UGANDA MARTYRS UNIVERSITY

FACULTY OF BUSINESS ADMINISTRATION AND MANAGEMENT

UNIVERSITY EXAMINATION

QUANTITATIVE METHODS

2022/2023

DATE: 19th July, 2022

TIME: 3 HOURS

Instructions:

i. Attempt question any four questions
ii. Do not write anything on the question paper
iii. Show all workings and they have to be clear and tidy

QUESTION ONE

(a) Using relevant examples, define the following terms in linear programming.

(ii) Constraints **[3 marks]**

(iii) Objective function **[3 marks]**

(b) The following table shows the components of making a cake for Jane's business

	Types of cakes		**Total available units**
	Fruit	**Ordinary**	
Floor (kg)	I	6	120
Sugar (kg)	1	3	66
Butter (kg)	3	2	86
Fruit cake earns a profit of Ugx 60,000 each while cake Ordinary earns a profit of Ugx 36,000 each.			

(i) Establish the LPP problem **[4 marks]**

(ii) Find the maximum profit using graphical or Simplex method **[12 marks]**

(iii) Determine the levels of utilization of each resource **[3 marks]**

QUESTION TWO

(a) Use exponential smoothing with $\alpha = 0.2$ and an initial value of demand forecast = 170 to get forecasts one period for the following time series

Month	1	2	3	4	5	6	7	8
Demand	178	180	156	150	162	158	154	132

(5 marks)

(b)

Year	2016	2017	2018	2019	2020	2021
Turnover '000,000	106	125	147	167	187	220
Profit before taxation '000,000	10	12	16	17	18	22

(i) Calculate the least squares regression line before taxation on turnover and predict the profit before taxation of 2022 and comment on the result **(12 marks)**

(ii) Comment on the likely accuracy of these predictions **(8 marks)**

QUESTION THREE

Five bus companies A, B, C, D and E compete for passengers travelling from Kampala to Nairobi. The table below shows the flow of passengers between 1st May and 30th June 2022.

			Gains from:					**Losses to:**					
			A	**B**	**C**	**D**	**E**	**A**	**B**	**D**	**E**		
Company	**PASSENGERS AS AT 1st February**	**Market share**										**PASSENGERS AS 31st March**	**Market share**
A	K		**F**	180	200	216	190					2732	
B	L		50	**G**	60	92	115					5232	
C	M		148	168	**H**	20	44					5836	
D	N		119	15	73	**I**	**64**					6400	
E	P		18	57	69	80	**J**					3800	

i. Write down the values of K, L, M, N, P **(10 marks)**

ii. Present the matrix of passenger losses **(3 marks)**

iii. Determine the values of F, G, H, I, J **(5 marks)**

iv. Set up a matrix of transition probabilities **(2 marks)**

v. Predict the market share February **(5 marks)**

QUESTION FOUR

(a) What will be the value of shs.450,000 compounded semi annually at 12% for 3 years **(2 marks)**

(b) A principal amount accrues to shs. 8,500,000. If it is compounded quarterly at 14.5% over 6 years. Find the value of the original amount **(4 marks)**

(c) A firm plans to invest an amount of money at the beginning of every year in order to accrue a sum of 1,000,000 at the end of a five year period. What is the value of the amount, if the investment rate is 14%? **(5 marks)**

(d) Suppose you are offered shs.5, 000,000 today but must make the following payments.

Year	Cash Flows (Ush)
0	5,000,000
1	2,500,0000
2	2,000,0000
3	1,000,0000
4	1,000,000

At a capital cost of 15 %

(i) What is the NPV of this offer? If the appropriate discount rate is 10 percent, should you accept this offer? **(4 marks)**

(ii) If the appropriate discount rate is 20 percent, should you accept this offer? **(4 marks)**

(iii) What is the IRR of the offer if the appropriate discount rate is 10 percent and 20 percent? **(5 marks)**

(iv) Are the decisions under the NPV rule in part (*i*) and (*ii*) consistent with those of the IRR rule? **(1 mark)**

