

UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL EXAMS FOR BSc. Economics and Statistics YEAR 1 [JAN. INTAKE]

SEMESTER I, 2022/2023

STA 1202: MATHEMATICAL STATISTICS 1

DATE: THURSDAY <sup>12<sup>th</sup></sup> ~~15<sup>th</sup>~~, DECEMBER, 2022

TIME: 9:30 am – 12:30 pm

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**Instructions**

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1. Attempt ANY FOUR questions
2. Ensure that your name and registration number is indicated on the cover page of your work.
3. Where applicable, leave your answer in fractional form or round it to 2dp
4. Only Non-Programmable calculators are allowed

## Question 2

(a) A discrete random variable  $X$  has the probability mass function given by:-

$$P(X = x) = \begin{cases} mx, & x = 2, 4, 6 \\ m(x - 2), & x = 7, 8 \\ 0, & \text{Otherwise} \end{cases}$$

Determine the:-

- (i) Value of  $m$  [2 marks]
- (ii) Standard deviation of  $X$  [3 marks]
- (iii) Median of  $X$  [3 marks]
- (iv) Mode of  $X$  [3 marks]

(b) A continuous random variable  $Y$  is described by the following probability density function:-

$$P(X = x) = \begin{cases} k_1 x; & 1 \leq y \leq 3 \\ k_2 (4 - y); & 3 \leq y \leq 4 \\ 0, & \text{Otherwise} \end{cases}$$

- (i) Find values of  $k_1$  and  $k_2$  ..... [3 marks]
- (iii) Find  $E(3Y)$  ..... [3 marks]
- (iv) Find  $F(Y)$  ..... [5 marks]
- (iv) Find median of  $Y$  ..... [3 marks]

#### Question 4

(a) The probability of having a girl in a family of 6 children is 0.75. Let the random variable  $X$  be the number of boys in such a family. Required:-

(i) Determine the probability distribution table for the number of boys [4 marks]

(ii) Determine the probability that at least three boys are in the family [3 marks]

(iii) Determine the standard deviation of boys [2 marks]

(b) A bicycle dealer imports 35% and 65% of spare parts from India and China respectively. The percentages of parts produced defective in the countries are 0.7% and 0.6% respectively. A spare part is drawn at random from a sample of spare parts imported from India and China. Required:-

(i) Construct a probability tree diagram for the problem [4 marks]

(ii) Determine the probability that it is defective [3 marks]

(ii) Determine the probability that it is defective and from china [3 marks]

(c) Two tetrahedrons are biased in a way that the chance of getting any one face is inversely proportional to the square of the number on it. If the two tetrahedrons are tossed, determine the probability that the faces show the same number [6 marks]

### Question 6

- (a) Define Baye's theorem and give one practical application [3 marks]
- (b) Let  $E_i$  be a partition of the sample space  $S$ . Let  $A$  be an event that occurred. Derive Baye's theorem general formula [7 marks]
- (c) Container 1 has 6 green and 4 black balls; container 2 has 4 green and 6 black balls and container 3 has 5 green and 5 black balls. A container is randomly picked and a ball drawn from it. If the ball drawn was: -
- (i) Green, find the probability that it was from container 1 [5 marks]
- (i) Black, find the probability that it was from container 2 [5 marks]
- (i) Green, find the probability that it was from container 3 [5 marks]

**END**



### Question 1

(a) A company is working on three independent projects P, Q and R. There is a 70% chance of finishing project P on time, 75% chance of finishing project Q and a 60% chance of finishing project R on time.

(i) Draw a probability tree diagram for the above projects [4 marks]

(ii) Determine  $P(\text{All the projects will be finished in time})$  [3 marks]

(ii) Determine  $P(\text{None of the projects will be finished in time})$  [3 marks]

(iii) Determine  $P(\text{Exactly one project will be finished in time})$  [4 marks]

(iv) Determine  $P(\text{At least one project will be finished in time})$  [4 marks]

(b) There are 3 black and 2 white balls in each of the two bags. A ball is taken from the first bag and put in the second bag, and then a ball is taken from the second bag and put in the first bag. Determine the probability that there are now the same number of black and white balls in each bag as there were to begin with [7 marks]

### Question 3

(a) A total of seven hundred students sat for English language exams for which the pass mark was 60. The marks were normally distributed. 30 students scored below 50 marks while 45 scored above 70 marks.

(i) Determine the mean mark and standard deviation of the students [4 marks]

(ii) Find the percentage of students who passed the examination [4 marks]

(iii) If the pass mark is lowered by 5 marks, how many more students will pass? [4 marks]

(b) The packets of OMO sold in a shop are of four categories, namely; small (S), medium (M), large (L) and giant (G). On a particular day, the stock is such that the ratio of S:M:L:G = 4:2:1:1. The costs of the packets are in the ratio S:M:L:G = 35:500:800:1,400 respectively. 30 packets are sold randomly on a particular day; the total cost of the sales being Y shillings. Determine:-

(i) The expected value of Y [4 marks]

(ii) The standard deviation of Y [5 marks]

(iii) 10 packets are picked at random. Find  $P(6 \text{ are medium size packets})$  [4 marks]



### Question 5

(a) For two events  $A$  and  $B$ ,  $P(A) = 0.26$ ,  $P(B) = 0.37$ , and  $P(A \cap B) = 0.11$ . Determine:-

(i)  $P(A|B)$  [2 marks]

(ii)  $P(B|A)$  [2 marks]

(iii) Whether or not  $A$  and  $B$  are independent [2 marks]

(b) Suppose that in an adult population the proportion of people who are both overweight and suffer hypertension is 0.09; those who are not overweight but suffer hypertension is 0.11; those who are overweight but do not suffer hypertension is 0.02 and those who are neither overweight nor suffer hypertension is 0.78. An adult is randomly selected from this population. Find the probability that the person selected suffers hypertension given that:-

(i) He is overweight [3 marks]

(ii) He is not overweight [3 marks]

(iii) Using the results find if overweight people tend to suffer from hypertension [2 marks]

(c) Under what circumstance can one use a normal distribution to approximate to binomial distribution? [2 marks]

(d) In a farm, there is a 53% chance that an animal is exposed to tick diseases. If a simple random sample of 300 animals is surveyed. Find the probability that:-

(i) Exactly 175 will be exposed to tick diseases [3 marks]

(ii) At least 150 will be exposed to tick diseases [3 marks]

(iii) More than 155 will be exposed to tick diseases [3 marks]