UGANDA MARTYRS UNIVERSITY, NKOZI

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

UNIVERSITY SPECIAL/SUPPLEMENTARY EXAMINATIONS

First Year Bachelor of Science (FM,ECON and GEN)

MTC 1201 Calculus II

Date : Thursday Aug 6^{th}, 2015

Time : 3 Hours (10:00 am - 1:00 pm)

Instructions

(i) *Read through the paper carefully and follow instructions on the answer booklet.*

(ii) *Attempt any* **Four** (4) *questions.*

(iii) *Do not write any thing on this question paper:*

(iv) *Calculators and mathematical tables may be used.*

(v) *Neat work is highly recommended.*

1. (a) Write out all terms and compute the sum

$\sum_{i=-2}^{1} 3i^2$ [5 marks]

(b) Evaluate $\int_1^2 (x^3+2)^4 x^2 dx$ [7 marks]

(c) Determine whether the following integrals converge or diverge

(i) $\int_2^\infty \frac{1}{(x-1)^2} dx$ [6 marks]

(ii) $\int_1^\infty xe^{-x^2} dx$ [7 marks]

2. a(i) Use Trapezoidal rule to evaluate $\int_0^1 \frac{1}{1+x} dx$ where the interval [0,1] is divided into n=5 equal parts [8 marks]

(ii) Estimate a maximum error in the approximation [5 marks]

b(i) Approximate $\int_1^2 \frac{1}{x} dx$ by using Simpson's rule with n=10. [7 marks]

(ii) Estimate the error in the approximation [5 marks]

3. (a) Define the following

(i) An infinite sequence [2 marks]

(ii) Limit of a sequence [2 marks]

b Evaluate each of the following using theorems on limits.

(i) $lim_{n \to \infty} \frac{3n^2+4n}{2n-1}$ [4 marks]

(ii) $lim_{n \to \infty} (\frac{3n-2}{3n+5})^3$ [5 marks]

c(i) Using the definition of limit, prove that: $lim_{n \to \infty} \frac{4-2n}{3n+2} = -\frac{2}{3}$ [6 marks]

(ii) Determine for which values of p the series

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ (a p-series) conveges [6 marks]

4. a(i) State properties of definite integrals [6 marks]

(ii) Express $\int_5^1 f(x)dx + \int_{-3}^5 f(x)dx$ as a single integral of the form

$\int_a^b f(x)dx$ [4 marks]

(iii) Express $\frac{x^2+2}{(x-2)^2(x+2)}$ into partial form.

hence find $\int_0^1 \frac{x^2+2}{(x-2)^2(x+2)}dx$ [8 marks]

(b) Evaluate $\int_0^1 x\sin 2x dx$ [7 marks]

5. a(i) State the Mean Value Theorem [3 marks]

(ii) Find a value C satisfying the conclusion of the Mean Value Theorem

for $f(x) = x^3 - x^2 - x + 1$ on the interval [0, 2] (6 marks)

b(i) State the Fundamental Theorem of Calculus. [3 marks]

(ii) Use the First Fundamental Theorem to evaluate $\int_0^{2\pi} \sin x\, dx$

and sketch the graph [7 marks]

(c) Compute the following integrals.

(i) $\int_0^2 (2x - 3)dx$ [4 marks]

(ii) $\int_0^4 (\sqrt{x} + 3x)dx$ [4 marks]

6. (a) Verify

(i) $\cosh x + \sinh x = e^x$ [5 marks]

(ii) $\cosh x - \sinh x = e^{-x}$ [5 marks]

(b) Compute the derivative of $f(x) = \sinh^2 x$ [5 marks]

(c) Find the limits if they exist

(i) $\lim_{x \to 0} \frac{e^{2x}-1}{x}$ [5 marks]

(ii) $\lim_{x \to 1} \frac{1+\cos \pi x}{x^2-2x+1}$ [5 marks]