QUESTION FIVE

a) With valid examples explain the relevance of calculus in business and Economics **(3 marks)**

b) Use the definition of calculus
To find the derivative of the following. **(1 mark each)**

i) $f(x) = -7x^2 + 4x$ ii) $f(x) = 5 - 2x$

Find the integrals of the functions **(2 marks each)**

i) $\int (8x^3 - 3x^2 + 6x - 10)dx$ ii) $\int (2.4x^2 - 8.6x - 3)dx$

c) A manufacturer knows that if 30,000 products are demanded in a particular week:
the total cost function (shs. '000'000) is 14 + 3x and
the total revenue function (shs.'000'000) is
$9x - 2x^2$ Derive the total profit function **(3 marks)**

d) A manufacturer has given you as a consultant the revenue function in shillings $R(x) = -3x^3 + 600x^2$ and the cost function in shillings $C(x) = 357x^2 + 1800x$; find:

i. The marginal profit at $x = 10$ units. Interpret the result. **(4 marks)**

ii. The marginal cost at x = 50 units. Interpret the result. **(4 marks)**

e) Given;

Total Revenue TR (x) = $800Q-7Q^2$.

Total Cost TC(x)= $2Q^2 - Q^2 + 80Q + 150$

Required;

i. Set up the profit function **(2 marks)**

ii. Find the critical values where profit is at relative maximum **(4 marks)**

QUESTION SIX

a) A man draws two balls at random from a bag containing three white and five black balls. If he is to receive Shs 14,000 for every white ball that he draws and Shs 7,000 for every black ball, what should be his expectation of earning in the game? **(5 marks)**

b) ABC company estimates the net profit on a new product, that it is launching, to be Shs 30,000,000 if it is successful, Shs 10,000,000 if it is moderately successful and a loss of Shs 10,000,000 if it is unsuccessful. The firm assigns the following probabilities to the different possibilities : Successful 0.15, moderately successful 0.25 and unsuccessful 0.60. Find the expected value of profits. **(6 marks)**

(c) It costs Shs 600 to test a machine. If a defective machine is installed, it costs Shs 12,000 to repair the damage resulting to the machine. Is it more profitable to install the machine without testing if it is known that 30% of all the machines produced are defective? Show by calculations.

(3 marks)

(d) A restaurant is about to install a machine for baking. Three suppliers have made bids to supply the machine. The first supplier offers BAS machine brand machine which automatically bakes immediately but does not outstand of quality. The output of the machine varies from the materials used and the settings of the machine and might be 1,000 loaves a week with a probability of 0.1, 2,000 a week with a probability of 0.7 or 3,000 with a probability 0.2. the second supplier offers a SUT machine which makes higher quality loaves. The output from this might be 700 loaves a week with 0.4 probability or 1,000 a week with a probability of 0.6. the third supplier offers a SWITCHOVER machine which mangers can set to produce either 1,300 high quality loaves a week at a profit 6000/= or 1,600 medium quality loaves at a profit of 5000/=.

Draw a decision tree for this problem and what should the management of the restaurant do to maximise profit. **(11 marks)**

GOOD LUCK

...ted formulas

...timisation using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$ax^2 + bx + c = 0$

a= coefficient of the x^2 term

b = Is the coeff. Of the x term

c= is the constant term number

Arithmetic progression

$a + (n-1)d$

Sum of Progressions

1st Formula; $Sn = n/2\ (a+l)$

2nd Formula ; $Sn = n/2\ ((2a + (n-1)d))$

Geometric progression

$t = ar^{n-1}$

Interest Calculations

$I = Prt$

$A = P(1 + rt)$

Linear Equation; Y=a+bx

$Y = b_0 + b_1x$

Coefficient of Determination = (Correlation Coefficient)^2

Formula

$$\text{Correlation Coefficient} = \frac{\Sigma[(X - X_m) * (Y - Y_m)]}{\sqrt{[\Sigma (X - X_m)^2 * \Sigma (Y - Y_m)^2]}}$$

Regression equation $y = b_0 + b_1 x$

$$b_1 = \frac{\sum(x_{1-\bar{x}})(x_{1-\bar{y}})}{\sum(x_{1-\bar{x}})^2} \text{ or } \sum xy - \frac{\frac{\sum x_1 \sum y_1}{n}}{\sum x_1^2 - \frac{\sum(x)^2}{n}}$$

$$b_0 = \bar{y} - b_1 x$$

Error of prediction $e_1 = y_{1-\bar{y}}$

Smoothing technique

$E_i = W \cdot Y_i + (1 - W) \cdot E_{i-1}$

PRESENT VALUE TABLE

Present value of $1, that is $(1+r)^{-n}$ where r = interest rate; n = number of periods until payment or receipt.

