UGANDA MARTYRS UNIVERSITY
NKOZI

UNIVERSITY EXAMINATIONS
January/February 2022
YEAR THREE

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS & STATISTICS
END OF SEMESTER ONE FINAL ASESSMENT

COMPLEX VARIABLES
MTC 3111

DATE: 17/01/2022

TIME: 9:30am - 1:00pm

DURATION: 3 HOURS

Instructions:

---

1. *Carefully read through ALL the questions before attempting*
2. ***ANSWER FIVE (4) Questions ONLY**. (Each question carries equal marks)*
3. *No **names** should be written anywhere on the examination book.*
4. *Ensure your work is **clear** and **readable**. Untidy work shall be penalized*
5. *Any type of examination Malpractice will lead to automatic disqualification*

QUESTION ONE

(a). Simplify the complex number below in the form z=x + iy,

$$Z = \frac{3-2i}{M}arks]2 + 2i + \frac{2+i}{5-6i}$$ **[03 Marks]**

(b) (i) State the two Cauchy Integral Formulas **[02 Marks]**

(ii) Evaluate;

$$\oint \frac{z^2+e^{3z}}{(z+1)^4}$$ **[06 Marks]**

(c) (i) Find the

$$\lim_{z\to\infty} \frac{5iz^2+20i+z+3}{z^2+4}$$ **[04 Marks]**

(ii) Using the definition of the limit of the function f(z) as $z \to z_0$, prove that

$$\lim_{z\to 3i} \frac{2(z^2-iz+6}{z-3i} = 10i$$ **[05 Marks]**

QUESTION TWO

(a) State and prove De-Moivre's Theorem **[05 Marks]**

(b) (i) Determine whether the function f(z) = cos z is analytic **[05 Marks]**

(ii) Find the roots of the polynomial $6z^4 - 47z^3 + 148z^2 - 167z + 52 = 0$, if $z = 3 + 2i$ is a root of the equation **[05 Marks]**

(c) (i) When is a function f(z) = u(x, y) + iv(x, y) said to be harmonic? **[02 Marks]**

(ii) Prove that the function $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic **[03 Marks]**

QUESTION THREE

(a) (i) Define the limit of a function f(z) at infinity **[01 Mark]**

(ii) Prove that

$$\lim_{z\to 1+i} \frac{z^2-z+1-i}{z^2-2z+2} = 1 - \frac{1}{2}i$$ [05 Marks]

(b) If $f(z) \equiv \frac{2z-1}{3z+2}$, prove that at $z = z_0$;

$$\lim_{h\to 0} \frac{f(z_0+h-f(z_0)}{h} = \frac{7}{(3z_0+2)} \text{ where } z_0 \neq \frac{-2}{3}$$ [05 Marks]

(c) (i) When is a complex function f(z) said to be continuous at a pointz = z_0 **[02 Marks]**

(ii) Find the points at which the function f(z) below is discontinuous.

$$f(z) = \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$$

Redefine the function at the points where the function is discontinuous to remove the discontinuity and show that the limit of the function at that point is $4 + 4i$ **[08 Marks]**

QUESTION FOUR

(a) Define the following terms:

(i) An Isolated singularity **[02 Marks]**

(ii) A pole of order n **[02 Marks]**

(b) Locate and name all the singularities of the following function

$$f(z) = \frac{z^2-3z}{(z^2+2z+2)(z+5)}$$ **[08 Marks]**

(c) Use L'Hopitals rule to evaluate:

$$\lim_{z\to i} \frac{z^{10}+1}{z^6+1}$$ **[04 Marks]**

(d) Let $z_1 and z_2$ be complex numbers. Prove that $z_1z_2 = z_1z_2$ **[04 Marks]**

QUESTION FIVE

Let $a_n, a_{n-1}, \ldots\ldots a_1 a_0$ be real numbers. If α is a complex root of the polynomial equation $a_n z^n + a_{n-1} z^{n-1} + \ldots\ldots a_1 z + a_0 = 0$ then show that $\bar{\alpha}$ is also a root of the equation **[03 Marks]**

(ii) Given that z = 2 + i is a root of the equation; $z^4 - 5z^3 + 3z^2 + 19z - 30 = 0$. Find the other roots **[04 Marks]**

(b) Prove the triangle inequality of complex numbers that says that;$z_1 + z_2 z_1 + z_2$ **[04 Marks]**

(c) Solve the equation $z^5 + 32 = 0$ using De-Moivre's Theorem **[05 Marks]**

(d) Find the sixth root of $z = 4 + 5i$ **[04 Marks]**

QUESTION SIX

(a) State the Residue Theorem **[02 Marks]**

(b) Find the residues of $f(z) = \frac{2z^2+5}{(z+2)(z^2+4)(z^2)}$. Hence evaluate $\oint \frac{2z^2+5}{(z+2)(z^2+4)(z^2)} dz$
using the residue theorem. **[08 Marks]**

(c) Solve the equation $z^6 + 729 = 0$ using De-Moivre's Theorem **[06 Marks]**

(d) Find the fourth roots of $z = 2 \quad (3) - 2i$ **[04 Marks]**

END