UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATIONS

May 2016

MTC3201 NUMENRICAL ANALYSIS II

YEAR 2015/2016: Third year (GEN & FM)

Date: 28th April 2016

Time: 9:30 – 12:30 PM

INSTRUCTIONS

(i) Attempt any **FIVE** questions

(ii) Read through the paper carefully and follow instructions on the answer booklet

(iii) Calculators and mathematical tables may be used.

1. (a) For the following orthogonal functions, state the weight function and interval of orthogonality.
   (i) Legendre functions [02 marks]
   (ii) Tchebyshev functions [02 marks]
   (iii) Laguerre functions [02 marks]
   (iv) Hermitian functions [02 marks]

   (b) Legendre polynomials can be generated using a recurrence relation. With conditions $P_0(x) = 1$, $P_1(x) = x$, generate
   (i) $P_2(x)$ [03 marks]
   (ii) $P_3(x)$ [03 marks]

   (c) Tchebyshev polynomials are denoted by $T_n(t)$. With conditions $T_0(t) = 1$ and $T_1(t) = t$, generate
   (i) $T_2(t)$ [03 marks]
   (ii) $T_3(t)$ [03 marks]

2. Find the fourth Taylor polynomial $P_4(x)$ for the function $f(x) = xe^{x^2}$ about x = 0. [06 marks]

(a) Find an upper bound for |f(x) - P4(x)|, for $0 \le x \le 0.4$. [04 marks]

(b) Approximate $\int_0^{0.4} f(x)dx$ using $\int_0^{0.4} P_4(x)dx$ [02 marks]

(c) Find an upper bound for the error in (b) using $\int_0^{0.4} P_4(x)dx$. [02 marks]

(d) Approximate f(0.2) and $P_4(0.2)$ and find the error [02 marks]

(e) Approximate $f'(0.2)$ and $P_4'(0.2)$ and find the error. [04 marks]

3. (a) Use appropriate Lagrange interpolating polynomial of degree one and two to approximate f(8.4), if f(8.1) = 16.94410, f(8.3) = 17.56492, f(8.6) = 18.59515, f(8.7) = 18.82091. [07 marks]

(b) The data for (a) were generated using the function f(x) = x ln x. Use the error formula to find a bound for the error, and compare the bound to the actual error for the cases n = 1 and n = 2. [07 marks]

(c) Consider $\int_{-1}^{1} \frac{T_n(x)T_m(x)}{\sqrt{1-x^2}}dx = \int_{-1}^{1} \frac{\cos(n\cos^{-1}x)\cos(m\cos^{-1}x)}{\sqrt{1-x^2}}dx$.

When n = m, show that $\int_{-1}^{1} \frac{[T_n(x)]^2}{\sqrt{1-x^2}}dx = \frac{\pi}{2}$ for each $n \ge 1$ [06 marks]

4. (a) Use the numbers $x_0 = 2$, $x_1 = 3.75$, and $x_2 = 4$ to find the second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$ [05 mark]

(b) Use this polynomial to approximate f(3) [02 marks]

(c) Determine the error bound for this polynomial when x = 2.45. [05 marks]

(d) Use the forward-difference formula to construct an interpolating polynomial of degree three for the following data and hence find f(0.25) [08 marks]

f(0.1) = -0.62049958, f(0.2) = -0.28398668, f(0.3) = 0.00660095, f(0.4) = 0.24842440

5. (a) The Newton forward divided-difference formula is used to approximate f(0.3) given the following data

x	0.0	0.2	0.4	0.6
f(x)	15.0	21.0	30.0	51.0

Suppose it is discovered that f(0.4) was understated by 10 and f(0.6) was overstated by 5. By what amount should the approximation to f(0.3) be changed? [13 marks]

(b) Use Newton backward-difference formula to find the interpolation polynomial that approximates the function with the following data. [07 marks]

x	0	1	2	3	4
f(x)	1	3	7	13	25

6. (a) Given that f(1) = 3, f(2) = 8, f(4) = 54 and f(5) = 107. Use Lagrange interpolation formula to find P_3. Hence estimate the value of f(3.5). [10 marks]

(b) Form a table of divided differences that fits the following data [10 marks]

x	0	3	5	8	10	13
f(x)	140	225	383	1623	2742	5993

7. Determine the values of n and h required to approximate $\int_0^2 \frac{1}{x+4}dx$ to within 10^{-5} and compute the approximation.
(a) Use the composite Trapezoidal rule [09 marks]
(b) Use the composite Simpson's rule [09 marks]
(c) By comparing results of (a) and (b) to the exact value, which of the two rules gives a value within the required error bound. [02 marks]

8. (a) Use the following values and five-digit rounding arithmetic to construct Hermite interpolating polynomial to approximate sin 0.34. [08 marks]

x	sin x	$D_x \sin x = \cos x$
0.30	0.29552	0.95534
0.32	0.31457	0.94924
0.35	0.34290	0.93937

(b) Determine the error bound for the approximation in part (a), and compare it to the actual error. [04 marks]
(c) Add sin 0.33 = 0.32404 and cos 0.33 = 0.94604 to the data, and redo the calculations. [08 marks]

END