Periods (n)	Interest rates (r)									
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909
2	0.980	0.961	0.943	0.925	0.907	0.890	0.873	0.857	0.842	0.826
3	0.971	0.942	0.915	0.889	0.864	0.840	0.816	0.794	0.772	0.751
4	0.961	0.924	0.888	0.855	0.823	0.792	0.763	0.735	0.708	0.683
5	0.951	0.906	0.863	0.822	0.784	0.747	0.713	0.681	0.650	0.621
6	0.942	0.888	0.837	0.790	0.746	0705	0.666	0.630	0.596	0.564
7	0.933	0.871	0.813	0.760	0.711	0.665	0.623	0.583	0.547	0.513
8	0.923	0.853	0.789	0.731	0.677	0.627	0.582	0.540	0.502	0.467
9	0.914	0.837	0.766	0.703	0.645	0.592	0.544	0.500	0.460	0.424
10	0.905	0.820	0.744	0.676	0.614	0.558	0.508	0.463	0.422	0.386
11	0.896	0.804	0.722	0.650	0.585	0.527	0.475	0.429	0.388	0.350
12	0.887	0.788	0.701	0.625	0.557	0.497	0.444	0.397	0.356	0.319
13	0.879	0.773	0.681	0.601	0.530	0.469	0.415	0.368	0.326	0.290
14	0.870	0.758	0.661	0.577	0.505	0.442	0.388	0.340	0.299	0.263
15	0.861	0.743	0.642	0.555	0.481	0.417	0.362	0.315	0.275	0.239
16	0.853	0.728	0.623	0.534	0.458	0.394	0.339	0.292	0.252	0.218
17	0.844	0.714	0.605	0.513	0.436	0.371	0.317	0.270	0.231	0.198
18	0.836	0.700	0.587	0.494	0.416	0.350	0.296	0.250	0.212	0.180
19	0.828	0.686	0.570	0.475	0.396	0.331	0.277	0.232	0.194	0.164
20	0.820	0.673	0.554	0.456	0.377	0.312	0.258	0.215	0.178	0.149

Periods (n)	Interest rates (r)									
	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833
2	0.812	0.797	0.783	0.769	0.756	0.743	0.731	0.718	0.706	0.694
3	0.731	0.712	0.693	0.675	0.658	0.641	0.624	0.609	0.593	0.579
4	0.659	0.636	0.613	0.592	0.572	0.552	0.534	0.516	0.499	0.482
5	0.593	0.567	0.543	0.519	0.497	0.476	0.456	0.437	0.419	0.402
6	0.535	0.507	0.480	0.456	0.432	0.410	0.390	0.370	0.352	0.335
7	0.482	0.452	0.425	0.400	0.376	0.354	0.333	0.314	0.296	0.279
8	0.434	0.404	0.376	0.351	0.327	0.305	0.285	0.266	0.249	0.233
9	0.391	0.361	0.333	0.308	0.284	0.263	0.243	0.225	0.209	0.194
10	0.352	0.322	0.295	0.270	0.247	0.227	0.208	0.191	0.176	0.162
11	0.317	0.287	0.261	0.237	0.215	0.195	0.178	0.162	0.148	0.135
12	0.286	0.257	0.231	0.208	0.187	0.168	0.152	0.137	0.124	0.112
13	0.258	0.229	0.204	0.182	0.163	0.145	0.130	0.116	0.104	0.093
14	0.232	0.205	0.181	0.160	0.141	0.125	0.111	0.099	0.088	0.078
15	0.209	0.183	0.160	0.140	0.123	0.108	0.095	0.084	0.079	0.065
16	0.188	0.163	0.141	0.123	0.107	0.093	0.081	0.071	0.062	0.054
17	0.170	0.146	0.125	0.108	0.093	0.080	0.069	0.060	0.052	0.045
18	0.153	0.130	0.111	0.095	0.081	0.069	0.059	0.051	0.044	0.038
19	0.138	0.116	0.098	0.083	0.070	0.060	0.051	0.043	0.037	0.031
20	0.124	0.104	0.087	0.073	0.061	0.051	0.043	0.037	0.031	0.026

Cumulative present value of $1 per annum, Receivable or Payable at the end of each year for n years $\frac{1-(1+r)^{-n}}{r}$

Periods (n)	Interest rates (r)									
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145
11	10.368	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495
12	11.255	10.575	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103
14	13.004	12.106	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367
15	13.865	12.849	11.938	11.118	10.380	9.712	9.108	8.559	8.061	7.606
16	14.718	13.578	12.561	11.652	10.838	10.106	9.447	8.851	8.313	7.824
17	15.562	14.292	13.166	12.166	11.274	10.477	9.763	9.122	8.544	8.022
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.372	8.756	8.201
19	17.226	15.679	14.324	13.134	12.085	11.158	10.336	9.604	8.950	8.365
20	18.046	16.351	14.878	13.590	12.462	11.470	10.594	9.818	9.129	8.514

