UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

University Examinations 2013-2014, Semester I

Third Year **Final Assessment Examination** for the Degree of Bachelor of Science in Financial Mathematics and General.

MTC 3101 NUMERICAL ANALYSIS I

Wednesday, 11^{th} December 2013 Time: 9:00am - 12:00 Noon

Instructions

(i) Answer **Five** questions

(ii) Write on both sides of the paper but begin a new question on a fresh page.

1. ***Question 1***

   (a) (i) **Theorem**: The n^{th} degree interpolating polynomial

$$P_n(x) = \sum_{k=0}^{n} L_k(x) f_k \ ,$$

for

$$L_k(x) = \prod_{\substack{i=0 \\ j \neq k}}^{n} \left(\frac{x - x_j}{x_k - x_j}\right)$$

What is the name of the given theorem?

[1 Mark]

(ii) Using Linear interpolation

$$(i.e\ the\ method\ of\ forming\ a\ system\ of\ algebraic\ polynomials)$$

, construct a cubic polynomial that passes through the points $(-2, 4)$ $(0, 10)$ $(1, 10)$ $(2, 16)$.

[8 Marks]

(b) Let $\{x_0, x_1, ..., x_n\}$ be $(n+1)$ distinct points on the interval $[a, b]$, and $f(x_k)$ is defined on $[a, b]$ such that

$$f(x_k) = \sum_{i=0}^{n} f_i L_i(x_k) + \frac{1}{(n+1)!} f^{n+1}(\psi) \prod_{\substack{i=0 \\ i \neq k}}^{n} (x_k - x_i)$$

, for some $x_k \in [a, b]$, Where $L_i(x_k)$ is the Lagrange's interpolation polynomial of degree i, Show that for $n = 2$ and $k = 0$

$$f'(x_0) = \frac{1}{h}\Big[-\frac{3}{2}f(x_0) + 2f(x_0 + h) - \frac{1}{2}f(x_0 + 2h)\Big] - \frac{h^2}{3}f'''(\psi) \ ,$$

for equally spaced data points $x_0, x_1 = x_0 + h, x_2 = x_0 + 2h$, and h is the spacing between the $x-$values.

[11 Marks]

Question 2

(a) What do you understand by the term polynomial interpolation?

[1 Mark]

(b) Given that $\triangle f(x_0) = f(x_0 + h) - f(x_0)$, by generating expressions for

$$f(x_0 + h),\ f(x_0 + 2h)\ and\ f(x_0 + 3h)$$

in terms of $\triangle$ *and* $f(x_0)$, verify that

$$f(x_0 + nh) = f(x_0) + n\triangle f(x_0) + \frac{n(n-1)}{2!}\triangle^2 f(x_0) + ... + \triangle^n f(x_0)$$

[6 Marks]

(c) Given the table below

x	2.95	3.05	3.15	3.25	3.35
$f(x)$	13.364875	14.665125	16.088375	17.640625	19.327875

(i) Construct a finite difference table from the table above and use it to evaluate the following with appropriate formulas.

[2 Marks]

(ii) $f(3.00)$, by using a polynomial of degree 3.

[4 Marks]

(iii) $f(3.12)$, using Bessel's formula.

[4 Marks]

(iv) $f(3.34)$, by using a polynomial of degree 3.

[3 Marks]

Question 3

(a) (i) From Newton Gregory forward difference formula, derive the Trapezoidal rule formula for evaluating

$$\int_{x_0}^{x_0+nh} f(x)dx ,$$

with $n = 1$.

[6 Marks]

(ii) Give an expression for the truncation error involved when evaluating

$$\int_a^b f(x)dx$$

using the Trapezoidal formula.

[1 Mark]

(b) (i) Use the composite Trapezoidal rule with 6$-$ strips to evaluate.

$$\int_0^{\frac{\pi}{2}} x^2 cos\ 2x\ dx ,$$

The cosine values expressed in radians to 4$-$ decimal places.

[8 Marks]

(ii) Find the value of the same integral in $b(i)$ above using the **closed Simpson's rule** formula.

[5 Marks]

Question 4

(a) (i) For a function $f(x)$, continuous on the interval [a, b] outline the **Bisection algorithm** for locating the roots to the equation $f(x) = 0$ on the interval [a,b].

[3 Marks]

(ii) The equation $f(x) = x^3 + 4x^2 - 10 = 0$,has a root in the interval $[1,2]$ use the Bisection method to find the value of this root approximated to two decimal places, after 6-iterations. Show your working clearly, and put the results of your iterations in a suitable table. During the computations the values should be rounded off to atleast 4-decimal places.

[10 Marks]

(b) Find the root to 2− decimal places to the equation $f(x) = x^3 - 7x^2 + 14x - 6 = 0$, using the Newton Raphson method on the interval $[3.2\ ,\ 4]$, beginning with $p_0 = 3.45$ and iterate up to p_3.

[7 Marks]

Question 5

(a) For $E, \triangle, \nabla, \delta$, the displacement, forward, backward and central difference operators respectively,prove that

(i) $\delta = E^{\frac{1}{2}} - E^{\frac{-1}{2}}$.

[3 Marks]

(ii) $\nabla = 1 - E^{-1}$.

[3 Marks]

(b) (i) From Newton Gregory backward difference formula derive the formulas for determining $f'(x_0 + ph)$,$f''(x_0 + ph)$ and $f'(x_0)$.

[7 Marks]

(ii) Using the table of values below, form a finite difference table out of it and use it to determine $f'(1.96)$, $f''(2.0)$ and $f'(2.0)$ using the derived formulas in $b(i)$ above.

[7 Marks]

x	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	14.5503	17.4726	22.1245	29.4157	40.6894	57.9336

Question 6

(a) Solve the linear system of equations below using the **Gauss Seidel method**.Beginning with the initial guess to the solution of $(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (1,1,1)$, do 4−iterations and present your results in a suitable table.The answers during the iterations should be rounded off to 4−decimal places and the final answers to the nearest whole.

$$\begin{aligned} 4x_1 + 3x_2 &= 24 \\ 3x_1 + 4x_2 - x_3 &= 30 \\ -x_2 + 4x_3 &= 24 \end{aligned}$$

[10 Marks]

(b) Solve the linear system of equations below using the **Jacobi method**.Beginning with the initial guess to the solution of $(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0, 0, 0)$, do 4−iterations and present your results in a suitable table.The answers during the iterations should be rounded off to 4−decimal places and the final answer to the nearest whole.

$$\begin{aligned} 10x_1 + 2x_2 + x_3 &= 25 \\ 3x_1 + 20x_2 - x_3 &= 23 \\ x_1 - 3x_2 + 10x_3 &= 29 \end{aligned}$$

[10 Marks]

END