UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

FINAL ASSESSMENT EXAMINATION SEMESTER 1 2012-2013

MTC: NUMERICAL ANALYSIS I

BSC GENERAL III and BSC FM III

DATE: Wednesday, 12th -December -2012

Time: 9:00am - 12:00 Noon

Instructions

Attempt any FIVE (5) questions.

Question 1

(a) (i) State the **Lagrange interpolation Theorem.** [2 Marks]

(ii) Derive an expression for the Lagrange's quadratic interpolation polynomial which interpolates the points $(x_0, f_0), (x_1, f_1)$ and (x_2, f_2). [2 Marks]

(iii) Find the Lagrange's quadratic polynomial that interpolates the points (-2 , 4), (0 , 10) and (1 ,10). [5 Marks]

(b) Prove that

(i) $\nabla = 1 - E^{-1}$ [2 Marks]

(ii) $\delta = E^{\frac{1}{2}} - E^{\frac{-1}{2}}$ [3 Marks]

(c) A function $f(x)$ has tabular values given as

x	1.0	1.2	1.4	1.6	1.8
$f(x)$	14.5503	17.4726	22.1245	29.4157	40.6894

State **Everett's Formula** and use the table to generate and give the values that you would use to substitute for the expressions $q, p, f_0, \delta^2 f_0, \delta^2 f_1$ in the formula to evaluate $f(1.35)$.

[6 Marks]

Question 2

(a) (i) Let $\{x_0, x_1, \ldots\ldots\ldots\ldots x_n\}$ be n+1 distinct points on the interval [a, b] and $f(x) \in [a,b]$ such that

$$f(x) = \sum_{i=0}^{n} f_i L_i(x) + \frac{1}{(n+1)!}\prod_{\substack{i=0 \\ i \neq k}}^{n}(x_k - x_i) f^{n+1}(\Psi_k)$$, for some $x_k \in [a,b]$, show that

$$f'(x_0 + 2h) = \frac{1}{h}\left[\frac{1}{2}f(x_0) - 2f(x_0+h) + \frac{3}{2}f(x_0+2h)\right] + \frac{h^2}{3}f'''(\psi_2)$$ where

$L_i(x)$ are the Lagrange's multipliers for equally spaced points $x_0,\ x_1 = x_0 + h \text{ and } x_2 = x_0 + 2h.$, h is the spacing and $\psi_2 \in (a,b)$. [9 Marks]

(ii) Given $f(x) = x^3 e^x - \tan x$, with h = 0.01 determine the value of $f'(2.21)$ by using an appropriate formula for numerical differentiation.($\tan x$ values should be in radians and $f(x)$ values rounded off to 4 decimal places). [4 Marks]

(b) Construct a finite difference table for the function defined in the table below

x	1.6	1.8	2.0	2.2	2.4
$f(x)$	0.0495	0.0605	0.0739	0.0903	0.1102

Use the finite difference table with an appropriate formula for numerical differentiation to find the value of $f'(2.37)$.

[4 Marks]

Question 3

(a) Derive the closed Trapezoidal formula for numerically determining the Integral $\int_a^b f(x)\,dx$, on the interval [a, b], using the **Newton Gregory Forward difference formula.** [8 Marks]

(b) (i) Given that the mean value of $f(x)$ over (a, b) is $\frac{7}{b-a}\int_a^b f(x)\,dx$, calculate the Mean value of $f(x) = 6x - x^2$ over (0, 6), by using the composite Simpson's rule with 4 strips. [7 Marks]

(ii) State the $\frac{3}{8}^{th}$ *Rule* for Numerical Integration, and use it to find $I(f) = \int_0^1 e^{-x^2}\,dx$, (values of $f(x)$ to be rounded off to 5 decimal places). [5 Marks]

Question 4

(a) State the **Intermediate value Theorem.** [1 Mark]

(b) (i) Determine approximately the number of iterations necessary to solve

$x^3 - 7x^2 + 14x - 6 = 0$ on the interval [3.2 , 4], with an accuracy of 10^{-6}.

[3 Marks]

(ii) Use the Bisection method to find a root to the equation $x^3 - 7x^2 + 14x - 6 = 0$ on the interval $[3.2, 4]$ to 2 decimal places. Show the working clearly and put the results in a suitable table, for 5-Iterations. [6 Marks]

(iii) Derive the Newton Raphson formula for locating the root P_n of a polynomial function $f(x) = 0$. [3 Marks]

(iv) Find the root to the same equation in b(ii) above using the Newton Raphson method with $P_0 = 3.45$ and iterate up to P_3. [4 Marks]

(c) In relation to the example above discuss the convergence of the Newton Raphson method in comparison to the Bisection method while locating the root to a given polynomial equation. [3 Marks]

Question 5

(a) (i) Discuss the Jacobi method procedure for solving the 3 x 3

linear system of equations below in the unknowns x_1, x_2 *and* x_3.

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \,.........(i)\\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \,.........(ii).\\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \,........(iii)
\end{aligned}
$$

[3 Marks]

(ii) Solve the Linear system of equations below by the Jacobi method beginning with an initial guess to the solution as $(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0,0,0)$.

$$
\begin{aligned}
4x_1 &+ x_2 &+ 3x_3 &= 17 \\
x_1 &+ 5x_2 &+ x_3 &= 14 \\
2x_1 &- x_2 &+ 8x_3 &= 12
\end{aligned}
$$

[8 Marks]

Present your results in a suitable table up to 3 iterations, and give your final answers rounded off to a whole number.

(iii) Solve the same system in a (ii) above by the Gaussian elimination method with pivoting. [8 Marks]

(b) Discuss the convergence of the Jacobi method compared to that of Gauss-Siedel when used to solve a linear system of equations. [1 Mark]

Question 6

(a) Find the cubic polynomial that passes through the following points (0,5), (1,8), (2,17), (3,44). using linear interpolation by reduction to row echelon form. [6 marks]

(b) Given that $\Delta f(x_0) = f(x_0 + h) - f(x_0)$,by generating expressions for $f(x_0 + h), f(x_0 + 2h)\ and\ f(x_0 + 3h)$, Prove by induction that

$$f(x_0 + nh) = f(x_0) + n\Delta f(x_0) + \frac{n(n-1)}{2!}\Delta^2 f(x_0) + \ldots.. + \Delta^n f(x_0).$$ [8 Marks]

(c) Solve the linear system of equations below by the Gauss-siedel method beginning with an initial guess to the solution of $(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0,0,0)$.

$$
\begin{aligned}
x_1 &+ 2x_2 &+ x_3 &= 4 \\
3x_1 &+ 8x_2 &+ 7x_3 &= 20 \\
2x_1 &+ 7x_2 &+ 9x_3 &= 23
\end{aligned}
$$

[6 Marks]

END