

UGANDA MARTYRS UNIVERSITY-NKOZI

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

ECONOMICS DEPARTMENT

END OF SEMESTER ONE FINAL ASSESSMENT 2014/15

ECO 2102: MATHEMATICS FOR ECONOMISTS

DATE: 5TH DECEMBER 2014

TIME: 2: 00- 5:00 PM

DURATION: 3HRS

Instructions:

1. Carefully read through ALL the questions before attempting
 2. **ANSWER FOUR (4) Questions ONLY.** (Each question carries equal marks)
 3. No **names** should be written anywhere on the examination booklet.
 4. Ensure that your **ID number** is indicated on all pages of the examination answer booklet.
 5. Ensure your work is **clear and readable**. Untidy work shall be penalized
 6. Any type of examination Malpractice will lead to automatic disqualification
 7. Do not write anything on the questions paper.
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Question One

- (a) What is meant by static analysis? (02marks)
- (b) Consider the following three commodity market model described by the equations below obtained using the equilibrium market conditions.

$$P_1 + 2P_2 + 3P_3 = 3$$

$$2P_1 + 4P_2 + 5P_3 = 4$$

$$3P_1 + 5P_2 + 6P_3 = 8$$

Given that the unknowns are the equilibrium prices in each commodity market, determine their values using matrix inversion (10marks)

- (c) The demand and supply functions for a two commodity market model are as follows

$$Q_{d1} = 410 - 5P_1 - 2P_2 \quad Q_{d2} = 295 - P_1 - 3P_2$$

$$Q_{s1} = -60 + 3P_1 \quad Q_{s2} = -120 + 2P_2$$

- (i) State the consumption relationship between the two commodities. (Give a reason for your answer) (03marks)
- (ii) Determine the equilibrium prices (\bar{P}_1 and \bar{P}_2) using crammers rule and the respective equilibrium quantities (\bar{Q}_1 and \bar{Q}_2) in the market (10marks)

Question Two

- (a) A firm's production function is given by $Q = 15KL - 4K^2 - 5L^2 + 6K + 4L$. Find the marginal products of capital (K) and labour (L) when $K = 1$ and $L = 2$ (6marks)
- (b) A whole seller at Nkozi trading center is involved in selling two types of products; Posho (X) and Beans (Y) measured in kilograms. The Profit function of the whole seller is given by $\pi = 30X - 3X^2 - 2XY + 20Y - 2Y^2 - 10$. As an expert in mathematics for economists, advise the whole seller on the quantities of Posho and Beans that must be sold in order to maximize profits. Hence determine His maximum profits (08marks)
- (c) A laundry firm is involved in the production of two products; Bars of soap (X) and Tins of omo (Y). The joint cost function of the firm is in such a way that $C = 45X^2 + 90XY + 90Y^2$. Given that the firm has to meet the production quota $2X + 3Y = 60$, determine the optimal number of Bars of soap and Tins of omo that can be produced in order to minimize the production costs. Hence show that the second order condition for cost minimisation is satisfied (11 marks)

Question Three

- (a) At an output level of 10 units, the firm has total costs of 330 units and fixed costs of 30 units. If the firm has a quadratic total cost function of the form $TC = \alpha + \beta Q^2$, determine the level of marginal cost of the firm at $Q = 10$ units (06marks)
- (b) The total cost function of the firm is given by $TC = -q^3 + \frac{9}{2}q^2 - 6q + 6$. Find the level of output that minimizes the total costs of production and show that the sufficient condition for cost minimization is satisfied. Hence determine the minimum costs of production (08marks)
- (c) Uganda Martyrs University owns a dairy farm along Kayabwe Nkozi road. The revenue and cost functions associated with producing and selling milk are given by $TC = \frac{1}{3}Q^3 - 10Q^2 + 600Q$ and $TR = 50Q - 5Q^2$ respectively, where Q is the quantity of milk in litres.
- (i) As a specialist in mathematics for economists, advise the university farm management on the quantity of milk that should be produced in order to maximize profits (06marks)
- (ii) Show that the second order condition for profit maximization is satisfied; hence determine the maximum profits the university can obtain from milk production (05marks)

Question Four

- (a) Ann's utility function is $U(X, Y) = 3\ln X + 2\ln Y$ where X and Y are weekly consumption levels of goods X and Y . The market prices are $P_X = 2$ euros and $P_Y = 1$ euro, and her weekly budget is $B = 100$ euros. Find the quantities that Ann should buy each week in order to maximize her utility. (08marks)
- (b) Suppose the firm wishes to produce 500 units of output per hour at the minimum cost. Machines cost \$40 per hour and Workers cost \$10 per hour. The firm's production function is $Q = X^{\frac{1}{3}}Y^{\frac{2}{3}}$ where Q denotes the quantity of output per hour, X is the number of machines and Y is the number of workers
- (i) Find the formula for the firm's marginal rate of substitution in production of workers for machines. (04marks)
- (ii) Compute the slope of the firm's Isocost line, with machines on the vertical axis and workers on the horizontal axis (03marks)
- (iii) Using (i) and (ii) above, obtain the equation of the expansion path of the firm (03marks)
- (iv) Find the number of machines and workers required to produce 500 units of output per hour at the minimum cost (05marks)
- (v) Compute the total cost of producing 500 units of output per hour. (2marks)

Question Five

A certain Telecom company can separate its customers into three distinct markets with the following demand functions; Weekdays: $P_1 = 63 - 4Q_1$, Weekends: $P_2 = 105 - 5Q_2$ and Nights: $P_3 = 75 - 6Q_3$

The cost function of the company is given by $C = 20 + 15Q + Q^2$ where $Q = Q_1 + Q_2 + Q_3$

- (a) Compute the profit maximizing levels of output Q_1 , Q_2 and Q_3 as well as their prices under the price discrimination regime [13marks]
- (b) Compute the elasticities in each sub-market and comment on the prices charged visa-vis the elasticities (06marks)
- (c) Suppose the Telecom company does not discriminate, compute the price charged and quantity sold in the market (06marks)

Question Six

- (a) Evaluate the following integrals

(i) $\int_0^2 \frac{3x^2}{(x^3 + 1)^2} dx$ (06marks)

(ii) $\int xe^{2x} dx$ (04marks)

- (b) Given that the marginal revenue of the firm is given by $MR = 3Q^2 + Q - 1$. Find the increase in total revenue (TR) when output increases from 40 units to 50 units (05marks)
- (c) Consider the following demand and supply functions in a competitive market
 $P_d = 25 - Q_d^2$ (Demand function); $P_s = 2Q + 1$ (Supply function)

Find the consumers' and producers' surplus at equilibrium market conditions. (10marks)

END (GOOD LUCK)