UGANDA MARTYRS UNIVERSITY
FACULTY OF SCIENCE
SPECIAL / SUPPLEMENTARY EXAMS
AUGUST 2015
STA 2201: ADVANCED PROBABILITY
BSc.FM II AND BSc. GEN II

Instructions

Attempt any FIVE questions in this paper
ONLY non programmable calculators are allowed to be used
Each question carries 20 marks

Question One

a) Show that the moment generating function for a gamma distribution given by: $f(x) = \frac{\beta^{\alpha}}{\Gamma\alpha.} . x^{\alpha-1} . e^{-x/\beta}$; $x = 0, 1, \ldots$ is:

$M(t) = \frac{\beta^{\alpha}}{(\beta - t)^{\alpha}}$. Hence determine the E(X) and V(X).

b) The joint distribution function of X and Y is defined as: $\frac{2}{3} y(\frac{x^2}{2} + x)$ for $0 \le x, y \le 1$ and zero elsewhere, determine the:

i. Marginal distribution and density functions
ii. Joint density function
iii. $P(X \le 0.5, Y \le 0.6)$
iv. $P(y \le 0.5 \mid x \le 0.8)$
v. $E(X \mid Y \le 0.7)$
vi. Independence or dependence of X and Y

c) Using the probability generating function, find the expected value and variance of a random variable whose probability mass function is: $(0.5)^{x+1}$; $x = 0, 1, 2, 3, \ldots$

Question Two

a) The following table shows a probability model of the demand (D) and supply (S) of a perishable commodity

$P(D=i, S=j)$	1	2	3
$j=0$	0.015	0.025	0.010
1	0.045	0.075	0.030
2	0.195	0.325	0.130
3	0.030	0.050	0.020
4	0.015	0.025	0.010

Prove that it is a genuine probability mass function and determine:

i. Expected demand of the commodity
ii. The expected demand of the commodity given that four were supplied
iii. The chance that at least two commodities are supplied given that at most three were demanded
iv. The chance that exactly two commodities were demanded
v. The covariance and correlation coefficient between demand and supply of the commodity. Comment on this relationship

b) Given that X and Y are iid exponentially distributed random variables. Find the p.d.f of $Z = X + Y$ using the moment generating function technique.

Question Three

a. State the probability mass function of a geometric distribution. Prove that its probability mass function is one.

b. Jobs arrive every fifteen seconds on average. What is the probability of waiting less than or equal to thirty seconds?

c. The joint density function of X and Y is defined as:

$2x^2y + 2y^2$ for $0 \leq x, y \leq 1$ and zero elsewhere, determine the:

i. Marginal density functions

ii. The conditional density functions

iii. $P(y \leq 0.5 | x \leq 0.8)$

iv. E(X, Y). Are X and Y independent?

Question Four

a) The joint density function of X, Y and Z is given by:

$$f(x, y, z) = \quad ; \qquad 0 < x, \; y, z < 1$$

Find

i. The value of the constant k

ii. The probability density function of x

iii. $f(x, z)$

iv. $E(X)$

b) Patty is a University basketball player from UMU. She is a 80% free throw shooter. During the competition with UCU, what is the probability that Patty makes:

i. Her third free throw on her fifth shot?

ii. Her first free throw on her fifth shot?

c) Derive the moment generating function of a uniform distribution described by the pdf:

$f(x) = \frac{1}{b-a}$; $\quad a, b \in \mathbb{R}$. Show that: $E(X) = \frac{a+b}{2}$ and $V(X) = \frac{(a-b)^2}{12}$

Question Five

a) Differentiate between a binomial distribution and a negative binomial distribution

b) If X_1, X_2, X_3, ……., X_n are n random variables for which $V(X_i)$ exists for all values of I =1, 2, 3, ……. n. Then:
$V(X_1 + X_2 + X_3 \ldots + X_n) = V(X_1) + V(X_2) + \ldots + V(X_n)$. Prove this for a case when n = 2.

c) Let the probability density function of X_1 and X_2 be given by

$f(x_1, x_2)$ = exponential $-(x_1+x_2)$; $x_1 \geq 0$, $x_2 \geq 0$ and zero otherwise.
Consider two random variables Y_1 and Y_2 defined as:

$Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1 + X_2)$. Determine using transformation technique the:

i. Joint density of Y_1 and Y_2
ii. Marginal densities

Question Six

a) Suppose that the events B_1; B_2; ....; B_k partition the sample space Ω for some experiment and that set A is an event defined on Ω. Suppose we know the probabilities $P(A/B_j)$, show that:

$$P(B_j/A) = \frac{P(A/B_1).P(B)}{P(A/B_1).P(B_1) + P(A/B_2).P(B_2) + \cdots + P(A/B_k).P(B_k)}$$

b) The entire output of a factory is produced on three machines. The three machines account for 20%, 30% and 50% of the output respectively. The fraction of defective items produced is as follows:-
1st machine 5%, 2nd machine 3% and 3rd machine 1%. If an item is chosen at random from the total output and found to be defective, what is the probability that it was produced by the 3rd machine?

c) Given $F(x, y) = \frac{2}{3}y\left(\frac{1}{2}x^2 + x\right)$; $0 \le x, y \le 1$ and zero otherwise.

Calculate the:

i. Joint probability density function of X and Y
ii. Marginal density and distribution functions
iii. The conditional density functions
iv. Are X and Y dependent?

END