UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL ASSESSMENT
SEMESTER I

THIRD YEAR EXAMINATIONS FOR BSc Economics & Statistics

STA 3104: Operations Research

DATE: Friday 15^{th} **July,** 2022
TIME: 2:00pm - 5:00pm
DURATION: 3 Hours

Instructions:

1. *Carefully read through ALL the questions before attempting the examination.*
2. ***ANSWER ANY FOUR Questions*** *(Each question carries a total of 25 marks).*
3. *No **names** should be written anywhere on the examination book.*
4. *Ensure that your **registration number** is indicated on all pages of the examination answer booklet.*
5. *Ensure your work is **clear** and **readable**. Untidy work shall be penalized.*
6. *Any type of examination Malpractice will lead to automatic disqualification.*
7. *Do not write anything on the questions paper.*

1. a. Draw a network representation of the following linear programming scenario and write a linear program for it:

A steel company has to ship the ore needed to make steel from the three different mines they are working on to the four different plants where the steel will be made. The amount of ore (in tons) that is available at the different mines, the minimum amount of ore (in tons) needed at the plants to process current orders, and the shipping cost (in US dollars) per ton from the various mines to the various plants are given in the following table:

	Plant 1	Plant 2	Plant 3	Plant 4	**Available at mine ↓**
Mine 1	500	300	600	500	**1800**
Mine 2	600	700	850	400	**1200**
Mine 3	900	600	700	300	**1500**
Needed at plant →	**100**	**1100**	**800**	**900**	

How must the shipments be made so that the demands are met at minimal shipping cost to the company? **[15 marks]**

b. Consider the following linear program:

$$\begin{aligned} \text{Maximize} \quad & Z = 2x_1 + 3x_2 \\ \text{subject to} \quad & x_1 + 3x_2 \leq 9 \\ & 2x_1 + 3x_2 \leq 12 \\ & x_1, x_2 \geq 0. \end{aligned}$$

(i) Which of the following points (x_1, x_2) are feasible and which ones are not feasible?

$(3, 0)$, $(7, 2)$, $(6, 0)$, $(3, 2)$ **[4 marks]**

(ii) Apart from the points in part (i) above, list three other points (x_1, x_2) which are feasible and three points (x_1, x_2) which are not feasible. **[6 marks]**

2. a. Explain the criteria followed in the Simplex algorithm when choosing the variable to:

(i) enter the basis. **[2 marks]**

(ii) leave the basis. **[2 marks]**

b. Use the *Big M* method to solve the following linear program.

$$\begin{aligned} \text{Maximize} \quad & Z = x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + x_2 = 8 \\ & x_1 + 2x_2 \leq 14 \\ & 2x_1 + x_2 = 14 \\ & x_1, x_2 \geq 0. \end{aligned}$$

[10 marks]

c. Given below is an LP program with it's initial Simplex tableau.

$$\begin{aligned} \text{Maximize} \quad & Z = 20x_1 + 10x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 40 \\ & 3x_1 + 2x_2 \leq 60 \\ & x_1, x_2 \geq 0. \end{aligned} \tag{1.1}$$

C_B	x_B	x_1	x_2	x_3	x_4	
0	x_3	1	2	1	0	40
0	x_4	3	2	0	1	60
	ΔZ_j	-20	-10	0	0	

Table 1: Initial tableau

Using the tableau,

(i) state the basic variables and the non-basic variables. [4 marks]

(ii) state the solution and mention if it is feasible or not. [5 marks]

(iii) Is the solution optimal? [2 marks]

3. a. Consider the following linear program (LP)

$$\begin{aligned} \text{Minimize} \quad & Z = 8x_1 + 4x_2 + 6x_3 \\ \text{subject to} \quad & 3x_1 + 2x_2 + x_3 \geq 6 \\ & 4x_1 + x_2 + 3x_3 \geq 7 \\ & 2x_1 + x_2 + 4x_3 \geq 8 \\ & x_1, x_2, x_3 \geq 0. \end{aligned} \tag{1.2}$$

(i) Determine the dual of LP (1.2) above, and [4 marks]

(ii) hence, solve the dual of LP (1.2) to give the optimal and feasible solutions. [10 marks]

(iii) hence, state the feasible solution of LP (1.2). [3 marks]

b. Using relevant examples, explain in detail the concepts of *standard* and *canonical* forms of a linear program and the differences between the two. [8 marks]

4. a. Find the optimal solution to the following linear programs using the Dual Simplex algorithm.

$$\begin{aligned} \text{Maximize} \quad & Z = -x_2 \\ \text{subject to} \quad & -x_1 - 5x_2 \leq -10 \\ & 6x_1 - 5x_2 \leq -3 \\ & x_1, x_2 \geq 0. \end{aligned}$$

[15 marks]

b. Using a specific example explain, clearly and in detail, one practical application of operations research in a university setting. [10 marks]

5. a. Using relevant examples, explain the difference between a *feasible solution* and an *optimal solution* of an LP problem? [5 marks]

b. Write the following linear program as a minimization problem in canonical form.

$$\begin{aligned} \text{Maximize} \quad & Z = x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + x_2 - x_3 \leq 8 \\ & x_1 + 2x_2 \leq 14 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

[5 marks]

c. Solve the following linear program using the graphical method

$$\begin{aligned} \text{Minimize} \quad & Z = 3x + 2y \\ \text{subject to} \quad & 5x + 2y \geq 20 \\ & x + y \leq 12 \\ & 6x - y \geq 18 \\ & 4x - 3y \leq 12 \\ & -5x + 2y \leq 10 \\ & x, y \geq 0. \end{aligned}$$

[15 marks]

END