

UGANDA MARTYRS UNIVERSITY  
NKOZI

UNIVERSITY EXAMINATIONS  
DECEMBER 2022

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

END OF SEMESTER ONE FINAL ASSESSMENT

SECOND YEAR EXAMINATION

BSC.GENERAL (II) & Bsc. EDU (II)

PROBABILITY THEORY

DATE: Friday, 16<sup>th</sup>/12/2022

DURATION: 3HRS (2:00- 5:00 PM)

---

---

**Instructions:**

- 
- 
1. Carefully read through ALL the questions before attempting
  2. **ANSWER FOUR (4) Questions ONLY.** (Each question carries equal marks)
  3. No **names** should be written anywhere on the examination book.
  4. Ensure that your **ID number** is indicated on all pages of the examination answer booklet.
  5. Ensure your work is **clear and readable**. Untidy work shall be penalized
  6. Any type of examination Malpractice will lead to automatic disqualification
  7. Do not write anything on the questions paper.
- 
-

### QUESTION 1

A and B are mutually exclusive events such that,  $\Pr(A \cup B) = 0.8$  and  $\Pr(A \cap B^c) = 0.5$

Find the (i)  $\Pr(A \cap B)$  (ii)  $\Pr(B)$  (iii)  $\Pr(A^c \cap B^c)$  (iv)  $\Pr(A^c \cup B)$  [03 marks @]

(b) Given two events A and B, explain what is meant by the following terms

(i) A and B are independent events

(ii) A and B are mutually exclusive events [02 Marks@]

© A loaded die with even numbers twice as likely to occur as odd numbers is tossed.

What is the probability of?

(i) Getting a 5? [04 marks]

(ii) Getting a 5 given that the number greater than 3 has occurred? [05 marks]

### QUESTION 2

A discrete random variable has a probability distribution

X	1	2	3	4
$\Pr(X=x)$	$1/2$	k	$1/8$	$1/8$

Find. (i) k (ii)  $E(X)$  (iii)  $\text{Var}(X)$  (iv)  $E(6X+2)$  (v)  $\text{Var}(3X-9)$  [03 marks @]

(b) A discrete random variable has a probability distribution

$P(X=x) = a/x$  for  $x=1, 2, 3, 4$  and 0 otherwise

Determine (i) value of a [02 marks]

(ii)  $\Pr(X \leq a)$  [02 marks]

(iii)  $E(X)$  [02 marks]

(iv) The variance and standard deviation of X [04 marks]

### QUESTION 3

(a) (i) Define decision tree [1 marks]

(ii) State the uses of decision tree [4 marks]

(b) Should we develop a new product or consolidate and why? [10 marks]



Stage I	Stage II	Stage III
New product	Through development (cost = \$150,000)	Good (pr = 0.4, \$ 1,000,000)
		Moderate (pr= 0.4, \$ 50,000)
		Poor (pr = 0.2,\$2000)
	Rapid development (cost= \$80,000)	Good (p = 0.1,\$1,000,000)
		Moderate (pr = 0.2,\$50,000)
		Poor (pr = 0.7, \$ 2000)
Consolidate	Strengthen the product (cost = \$30,000)	Good (pr = 0.3, \$ 400,000)
		Moderate (pr= 0.4, \$ 20,000)
		Poor (pr = 0.3,\$6000)
	Reap products (cost = \$0)	Good (pr = 0.6, \$ 20,000)
		Poor (pr = 0.4,\$2000)

© The NIC insurance company insures 100,000 cars. Their records indicate that during a year they will pay out the following for accidents;

X in \$ (Dollars)	100,000	50,000	25,000	5,000	1,000
Pr(X= x)	0.0001	0.001	0.002	0.008	0.02

What amount of money would the company expect to pay per car for accidents?

[10 marks]

#### QUESTION 4

- Briefly define the following
  - Sample space[2 marks]
  - Independent Events[2 marks]
  - Mutually exclusive[2 marks]
  - Conditional probability[2 marks]
- State De Morgan's laws of probability[2 marks]
- A and B are events associated with an experiment such that  $P(B)=1/6$ ,  
 $P(B/A)=1/3$ ,  
 $P(A \cap B)=1/12$ . Find:

- i.  $P(A)$  [3 marks]
- ii.  $P(A/B)$  [3 marks]
- iii.  $P(A/B')$  [5 marks]
- iv. State whether events  $A$  and  $B$  are independent or mutually exclusive [4 marks]

### QUESTION 5

- a) Define the terms:
  - i. Random Variables [2 marks]
  - ii. Continuous random variable [3 marks]
- b) A box contains 9 blue balls and 5 green balls. 2 balls are picked from the box one after the other. Find the probability of picking one blue and one green in any order given:
  - i. The first ball is not replaced [4 marks]
  - ii. The first ball is replaced [4 marks]
- c) A continuous random variable has probability density function as
 
$$f(x) = \begin{cases} kx^3 & 0 \leq x \leq 4 \\ 0 & \text{Elsewhere} \end{cases}$$
  - i. Find the value of  $k$  [4 marks]
  - ii. Calculate the  $E(X)$  and  $\text{Var}(X)$  [8 marks]

### QUESTION 6

- (a) (i) Define probability of an event [2marks]
- (ii) Three men  $P$ ,  $Q$  and  $R$  pack biscuits in a factory. From the batch allocated to them,  $P$  packs 55%,  $Q$  (30%) and  $R$  (15%). The probability that  $P$  breaks some biscuits in a packet is 0.7 and the respective probabilities for  $Q$  and  $R$ , are 0.2 and 0.1 respectively. What is the probability that a packet with broken biscuits found by the checker was packed by  $P$ ? [6marks]

(b) A business firm from Masaka District saved money for investment. The management wanted to start two projects (I & II). Below are expected sales from the projects.

PROJECT I		PROJECT II	
SALES (\$)	Probability	SALES (\$)	Probability
8,000	0.25	10,000	0.15
8,500	0.60	9,000	0.60
9,500	0.15	5,500	0.25

The firm's profit is 85% of the sales.

(i) Calculate the expected profit under each project. **[4 marks]**

(ii) Which project would you recommend to the management for the better returns? **[2 Marks]**

(iii) Calculate the standard deviation of the distribution of the profits for each project. **[4 Marks]**

(iv) What is the coefficient of variation for each project? **[4 Marks]**

(v) As a risk expert, which project would you recommend to the management? **[3 marks]**

**END**