UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

University Supplementary/ Special Examinations 2013-2014, Semester II

First Year **Supplementary/ Special Examination** for the Degrees of Bachelor of Science **Financial Mathematics** and **Bachelor of Science General**.

STA 1202 ELEMENTS OF PROBABILITY

WEDNESDAY 13th August 2014 Time: 2:00am - 5:00 pm

Instructions

(i) Answer **Five** questions

(ii) Write on both sides of the paper but begin a new question on a fresh page.

1. ***Question 1***

(a) Give the three rules of Probability.

[3 Marks]

(b) Define the following as associated to probability

(i) Sample points.

[1 Mark]

(ii) Mutually exclusive events.

[1 Mark]

(c) (i) If A and B are two events which are not disjoint in that $P(A \cap B) \neq 0$, then

$$P(A \cup B) = ?$$

[1 Mark]

(ii) Let S be a sample space and E is an event from S, then show that

$$P(E') = 1 - P(E).$$

[3 Marks]

(d) Let A and B be events in a sample space S, if

$$P(A) = 0.40, P(A \cap B') = 0.1, \text{ and } P(A \cup B) = 0.8,$$

Represent the information on a venn diagram and use it to compute the following probabilities

(i) $P(A \cap B)$.

[2 Marks]

(ii) $P(A \cup B)'$.

[2 Marks]

(e) Let A, B *and* C be events in a sample space S with

$$P(A) = 0.53, P(B) = 0.34, P(C) = 0.28, P(A \cap B') = 0.33,$$

$$P(B \cap C) = 0.13, P(A \cap C) = 0.1 \text{ and } P(A' \cap B \cap C) = 0.06.$$

Represent the given information on a venn diagram and use the venn diagram to compute the following probabilities.

(i) $P(A \cap B)$.

[5 Marks]

(ii) $P(A' \cap B' \cap C')$

[2 Marks]

Question 2

(a) Two wheels of chance are spun. The first is marked with $\{1, 2, 3, 4\}$ and the second with $\{1, 2, 3\}$, on both wheels each number is equally likely to occur. Construct a suitable sample space to describe the outcomes of this random phenomenon.

[2 Marks]

Then use the probability formula to compute the probability of the following events

(i) Exactly one of the numbers is odd.

[2 Marks]

(ii) The sum of the numbers which show up is at most 5.

[3 Marks]

(b) Three distinguishable dice are rolled.

(i) What is the probability of not getting a 1 on any of the dice?

[2 Marks]

(ii) What is the probability of getting atleast 1, from the three dice?

[3 Marks]

(iii) What is the probability of getting exactly two 6's or all the three digits that show up are the same?

[3 Marks]

(c) A student meets the prerequisites for 11 English, 9 History, 4 mathematics, and 3 chemistry courses. Suppose he decides to take one course in each of these subjects,

(i) How many different programs are available for this student?

[2 Marks]

(ii) How many programs are available to him if he decides to take one course each in three of these four subjects?

[3 Marks]

Question 3

(a) (i) Define a combination of $n-$ objects taking $r-$ of them at a time.

[1 Mark]

(ii) Use the Binomial theorem and Pascal's triangle to expand $(2x - 3y)^6$.

[4 Marks]

(b) Determine the number of thirteen letter words that can be formed using the letters of the word "**DENOMINATIONS**".

[4 Marks]

(c) A student wants to arrange his 9 different record albums on a shelf.

(i) In how many ways can this be done?

[2 Marks]

(ii) In how many ways can this be done if his **Wilson Bugembe Album** and **Judith Babirye's album** are separated by exactly one other album.

[3 Marks]

(d) (i) How many 3-digit numbers can be formed using 3 different digits from $\{1, 2, 3, 4, 5, 6\}$

[2 Marks]

(ii) How many of the numbers in $d(i)$ above are divisible by 5?

[2 Marks]

(iii) How many of the numbers in $d(i)$ above are less than 300?

[2 Marks]

Question 4

(a) Given that $A_1, A_2, ..., A_n$ are mutually exclusive events and that A is any event, state and prove Baye's Theorem.

