

## Uganda Martyrs University Faculty of Science Department of Natural Sciences

## Statistical Inference

End of Semester Exam, Date: December 13, 2022

Timing: 09:30am to 12:30 pm

Academic Year 2022/2023, Semester 1

Maximum mark: 100

## Instructions:

1. Carefully read through ALL the questions before attempting them.

- 2. ANSWER ANY FIVE Questions (Each question is 20 marks)
- 3. No names should be written anywhere on the examination book.
- 4. Ensure that your **Reg number** is indicated on all pages of the examination answer booklet.
- 5. Ensure your work is clear and readable. Untidy work shall be penalized
- 6. Any type of examination Malpractice will lead to automatic disqualification
- 7. Do not write anything on the questions paper.

1. (a) What is "Biased Sampling" and how can it be eliminated?

(02 mark)

(b) The weights of a group of students (in pounds, lbs) sampled from a population

135 105 118 163 172 183 122 150 121 162

- (i) Find the sample mean and standard Deviation.
- (ii) What is the sample mean.
- (iii) If another student joins in the group and his weight is 250 lbs, what would be the new sample mean? (07 marks)
- (c) What is meant by a "Sampling Distribution"?

(01 mark)

- (d) A patient is classified as having gestational diabetes if the glucose level is above 140 milligrams per deciliter (mg/dl) one hour after a sugary drink is ingested. Sheilas measured glucose level one hour after ingesting the sugary drink varies according to the normal distribution with  $\mu=125$  mg/dl and  $\sigma=10$  mg/dl.
  - (i) If a single glucose measurement is made, what is the probability that Sheila is diagnosed as having gestational diabetes?
  - (ii) If measurements are made on three separate days and the mean result is compared with the criterion 140 mg/dl, what is the probability that Sheila is diagnosed as having gestational diabetes?
  - (iii) What is the level L such that there is probability only 5% that the mean glucose level of three test results fall above L for Sheilas glucose level distribution. (10 marks)
- 2. (a) A soft-drink machine is regulated so that the amount of drink dispensed averages 240 milliliters with a standard deviation of 15 milliliters. Periodically, the machine is checked by taking a sample of 40 drinks and computing the average content. If the mean of the 40 drinks is a value within the interval μ<sub>X</sub> ± σ<sub>X</sub>, the machine is thought to be operating satisfactorily; otherwise, adjustments are made. The company official found the mean of 40 drinks to be x = 236 milliliters and concluded that the machine needed no adjustment. Was this a reasonable decision? (06 marks)
  - (b) State the Central Limit Theorem, CLT?

(01 mark)

- (c) The entrance exam scores of Uganda Martyrs' University has a mean of 74 and a standard deviation of 6.8. The exam scores follow a normal distribution:
  - If a student is selected at random, what is the probability that his exam score is less than 65.
  - (ii) If a sample of 50 students is selected at random, what is the probability that them mean exam score of this group is greater than 75?
  - (iii) What is the distribution for the mean score of 50 students.
  - (iv) Find the 80<sup>th</sup> percentile for the mean score of the 50 students. (09 marks)
- (d) Show that if independent samples of sizes n<sub>1</sub> and n<sub>2</sub> are drawn at random from two populations with means μ<sub>1</sub> and μ<sub>2</sub>, then the sampling distribution of means X

  1 - X

  2 is approximately normally distributed such that:

