UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL ASSESSMENT
SEMESTER I

FIRST YEAR EXAMINATIONS FOR BSc (GEN), BSc (IT), BSc (CS)

MTH 1102: Elements of Mathematics

DATE: Thursday 15th December, 2022
TIME: 9:30am - 12:30pm
DURATION: 3 Hours

Instructions:

1. *Carefully read through ALL the questions before attempting the examination.*
2. ***ANSWER ANY FOUR Questions*** *(Each question carries a total of 25 marks)*
3. *No* ***names*** *should be written anywhere on the examination book.*
4. *Ensure that your* ***Reg number*** *is indicated on all pages of the examination answer booklet.*
5. *Ensure your work is* ***clear*** *and* ***readable****. Untidy work shall be penalized*
6. *Any type of examination Malpractice will lead to automatic disqualification*
7. *Do not write anything on the questions paper.*

1. (a) Given $A = \{1, 2\}$, $B = \{a, b\}$ and $C = \{x, y\}$, find:

   (i) $A \times B$ [2 marks]

   (ii) $A \times B \times C$ [4 marks]

   (b) Given that $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$ and $R : A \to B$ where $R = \{(1, b), (1, c), (3, b), (4, a), (4, c)\}$. Determine:

   (i) the domain or R [2 marks]

   (ii) the range of R [2 marks]

   (iii) R^{-1} [4 marks]

   (c) Given that the sets $A = \{1, 2, 3\}$, $B = \{a, b, c\}$, $C = \{p, q, r\}$; and relations $R = \{(1, b), (2, a), (2, c)\}$ and $S = \{(a, q), (b, p), (c, q), (c, r)\}$. Find the relation composition $S \circ R$. [6 marks]

   (d) Let $X = \{1, 2, 3, 4\}$. Determine whether or not the following relations are functions on X

   (i) $R = \{(2, 3), (1, 4), (2, 1), (3, 2), (4, 4)\}$ [2 marks]

   (ii) $R = \{(3, 1), (4, 2), (1, 1)\}$ [1 mark]

   (iii) $R = \{(2, 1), (3, 4), (1, 4), (2, 1), (4, 4)\}$ [2 marks]

2. (a) A *proposition* is known to be a statement which can be classified as either true or false. Classify each of the following statements as a proposition or not

   (i) Every mathematics student can be able to do other subjects. [2 marks]

   (ii) $3 + 2 = 8$. [2 marks]

   (iii) John works in a bank [2 marks]

   (b) Given the set $V = \{1, 2, 3, 4\}$; and functions $f(n) = 6 - n$, $g(n) = 3$ and $h = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$ defined on V. For each of the functions check (and give a reason) which one is:

   (i) one to one [6 marks]

   (ii) onto [6 marks]

   (iii) both one to one and onto [1 mark]

   (iv) neither one to one nor onto [1 mark]

   (b) With the help of venn diagrams or otherwise, state whether each of the following is true of false for sets A, B and empty set Φ.

   (i) $B \cup A \subset A$

   (ii) $\emptyset \subset A$

   (iii) $B \cap A \subset A \cup B$

   (iv) $0 \notin \emptyset$

   (v) $\emptyset \subset \{\emptyset\}$

[5 marks]

3. (a) Define the following;

(i) a set [2 marks]

(ii) a subset of a set [2 marks]

(iii) Cardinality of a set [2 marks]

b (i) Define a power set of a set. [1 mark]

(ii) Given set $A = \{a, b, c, d\}$, state the power set of A. [3 marks]

(iii) State the different ways of describing a set. [3 marks]

(c) A survey of a sample of 25 new cars sold at a local auto dealer was conducted to see which of the three popular option, Air-conditioning (A), Radio (R) and Power Window (P), were already installed. The survey found: 15 had Air Conditioning, 12 had Radio, 11 had Air Power Window, 5 had Air Conditioning and Power Window, 4 had Radio and power Window, 9 had Air conditioning and Radio , 3 had all the three options.Find the number of cars that had

(i) only Power Window [5 marks]

(ii) only Radio [4 marks]

iii Air-Conditioning and Radio but no Power windows [3 marks]

4. (a) Using relevant examples, define the following terms:

(i) a function [3 marks]

(ii) Domain of a function [3 marks]

(b) Given that $f(x) = x^2 - 2x + 3$ and $g(x) = x + 2$

Evaluate the following;

(i) $f(5)$ [3 marks]

(ii) $g(3)$ [2 marks]

(iii) $f(g(x))$ [3 marks]

(iv) $gof(-2)$ [3 marks]

(c) Sketch the graphs of the following funcions.

(i) $f(x) = x - 3$ [3 marks]

(ii) $f(x) = 2 - 5x$ [3 marks]

5. (a) Given the matrices $\mathbf{P} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & -1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 1 & 0 & 7 \end{pmatrix}$

Evaluate the following

(i) $4\mathbf{P} + \mathbf{Q}$ [5 marks]

(ii) $\mathbf{PQ}$ [5 marks]

(iii) $\mathbf{P^2}$ [5 marks]

(b) Find the inverse of the following matrices

(i) $\mathbf{M} = \begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix}$ [3 marks]

(ii) $\mathbf{N} = \begin{pmatrix} 8 & -2 \\ -11 & 3 \end{pmatrix}$ [3 marks]

(c) Find the determinant of the following matrices

(i) $\mathbf{A} = \begin{pmatrix} 9 & 2 \\ 8 & 4 \end{pmatrix}$ [2 marks]

(ii) $\mathbf{B} = \begin{pmatrix} -3 & 5 \\ -6 & -1 \end{pmatrix}$ [2 marks]

END