UGANDA MARTYRS UNIVERSITY, NKOZI

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

UNIVERSITY SPECIAL/SUPPLEMENTARY EXAMINATIONS

Second Year Bachelor of Science (FM and GEN)

MTC 2101 Calculus III

Date : Friday Aug 7^{th}, 2015

Time : 3 Hours (10:00 am - 1:00 pm)

Instructions

(i) *Read through the paper carefully and follow instructions on the answer booklet.*

(ii) *Attempt any* **Four** (4) *questions.*

(iii) *Do not write any thing on this question paper:*

(iv) *Calculators and mathematical tables may be used.*

(v) *Neat work is highly recommended.*

1. (a) Given two vectors A and B, define the following:

(i) Dot product of A and B (2 marks)

(ii) Cross product of A and B. (2 marks)

(b) The diagonals of a parallelogram are given by $A = 3i - 4j - k$ and $B = 2i + 3j - 6k$. Show that the parallelogram is a Rhombus and determine the length of it's sides and it's angles [6 marks]

c(i) If $\mathbf{A} = \mathbf{A_1 i} + \mathbf{A_2 j} + \mathbf{A_3 k}.\mathbf{B} = \mathbf{B_1 i} + \mathbf{B_2 j} + \mathbf{B_3 k}$.

Prove that $\mathbf{A}.\mathbf{B} = A_1B_1 + A_2B_2 + A_3B_3$ [5 marks]

(ii) If $\mathbf{A} = \mathbf{2i} - \mathbf{3j} + -\mathbf{k}$ and $\mathbf{B} = \mathbf{i} + \mathbf{4j} - \mathbf{2k}$,

find

(.) $\mathbf{A} \times \mathbf{B}$ [2 marks]

(..) $(\mathbf{A} + \mathbf{B}) \times (\mathbf{A} - \mathbf{B})$ [4 marks]

(d) Determine the value of $\mathbf{a}$ so that $\mathbf{A} = \mathbf{2i} + \mathbf{aj} + \mathbf{k}$ and $\mathbf{B} = \mathbf{4i} - \mathbf{2j} - \mathbf{2k}$

[4 marks]

2. (a) Plot the points with the given polar coordinates and then find the rectangular coordinates

(i) $A(3, \frac{\pi}{3})$ [4 marks]

(ii) $B(-1, \frac{\pi}{4})$ [4 marks]

(b) Convert the following polar equations to rectangular forms and sketch the resulting graphs

(i) $sin\theta = \frac{r}{4}$ [6 marks]

(ii) $r^2 sin2\theta = 8$ [6 marks]

(c) Find the area of the region enclosed by the Limacon with equation $r = 3 + 2sin\theta;\ 0 \leq \theta \leq 2\pi$ [5 marks]

3. (a) State what is meant by curl and divergence of a vector field **F**. Hence given $\mathbf{F}(x, y, z) = xz\mathbf{i} + xyz\mathbf{j} - y^2\mathbf{k}$ find curl **F**. div **F** and div curl **F**. [8 marks]

(b) If $A = xy^2\mathbf{i} - 3yx^2\mathbf{j} + 2xy^2z\mathbf{k}$ and $\phi = 2y^3 + xz$ Find:

(i) $A \times \nabla(\phi)$ [04 marks]

(ii) $(A \times \nabla)\phi$ [04 marks]

(iii) $\nabla.(\phi A)$ [03 marks]

(c) If $B = zy^2\mathbf{i} - 2yx^2\mathbf{j} + 3xyz^3\mathbf{k}$: Find $B \times (\nabla\phi)$ [06 marks]

4. a(i) Define gradient of a scalar ϕ [2 marks]

(ii) If $\phi = 3x^2 - y^2z^3 + 4x^3y + 2x - 3y - 5$, find $\nabla^2\phi$ [6 marks]

b A particle moves so that its position vector is given by $\mathbf{r} = cos\omega t\mathbf{i} + sin\omega t\mathbf{j}$ where ω is a constant. Show that

(i) the velocity **v** of the particle is perpendicular to **r** [4 marks]

(ii) the acceleration **a** is directed toward the origin and has magnitude proportional to the distance fromthe origin [4 marks]

(iii) $\mathbf{r} \times \mathbf{v} =$ constant [4 marks]

(c) If $B = zy^2\mathbf{i} + 2yx^2\mathbf{j} + 3xyz^3\mathbf{k}$ Find $B \times \nabla\varphi$ [5 marks]

5. a(i) Given the region R in the xy plane bounded by $x + y = 6$, $x - y = 2$ and $y = 0$. Find the area bounded by R [7 marks]

(ii) Sketch the region represented by $\int_0^1 \int_y^1 dxdy$ [4 marks]

(b) If $\phi = 2xyz^2$, $F = xy\mathbf{i} - z\mathbf{j} + x^2\mathbf{k}$ and C is the curve $x = t^2$, $y = 2t$, $Z = t^3$ from $t = 0$ to $t = 1$, evaluate the line integrals

(i) $\int_C \phi dr$ (05 marks)

(ii) $\int_C F \times dr$ (05 marks)

(c) Given $5x^3z - x^2y + 4z^2 - 2yz - 5x = 0$: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ (04 marks)

6. a(i) Define arc length of a curve [2 marks]

(ii) Find the length of $r(t) = 4cost\mathbf{i} + 4sint\mathbf{j} + 3t\mathbf{k}$, $0 \leq t \leq \frac{\pi}{2}$ [5 marks]

b(i) Evaluate the integral $\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 (x + y + z)dydxdz$ [6 marks]

(b) Compute the value of triple integral $\int\int\int_D f(x, y, z)dv$ for the following:

(i) $f(x, y, z) = xysinz$; D is a cube bounded by $0 \leq x \leq \pi$, $0 \leq y \leq \pi$ and $0 \leq z \leq \pi$ [6 marks]

(ii) $f(x, y, z) = x + y$; D is the region between the surfaces $z = 2 - x$ and $z = x^2$ for $0 \leq y \leq 3$, $-1 \leq y \leq 1$ [6 marks]

Success

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