

UGANDA MARTYRS UNIVERSITY NKOZI & MASAKA

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL SEMESTER ASSESSMENT
SEMESTER I

FIRST YEAR EXAMINATIONS FOR BSc, BSc Econ & Stat, BSc (Ed)

MAT 1101: Calculus I

DATE: Monday 11th December, 2023

DURATION: 3 Hours

Instructions:

1. *Carefully read through ALL the questions before attempting*
2. *ANSWER ANY FIVE Questions (Each question is 20 marks)*
3. *No names should be written anywhere on the examination book.*
4. *Ensure that your Reg number is indicated on all pages of the examination answer booklet.*
5. *Ensure your work is clear and readable. Untidy work shall be penalized*
6. *Any type of examination Malpractice will lead to automatic disqualification*
7. *Do not write anything on the questions paper.*



1.
 - a. Using relevant examples, give the difference between an *odd function* and an *even function*. 4 marks
 - b. Give an example of a function that is neither odd nor even. 1 mark
 - c. Classify the following functions as odd, even or neither
 - i. $f(x) = 2$ 2 marks
 - ii. $f(x) = \frac{g(x)}{h(x)}$ where g is even, h is odd and $h(x) \neq 0$ for all x . 4 marks
 - d. Functions g and h are such that g has a domain $\{-1, 0, 1, 2, 3\}$, $g(x) = x^2$ and $h(x) = 1 - 2x$. Find the:
 - i. range of g 3 marks
 - ii. domain and range of $h \circ g$. 6 marks
2.
 - a. What is meant by *continuity of a function on an interval*? 2 marks
 - b. Give an example of a function that is continuous. 1 mark
 - c. Determine the values of the parameters k and m for which the function h defined below is differentiable at the point $x = 3$.

$$h(x) = \begin{cases} kx + 1, & x \leq 3 \\ 10 - mx^2, & x > 3 \end{cases}$$

4 marks

- d. Given the function g , where $g(x) = x(x^2 + 2x - 1) - 2$. Find:
 - i. the domain of g 1 mark
 - ii. the points where the graph of g crosses the axes. 4 marks
 - iii. the coordinates $(x, g(x))$ where g' and g'' are zero. 6 marks
 - iv. $\lim_{x \rightarrow -\infty} g(x)$ and $\lim_{x \rightarrow \infty} g(x)$. 2 marks
3.
 - a. Use the $\epsilon - \delta$ definition of a limit to prove that $\lim_{x \rightarrow 1} -x + 2 = 1$. 3 marks
 - b. Given the function

$$g(x) = \begin{cases} 1 - x, & x < 1 \\ -2 + 2x, & x \geq 1. \end{cases}$$

Compute the limits:

- i. $\lim_{x \rightarrow 1^+} g(x)$ 2 marks
 - ii. $\lim_{x \rightarrow 1^-} g(x)$ 2 marks
- Hence state $\lim_{x \rightarrow 1} g(x)$ 1 mark

- c. Give an example of a function g and a point c such that the limit of g does not exist at c . 2 marks
- d. Find the following limits:
- i. $\lim_{x \rightarrow -1} x^4 - 2x$ 2 marks
 - ii. $\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{x - 4}$ 3 marks
 - iii. $\lim_{x \rightarrow 1} g(x)$ over an interval where $3 - x^2 - x \leq g(x) \leq x^2$ 3 marks
- e. Explain one application of limits in real life situations. 2 marks
4. a. Using relevant examples, differentiate between a *relation* and a *function*. 3 marks
- b. A function h is given by $h(x) = \frac{2x - 1}{x - 1}$. Find the inverse of h . 2 marks
- c. Solve for x and represent the solution on a number line.
- i. $x - x^2 < 0$ 3 marks
 - ii. $-2 < 4(1 - x) < 5$ 3 marks
- d. Show that
- i. the product of two odd functions is even. 3 marks
 - ii. $f(x) = \frac{3x}{2x - 1}$ is a one to one function. 3 marks
- e. If $y = f(x) = \frac{3 + 5x}{4x - 5}$, show that $x = f(y)$ 3 marks
5. a. Find the domain of each of the following functions.
- i. $g(x) = \sqrt{2 - x}$. 2 marks
 - ii. $f(x) = \frac{2x}{x^2 - 4}$. 3 marks
- b. A function f defined on \mathbb{R} is such that $f(x) = 2x$. Find a formula for:
- i. $f^2 = f \circ f$ 2 marks
 - ii. $\frac{d}{dx} f^2(x)$ 1 mark
 - iii. $f^3 = f \circ f \circ f$ 2 marks
 - iv. $\frac{d}{dx} f^3(x)$ 1 mark
- Hence state the formula for f^n and $\frac{d}{dx} f^n(x)$, where n is a positive integer. 3 marks
- c. Using relevant examples, distinguish between an *increasing function* and a *decreasing function*. 4 marks

d. Sketch the graph of the function $g(x) = |5x + 1|$.

2 marks

6. a. i. Give the limit definition of a derivative, f' , of a function f at a point x .
1 mark

ii. Use the definition in part a.i. above to show that

$$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}.$$

4 marks

b. Compute the derivatives of the following functions:

i. $f(x) = \frac{x^3 + 3x^2}{3 - x}$

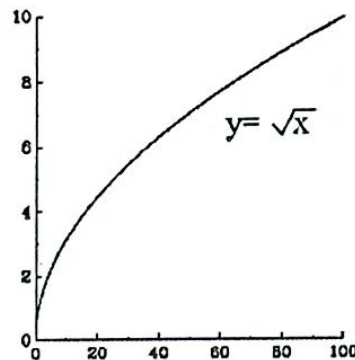
4 marks

ii. $g(x) = x^4(2 + x)^2$

4 marks

c. Find the equation of the tangent line to the curve $h(x) = x^3$ at the point $x = 1$.
3 marks

d. The graph of function f , where $y = f(X) = \sqrt{X}$ is shown below.



On the same axes sketch the graphs of f and $g(X) = 1 - \sqrt{X - 2}$. 4 marks

END