UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS/ STATISTICS

UNIVERSITY EXAMINATIONS
SEMESTER I, 2013/14

THIRD YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE (GEN)

EXPERIMANTAL DESIGN

DATE: 16TH DECEMBER 2013

TIME: 10:00 - 1:00 PM

Instructions:
i) Attempt any four questions.

ii) Write on both sides of the answer booklet paper but each question should be answered starting on a new sheet of paper.

iii) Start with questions you find easiest and not necessarily those that carry most marks.

Instructions :

(i) Attempt any four questions.

(ii) Write on both sides of the answer booklet paper but each question should be answered starting on a new sheet of paper.

(iii) Start with questions you find easiest and not necessarily those that carry most marks.

1. (a) State the underlying assumptions of the ANOVA. **(3 marks)**

   (b) In a biological experiment, three different fertilisers are used to enhance the growth of beans over a period of twenty days. The following growth data in *cm* were recorded for the plants that survived.

concentration

1	2	3
6.2	5.7	4.8
6.7	6.4	5.3
	6.6	4.3
		4.9

   Is there a significant difference in the average growth of these plants for the different fertilisers? Use $\alpha = 0.1$ **(9 marks)**

   (c) The table below gives the yields in thousands of kgs/hectare of 3 varieties of beans grown using 3 different types of fertilisers.

Varieties

		V_1	V_2	V_3
	F_1	29	37	39
Fertilisers	F_2	20	22	12
	F_3	24	31	23

Assuming there is no interaction between the fertilisers and the varieties, test the hypotheses (at the 5% level of significance)

i) There is no difference in the average yield of beans when different kinds of fertilisers are used. **(7 marks)**

ii) There is no difference in the average yields of the 3 varieties of beans. **(6 marks)**

2. (a) Define *interaction* as used in experimental design. **(2 marks)**

(b) Write a statistical model for the Randomized Block design with interaction **(2 marks)**

(c) The following data represent the final grades obtained by three students Tom, Jane and Harry in Statistical Inference (SI), Experimental Design (ED) and Statistical Computing (SC).

		Course Unit		
		SI	ED	SC
students	Tom	3.0	3.5	4.0
		3.5	4.0	2.5
	Jane	3.5	4.0	5.0
		3.5	2.0	3.5
	Harry	2.5	2.5	3.0
		2.0	2.0	3.5

Use a 10% level of significance to test the hypothesis that:

i) the courses are of equal difficulty; **(7 marks)**

ii) the students have equal ability; **(7 marks)**

iii) the students and the courses do not interact. **(7 marks)**

3. (a) Define the following terms as used in Experimental Design and Statistical modelling:

i) an experiment **(2 marks)**

ii) a treatment **(2 marks)**

ii) main effect **(2 marks)**

(b) The Total sum of squares SST in the two-way ANOVA is given by

$$SST = \sum_i \sum_j \sum_k (y_{ijk} - \bar{y}_{+++})^2$$

Show that this is equal to

$$\sum_i \sum_j \sum_k {y_{ijk}}^2 - \frac{{y_{+++}}^2}{N}$$

(4 marks)

(c) Write a statistical model which can be used for each of the following experimental designs. (Clearly explain each term used)

i) Completely Randomised Design **(2 marks)**

ii) Latin Squares Design **(2 marks)**

With reference to these models explain why a statistician would prefer LSD over CRD. **(2 marks)**

(d) Explain your understanding of the practice of *EXPERIMENTAL DESIGN* as used by Statisticians. (Use a relevant example). **(9 marks)**

4. (a) *In an experiment to study the effect of a fertilizer on the growth of tomato plants, four different fertilizers and one nursery bed are used.*
Explain each of the following terms as used in Experimental Design citing the application of each term in the experiment above where possible.
   i) Experimental unit **(2 marks)**
   ii) Factor **(2 marks)**
   iii) Blinding **(2 marks)**
   iv) Placebo **(2 marks)**

(b) Explain the steps followed in designing an experiment. **(10 marks)**

(c) Write brief notes about the basic principles of experimental design. **(7 marks)**

5. (a) A study on the academic performance of four students A B C and D before (pre-test) and after (post-test) subjecting them to a teaching technique gave the following GPA scores.

