UGANDA MARTYRS UNIVERSITY
NKOZI

UNIVERSITY EXAMINATION
FACULTY OF SCIENCE

SEMESTER I, 2023/ 2024

THIRD YEAR EXAMINATION FOR BACHELOR OF SCIENCE WITH EDUCATION

PHY 3103: STATISTICAL MECHANICS

DATE: 12 DEC 2023
TIME: 9:30 – 12:30pm
DURATION: 3HRS

Instructions:

1. *Carefully read through ALL the questions before attempting*
2. *Attempt ANY five questions*
3. *All Questions carry equal marks*
4. *No **names** should be written anywhere on the examination book.*
5. *Ensure that your **Reg number** is indicated on all pages of the examination answer booklet*
6. *Ensure your work is **clear** and **readable**. Untidy work shall be penalized*
7. *Any type of examination Malpractice will lead to automatic disqualification*
8. *Do not write anything on the questions paper.*

Where necessary assume

Planck's constant	$h = 6.63 \times 10^{-34}\ J\,s$
Boltzmann's constant	$K_B = 1.38 \times 10^{-23} JK^{-1}$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ kg$
Electronic charge	$e = 1.60 \times 10^{-19}\ C$
Speed of light	$c = 3.0 \times 10^{8}\ ms^{-1}$
Avogadro's number	$NA = 6.02 \times 10^{23}\ mol^{-1}$
Universal gas constant	$R = 8.31\ JK^{-1}mol^{-1}$
Acceleration due to gravity	$g = 9.81 ms^{-2}$
1 standard atmosphere	$= 1{:}01 \times 10^{5} Nm^{-2}$
Radius of Earth	$R_e = 6.38 \times 10^{6}\ m$
Solar constant	$S = 1.37 \times 10^{3}\ Js^{-1}m^{-2}$
Universal constant	$G = 6.67 \times 10^{-11} Nm^2kg^{-2}$
Coulomb constant	$K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}$
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12} Fm^{-1}$

QUESTION ONE

a. i. What is statistical physics? **(2 marks)**

ii. Differentiate between microscope properties and macro properties, giving two examples of each. **(4 marks)**

iii. Show how macroscopic properties depend on microscopic properties? **(3 marks)**

iv. Explain why macro state of a system remains the same while micro state changes. **(3 marks)**

b. i. What is the difference between Cartesian space and phase space? **(2 marks)**

ii. What is the difference between a molecule phase space and a gas phase space? **(2 marks)**

iii. Derive the general formula of the ensemble average of a system of particles. **(4 marks)**

QUESTION TWO

a. i. State the Liouville's theorem and derive the Second form of Liouville's theorem. **(4 marks)**

ii. Using Liouville's theorem, derive the equation that describes the implicit dependence of P (q, p, t) on time. **(3 marks)**

b. i. Define a stationary ensemble writing down its condition. **(4 marks)**

ii. What is an accessible phase space? **(2 marks)**

iii. Outline the two methods of making the ensemble of a system in equilibrium. **(2 marks)**

ii. Outline any five characteristics of entropy according to thermodynamics. **(5 marks)**

QUESTION THREE

a. i. what are the three properties of entropy according to thermodynamics. **(3 marks)**

ii. Show that entropy is a dimensionless quantity? **(5 marks)**

iii. Prove that is statistical mechanics, the additive property of entropy is given as, $\sigma = \sigma_1 + \sigma_2$. **(3 marks)**

b. i. What are the three types of equilibrium that qualify a system to be in equilibrium? **(3 marks)**

ii. Derive the internal energy expression, $U = F + \sigma\tau$, given that, $\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_V$, and $F = -\tau \log Z$ **(6 marks)**

QUESTION FOUR

a. i. Show that for a perfect gas, $\pi = \rho$, if $\sigma = \log \Delta\Gamma$ and $\frac{\pi}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_{U,N}$ **(5 marks)**

ii. What is chemical potential? **(2 marks)**

iii. Show that for a non-equilibrium system, particles tend to move from a region of higher chemical potential (μ) to region of lower chemical potential(μ) as the system approaches equilibrium. **(3 marks)**

b. i. Describe the characteristics of a micro canonical ensemble. **(5 marks)**

ii. Describe the Gibbs Paradox. **(5 marks)**

QUESTION FIVE

a. i. Show that the probability of a canonical ensemble is a combined probability of multiplying the probability of each of the systems. **(5 marks)**

ii. Sketch and discuss the probability density graph of a canonical ensemble. **(5 marks)**

b. Show that,

i. $\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_V$, if Gibbs free energy is given as, $F = U - \sigma\tau$. **(5 marks)**

ii. $U = -\tau^2 \frac{\partial}{\partial \tau}\left(\frac{F}{\tau}\right)_V$, if Gibbs free energy is given as, $F = U - \sigma\tau$. **(5 marks)**

QUESTION SIX

(a) Define the following terms?

(i) Phase space **(1 mark)**

(ii) Number of degree of freedom **(1 mark)**

(b) The Max-Wellian distribution of velocities for a gas is

$$P(v) = 4\pi\left(\frac{m}{2\pi K_B T}\right)^{3/2} v^2 exp\left(\frac{-mv^2}{2K_B T}\right)$$

(i) Find the most probable speed v_m of the Max-Wellian gas **(6 marks)**

(ii) Sketch the variation of $P(v)$ with v, indicating v_m, the mean speed $< v >$ and the root mean square velocity v_{rms} **(4 marks)**

(c) Consider a gas composed of Oxygen molecules. If the volume of the gas is 10^2 cm^3 at a pressure of 1.013 x 10^5 Pa and temperature of 300K, find

(i) The number of molecules in the system **(3 marks)**

(ii) Average speed of an Oxygen molecule at 300 K. (Assume the gas obeys the ideal gas equation, molar mass of Oxygen molecules = 32g) **(5 marks)**

END