Periods (n)	Interest rates (r)									
	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833
2	1.713	1.690	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528
3	2.444	2.402	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106
4	3.102	3.037	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589
5	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991
6	4.231	4.111	3.998	3.889	3.784	3.685	3.589	3.498	3.410	3.326
7	4.712	4.564	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605
8	5.146	4.968	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837
9	5.537	5.328	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031
10	5.889	5.650	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.192
11	6.207	5.938	5.687	5.453	5.234	5.029	4.836	4.656	4.486	4.327
12	6.492	6.194	5.918	5.660	5.421	5.197	4.988	7.793	4.611	4.439
13	6.750	6.424	6.122	5.842	5.583	5.342	5.118	4.910	4.715	4.533
14	6.982	6.628	6.302	6.002	5.724	5.468	5.229	5.008	4.802	4.611
15	7.191	6.811	6.462	6.142	5.847	5.575	5.324	5.092	4.876	4.675
16	7.379	6.974	6.604	6.265	5.954	5.668	5.405	5.162	4.938	4.730
17	7.549	7.120	6.729	6.373	6.047	5.749	5.475	5.222	4.990	4.775
18	7.702	7.250	6.840	6.467	6.128	5.818	5.534	5.273	5.033	4.812
19	7.839	7.366	6.938	6.550	6.198	5.877	5.584	5.316	5.070	4.843
20	7.963	7.469	7.025	6.623	6.259	5.929	5.628	5.353	5.101	4.870

FORMULAE

PROBABILITY

$A \cup B$ = **A or B**. $A \cap B$ = **A and B** (overlap).

$P(B \mid A)$ = probability of B, **given** A.

Rules of Addition

If A and B are mutually exclusive: $P(A \cup B) = P(A) + P(B)$

If A and B are **not** mutually exclusive: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Rules of Multiplication

If A and B are *independent*: $P(A \cap B) = P(A) * P(B)$

If A and B are **not** *independent*: $P(A \cap B) = P(A) * P(B \mid A)$

$E(X) = \Sigma$ (probability * payoff)

DESCRIPTIVE STATISTICS

Arithmetic Mean

$$\bar{x} = \frac{\Sigma x}{n} \qquad \bar{x} = \frac{\Sigma fx}{\Sigma f} \quad \text{(frequency distribution)}$$

Standard Deviation

$$SD = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}} \qquad SD = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x^2}} \quad \text{(frequency distribution)}$$

INDEX NUMBERS

Price relative = $100 * P_1/P_0$ Quantity relative = $100 * Q_1/Q_0$

Price:
$$\frac{\Sigma w * \left(\frac{P_1}{P_0}\right)}{\Sigma w} \times 100$$

Quantity:
$$\frac{\Sigma w * \left(\frac{Q_1}{Q_0}\right)}{\Sigma w} \times 100$$

TIME SERIES

Additive Model

Series = Trend + Seasonal + Random

Multiplicative Model

Series = Trend * Seasonal * Random

FINANCIAL MATHEMATICS

Compound Interest (Values and Sums)
Future Value *S*, of a sum of *X*, invested for *n* periods, compounded at *r*% interest

$$S = X[1 + r]^n$$

Annuity
Present value of an annuity of £1 per annum receivable or payable for *n* years, commencing in one year, discounted at *r*% per annum:

$$PV = \frac{1}{r}\left[1 - \frac{1}{[1+r]^n}\right]$$

Perpetuity
Present value of £1 per annum, payable or receivable in perpetuity, commencing in one year, discounted at *r*% per annum:

$$PV = \frac{1}{r}$$

LEARNING CURVE

$$Y_x = aX^b$$

where:
Y_x = the cumulative average time per unit to produce X units;
a = the time required to produce the first unit of output;
X = the cumulative number of units;
b = the index of learning.

The exponent b is defined as the log of the learning curve improvement rate divided by log 2.

INVENTORY MANAGEMENT

Economic Order Quantity

$$EOQ = \sqrt{\frac{2C_oD}{C_h}}$$

where: C_o = cost of placing an order
C_h = cost of holding one unit in Inventory for one year
D = annual demand