UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL EXAMINATIONS FOR BACHELOR OF SCIENCE ECONOMICS AND STATISTICS

SEMESTER II, 2022/2023

YEAR II [AUGUST INTAKE]

STA 2202: MATHEMATICAL STATISTICS II

DATE: MAY 22, 2023

TIME: 2:00-5:00PM

DURATION: 3 hours

Instructions

1. Carefully read through ALL the questions before attempting
2. Attempt FOUR questions
3. Indicate the numbers you have done on the front page in their order
4. Ensure that ONLY your Registration number is indicated on the front page

Question 1

The joint probability mass function of a random variable X and Y is given by:-

$f(x, y) = kx^2 y; \quad x = 1, 2, 3; \quad y = 1, 2, 3, 4.$ *Find* : –

(*i*) k [4 *Marks*]
(*ii*) $P(X \leq 2, Y \geq 3)$ [3 *Marks*]
(*iii*) $P(X \geq 2 / Y = 3)$ [3 *Marks*]
(*iv*) $E(X)$ [4 *Marks*]
(*v*) $E(X^2)$ [4 *Marks*]
(*vi*) $V(X)$ [3 *Marks*]
(*vii*) $F(X, Y)$ [4 *Marks*]

Question 2

(a) Explain one application of a probability generating function [3 Marks]

(b) The probability distribution of a Geometric random variable is given by:-

$f(x) = pq^{x-1}; x = 1, 2, 3, \cdots$ determine : –
(*i*) $G_X(t)$ ..... [4 *Marks*]
(*ii*) $P_X(t)$ ..... [4 *Marks*]
(*iii*) $E(X)$ ..... [4 *Marks*]
(*iv*) $V(X)$ ..... [4 *Marks*]

(c) A random variable X is defined by the following probability mass function:-

$$f(x) = \begin{cases} 1/4; & X = -2, 1, 2 \\ 1/8; & X = -1, 0 \end{cases}$$

Obtain the probability mass function of $Y = (X + 1)^2$ [6 Marks]

Question 3

(a) A sugar factory sells sugar in bags of mean weight 50 kg and STD of 2.5 kg. Given that the weights the bags are normally distributed; determine:-

i. $P(51.5 < X < 53)$ [4 Marks]

ii. The percentage of bags whose weight exceeds 54 kg [3 Marks]

iii. Number of bags that will be rejected out of 1,000 bags that were purchased for weighing below 45 kg [4 Marks]

(b) The length of a phone call is exponentially distributed with a mean of 0.5 minutes. Find the probability of waiting:-

(i) For over 20 minutes to receive a phone call [3 Marks]

(ii) Between 10 and 20 minutes to receive a phone call [3 Marks]

(iii) Less than 15 minutes to receive a phone call [3 Marks]

(c) The time a person waits for a service is uniformly distributed between 0 and 15 minutes. Determine:-

i. The probability that a person will wait for fewer than 12.5 minutes [2 Marks]

ii. The mean of the waiting time [3 Marks]

Question 4

A farm produces two types of eggs; organic and in-organic eggs. Let X be organic and Y be in-organic. Given that the cumulative joint pdf of X and Y is given by:-

$$F(x, y) = \frac{xy}{k}(x + y); \quad 0 \le x, y \le 2$$
$$F(x, y) = 1; \quad x, y > 2$$

(i) Determine the pdf of X and Y [5 Marks]

(ii) Determine the value of k [5 Marks]

(iii) The marginal cumulative distribution function of X [5 Marks]

(iv) The conditional density function of X given Y = y [5 Marks]

(v) The marginal density of Y [5 Marks]

Question 5

(a) A Poisson distribution is given by the pdf $f(x) = e^{-\lambda}.\frac{\lambda^x}{x!}; x = 0, 1, \cdots$ determine the:-

(i) Moment generating function [4 Marks]

(ii) Variance using the moment generating function [3 Marks]

(b) The joint probability density function of two random variables X and Y is given by:-

$f(x, y) = k\{x^2 + y^2\}; \quad 0 \le x \le 1; \quad 0 \le y \le 1$. *Find* :–

(*i*) k [5 *Marks*]

(*ii*) $P(X \le \frac{1}{4}, Y < \frac{1}{5})$ [5 *Marks*]

(*iii*) $P(X < 0.7)$ [5 *Marks*]

(*iv*) $f_Y(y)$ [3 *Marks*]

END