- 3. (a) Suppose that for Ugandans between 20 to 30 years, male heights are approximately normally distributed with μ<sub>M</sub> = 177.7 cm and σ<sub>M</sub> = 5.6 cm. Female heights are approximately distributed with mean μ<sub>F</sub> = 163.0 and σ<sub>F</sub> = 5.1 cm.
  - (i) If 20 males and 15 females in this age group are randomly selected, what is the sampling distribution of  $\overline{X}_M \overline{X}_F$ .
  - (ii) If 20 males and 15 females in this age group are randomly selected, what
    is the probability that the average height of males is at least 10 cm greater
    than the average height of females.
     (07 marks)
  - (b) What is meant by the following:
    - (i) Point Estimate
    - (ii) Interval Estimate (04 marks)
  - (c) Derive a  $100(1-\alpha)\%$  confidence interval for population mean  $\mu$ , given that  $\overline{x}$  is the mean of a random sample of size n obtained from a population of standard deviation  $\sigma$ . (03 marks
  - (d) High school students who take the SAT mathematics exam a second time generally score higher than on their first try. The change in score has a normal distribution with variance  $\sigma^2 = 2500$ . A random sample of 1000 students gains an average of  $\overline{x} = 22$  points on their second try.
    - (i) Construct a 95% confidence interval for the mean score gain  $\mu$  in the population of all students.
    - (ii) Interpret the C.I. in part (i) above. (06 marks)
- 4. (a) A community health nutritionist wishes to conduct a survey among a population of teenage girls to determine their average daily protein intake (measured in grams). Assume that the population of protein intakes is normally distributed with a standard deviation of 20 grams. If she wants a 95% confidence interval with an error of no more than 5 grams, how many teenage girls should be interviewed?
  - (b) An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 98% lower one-sided confidence interval for the population mean of all bulbs produced by this firm.

    (05 marks)
  - (c) Given that  $\overline{x}$  and s are the mean and standard deviation of a random sample of size n obtained from a population with unknown standard deviation  $\sigma$ , show that the  $100(1-\alpha)\%$  confidence interval for population mean  $\mu$  is such that:

$$1 - \alpha = P\left(\overline{X} - t_{\alpha/2} \frac{s}{\sqrt{n}} < \mu < \overline{X} + t_{\alpha/2} \frac{s}{\sqrt{n}}\right)$$
 (05 marks)

(d) The data bellow represents monthly fees (in dollars) paid by a random sample of 50 users of commercial Internet service providers in August 2000:

- (i) Is it appropriate to use t confidence interval to analyze the data? Briefly explain.
- (ii) Give a 95% confidence interval for the mean monthly cost of Internet access in August 2000. (07 marks)
- 5. (a) We would like to compare the mean tar content in regular cigarettes and light cigarettes. We take simple random samples of regular and light cigarettes of a particular brand and measure the tar content (in mg) of each cigarette. The data are as follows:

 Regular
 11.3
 12.1
 12.6
 11.5
 12.2
 12.8

 Light
 9.5
 9.8
 9.3
 8.9
 10.0

It is known that tar content for regular cigarettes of this brand follows a normal distribution with standard deviation 0.4 mg and tar content for light cigarettes of this brand follows a normal distribution with standard deviation 0.3 mg. Find a 95% confidence interval for the difference in mean tar content for all regular cigarettes and all light cigarettes of this brand.

(05 marks)

- (b) If  $\overline{x}_1$  and  $\overline{x}_2$  are means of independent random samples of sizes  $n_1$  and  $n_2$  from populations with unknown but equal variances, state the  $100(1-\alpha)\%$  confidence interval for  $\mu_1 \mu_2$ , clearly stating the pooled estimate of standard deviation. (02 marks)
- (c) An insurance company would like to know if men drive faster on average than women. The company took a random sample of 52 cars driven by men on a highway and found the mean speed to be 114 km/h with a standard deviation of 10 km/h. Another sample of 30 cars driven by women on the same highway gave a mean speed of 108 km/h with a standard deviation of 7 km/h. Construct a 98% confidence interval for the true difference between the mean speeds of cars driven by men and women on this highway. (06 marks)
- (d) The gasoline prices (in cents/litre) for a random sample of 8 Winnipeg gas stations and 5 Calgary gas stations are recorded one day and are shown below:

Winnipeg 119.9 122.4 121.7 120.9 121.0 122.9 119.9 121.7 Light 117.9 120.4 118.4 122.9 117.0

Find a 95% confidence interval for the difference in mean gas prices for the two cities. (07 marks)

6. (a) If  $\hat{p}$  is the proportion of successes in a random sample of size n, state the approximate  $100(1-\alpha)\%$  confidence interval, for the binomial parameter p.

