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# UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

UNIVERSITY EXAMINATIONS

May 2016

**Second Year Examination for Bachelor of Science**

**(General and Financial Mathematics)**

STA 2201: Advanced Probability Theory

Date: Monday $2^{nd}$ May, 2016

Time: 9:30 AM - 12:30 PM

## Instructions

1. *Do not write any thing on this question paper.*
2. Attempt any **FIVE** (5) questions.
3. Begin answering each question from a fresh page of the Answer Booklet.

1. a) Given that two events A and B are independent with P(A and B)= 2/15 and P(A or B) = 3/5. Find P (A) and P (B). [5 marks]

   b) Bag X contains 3 red and 7 blue balls. Bag Y has 6 red and 4 blue balls while bag Z has 4 red and 8 blue balls. A bag is chosen at random and a ball is drawn from it.

   i) What is the probability that the ball is red? [4 marks]

   ii) Given that the ball is blue, what is the probability that it came from bag Z? [5 marks]

   c) $A$ and $B$ are events such that P(A) = 1/3, P(B) = 1/5 and P(A/B) + P(B/A) = 2/3.

   i) Calculate $P(A \cap B)$. [2 marks]

   ii) Find the value of $P(\bar{A} \cap \bar{B})$. [4 marks]

2. a) The probability that a golfer hits the ball on to the green if it is windy as he strikes the ball is 0.4, and the corresponding probability if it is not windy is 0.7. The probability that the wind will blow as he strikes the ball is 0.3. Find the probability that;

   i) he hits the ball on to the green [5 marks]

   ii) it was not windy, given that he does not hit the ball on to the green [6 marks]

   b) i) Given that $X \sim U(0,\ \frac{\pi}{2})$, find $P(0.5 \leq X \leq 1)$. [3 marks]

   ii) The random variable $X$ is exponential with parameter $\lambda$. Find E(X) and Var(X) [6 marks]

3. a) Let $\{X_i\}_{i=1}^{n}$ be a random sample from $\exp(\theta)$. Use the moment generating function (mgf) to determine the distribution of $Y=\sum_{i=1}^{n} X_i$ [8 marks]

   b) A random variable $X$ has pdf given as,

$$f(x) = \begin{cases} \frac{1}{4}xe^{-\frac{1}{2}x}; & x > 0, \\ 0; & \text{elsewhere.} \end{cases}$$

   Find;

   i) the mgf of $X$ [5 marks]

ii) E(X) [3 marks]

iii) Var(X) [4 marks]

4. a) A random variable $X \sim bin(n, p)$ i.e.

$$P(X=x) = \begin{cases} \binom{n}{x} p^x q^{n-x}; & x = 0, 1, \cdots, n, \\ 0; & \text{elsewhere.} \end{cases}$$

i) Derive the probability generating function of $X$, $G_X(t)$. [3 marks]

ii) If n = 5, p = 0.3, find P(X = 2) using your $G_X(t)$. [4 marks]

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from Bernoulli(p). Use the moment generating function, mgf, technique to determine the distribution of $Y = \sum_{i=1}^{n} X_i$. [4 marks]

c) i) If X is a random variable and Y is defined as Y = aX + b, $a, b \in \mathbb{R}$, derive the mgf of Y in terms of that of X. [3 marks]

ii) Given that the mgf, of a random variable $X$ is given by $M_X(t) = e^{3t+8t^2}$. Find the mgf of a random variable $Z = \frac{1}{4}(X-3)$ and use it to deduce E(Z) and Var(Z). [6 marks]

5. a) The joint probability density function, pdf of two random variables $X$ and $Y$ is given as,

$$f(x, y) = \frac{3}{5}x(y+x); 0 < x < 1; 0 < y \leq 2.$$

Given that $\mathbb{A}$ is the region $\{(x, y) : 0 < x < 1/2, 1 < y < 2\}$, find $P[(X, Y) \in \mathbb{A}]$. [6 marks]

b) Two random variables have joint pdf given below:

$$f(x, y) = \begin{cases} cx^2y & ; x^2 \leq y \leq 1, \\ 0 & ; elsewhere. \end{cases}$$

Find;

i) the value of $c$ [4 marks]

ii) $P(X \geq Y)$ [4 marks]

c) The joint pdf of random variables $X_1$ and $X_2$ is given as;

$f(x_1, x_2) = e^{-(x_1+x_2)}$; $x_1 > 0$, $x_2 > 0$.

Find the joint pdf of $Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1 + X_2)$. [6 marks]

6. a) The joint probability mass function, pmf of two discrete random variables $X$ and $Y$ is given as shown below:

$$f(x,y) = \begin{cases} kxy; & x = 1,2,3;\ y = 1,2,3, \\ 0; & \text{otherwise.} \end{cases}$$

Calculate:

i) the value of the constant k. [4 marks]

ii) the Covariance between X and Y, Cov(X, Y). [8 marks]

b) Given that two random variables $X$ and $Y$ have a joint probability mass function given by

$$f(x,y) = \begin{cases} c|x+y|; & ;\ x = 1,2,3;\ y = -1,0,1,2 \\ 0; & \text{otherwise.} \end{cases}$$

Calculate:

i) the value of the constant c. [2 marks]

ii) P($X > 2$) [3 marks]

iii) the marginal pdf of X. [3 marks]

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