UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS & STATISTICS

UNIVERSITY EXAMINATIONS
SEMESTER I, 2013/14

SECOND YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE
(FM & GEN)

LINEAR ALGEBRA

DATE: 18^{TH} DECEMBER 2013

TIME: 2:00 - 5:00 PM

Instructions:
i) Attempt any five questions.

ii) Write on both sides of the booklet paper but each question should be answered starting on a new sheet of paper.

iii) Start with questions you find easiest and not necessarily those that carry most marks

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1. i) Let $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 4 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 3 & 0 & 0 \\ 1 & 1 & 2 \end{pmatrix}$.

   a) Compute $B(A + C)$ **(3 marks)**

   b) Verify that $(A - C)^T = A^T - C^T$ **(3 marks)**

   ii) If $A = \begin{pmatrix} -1 & 0 & 2\lambda \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, compute A^2, A^3, A^4 and hence state A^n where n is a positive integer. **(7 marks)**

   iii) Let A and B be matrices of size $m \times n$ and $n \times p$ respectively. Prove that $(AB)^T = B^T A^T$ **(4 marks)**

   iv) Write down a 2×3 matrix whose entries are given by $x_{ij} = \frac{i^2+1}{2j}$ **(3 marks)**

2. i) Let V be a vector space. Define

   a) a linearly independent subset of V **(3 marks)**

   b) a basis for V **(2 marks)**

   c) dimension of V. **(2 marks)**

   ii) Give an example of a subset U of vectors in $\mathbb{R}^2$ that is linearly dependent. **(3 marks)**

   iii) Show that the subset $W = \{2, 1 + 3x, x^2\}$ is a basis of vector space $\mathbb{P}^2$. Hence state the dimension of $\mathbb{P}^2$. **(6 marks)**

   iv) Let $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $M_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $M_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Describe the space generated by $\{M_1, M_2, M_3\}$. **(4 marks)**

3. Let $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & -2 & 1 \\ -2 & 2 & 1 \end{pmatrix}$. Compute A^{-1} using

   i) the Gauss-Jordan elimination method **(10 marks)**

   ii) the minor - cofactors method **(10 marks)**

4. Given the following system of linear equations

$$
\begin{aligned}
2x + y - 2z &= -2 \\
x - y + z &= 2 \\
-x - 3y + 2z &= -1
\end{aligned}
$$

i) Express the system in the form $AX = b$ **(3 marks)**

ii) Hence solve the system by the row reduction method. **(9 marks)**

iii) Find the values of t for which the following system of linear equations has

a) no solution **(4 marks)**

b) infinitely many solutions **(4 marks)**

$$x - ty = -2$$
$$-2x + y = 4$$

5. i) Define a subspace W of vector space $(V, +, \times)$. **(3 marks)**

ii) Prove that the neutral (identity) element of a group $(V, +)$ is unique. **(3 marks)**

iii) Let $V = \mathbb{R}^3$. Determine whether or not $W = \{(x_1, 0, 0) \in \mathbb{R}^3\, x_1, x_2, x_3 \in \mathbb{R}\}$ is a vector subspace of V. **(5 marks)**

iv) Let $(V, +, \times)$ be a vector space and U, W be subspaces of V. Show that $U \cap W$ is a subspace. **(5 marks)**

v) Give an example of a vector space V and two subsets U and W of V. Where U is a subspace of V and W is not a subspace of V. **(4 marks)**

6. i) Define a linear transformation T from vector space V into vector space W. **(3 marks)**

ii) Give an example of a transformation that is not linear. **(2 marks)**

iii) Let $T : V \to W$ be a linear transformation. Show that

a) $T(0) = 0$. **(3 marks)**

b) the image $T(V) = \{T(v) : v \in V\}$ of V is a subspace of W. **(6 marks)**

iv) Show that the mapping $T : \mathbb{P}_3 \to \mathbb{P}_2$ given by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1 + 2a_2x + 3a_x^2$ is linear where $a_i \in \mathbb{R}$ for all $i = 1, 2, 3$. **(6 marks)**

END