(03 marks)

- (b) A question in a Christmas tree market survey was Did you have a Christmas tree last year? Of the 500 respondents, 421 answered Yes.
  - (i) Find the sample proportion and its standard error.
  - (ii) Give a 90% confidence interval for the proportion of Indiana households who had a Christmas tree this year. (05 marks)
- (c) What is meant by a "Statistical Hypothesis" as used in Non-parametric Methods. (01 marks)
- (d) A health advocacy group suspects that cigarette manufacturers sell cigarettes with a nicotine content higher than what they advertise (1.4 mg per cigarette) in order to better addict consumers to their products and maintain revenues. The health advocacy group took a sample random sample of 12 cigarettes and found that the sample mean was 1.6 mg with a standard deviation of 0.3 mg. Suppose that the measurement of nicotine in cigarettes is normally distributed. Is there any evidence for the suspicion of the health advocacy group at  $\alpha = 5\%$ ? State the hypotheses, calculate the value of the test statistic, write the rejection region and draw the statistical conclusion. (07 marks)
- (e) Explain the two types of errors applied in Hypothesis testing.

(04 marks)

Best wishes

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2.0	0.3814	3802	3790	3778	3765	3867	3857	3847	3836	3825	1	2 2	3	4 5	5	5 7 7	6	7	
.4	0.3683	3668	3653	3637	3621	3752 3605	3739 3589	3725 3572	3712 3555	3697 3538	1 2	3	4 5	6	7	8	10	10 11	
.5	0.3521	3503	3485	3467	2440	2420						-		1	Ü	10	11	13	
.6	0.3332	3312	3292		3448	3429	3410	3391	3372	3352	2	4	6	8	10	11	13	15	
.7	0.3123	3101	3079	3271	3251	3230	3209	3187	3166	3144	2	4	6	8	10	13	15	17	
.8	0.2897	2874		3056	3034	3011	2989	2966	2943	2920	2	5	7	9	11	14	16	18	
9	0.2661	2637	2850	2827	2803	2780	2756	2732	2709	2685	2	5	7	10	12	14	17	19	
1	0.2004	2007	2613	2589	2565	2541	2516	2492	2468	2444	2	5	7	10	12	14	17	19	
.0	0.2420	2396	2371	2347	2323	2299	2275	2251	2227	2207	1		-						
I.	0.2179	2155	2131.	2107	2083	2059	2036		2227	2203	2	5	7	10	12	14	17	19	
2	0.1942	1919	1895	1872	1849	1826		2012	1989	1965	2	5	7	10	12	14	17	19	
.3	0.1714	1691	1669	1647	1	3777	1804	1781	1758	1736	2	5	7	9	12	14	16	18	
4	0.1497	1476	1456	1435	1626	1604 1394	1582 1374	1561 1354	1539 1334	1518 1315	2 2	4	7	9	11 10	13 12	15	18 16	
.5	0.1295						13, (	1331	2551	1313	-	,			10	12	1.4	10	
		1276	1257	1238	1219	1200	1182	1163	1145	1127	2	4	6	8	9	11	13	15	
.6	0.1109	1092	1074	1057	1040	1023	1006	0989	0973	0957	2	3	5	7	8	10	12	14	
.7	0.0940	0925	0909	0893	0878	0863	0848	0833	0818	0804	2	3	5	6	8	9	11	12	
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0	0.0540	0529	0519	0508	0498	0488	0478	0468	0459	0449	1	2	3	4	5	6	7	8	
1	0.0440	0431	0422	0413	0404	0396	0387	0379	0371	0363	1	2	3	4	4	5	6	7	
	0.0355	0347	0339	0332	0325	0317	0310	0303	0297	0290	1	1	2	3	4	4	5	6	
3	0.0283	0277	0270	0264	0258	0252	0246	0241	0235	0229	1	1	2	2	3	4	4	5	
4	0.0224	0219	0231	0208	0203	0198	0194	0189	0184	0180	0	1	1	2	2	3	3	4	
5	0.0175	0171	0167	0163	0158	0154	0151	0147	0143	0139	0	1	1	2	2	2	3	3	
	0.0136	0132	0129	0126	0122	0119	0116	0113	0110	0107	0	1	1	1	2	2	2	2	
7	0.0104	0101	0099	0096	0093	0091	0088	0086	0084	0081	0	1	1	1	1	2	2	2	
8	0.0079	0077	0075	0073	0071	0069	0067	0065	0063	0061			- [						
9	0.0060	0058	0056	0055	0053	0051	0050	0048	0047	0046									
.	0.0044	0033									1	2	3	4	5	6	7	8	
	0.50-	0000	0024	0017						1	1	1	2	2	3	4	4	5	
					0012	0009	0006	0004	0003	0002									

