UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

UNIVERSITY EXAMINATIONS
SEMESTER I, 2012/13

THIRD YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE
(FM)

MTF 3102: FINANCIAL MATHEMATICS

DATE: 21ST DECEMBER 2012

TIME: 9:00 – 12:00 NOON

Instructions:

i) Attempt any five questions in this paper.
ii) Show all the necessary workings.
iii) All questions carry equal marks.
iv) Where applicable, round off your answers to 2 decimal places
v) Only non programmable calculators are to be used.

Question One

a) Describe the major differences between the European option and the American option in stock market dynamics.

b) Compute the expected return and standard deviation of a portfolio consisting of three securities with the following information

W1 = 40%, W2 = -20%, W3 = 80%, $\mu_1 = 8\%$, $\mu_2 = 10\%$, $\mu_3 = 6\%$, $\delta_1 = 1.5$, $\delta_2 = 0.5$, $\delta_3 = 1.2$, $r_{12} = 0.3$, $r_{23} = 0.0$ and $r_{31} = -0.2$

c) At the end of a given day, the price of stock is recorded. Suppose that the stock going up tomorrow depends on whether it increased today and yesterday. If the stock has decreased for the past two days, it will decrease tomorrow with a probability of 0.4. If the stock decreased today but increased yesterday, then it will increase tomorrow with a probability of 0.8. If the stock increased today but decreased yesterday, then it will increase tomorrow with probability 0.1. Finally, if stock increased today and increased yesterday then it will decrease tomorrow with probability 0.3. Construct a Markov chain and calculate the different probabilities in the matrix.

Question Two

a) Define and state six assumptions of Markowitz model. Prove that Markowitz optimization solver uses the products of the matrix of weights arranged in rows by the covariance square matrix and by the matrix of weights arranged in columns (assume n = 3)

b) Define the following terms as applied in stock markets:-

i. *Underlying*
ii. *Financial instrument*
iii. *Stock market index*
iv. *Option valuation*
v. *Long CALL*
vi. *Short PUT*
vii. *Moneyness of an option*

Question Three

a) Suppose that the selling price of stock will be measured daily and that the starting measurement will be S(0). Given that the stock volatility and drift are 0.08 and 0.02 respectively; calculate:-

I. The probability that the selling price of the stock on the next day will be higher than the price on the previous day.

II. The probability that the selling price of stock two days will be higher than the present selling price.

b) Define the following terms:-

i. Underlying
ii. Jensen Index
iii. Stock market index
iv. Short selling
v. Markov chain
vi. Security market line
vii. Lognormal random variable

Question Four

a) Differentiate between long call and long put as applied to stock markets.

b) Consider the following advertisement from a Microsoft (MSFT) company

Type: MSFT

Position: Long

Strike Price: $25

Expiry date: 27th – April – 2013

Share Price: 15 cents

Suppose you bought 8 MSFT share options:-

i. How much would you pay to MSFT?

ii. If you exercise your right, how much would you pay MSFT?
iii. If you let the option expire worthless, how much would you pay MSFT?
iv. If you sold share in Open Market Operation (OMO) at $38, how much profit would you get?

b) A stock with a current value S(0) = £4500 has an expected growth rate of its logarithm of v being 12% and a volatility of that growth rate of $\sigma = 23\%$.

i. Find suitable parameters of a binomial lattice representing this stock with a basic elementary period of six months.
ii. Draw the lattice and enter the mean stock values for one year.
iii. What are the probabilities at the final nodes?

Question Five

a) Define Capital Asset Pricing Model (CAPM) and state ONLY one of its application
b) Given the following information about a certain investment portfolio

Year	Z	Barclays (Market Portfolio)	Treasury Bills
1	14	12	7.0
2	12	8	7.5
3	20	20	7.7
4	-10	-4	7.5
5	25	10	8.0
6	24	13	7.6
7	26	16	7.5

Compute the following parameters:

I. Covariance between Z and the market portfolio
II. The beta (β) index
III. The Jensen (J) index
IV. Comment on whether the fund will perform better than predicted by CAPM.

Question Six

a) Mention any five assumptions of The Capital Asset Pricing Model (CAPM).
b) Assume a CAPM with risk free lending but no risk free borrowing. Given the following:
Return on the market portfolio =10%.
Return on the zero beta portfolio = 6%.
Market's standard deviation = 30%.

Stock	Expected return	Standard Deviation (σ)	Beta	Residual Variance
A	0.16	-	-	0.0375
B	0.08	-	-	0.0775

i) Calculate the values of the standard deviations and beta factors for stock A and B.
ii) Given that the covariance between the expected return of security A and B is zero, what will be the expected return of the portfolio consisting of equal portions of each stock?
iii) Hence determine the variance of the portfolio

END

GOOD LUCK