	Student			
	A	B	C	D
pre − test	4.0	3.5	4.5	3.0
post − test	3.5	4.0	4.5	3.5

Using a 5% level of significance check whether or not the teaching technique was effective. **(10 marks)**

(b) The table below gives the quantity of eggs laid by 3 different hens being fed on 3 different types of feeds.

		hens		
		h_1	h_2	h_3
	F_1	6	2	4
Feeds	F_2	2	y	4
	F_3	4	5	6

The quantity y was lost in the process of carrying out the experiment.

i) estimate the value of y **(8 marks)**

ii) hence using a 5% level of significance test the hypothesis that there is no difference in the quantity of eggs laid when different feeds are used. **(7 marks)**

List of selected formulae

(A) Consider k samples containing a total of N observations. If a sample i has n_i observations, then

$$SSB = \sum_{i=1}^{k} \frac{y_{i+}^2}{n_i} - \frac{y_{++}^2}{N} \tag{1}$$

$$SST = \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 - \frac{y_{++}^2}{N} \tag{2}$$

$$SST = SSE + SSB \tag{3}$$

where;

$$y_{i+} = \sum_{j=1}^{n_i} y_{ij} \qquad y_{++} = \sum_{i=1}^{k} \left\{ \sum_{j=1}^{n_i} y_{ij} \right\}$$

y_{ij} is the $j-th$ observation in sample i.
N is the total number of observations.

(B) Consider a set of observations classified according to two criteria (without interaction) at once with a total number of N observations.

$$SST = \sum_{i=1}^{r} \sum_{j=1}^{c} y_{ij}^2 - \frac{y_{++}^2}{rc} \tag{4}$$

$$SSR = \sum_{i=1}^{r} \frac{y_{i+}^2}{c} - \frac{y_{++}^2}{rc} \tag{5}$$

$$SSC = \sum_{j=1}^{c} \frac{y_{+j}^2}{r} - \frac{y_{++}^2}{rc} \tag{6}$$

$$SST = SSR + SSC + SSE \tag{7}$$

where;

$$y_{i+} = \sum_{j=1}^{c} y_{ij} \qquad y_{+j} = \sum_{i=1}^{r} y_{ij} \qquad y_{++} = \sum_{i=1}^{r} \left\{ \sum_{j=1}^{c} y_{ij} \right\}$$

y_{ij} is an observation in $i-th$ row and $j-th$ column.
r is the number of rows.
c is the number of columns.
$N = rc$ is the total number of observations.

(C) Consider a set of observations classified according to two criteria with interaction at once with a total number of N observations.

$$SST = \sum_{i=1}^{r} \sum_{j=1}^{c} \sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{+++}^2}{rcn} \tag{8}$$

$$SSR = \sum_{i=1}^{r} \frac{y_{i++}^2}{cn} - \frac{y_{+++}^2}{rcn} \tag{9}$$

$$SSC = \sum_{j=1}^{c} \frac{y_{+j+}^2}{rn} - \frac{y_{+++}^2}{rcn} \tag{10}$$

$$SSI = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{y_{ij+}^2}{n} - \sum_{i=1}^{r} \frac{y_{i++}^2}{cn} - \sum_{j=1}^{c} \frac{y_{+j+}^2}{rn} + \frac{y_{+++}^2}{rcn} \tag{11}$$

where;

$$y_{i+k} = \sum_{j=1}^{c} y_{ijk} \qquad y_{+jk} = \sum_{i=1}^{r} y_{ijk} \qquad y_{+++} = \sum_{i=1}^{r} \left\{ \sum_{j=1}^{c}\sum_{k=1}^{n} y_{ijk} \right\}$$

y_{ijk} is the k^{th} observation in row i and column j.
r is the number of rows.
c is the number of columns.
k is the number of observations in cell $i - j$.
$N = rcn$ is the total number of observations.

END

F Values for $\alpha = 0.01$

d_2 \ d_1	1	2	3	4	5	6	7	8	9
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	10.04	7.56	6.55	5.99	5.64	5.39	5.2	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.14
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
inf	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41

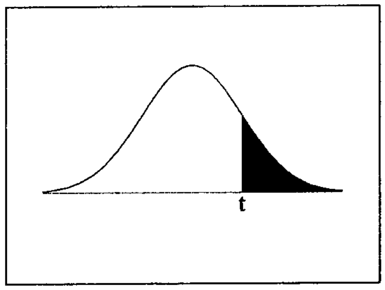
F Values for $\alpha = 0.05$

d_2 \ d_1	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.3	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
inf	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

F Values for $\alpha = 0.10$

					d_1				
d_2	1	2	3	4	5	6	7	8	9
1	39.86	49.5	53.59	55.83	57.24	58.2	58.91	59.44	59.86
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.3	2.27
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
inf	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63

t-Distribution Table



The shaded area is equal to α for $t = t_\alpha$.

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
∞	1.282	1.645	1.960	2.326	2.576