UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

END OF SEMESTER ONE FINAL ASSESSMENT

SEMESTER 1, 2021/2022

Third Year for Bachelor of Science (EDUC, General)

MULTIVARIATE CALCULUS / CALCULUS III

DATE: 17th January 2022

TIME: 9:30 – 12:30 PM

DURATION: 3 hours

Instructions

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1. Carefully read through ALL the questions before attempting
2. Attempt any **TEN** from the twelve questions
3. No names should be written anywhere on the examination booklet
4. Ensure that your Reg. number is indicated on all pages of the examination answer booklet
5. Ensure your work is clear and readable. Untidy work shall be disqualified.
6. Any type of examination malpractice will lead to automatic disqualification
7. Do not write anything on the question paper
8. Calculators and mathematical tables may be used.

1. [10 marks] Find the length of the curve on the specified interval $x = t^3 - 1$, $y = 2t^2 + 1$, $0 \le t \le 1$.

2. (a) [5 marks] Find the area of the triangle with vertices A(2, 1, 4), B(3, -1, 7) and C(-1, 2, 5)

   (b)[5 marks] Given the points A(3, -1, 1), B(2, 3, -2), C(0, 1, 3) and D(-1, 2, 4), find the volume of the parallelepiped determined by the vectors $A\bar{B}$, $A\bar{C}$ and $A\bar{D}$.

3. (a) [5 marks] Find the angle between the plane $2x - 3y + 4z - 7 = 0$ and $x + y - 2z + 4 = 0$.

   (b) [5 marks] Find the distance between the plane $3x - 4y + 5z - 8 = 0$ and the point (2, 1, -1).

4. [10 marks] Find the area of the surface generated by revolving the curve C defined by $x = t^2 - 1$, $y = 3t$. $0 \le t \le 2$ about the x-axis.

5. (a) [5 marks] Find the equation of the tangent line to the curve C defined by $x = \frac{4}{t}$, $y = \sqrt{t}$, $1 \le t \le 9$ at the point (1, 2).

   (b) [5 marks] Find y'' for the function defined in part (a)

6. (a) [5 marks] Find the points of intersection of the curves $r = 2 + \sin\theta$ and $r = 5\sin\theta$

   (b) [5 marks] Find the area outside the circle r = 1 and inside the circle $r = 2\cos\theta$.

7. (a) [4 marks] Find the length of the arc of the circular helix r(t) = <cos t, sin t, t> from t varying from t = 0 to $t = 2\pi$

   (b) [6 marks] Find the length of the curve $r(t) = 3t^2 i + (1 - 4t^2) j + 2t^3 k$ from the point given by t = 0 to the point given by t = 4.

8. (a) [5 marks] Fin the angle between the line l_1 and l_2 defined by $r = <1 - 2t, 3 + t, -2 + 3t>$ and $r = <-2 + t, 4, 3 - t>$ respectively.

   (b) [5 marks] Find the equation for the plane that contains the points P(1, 0, -3), Q(2, -5, -6) and R(6, 3, -4).

9. [10 marks] Find the angle from $r_1 = 2 - \cos\theta$ to $r_2 = 2(1 + \cos\theta)$ at the point of intersection

10. (a) [5 marks] Let $f(x,y) = \tan^{-1}\dfrac{y}{x}$. Find (i) $f_x(4,-3)$, (ii) $f_y(4,-3)$.

(b) [5 marks] If $z = \dfrac{xy}{x^2+y^2}$, show that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = z$

11. (a) [5 marks] Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist.

(b) [5 marks] Let $f(x,y) = \begin{cases} \dfrac{x^4-y^4}{x^2+y^2}, \ldots (x,y) \neq (0,0) \\ 0, \ldots\ldots\ldots\ldots (x,y) = (0,0) \end{cases}$

Determine if f is continuous at (0, 0).

12. (a) [5 marks] Let **u** = **i** - 2**j** + 2**k** and **v** = -3**i** -4**j** + 5**k**. Find the angle between **u** and **v**.

(b) [5 marks] If A = 2i – 3j + 4k, B = -8i – 2j – 4k, and C = -15i + 3j -3k. Find (AxB).C