# UGANDA MARTYRS UNIVERSITY
# NKOZI

## FACULTY OF SCIENCE

## DEPARTMENT OF MATHEMATICS AND STATISTICS

### SUPPLEMENTARY/SPECIAL EXAMINATIONS

### UNIVERSITY EXAMINATIONS
### AUGUST 2014

### YEAR ONE- ECON, GEN &FM

CALCULUS 1I

DATE: 7TH AUGUST 2014

TIME: 2:00 - 5:00 PM

DURATION: 3HRS

***Instructions:***

*i) Attempt **FIVE** Questions*
*ii) Write on both sides of the paper but begin a new question on afresh page*

1. (a) Find the following integrals

(i) $\int x\sqrt{x^2+1}\,dx$

**[4 marks]**

(ii) $\int \frac{\sin x}{\sqrt{x}}dx$

**[4 marks]**

(iii) $\int \frac{7}{x^2-6x+25}dx$

**[4 marks]**

(b) Let $u = u(x)$ and $v = v(x)$ be differentiable real valued functions. By using the Chain rule of differentiation, show that

$$\int udv = uv - \int vdu$$

Hence or otherwise, find $\int e^x \sin x\,dx$.

**[8 marks]**

2. (a) Plot the following points given in polar coordinates and give the rectangular coordinates of each.

(i) $A(2, \frac{\pi}{2})$

**[3 marks]**

(ii) $B(-2, \frac{2\pi}{3})$

**[3 marks]**

(b) Convert the following polar equations to rectangular forms and sketch the resulting graphs

(i) $\frac{6}{r} = \sin\theta$

**[4 marks]**

(ii) $r^2 \sin 2\theta = 8$

**[4 marks]**

(c) Express the following rectangular equations in polar form

(i) $y = x^2$

**[3 marks]**

(ii) $(x^2+y^2)^5 = 4(xy)^2$

**[3 marks]**

3. (a) By the method of partial fractions, find;

(i) $\int \frac{5x+3}{x^3-2x^2-3x}\,dx$

**[5 marks]**

(ii) $\int \frac{6x^2-15x+22}{(x+3)(x^2+2)^2}\,dx$

**[5 marks]**

(b) Using the substitution $t = \tan\frac{x}{2}$, find;

(i) $\int \frac{1}{\cos x - \sin x - 1}dx$

**[5 marks]**

(ii) $\int \frac{1}{3\sin x + 4\cos x}\,dx$ [5 marks]

4. (a) Using the method of approximating sums, find $\int_0^3 (x^2+2)\,dx$ [10 marks]

Hint:
$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(b) Investigate the following improper integrals for convergence

(i) $\int_0^{\infty} \frac{1}{1+x^2}\,dx$ [5 marks]

(ii) $\int_{-\infty}^{\infty} xe^x\,dx$ [5 marks]

5. (a) Let $e$ be a given positive number. Determine the locus of a point P(x, y) if its distance from the fixed point F(p, 0) is $e$ times its distance from the vertical line L whose equation is x = -p . Hence discuss the locus of P(x, y) when $e = 1$ and when $e > 1$. [7 marks]

(b) Find the focus, directrix, axis and the vertex of the Parabola $x^2 = 24y$. [5 marks]

(c) Determine the graph of the equation

$$y^2 - 2x + 6y + 15 = 0$$

[8 marks]

6. (a) Find the following integrals

(i) $\int \cosh 3x\,dx$ [4 marks]

(ii) $\int_0^1 \tanh^2 x\,dx$ [4 marks]

(b) Evaluate the following limits

(i) $\lim_{x \longrightarrow 1} \frac{1-x+\ln x}{1+\cos \pi x}$ [3 marks]

(ii) $\lim_{x \longrightarrow \infty} \frac{e^x}{x^2+x}$ [3 marks]

(c) Find the derivatives of the following functions

(i) $\cosh(3x-2)$ [3 marks]

(ii) $sec^{-1}(e^{2x})$ [3 marks]

END