UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

CALCULUS I

UNIVERSITY EXAMINATIONS

END OF SEMESTER ONE 2012/2013

FIRST YEAR EXAMINATIONS FOR; BACHELOR OF SCIENCE BUSINESS ECONOMICS

BACHELOR OF SCIENCE –GENERAL

BACHELOR OF FINANCIAL MATHEMATICS

DATE: 11th December 2012

Time: 9.00am-12.00pm

Instructions:

- Attempt Question 1 and any other four questions
- Question 1 is compulsory

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Question 1 (Compulsory)

(a) Distinguish between the following terms as used in Calculus:

i) a relation and a function (2 marks)

ii) an even function and an odd function (2 marks)

iii) an increasing function and a strictly increasing function (2 marks)

(b) Solve for x:

i) $x^2 - 3x + 2 < 0$ (2 marks)

ii) $3|x| = x + 2$ (3 marks)

(c) Compute the following limits:

i) $\lim_{x \to 1} 2x - x^2$ (2 marks)

ii) $\lim_{x \to 0} f(x)$ if $f(x) = \begin{cases} x + 2.2, & for\ x \geq 0 \\ -x + 2, & for\ x < 0. \end{cases}$ (3 marks)

(d) Compute the derivatives of the following functions:

i) $f(x) = \frac{x^3 - 2}{x^2}$ (2 marks)

ii) $f(x) = \sin(1 - 2x)$ (2 marks)

Question 2

(a) Define the following concepts as used in Calculus:

i) range of a function (1 mark)

ii) inverse of a function f (1 mark)

iii) graph of a function (1 mark)

(b) State the domains and ranges (as subsets of $\mathbb{R}$) of the following functions:

i) $f(x) = \frac{1}{x-2}$ (2 marks)

ii) $f(x) = \sqrt{16 - x^2}$ (3 marks)

(c) A function $f : [0, \infty) \to \mathbb{R}$ is defined as $f(x) = 1 - 2x^2$. Determine whether or not f is:

i) injective (2 marks)

ii) surjective (2 marks)

(d) Functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R}/\{0\} \to \mathbb{R}$ are defined as $f(x) = 3 - 5x$ and $g(x) = \frac{x-2}{x}$ respectively. Determine:

i) $f \circ g$ (2 marks)

ii) $(f \circ g)^{-1}$ (2 marks)

iii) the range of $f \circ g$ (1 mark)

Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$ (3 marks)

Question 3

(a) *The identity function on a set X is a bijective function.* What is meant by:

i) identity function (1 mark)

ii) bijective function (1 mark)

(b) Two functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined as

$$f(x) = \begin{cases} x - 3, & for\, x \geq 2 \\ 1 - x, & for\, x < 2. \end{cases}$$

and

$$g(x) = \begin{cases} 5 - x, & for\, x \geq 3 \\ x - 1, & for\, x < 3. \end{cases}$$

respectively.

i) Draw the graphs of f and g on the same axes, and (4 marks)

ii) hence solve the inequality $f(x) \leq g(x)$. (2 marks)

(c) For the functions $f(x) = 4 + 2x$, $g(x) = \sqrt{x+1}$ compute:

i) $(f + g)(0)$ (2 marks)

ii) $\left(\frac{f-g}{fg}\right)(3)$ (4 marks)

(d) Classify the following functions as even, odd or neither

i) $h(x) = 2x^3 - 5x$ (2 marks)

ii) $g(x) = x^2 + \frac{1}{x}$ (2 marks)

iii) $f(x) = |x| + 1$. (2 marks)

Question 4

(a) i) What is the difference between *continuity at a point* x_0 and *continuity on an interval* $[a, b]$? (2 marks)

ii) Give the formal $(\epsilon - \delta)$ definition of the limit (l) of a function f at a point say c. (2 marks)

(b) Find the values of a and b so that the function

$$f(x) = \begin{cases} x + 3a, & for\, x < -2 \\ bx^2, & for\, -2 \leq x \leq 2 \\ 3a - bx, & for x > 2. \end{cases}$$

is continuous at the points -2 and 2. (4 marks)

(c) Use the formal definition (in part (a) ii) above) of a limit to check the stated limits:

i) $\lim_{x\to 2} 5 = 5$ (2 marks)

ii) $\lim_{x\to 0} 3 - x = 3$ (3 marks)

(d) Compute the following limits:

i) $\lim_{x\to 0} 2 - 5x$ (2 marks)

ii) $\lim_{x\to -1} \frac{x^2-2x-3}{x+1}$ (3 marks)

iii) $\lim_{x\to 0} f(x)$ if $f(x) = \begin{cases} 2, & for\ x \geq 1 \\ 0, & for\ x < 1. \end{cases}$ (2 marks)

Question 5

(a) i) Distinguish between a *local maximum* and a *global maximum* of a function (2 marks)

ii) Use the linear approximation method to approximate the value of $\sqrt{36.02}$ correct to 4 decimal places. (3 marks)

(b) Define a *left hand limit* and a *right hand limit* of $f(x)$ as x tends to x_0.

(2)

Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x, & for\, x \leq 2 \\ 2 - x, & for\, x > 2. \end{cases}$$

i) Sketch the function $f(x)$. (3 marks)

ii) Find $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$. (2 marks)

ii) Does $\lim_{x\to 2} f(x)$ exist? Justify your answer. (2 marks)

(c) The equation of an implicit function is

$$(1 + yx^2)^2 = 5 - y^3x$$

Find:

i) the gradient function (4 marks)

ii) the equation of the tangent line at the point $(1, 1)$ (2 marks)

Question 6

(a) What is the meaning of the term *differentiable function* as applied in Calculus? (1 mark)

i) Using the limit definition of a derivative of a function compute f' where $f(x) = 1 - 3x^2$ (3 marks)

ii) Two functions f and g are such that $f(1) = 1, g(1) = 2, f'(2) = 5,$ and $g'(1) = 2$. Find:

a) $(f \circ g)'(1)$ (2 marks)

b) $(g \circ f)'(1)$ (2 marks)

(b) A continuous real valued function f defined on $\mathbb{R}$ is such that $f(x) = x^3 - 2$.

i) Is f differentiable on $(1, 7)$? (give a reason for your answer) (2 marks)

ii) Find a value $c \in [1, 7]$ such that

$$f'(c) = \frac{f(7) - f(1)}{6}.$$

(2 marks)

iii) What is the common name for the Theorem portrayed in parts (i) and (ii) above? (1 mark)

(c) Give an example of a function that is:

i) differentiable but not continuous (1 mark)

ii) continuous but not differentiable. (2 marks)

(d) The population $P(t)$ of bacteria is function of time given by

$$P(t) = \frac{t}{t^2 + 1}$$

(millions) at a time t hours.

i) What is the rate of growth $\frac{dP}{dt}$ of the population after 15 minutes? (2 marks)

ii) When is the population maximum? (2 marks)