The functions tabled are:

 $\phi(Z) = \sqrt{(\frac{1}{2\pi})} \exp(-\frac{1}{2}) Z^2$ ), where  $\phi(Z)$  is the probability density of the standardized normal distribution N(0,1)

CRITICAL POINTS OF THE NORMAL DISTRIBUTION ZD

P	9	,	P	Q	z	P	Q	
		0.000	.460	.040	1.751	.490	.010	2.32
.00	.50	0.126	.462	.038	1.774	.491	.009	2.36
.05	.45	0.253	.464	.036	1.799	.492	.008	2.40
.10	.40	0.385	.466	.034	1.825	.493	.007	2.45
.15	.35		.468	.032	1.852	.494	.006	2.51
.20	.30	0.524	,408	.032	1.032	1		
			4710	020	1.881	.495	.005	2.57
.25	25	0.674	.470	.030		.496	.004	2.65
.30	.20	0.842	.477	.028	1.911	.497	.003	2.748
.35	.15	1.036	.474	.026	1.943	.498	.002	2.878
.40	.10	1.282	.476	.024	1.977	.499	.001	3.090
.45	.05	1.645	.478	.022	2.014	,499		
						.4995	.0005	3.291
.450	.050	1.645	.480	.020	2.054		.0001	3.719
.452	.048	1.665	.482	.018	2.097	.4999	.00005	3.891
.454	.046	1.685	484	.016	2.144	,49995	.00001	4.265
.456	.044	1.706	.486	.014	2.197	.49999	.000005	4.417
.458	.042	1.728	.488	.012	2.257	499995	.000003	



## t-distribution

Areas in the upper tail are given along the top of the table. Critical t\* values are given in the table.

