UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

University Examinations 2013-2014, Semester I

Third Year **Final Assessment Examination** for the Degree of Bachelor of Science Financial Mathematics and Bachelor of Science General.

MTC 3102 COMPLEX VARIABLES

Tuesday 17^{th}, December 2013 Time: 9:00am - 12:00 Noon

Instructions

(i) Answer **Five** questions

(ii) Write on both sides of the paper but begin a new question on a fresh page.

1. ***Question 1***

(a) Simplify the complex number below in the form of $z = x + iy$,

$$z = \frac{3-2i}{2+2i} + \frac{2+i}{5-6i}$$

[3 Marks]

(b) (i) State the two Cauchy integral formulas.

[2 Marks]

(ii) Evaluate

$$\oint_C \frac{z^2 + e^{3z}}{(z+1)^4},$$

using one of the Cauchy integral formulas.

[6 Marks]

(c) (i) Find the

$$\lim_{z \longrightarrow \infty} \frac{5iz^2 + 20i + z + 3}{z^2 + 4}.$$

[4 Marks]

(ii) Using the definition of the limit of function$f(z)$ as $z \to z_0$, prove that

$$\lim_{z \longrightarrow 3i} \frac{2(z^2 - iz + 6)}{z - 3i} = 10i$$

[5 Marks]

Question 2

(a) State and prove De-Moivre's theorem.

[5 Marks]

(b) (i) Determine whether the function $f(z) = cos\ z$ is analytic.

[5 Marks]

(ii) Find the roots of the polynomial $6z^4 - 47z^3 + 148z^2 - 167z + 52 = 0$, if $z = 3 + 2i$ is a root of the equation.

[5 Marks]

(c) (i) When is a function $f(z) = u(x, y) + iv(x, y)$ said to be harmonic?

[2 Marks]

(ii) Prove that the function $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is Harmonic.

[3 Marks]

Question 3

(a) Find all the fifth roots to the complex number $Z = -3 + 2i$, sketch them out on an argand diagram and indicate the principal root.

[8 Marks]

(b) (i) Define the derivative of a single valued function $f(z)$ at a point z_0 in the complex plane.

[1 Marks]

(ii) By using the definition of the derivative of a function $f(z)$ at a point z, find $f'(z)$, for $f(z) = z^2 + 4z + 3$.

[5 Marks]

(c) Prove that if a function $f(z) = u(x, y) + iv(x, y)$ is analytic in the domain D,then u and v are Harmonic in D.

[6 Marks]

Question 4

(a) When is a singular point $z = z_0$ to a function $f(z)$ said to be

(i) A pole of order n.

[1 Mark]

(ii) An essential singularity.

[1 Mark]

(b) (i) Determine the singular points and their nature for the function

$$f(z) = \frac{z}{(z^2 + 25)^2(z + 2)} .$$

[8 Marks]

(ii) Determine the nature and type of singularity at infinity for the function

$$f(z) = \frac{(z-1)^3 z^2}{(z^2 + z + 2)} .$$

[4 Marks]

(iii) Determine the singular points and their nature for the function

$$f(z) = \frac{ln\ (z-3)}{(z+4)^3} .$$

[4 Marks]

Question 5

(a) Let $M(x, y)$ and $N(x, y)$ be continuous and have continuous partial derivatives in the region D and along its boundary c, then the theorem

$$\oint_c M dx + N dy = \int\int_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\ dy ,$$

holds.

(i) What name is given to the theorem in a above.

[1 Mark]

(ii) Use the stated theorem in (a) above to evaluate $\oint_c 5xy dx + x^3 dy$ for the

[6 Marks]

(b) For a function $f(z)$ which is analytic on and inside a simple closed curve C, Cauchy's Theorem states that

$$\oint_C f(z) = 0 ,$$

Prove the theorem.

[6 Marks]

(c) Find the value of

$$\int_C (2y + x^2)dx + (3y - x)dy ,$$

along the straight line joining the points $(0, 3)$ *to* $(2, 4)$.

[7 Marks]

Question 6

(a) Determine the Laurent's series expansion for the function

$$f(z) = \frac{1}{z^2(z-3)^2}$$

along the singular point $z = 3$, use it to determine the residue of the function at $z = 3$.

[6 Marks]

(b) With a clearly labelled diagram state the **Residue theorem.**

[2 Marks]

(c) Find the residues of the function

$$f(z) = \frac{3+2z}{z(z-2)^2(z-4)} ,$$

at all its poles in the finite plane and hence evaluate

$$\oint_C \frac{3+2z}{z(z-2)^2(z-4)} dz ,$$

Using the residue theorem where C is the circle $|z| = 3.5$.

[12 Marks]

END