UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

END OF SEMESTER FINAL ASSESSMENT

SEMESTER II 2021/22

THIRD YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE GENERAL

(BSc GEN. 3)

Biomathematics and Modelling

MTH 3205

DATE : WED 11^{th} July 2022

TIME : 2:00 PM - 5:00 PM

DURATION: 3 Hrs

Instructions

1. *Carefully read through ALL the questions before attempting.*
2. *ANSWER FOUR (4)Questions (All questions carry equal marks).*
3. *No* ***names*** *should be written anywhere on the examination booklet.*
4. *Ensure that your* ***Reg. number*** *and* ***Course*** *are indicated on all pages of your work.*
5. *Ensure that your work is* ***clear*** *and* ***readable****. Untidy work will be penalized.*
6. *Any type of examination Malpractice will lead to automatic disqualification.*

Question 1

In a pure death process, let μ be the death rate and define $\mu\delta t$ as the probability that an individual alive at time t dies within an interval $(t, t+\delta t)$. Assume that we have the population $N(t)$ at time t and let $P_n(t) = Pr(N(t) = n)$; for $P_n(0) = 0$ when $n \neq N_0$ and $P_n(0) = 1$ when $n = N_0$

(a) Show that the probability function satisfies the equation

$$P_n'(t) = (n+1)\mu P_{n+1}(t) - n\mu P_n(t).$$

[10 **Marks**]

(b) Using $G(x,t) = \sum_{n=0}^{\infty} x^n P_n(t)$ show that $G(x,t)$ satisfies the PDE

$$\frac{\partial G}{\partial t} = \mu(1-x)\frac{\partial G}{\partial x}$$

[5 **Marks**]

(c) Subject to $G(x,0) = x^{N_0}$ obtain the expression for $G(x,t)$ and hence find the expression for $E[N(t)]$.

[10 **Marks**]

Question 2

The interaction of two birds fiercely competing for the same ecological resources on an Island is described as

$$\frac{dN_1}{dt} = N_1(2 - 3N_1 - 4N_2)$$

$$\frac{dN_2}{dt} = N_2(5N_1 - 3)$$

where N_1 and N_2 densities of species measured at appropriate time

(a) What kind of interaction is represented by the system?

[4 **Marks**]

(b) Classify all the critical points of the system.

[11 **Marks**]

(c) Draw a phase diagram to illustrate the behaviour of the system and indicate all the critical points.

[10 **Marks**]

Question 3

(a) Fishing in Lake Victoria is now under intensive scrutiny because it is believed that over fishing has brought stocks of several species of fish to dangerously low levels. Give three strategies that should be used to remedy the situation giving possible reactions by the fishermen.

(b) A simple model or logistically changing population of fish in a pond undergoing a constant- rate harvesting or restocking is given by

$$P' = r(1 - \frac{P}{K}) + Q, \qquad P(0) = P_0$$

where $P(t)$ is the fish stock at any time t, $r > 0$, $K > 0$, $P_0 > 0$ and $Q > 0$ are constants. [6 Marks]

(a) Describe all parameters and variables in the model. [5 Marks]

(b) Taking $r = 1$ and $K = 1$, find the equilibrium levels of the system and examine how this equilibrium changes with Q. [7 Marks]

(c) Plot the function $f(P, Q)$ for $Q = 0$ and explain the solution behaviour over time as the population approaches the carrying capacity of the system. [7 Marks]

Question 4

Consider an ecosystem that has dynamics

$$\frac{dM}{dt} = M\left[\frac{20 - M - N}{20}\right]$$

$$\frac{dN}{dt} = N\left[\frac{10 + M - N}{10 + M}\right]$$

(a) Find and analyse the stability the steady states of the system (using the Jacobian method). [17 Marks]

(b) Draw a phase diagram and deduce from it the nature of the non-zero positive steady state. [8 Marks]

Question 5

(a) (i) Outline clearly (using compartments / boxes) all the steps involved in mathematical modelling. [5 Marks]

(ii) What is the use of modelling as applied in ecological and biological systems? [2 Marks]

(b) Give a brie explanation of the following terms as applied in ecology and environment.

(i) Competition

(ii) Predation

(iii) Mutualism

(iv) Parasitism

[8 Marks]

(c) A simple disease is spreading according to the equations

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta SI - \mu S \\
\frac{dI}{dt} &= \beta SI - \mu I - \alpha I \\
\frac{dR}{dt} &= -\mu R + f\alpha I
\end{aligned}
$$

where f is the probability of effective recovery from the disease.

(i) What assumption does the dynamics represent? [2 Marks]

(ii) What does each parameter stand for in the model? [8 Marks]

End