OR

Suppose A an B are mutually exclusive events of an experiment, state and prove **Baye's Theorem**.

[6 Marks]

(b) In a certain city 40% of the people are conservatives(C), 35% are Liberals(L) and 25% are Independents (I). During a particular elections 45% of the conservatives voted, 40% of the Liberals voted and 60% of the Independents voted. Suppose a person is randomly selected.

(i) Find the probability that the person voted.

[4 Marks]

(ii) If the person voted find the probability that the voter is a conservative.

[3 Marks]

(c) (i) Give the formula defining the binomial probability of an event **E**, happening **r**− times exactly in **n**− trials, with **p**− as the probability of success of the event **E**, and **q**− the probability of failure.

[1 Mark]

(ii) The probability that a man aged 60 years will live to be 70 years is 0.65, what is the probability that out of 10 men now 60 years old atleast 7 will live to be 70 years.

[4 Marks]

(iii) Determine the mean of 10 men, now 60 years old who will live to be 70 years.

[2 Marks]

Question 5

(a) Let S be a sample space, define **a discrete random variable** as related to S.

[1 Mark]

(b) The following table gives the values and corresponding probabilities of a random variable **X**

X_i	-4	-2	0	1	3
$P(X = x_i)$	0.2	0.25	0.15	0.10	0.30

(i) Determine the expectation of **X**, **E(X)**.

[3 Marks]

[2 Marks]

(iii) Compute $\mathbf{E(2X+10)}$.

[3 Marks]

(c) The faces of a fair twelve sided (*Duo decahedron*) die are numbered from 1 to 12. Let X be the remainder obtained from dividing the outcome of roll of this die by 6. Compute the probability function of X.

[3 Marks]

(d) Box 1, contains 14 blue marbles and 10 red marbles and Box 2 contains 10 blue marbles and 2 red marbles. A fair die is rolled, if the outcome of the roll on the die is 1, then Box 1 is selected, otherwise if the outcome on the die is a $2, 3, 4, 5,$ *or* 6 then Box 2 is selected. After selecting the box, a marble is randomly selected from it.

(i) Given that a red marble is selected, what is the probability that it came from Box 1.

[6 Mark]

(ii) What is the probability of rolling a 1, and a blue marble is selected?

[3 Marks]

Question 6

(a) Give three geometric properties of a standard normal density curve.

[1 Marks]

(b) (i) Let $A(t)$ denote the area beneath the standard normal curve to the left of t so that $P(Z < t) = A(t)$. Show that $P(-t < Z < t) = 2A(t) - 1$.

[3 Marks]

(ii) The Weight of new born babies are approximately normally distributed with mean $\mu = 7.5\ Kgs$ and a standard deviation $\sigma = 1.25\ Kgs$, what is the probability that a new born baby will weigh more than 9 kgs.

[4 Marks]

(iii) From b(ii) above what is the probability that a new born baby will weigh between 7.9 and 8.6 Kgs.

[4 Marks]

(b) A box contains 9 balls 2 of which are red, 3 blue and 4 black. 3 balls are drawn from the box at random. what is the probability that

(i) The three balls are of different colors.

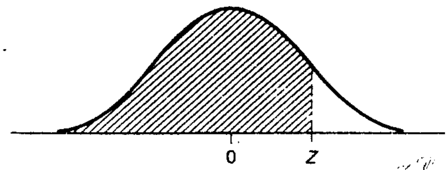
[4 Marks]

(ii) The 3 balls are of the same color.

[4 Marks]

END

TABLE A.1 CUMULATIVE AREAS UNDER THE STANDARD NORMAL DISTRIBUTION



Z	0	1	2	3	4	5	6	7	8	9
−3.	0.0013	0.0010	0.0007	0.0005	0.0003	0.0002	0.0002	0.0001	0.0001	0.0000
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0238	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0300	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0570	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

TABLE A.1 CUMULATIVE AREAS UNDER THE STANDARD NORMAL DISTRIBUTION (cont.)

Z	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

SOURCE: B. W. Lindgren, *Statistical Theory* (New York: Macmillan, 1962), pp. 392–393.