UGANDA MARTYRS UNIVERSITY
NKOZI

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

END OF SEMESTER ONE FINAL ASESSMENT
SEMESTER I, 2014/15

SECOND YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE GENERAL & FINANCIAL MATHEMATICS

Linear Algebra

MTH 2101

DATE: 10TH DECEMBER 2014

TIME: 2: 00- 5:00 PM

DURATION: 3HRS

Instructions:

1. *Carefully read through ALL the questions before attempting*
2. ***ANSWER FIVE (5) Questions** ONLY. (Each question carries equal marks)*
3. *No **names** should be written anywhere on the examination book.*
4. *Ensure that your **Reg number** is indicated on all pages of the examination answer booklet.*
5. *Ensure your work is **clear** and **readable**. Untidy work shall be penalized*
6. *Any type of examination Malpractice will lead to automatic disqualification*
7. *Do not write anything on the questions paper.*

1. a) Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$ C = $\begin{bmatrix} 3 & 0 & 4 \\ 0 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$. Verify that:

i) $(A+C)B = AB + CB$ **(3 marks)**

ii) $(AB)^T = B^TA^T$. **(3 marks)**

b) Let A and B be mxn matrices and C be an nxp matrix. Prove that
$(A+B)C = AC + BC$. **(3 marks)**

c) Let A = $\begin{bmatrix} 2 & 0 & t \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ where t is a scalar. Find A^2 and A^3. Hence state A^n for n a positive integer. **(5 marks)**

d) i) Let A be a non singular matrix. Show that the inverse of A is unique. **(3 marks)**

ii) Show that no positive power of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ equals the identity I_2. **(3 marks)**

2. a) Find the condition on r and t such that the system

$$-2x + ry = 2$$
$$x - y = t$$

i) has no solution

ii) has one solution

iii) has infinitely many solutions. **(6 marks)**

b) i) Explain the three row operations of the Gaussian elimination process. **(3 marks)**

ii) Solve the following system of linear equation by Gaussian elimination process.

$$2x + 3y - z = 3$$
$$x - 2y + 3z = -2$$
$$3x + y + z = 2$$

(5 marks)

c) The UMU library owns 20,000 books. Each month 20% of the books in the library are lent out, and 80% of the books lent out are returned, while 10%

remain lent out and 10% are reported lost. 25% of the books are listed as lost the previous month are found and returned to the library. At present 18,000 books are in the library, 2,000 are lent out, and none are lost. How many books will be in the library, lent out and lost after two months? **(6 marks)**

3. a) Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 4 & 0 \end{bmatrix}$ and I_3 be the 3x3 identity matrix.

i) Find the characteristic polynomial P(t) = det(A-tI_3), where t is a scalar. **(3 marks)**

ii) Find the values of t for which det(A-tI_3) = 0. **(2 marks)**

iii) For each of the values of t in part ii) above, find a vector v = $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ such that Av = tv. **(3 marks)**

b) Let A = $\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$. Find all 3 x 2 matrices B such that BA = 0. **(5 marks)**

c) Show that for any 2 x 2 matrices A and B, det(AB) = det (A) det(B). **(3 marks)**

d) i) Show that det(I_n) = 1. **(2 marks)**

ii) Using part 3c) and 3di) above, show that det(A) = $\frac{1}{\det(A^{-1})}$. **(2 marks)**

4. a) Let A be a square matrix. What is meant by "A is a non singular matrix"? **(2 marks)**

b) Find the inverse of the matrix $\begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \\ 5 & 4 & -3 \end{bmatrix}$ by row reduction method. **(5 marks)**

c) Let A = $\begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \\ 5 & 4 & -3 \end{bmatrix}$ Find:

i) the co-factor matrix **C** of A. **(4 marks)**

ii) the determinant of A. **(2 marks)**

Hence state A^{-1}. **(2 marks)**

d) Let A = $\begin{bmatrix} 1 & -1 & t \\ 3 & 0 & r \\ -2 & 2 & 1 \end{bmatrix}$. Find the values of t and r for which:

i) A is singular.
ii) A is non singular. **(5 marks)**

5. a) Define the following terms as used in Linear algebra:
i) vector subspace **(1 marks)**
ii) span of a set **(1 marks)**
iii) linear combination of vectors $v_1, v_2, \dots, v_n$. **(1 marks)**

b) A subset L of $\mathbf{R}^3$ is given by $L = \{(x,y,z) : rx+ty+sz = 0\}$ for r,t,s scalars. Show that L is a subspace of $\mathbf{R}^3$. **(6 marks)**

c) i) Let U_1, U_2 and U_3 be subspaces of vector space V. Show that $U_1 \cup U_2 \cup U_3$ is not necessarily a subspace of V. **(3 marks)**

ii) Vectors v_1, v_2, v_3 are such that $v_3 = 2v_2 - v_1$. Comment about the linear independence of the set $\{ v_1, v_2, v_3\}$. **(2 marks)**

d) Determine which of the following sets are linearly independent:

i. $S = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 3 & 3 \end{pmatrix} \right\}$ **(3 marks)**

ii. $P = \{x^2 + 2x, x + 5, x^2 - 3\}$ **(3 marks)**

6. a) Let V be a vector space. What is meant by a "basis for V"? **(2 marks)**

b) A subset $W = \left\{ \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \middle| a = e = c,\ f = 0 \right\}$. Find a basis for W. Hence state the dim(W). **(5 marks)**

c) Show that $S = \{(2,1,0), (0,1,2), (0,0,-1)\}$ generates $\mathbf{R}^3$. **(3 marks)**
d) i) What is a linear transformation? **(2 marks)**
ii) Which of the following functions is a linear transformation?

$T:\mathbf{R}^3\ \mathbf{R}^3$ where $T(x_1,x_2,x_3) = (1,x_2,x_3)$ **(2 marks)**
$T:\mathbf{P_2}\ \mathbf{P_1}$ where $T(a_0 + a_1x + a_2x^2) = a_1 + 2a_2x$ **(3 marks)**
$T:\mathbf{P_2}\ \mathbf{P_3}$ where $T[p(x)] = p(0) + xp(x)$. **(3 marks)**

END