50	49	40	47	46	45	44	43	42	41	40	39	8	37	36	Si	φ ;	3	8 9	2 6	3 23	28	27	26	25	24	23	2 !	3 8	20 15	18	17	16	र्ज :	£ 7	t 15	1	70	9 (	D -	4 0	, 0	1 4	ω	N	-	ř
1.299																																								1445	1410	1533	1638	1.886	3 078	0.1
1.676																																													- 1	
2.009	2.010	2.011	2.012	2.013	2.014	2.015	2.017	2.018	2.020	2.021	2.023	2.024	2.026	2.028	2.030	2.032	2 035	2.037	0 040	2.045	2.048	2.052	2.056	2.060	2.064	2.069	2.074	2.080	2 086	2002	2.110	2.120	2.131	2.145	2 160	2.201	2.228	2.262	2.306	2.365	2 447	2 571	2 776	3 192	12,706	0.025
2.109	2.110	2 111	2.112	2.114	2.115	2.116	2.118	2.120	2.121	2.123	2.125	2.127	2.129	2.131	2 133	2.136	2 138	2.141	2 1	2.150	2.154	2.158	2.162	2.167	2.172	2.177	2.183	2.189	2.197	2005	2.224	2.235	2.249	2.264	2.282	2.320	2.359	2.398	2.449	2.517	2.612	2.757	2.999	3 482	15, 835	0.02
2.403	2.405	2.407	2.408	2.410	2.412	2.414	2.416	2.418	2.421	2.423	2.426	2.423	2.431	2.434	2.438	2.441	2445	2.443	3 40	2087	2.467	2.473	2.479	2.485	2.492	2.500	2.508	2.518	2.528	2539	2.567	2.583	2.602	2.624	2.650	2 681	2.764	2.821	2.896	2.998	3.143	3.365	3.747	4.541	2000	0.01
2.678	2.680	2.682	2.685	2.687	2.690	2.692	2.695	2.698	2 701	2.704	2.708	2.712	2.715	2.719	2.724	2.728	2.733	2.738	2000	2 7 SO	2.703	2771	2.779	2.787	2.757	2.807	2.819	2.831	2.845	2881	2.898	2.921	2.947	2.977	3.012	3.055	2 100	3.250	3.355	3.499	3.707	4.032	4.604	5 841	9 925	0.005

																																														19	<b>±</b>	
100	99	98	9 6	95	2	93	92	91	90	89	00	87	36	28	90 44	ea Eu	çu fu	63.9 9.45	25	5		ZI.	C. I	(3)	0 1	ids p	7 2	7.5	7 5	n 6	20 0	9	2 65	2	63	62	61	5 4	5 %	5	36	55	2	53	52	51	_	
1.290	1.290	1290	1.290	1.291	1.291	1.291	1291	1291	1.291	1291	1.291	1.291	1.291	1.292	1.292	1.292	1.292	1.292	1.292	1 292	1.292	1.293	1.293	1293	1293	1293	1293	1294	1294	1294	1294	200	1.295	1.235	1.295	1.235	1.236	1296	1296	1000	1.237	1.237	1821	1298	1,238	1.238	0.1	
1.660	1 660	1 661	1.661	1.661	1.661	1.661	1 662	1 662	1.662	1.662	1.662	1.663	1.663	1.663	1.663	1.663	1.664	1.62	1.664	1664	1 665	1.665	1.665	1.665	1.666	1.666	1.666	1.667	1.667	1.667	1.668	1 660	1.669	1.669	1.669	1.670	1.670	1.671	1.671	1 672	1 677	1 672	1 673	1 674	1.674	1.675	0.05	
1.984	1 984	1.985	1.985	1.985	1.986	1.986	1.986	1.986	1.987	1.987	1.987	1.988	1.988	1.988	1.989	1.989	1.989	1.990	1.990	1.990	1.991	1.991	1.992	1.992	1.993	1.993	1.993	1.994	1.994	1.995	1.995	1 006	1 997	1.998	1.998	1.999	2.000	2.000	2.001	2.002	2.002	2 003	2.004	2 005	7 006	7 007	0.025	
2.081	2.061	2.082	2.082	2.082	2.083	2.083	2.083	2.084	2.084	2.084	2.085	2.085	2.085	2.086	2.086	2.087	2.087	2.087	2.088	2.088	2.089	2.089	2.090	2.090	2.091	2.091	2.092	2.092	2.093	2.093	2.094	2 095	2.095	1,006	1,60.2	2.098	2.099	2.099	2.100	2.101	2.102	2.103	2.104	2.105	2 106	2 107	7 108	
2.364	2 365	2.365	2.366	2.366	2.367	2.367	2.368	2.368	2.368	2.369	2.369	2.370	2.370	2.371	2.372	2.372	2.373	2.373	2.374	2374	2.375	2.376	2.376	2.377	2.378	2379	2.379	2.380	2.381	2.382	2.382	2.383	2384	7 395	7 395	2.388	2.389	2.390	2.391	2.392	2394	2.395	2.396	2.397	2399	2.400	2.402	2
2.626	2636	2.627	2.628	2.629	2.629	2.630	2.630	2.631	2.632	2.632	2.633	2.634	2.634	2.635	2.636	2.636	2.637	2.638	2.639	2.540	2.640	2.641	2.642	2.643	2.544	2.645	2.646	2.647	2.648	2.649	2.650	2.651	2.652	7654	2.655	1 656	2.659	2.660	2.662	2.663	2.665	2.667	2.668	2.670	2.672	2.674	2.676	0.005

