UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL ASSESSMENT
SEMESTER II

THIRD YEAR EXAMINATIONS FOR BSC ECON & STAT

Statistical Methods

DATE: Tuesday 23rd May, 2023
TIME: 9:30am - 12:30pm
DURATION: 3 Hours

Instructions:

1. Carefully read through ALL the questions before attempting the examination.

2. **ANSWER ANY FOUR Questions** (Each question carries a total of 20 marks).

3. No **names** should be written anywhere on the examination book.

4. Ensure that your **registration number** is indicated on all pages of the examination answer booklet.

5. Ensure your work is **clear** and **readable**. Untidy work shall be penalized.

6. Any type of examination malpractice will lead to automatic disqualification.

7. Do not write anything on the questions paper.

1. a. Using relevant examples, define the concept of *time series analysis* and give it's components. **[5 marks]**

    b. Do we need time series analysis? Explain. **[5 marks]**

    c. What are the problems associated with time series analysis? **[5 marks]**

    d. The following data shows a farm's production (in tonnes) of cassava and potatoes from the years 2018 to 2022.

Year	2018	2019	2020	2021	2022
Cassava	19	16	15	16	19
Potatoes	25	34	41	34	25

   Taking 2020 as the year of origin,

   i. Fit the trend of the type $\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \lambda_1 X_1 \\ \lambda_2 X_2 \end{pmatrix} + \begin{pmatrix} \alpha_1 X_1^2 \\ \alpha_2 X_2^2 \end{pmatrix}$ to the above. **[8 marks]**

   ii. What will the production be in 2024? **[2 marks]**

2. The annual production (in thousands) of a firm by wood products company Ltd, since 2001 follows;

Year	2001	2002	2003	2004	2005	2006	2007	2008
Production	8	11	9	13	10	12	15	14

    a. plot the production data **[6 marks]**

    b. Determine the least squares equation **[10 marks]**

    c. Determine the points on the least squares for 2001 and 2007 connect the two points to arrive at the straight line **[4 marks]**

    d. Based on the equation for the straight line, what was the estimated production for 2012? **[5 marks]**

3. a. Describe non-linear trend analysis. **[2 marks]**

    b. It appears that the imports (in thousands of tons) of a product have been increasing by about 10 percent annually.

Year	2005	2006	2007	2008	2009	2010	2011	2012
Imports	90	100	124	135	143	155	167	178

    i. Determine the logarithmic straight line equation **[15 marks]**

ii. By what percentage did imports increase on average, during the period? [3 marks]

iii. Estimate imports for the year 2018? [5 marks]

4. a. Using relevant examples, explain the relevance of matrix theory in multivariate analysis? [3 marks]

   b. Matrix A is defined as
   $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

   Find:

   i. the eigenvalues and eigenvectors of A. [8 marks]

   ii. $rank(A)$. [1 mark]

   iii. A^4 using Jordan decomposition techniques. [8 marks]

   c. Find the quadratic form $Q(x)$ of matrix
   $$B = \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix},$$

   where x is a vector. [5 marks]

5. a. The *sample covariance* of data is said to be unbiased. What does this mean? [1 mark]

   b. The sample covariance of data, s_{jk} for variables j and k, takes on a variety of values. What does each of the following values imply:

   i. $s_{jk} < 0$? [1 mark]

   ii. $s_{jk} = 0$? [1 mark]

   c. The following observations were made for three varieties V_1, V_2 and V_3 of potatoes.

V_1	V_2	V_3
7	9	9
10	7	10
13	5	16

   i. Find the variances s_1^2, s_2^2, s_3^2. [6 marks]

   ii. Find the covariances s_{jk} where $j, k = 1, 2, 3$. [12 marks]

iii. Write the covariance matrix. [2 marks]

iv. Comment about the pairwise associations of the varieties. [2 marks]

6. a. Most researchers prefer the *correlation matrix* to the *covariance matrix*. Why is this the case? [2 marks]

    b. The following covariance matrix shows the pairwise covariances for the variables: *Height, Left forearm, Left foot* and *Head circumference* measurements of $n = 30$ female university students.

Height	Left arm	Left foot	Head circ
8.7	3.0	2.8	0.3
3.0	2.3	1.2	0.2
2.8	1.2	1.9	0.1
0.3	0.2	0.1	3.4

    i. Write the correlation matrix. [10 marks]

    ii. Make comments about the associations amongst the variables. [3 marks]

    c. The scores of $n = 6$ university students who have taken semester tests in Calculus and Accounting are shown in the following table.

Calculus	Accounting
22	33
28	33
25	30
20	50
30	40
20	42

    Write a:

    i. data matrix for the table above. [1 mark]

    ii. mean vector for the scores. [6 marks]

    d. Explain why it is advisable to incorporate graphs in the process of analyzing multivariate data. [3 marks]

END

Selected formulas

i. Sample variance of n observations of variable j is

$$s_j^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_{ij} - \bar{x}_j)^2$$

ii. Sample covariance of n observations of variables j and k is

$$s_{jk} = \frac{1}{n-1} \sum_{i=1}^{n} (X_{ij} - \bar{x}_j)(X_{ik} - \bar{x}_k)$$

iii. Sample correlation of n observations of variables j and k is

$$r_{jk} = \frac{s_{jk}}{s_j s_k}$$