UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

University Examinations 2013/2014

Final Examination for the degree of Bachelor of Science Financial Mathematics and for Bachelor of Science General

Monday, December 16, 2013

MTC 2103: REAL ANALYSIS 1

Time allowed: 3 hours

Instructions

(i) Answer **FIVE** questions.

(ii) Begin a new question on a fresh page.

(iii) Only approved basic scientific calculators should be used

Question one

Define the following:
(a) A function
(b) Greatest lower bound (g.l.b) of a set
(c) A limit point
(d) Set X is a Metric space
(e) Denumerable set
(f) Monotone increasing function
(g) Archimedean property
(h) A sequence
(i) Cauchy criterion of uniform convergence
(j) Uniform continuity of a function f on a set E

Question two

a) State the following series checks of convergence

I. D' Alembert's test
II. Integral test
III. Limit Comparison test
IV. Alternating series test
V. Weierstrass-M test of uniform convergence

b) If b, B > 0 and $\frac{a}{b} < \frac{A}{B}$. Prove that $aB < bA$. Therefore deduce that:
$\frac{a}{b} < \frac{a+A}{b+B} < \frac{A}{B}$

c) Let **a**, **b** be any two real numbers such that **a** < **b.** Then there is a rational number $\frac{p}{q}$ such that $a < \frac{p}{q} < b$

Question three

Determine if the convergence and divergence of each of the series below

a) $\sum_{n=0}^{\infty} \frac{1}{3^n - n}$

b) $\sum_{n=2}^{\infty} \frac{4n^2 + n}{\sqrt[3]{n^7 + n^3}}$

c) $\sum_{n=1}^{\infty} n$

d) $\sum_{n=0}^{\infty} \frac{4n^2 - n^3}{10 + 2n^3}$

e) $\sum_{n=0}^{\infty} (-1)^n$

f) Use the limit comparison test to show that the following series diverges. $\sum_{n=1}^{\infty}$.[$1/n^{(1+1/n)}$]

Question four

a) For a bounded function f defined on the interval [a, b], a partition P of [a, b] is a finite ordered set P = $\{a = x_0 < x_1 < \dots\dots\dots < x_n = b\}$. define the following

   I. Right Riemann sum
   II. Left Riemann sum
   III. Middle Riemann sum

b) Given that $f(x) = 3x^2 + x$. Use the general formulae for Right and Left Riemann sums to evaluate $\int_1^4 f(x)dx$ to five decimal places

Question five

Let X be a nonempty set:

(a) (i) Define a metric d on X and explain what it means to say that (X, d) is a metric space.
(ii) Let R be a set of all real numbers and define the usual metric d(x, y) by d(x, y) = |x − y| where x, y ϵ R.
Show that R with the usual metric is a metric space.

(b) Let x be a point in X and E be a subset of X. Define the following terms:
(i) a set U is a neighborhood of x,
(ii) x is an accumulation point of E,
(iii) x is an isolated point of E,
(iv)The set E is closed.

(c) Let E be a subset of R and x be a limit point of E. Prove that every neighborhood of x contains infinitely many points of E.

Question six

Evaluate the following limits to infinity

a) $\lim \frac{2sinx-sin2x}{x-sinx}$

b) $\lim(\frac{x}{x-1} - \frac{1}{lnx})$

c) Prove directly from definition of convergence of a sequence that the sequence $a_n = \frac{3n+1}{n+2}$ converges to 3.

d) Consider the sequence $\{f_n\}$ of functions defined by:

$f_n(x) = (nx + x^2) / n^2$ show that $\{f_n\}$ converges pointwise

END