TABLE A-3		tion: Critical			
	0.005	0.01	Area in One T 0.025	0.05	0.10
- or of			rea in Two Ta	ils	0.00
Degrees of Freedom	0.01	0.02	0.05	0.10	0.20
	63.657	31.821	12.706	6.314	3.078
1 2 3	9.925	6.965	4.303	2.920	1.886
3	5.841	4.541	3.182	2.353	1.638
4	4.604	3.747	2.776	2.132	1.533
5	4.032	3.365	2.571	2.015	1.470
6	3.707	3.143	2.447	1.943	1.440
7	3.499	2.998	2.365	1.895	1.415
8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	2.228	1.812	1.372
	3.106	2.718	2.201	1.796	1.363
11	3.055	2.681	2.179	1.782	1.356
12	3.012	2.650	2.160	1.771	1.350
13	2.977	2.624	2.145	1.761	1.345
14		2.602	2.131	1.753	1.341
15	2.947	2.583	2.120	1.746	1.337
16	2.921	2.567	2.110	1.740	1.333
17	2.898	2.552	2.101	1.734	1.330
18	2.878	2.539	2.093	1.729	1.328
19	2.861	2.528	2.086	1.725	1.325
20	2.845	2.518	2.080	1.721	1.323
21	2.831	2.508	2.074	1.717	1.321
22	2.819	2.500	2.069	1.714	1.319
23	2.807	2.492	2.064	1.711	1.318
24	2.797	2.485	2.060	1.708	1.316
25	2.787	2.479	2.056	1.706	1.315
26	2.779 2.771	2.473	2.052	1.703	1.314
27	2.763	2.467	2.048	1.701	1.313
28	2.756	2.462	2.045	1.699	1.311
29 30	2.750	2.457	2.042	1.697	1.310
	2.744	2.453	2.040	1.696	1.309
31 32	2.738	2.449	2.037	1.694	1.309
34	2.728	2.441	2.032	1.691	1.307
36	2.719	2.434	2.028	1.688	1.306
38	2.712	2.429	2.024	1.686	1.304
40	2.704	2.423	2.021	1.684	1.303
45	2.690	2.412	2.014	1.679	1.301
50	2.678	2.403	2.009	1.676	1.299
55	2.668	2.396	2.004	1.673	1.297
60	2.660	2.390	2.000	1.671	1.296
65	2.654	2.385	1.997	1.669	1.295
70	2.648	2.381	1.994	1.667	1.294
75	2.643	2.377	1.992	1.665	1.293
80	2.639	2.374	1.990	1.664	1,292
90	2.632	2.368	1.987	1.662	1.291
100	2.626	2.364	1.984	1.660	1.290
200	2.601	2.345	1.972	1.653	1.286
300	2.592	2.339	1.968	1.650	1.284
400	2.588	2.336	1.966	1.649	1.284
500	2.586	2.334	1.965	1.648	1.283
750	2.582	2.331	1.963	1.647	1.283
1000	2.582	2.330	1.962	1.646	1.282
2000	2.578	2.328	1.961	1.646	1.282
		2.326	1.960	1.645	1.282
Large	2.576	2.320	1.700	1.0	1 14004

