numbers. If α is a complex root of the polynomial equation $a_n z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0 = 0$, then show that $\overline{\alpha}$ is also a root of the equation.

(3 marks)

(ii) Given that $z = 2 + i$ is a root of the equation $z^4 - 5z^3 + 3z^2 + 19z - 30 = 0$. Find the other roots.

(4 marks)

(b) Prove the triangle inequality of complex numbers that says that: $|z_1 + z_2| \leq |z_1| + |z_2|$

(4 marks)

(c) Solve the equation $z^5 + 32 = 0$ using De-Moivre's theorem.

(5 marks)

(d) Find the sixth root of $z = 4 + 5i$.

(4 marks)

2 (a) (i) Define the limit of a function $f(z)$ at infinity.

(1 mark)

(ii) Show that $\lim\limits_{z \to \infty} \frac{z^3+4z^2-2}{(z-3)(2z^2-3z+5)} = \frac{1}{2}$.

(4 marks)

(b) If $f(z) = \frac{2z-1}{3z+2}$, prove that, at $z = z_0$, $\lim\limits_{z \to 0} \frac{f(z_0+h)-f(z_0)}{h} = \frac{7}{(3z_0+2)^2}$ where $z_0 \neq \frac{-2}{3}$.

(5 marks)

(c) (i) When is a complex function $f(z)$ said to be continuous at a point $z = z_0$.

(2 marks)

(ii) Find the points at which the function $f(z)$ below is discontinuous.

$$f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i}$$

Redefine the function at the points where the function is discontinuous to remove the discontinuity and show that the limit of the function at that point is $4 + 4i$.

(8 marks)

3 (a) What does it mean to say that $f(z) = u(x, y) + iv(x, y)$ is analytic? When is such a function said to be harmonic?

(3 marks)

(b) Prove that a necessary condition for $f(z)$ to be analytic is that it must satisfy the Cauchy-Reimann equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

(7 marks)

(c) Determine whether the function $f(z) = \cos\ z$ is analytic where z is a complex number.

(4 marks)

(d) Prove that the function $u = e^{-x}(x \sin\ y - y\ \cos\ y)$ is harmonic.

(6 marks)

4 (a) Explain the following terms:

(i) a branch point,

(ii) an isolated singularity,

(ii) a pole of order n.

(5 marks)

(b) Locate and name all the singularities of the function

$$f(z) = \frac{(z+4)^3 \ln(z^2 - 7z + 12)}{(z^2+4)^2(z^2+3z-4)^3(z+i)}$$

(12 marks)

(c) Using the definition of a derivative of a function, show that the derivative of $f(z) = 2z^2 + 3z + 1$ at $z = z_0$ is $f'(z_0) = 4z_0 + 3$.

(3 marks)

5 (a) (i) Define the complex line integral of a function $f(z) = u(x, y) + iv(x, y)$ along a curve C.