Uganda Martyrs University
Faculty of Science
Department of Natural Sciences

Quantitative Methods II

End of Semester Exam, Date: December 13, 2022

Timing: 09:30am to 12:30 pm

Academic Year 2021/2022, Semester 11

Maximum mark: 100

Instructions:

1. *Carefully read through ALL the questions before attempting them.*
2. ***ANSWER ANY FIVE Questions*** *(Each question is 20 marks)*
3. *No* ***names*** *should be written anywhere on the examination book.*
4. *Ensure that your* ***Reg number*** *is indicated on all pages of the examination answer booklet.*
5. *Ensure your work is* ***clear*** *and* ***readable****. Untidy work shall be penalized*
6. *Any type of examination Malpractice will lead to automatic disqualification*
7. *Do not write anything on the questions paper.*

1. (a) What is meant by the term Annuities. (06

(b) With examples, distinguish between annuity due and ordinary annuity. (03

(c) Mrs Musoke wants to save money for her child's college expenses. Suppose she deposits \$1000 at the beginning of each year for 18 years at the interest rate of 5%. How much will be available for her child's fund at the end of this period.

(04 marks)

(d) What do you understand by "Sinking funds". (01 mark)

(e) A business man borrows \$15000 at 18% payable monthly and makes monthly deposit into a Sinking fund so that his debt may be paid off at the end of one year. The Sinking fund earns 9% Compounded monthly.

(i) What is the monthly expense on the debt?

(ii) What is the book value of the debt at the end of 6 month.

(05 marks)

(f) Define "Amortization". **(01 mark)**

(g) Mohamed buys a car costing \$19,300. He agrees to make payment at the end of each month for a period of 5 years. He pays 6% interest compounded monthly.

(i) How much money does he pay in each monthly Installment?

(ii) Find the total amount of interest paid. **(05 marks)**

2. (a) A manufacturer produces three models of bicycles. The time (in hours) required for assembling, painting, and packaging each model is as follows.

	Model A	Model B	Model C
Assembling	2	2.5	3
Painting	1.5	2	1
Packaging	1	0.75	1.25

The total time available for assembling, painting, and packaging is 4006 hours, 2495 hours and 1500 hours, respectively. The profit per unit for each model is \$45 (Model A), \$50 (Model B), and \$55 (Model C). How many of each type should be produced to obtain a maximum profit? **(10 marks)**

(b) A small petroleum company owns two refineries. Refinery 1 costs \$20,000 per day to operate, and it can produce 400 barrels of high-grade oil, 300 barrels of medium-grade oil, and 200 barrels of low-grade oil each day. Refinery 2 is newer and more modern. It costs \$25,000 per day to operate, and it can produce 300 barrels of high-grade oil, 400 barrels of medium-grade oil, and 500 barrels of low-grade oil each day. The company has orders totaling 25,000 barrels of high-grade oil, 27,000 barrels of medium-grade oil, and 30,000 barrels of low-grade oil. How many days should it run each refinery to minimize its costs and still refine enough oil to meet its orders? **(10 marks)**

3. (a) The following data gives the quarterly sales, in 10000's, of gardening equipment at the Green Fingers Garden Centre over a period of four years.

	Quarter			
	1st	2nd	3rd	4th
1992	20	26	24	18
1993	24	30	27	23
1994	26	34	31	25
1995	30	36	35	29

Figure 1:

(i) Plot these values on a graph, joining the points with straight lines.

(ii) Suggest a reason for the seasonal variation shown by your graph.

(iii) Calculate the four-point moving averages for these data and enter these values a table.

(iv) Plot these moving averages on the graph.

(v) On your graph, draw a trend line by eye.

(vi) Use your graph to estimate the sales during the first quarter of 2022. **(20 marks)**

4. (a) (i) What is a Matrix?

(ii) State three Types of Matrices.

(04 marks)

(b) Given the Matrices below;

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 4 \\ -5 & 8 \end{bmatrix}$$

Determine;

(i) $A + B$.

(ii) $B^T \times A$. **(04 marks)**

(c) Using Crammer's rule, compute the determinant of Matrix A, such that:

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix}.$$

(04 marks)

(d) A company has products A and B at two locations P and Q. Full sales at the end of the year are given as follows: Product A: 50 units at location P and 45 units at location Q Product B: 60 units at location P and 70 units at location Q. Sales for the first quarter were given as follows:

- Product A: 30 units at location P and 15 units at location Q.

- Product B: 20 units at location P and 25 units at location Q.

Determine the sales position for the last nine months. (03

(e) Three processes A, B and C require inputs x, y and z (in kgs) to produce a particular product in the following proportions:

- Process A requires 3 of x, 2 of y & 1 of z for a total of 85 kg.
- Process B requires 1 of x, 3 of y & 2 of z for a total of 100 kg.
- Process C requires 4 of y & 3 of z for a total of 125 kg.

Determine the total amount of x, y and z needed to produce the product. **(05 marks)**

5. (a) Differentiate the following with respect to x:

(i) $y = 2x^2 + 3x$

(ii) $y = (4x^3 - 3x + 2)(2x^2 + 4x)$

(iii) $y = (4x^3 + 3x - 7)^4$ **(07 marks)**

(b) A firms total cost curve is given by,

$$TC = Q^3 - 4Q^2 + 12Q.$$

(i) Find an expression for AC in terms of Q.

(ii) Find an expression for MC in terms of Q.

(iii) When does AC=MC?

(iv) When does the slope of AC=0? **(06 marks)**

(c) Differentiate the functions with respect to x:

(i) $y = e^{-7x}$

(ii) $y = ln(x^2 + 2x + 1)$ **(04 marks)**

(d) If the (inverse) Demand equation is,

$$P = 20040ln(Q + 1).$$

Calculate the price elasticity of demand when $Q = 20$. **(03 marks)**

6. (a) Distinguish between "Discrete" and "Continuous" data. **(02 mark)**

(b) Categories each of the following Random Variables as continuous or Discrete:

(i) A is "the age in years of the first person i see wearing a hat".

(ii) B is "the length of the next car to enter the car park".

(iii) C is "the number of cows i see on my way to work". **(03 marks)**

(c) Let X represent the number of heads obtained when tossing a fair coin 3 times.

(i) What are the possible values of X.

(ii) What are the associated probabilities.

(iii) Determine the mean value of X. **(05 marks)**

(d) For the discrete random variable X, the probability distribution is given by,

$$P(X = x) = \begin{cases} \text{kx} & x = 1, 2, 3, 4, 5 \\ \text{k(10-x)} & x = 6, 7, 8, 9 \end{cases}$$

Find:

(i) the value of constant k.

(ii) E(X).

(iii) V(X).

(iv) Standard Deviation, σ.

(09 marks)

Best wishes