UGANDA MARTYRS UNIVERSITY
NKOZI & MASAKA

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL SEMESTER ASSESSMENT
SEMESTER I

FIRST YEAR EXAMINATIONS FOR BSc, BSc Econ & Stat, BSc (Ed)

MAT 1101: Calculus I

DATE: Monday 11th December, 2023
DURATION: 3 Hours

Instructions:

1. *Carefully read through ALL the questions before attempting*
2. ***ANSWER ANY FIVE Questions*** *(Each question is 20 marks)*
3. *No* ***names*** *should be written anywhere on the examination book.*
4. *Ensure that your* ***Reg number*** *is indicated on all pages of the examination answer booklet.*
5. *Ensure your work is* ***clear*** *and* ***readable****. Untidy work shall be penalized*
6. *Any type of examination Malpractice will lead to automatic disqualification*
7. *Do not write anything on the questions paper.*

1. a. Using relevant examples, give the difference between an *odd function* and an *even function.* **4 marks**

b. Give an example of a function that is neither odd nor even. **1 mark**

c. Classify the following functions as odd, even or neither

i. $f(x) = 2$ **2 marks**

ii. $f(x) = \dfrac{g(x)}{h(x)}$ where g is even, h is odd and $h(x) \neq 0$ for all x. **4 marks**

d. Functions g and h are such that g has a domain $\{-1, 0, 1, 2, 3\}$, $g(x) = x^2$ and $h(x) = 1 - 2x$. Find the:

i. range of g **3 marks**

ii. domain and range of $h \circ g$. **6 marks**

2. a. What is meant by *continuity of a function on an interval*? **2 marks**

b. Give an example of a function that is continuous. **1 mark**

c. Determine the values of the parameters k and m for which the function h defined below is differentiable at the point $x = 3$.

$$h(x) = \begin{cases} kx + 1, & x \leq 3 \\ 10 - mx^2, & x > 3 \end{cases}$$

4 marks

d. Given the function g, where $g(x) = x(x^2 + 2x - 1) - 2$. Find:

i. the domain of g **1 mark**

ii. the points where the graph of g crosses the axes. **4 marks**

iii. the coordinates $(x, g(x))$ where g' and g'' are zero. **6 marks**

iv. $\lim\limits_{x \to -\infty} g(x)$ and $\lim\limits_{x \to \infty} g(x)$. **2 marks**

3. a. Use the $\epsilon - \delta$ definition of a limit to prove that $\lim\limits_{x \to 1} -x + 2 = 1$. **3 marks**

b. Given the function

$$g(x) = \begin{cases} 1 - x, & x < 1 \\ -2 + 2x, & x \geq 1. \end{cases}$$

Compute the limits:

i. $\lim\limits_{x \to 1^+} g(x)$ **2 marks**

ii. $\lim\limits_{x \to 1^-} g(x)$ **2 marks**

Hence state $\lim\limits_{x \to 1} g(x)$ **1 mark**

c. Give an example of a function g and a point c such that the limit of g does not exist at c. **2 marks**

d. Find the following limits:

i. $\lim_{x \to -1} x^4 - 2x$ **2 marks**

ii. $\lim_{x \to 4} \dfrac{2 - \sqrt{x}}{x - 4}$ **3 marks**

iii. $\lim_{x \to 1} g(x)$ over an interval where $3 - x^2 - x \leq g(x) \leq x^2$ **3 marks**

e. Explain one application of limits in real life situations. **2 marks**

4. a. Using relevant examples, differentiate between a *relation* and a *function*. **3 marks**

b. A function h is given by $h(x) = \dfrac{2x - 1}{x - 1}$. Find the inverse of h. **2 marks**

c. Solve for x and represent the solution on a number line.

i. $x - x^2 < 0$ **3 marks**

ii. $-2 < 4(1 - x) < 5$ **3 marks**

d. Show that

i. the product of two odd functions is even. **3 marks**

ii. $f(x) = \dfrac{3x}{2x - 1}$ is a one to one function. **3 marks**

e. If $y = f(x) = \dfrac{3 + 5x}{4x - 5}$, show that $x = f(y)$ **3 marks**

5. a. Find the domain of each of the following functions.

i. $g(x) = \sqrt{2 - x}$. **2 marks**

ii. $f(x) = \dfrac{2x}{x^2 - 4}$. **3 marks**

b. A function f defined on $\mathbb{R}$ is such that $f(x) = 2x$. Find a formula for:

i. $f^2 = f \circ f$ **2 marks**

ii. $\dfrac{d}{dx} f^2(x)$ **1 mark**

iii. $f^3 = f \circ f \circ f$ **2 marks**

iv. $\dfrac{d}{dx} f^3(x)$ **1 mark**

Hence state the formula for f^n and $\dfrac{d}{dx} f^n(x)$, where n is a positive integer. **3 marks**

c. Using relevant examples, distinguish between an *increasing function* and a *decreasing function*. **4 marks**

d. Sketch the graph of the function $g(x) = |5x + 1|$. **2 marks**

6. a. i. Give the limit definition of a derivative, f', of a function f at a point x. **1 mark**

ii. Use the definition in part a.**i.** above to show that

$$\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}.$$

4 marks

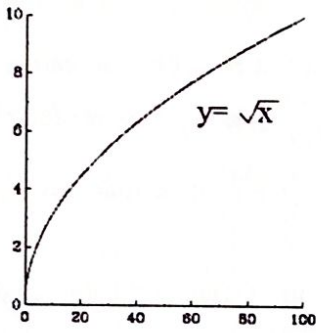
b. Compute the derivatives of the following functions:

i. $f(x) = \dfrac{x^3 + 3x^2}{3 - x}$ **4 marks**

ii. $g(x) = x^4(2 + x)^2$ **4 marks**

c. Find the equation of the tangent line to the curve $h(x) = x^3$ at the point $x = 1$. **3 marks**

d. The graph of function f, where $y = f(X) = \sqrt{X}$ is shown below.



On the same axes sketch the graphs of f and $g(X) = 1 - \sqrt{X - 2}$. **4 marks**

END