

# UGANDA MARTYRS UNIVERSITY

## FACULTY OF SCIENCE

### DEPARTMENT OF MATHEMATICS

#### DIFFERENTIAL EQUATIONS I : FINAL ASSESSMENT

YEAR 2015/2016: First Year (BSc GEN, ECON & FM)

Date: May 11, 2016

Time: 9:30 AM - 12:30 PM

#### INSTRUCTIONS

- (i) Attempt ALL questions
- (ii) Read through the paper carefully and follow the instructions on each respective questions.
- (iii) Calculators and mathematical tables may be used.

1. (a) [2 marks] Define a differential equation.  
(b) [2 marks] What is meant by the order of a differential equation.
2. [7 marks] Chose (a) OR (b). Use the Wronskian to prove that the given functions are linearly independent on the real line  
(a)  $f(x) = e^x$ ,  $g(x) = e^{2x}$ ,  $h(x) = e^{3x}$   
(b)  $f(x) = e^{2x}$ ,  $g(x) = \cos 2x$ ,  $h(x) = \sin 2x$
3. [8 marks] Choose (a) OR (b). Verify by substitution that each of the given functions is a solution of the given differential equation  
(a)  $y'' + 4y' + 4y = 0$ ;  $y_1 = e^{-2x}$ ,  $y_2 = xe^{-2x}$   
(b)  $y'' - 2y' + 2y = 0$ ;  $y_1 = e^x \cos x$ ,  $y_2 = e^x \sin x$
4. [8 marks] Choose (a) OR (b). Verify that  $y(x)$  is a solution of the given differential equation. Then determine a value of the constant  $C$  so that  $y(x)$  satisfies the initial condition.  
(a)  $xy' + 3y = 2x^5$ ;  $y(x) = \frac{1}{4}x^5 + Cx^{-3}$ ;  $y(2) = 1$   
(b)  $xy' - 3y = x^3$ ;  $x^3(C + \ln x)$ ;  $y(1) = 17$

5. [12 marks] Choose (a) OR (b). Verify that  $y_1$  and  $y_2$  are solutions of the given differential equation, then find a particular solution of the form  $y = c_1 y_1 + c_2 y_2$  that satisfies the given initial solution
- (a)  $y'' - 3y' + 2y = 0$ ;  $y_1 = e^x$ ,  $y_2 = e^{2x}$ ;  $y(0) = 1$ ,  $y'(0) = 0$
- (b)  $y'' + y' - 6y = 0$ ;  $y_1 = e^{2x}$ ,  $y_2 = e^{-3x}$ ;  $y(0) = 7$ ,  $y'(0) = -1$
6. [9 marks] Choose (a) OR (b). Verify that the given differential equation is exact and then solve it.
- (a)  $(4x - y)dx + (6y - x)dy = 0$
- (b)  $(3x^2 + 2y^2)dx + (4xy + 6y^2)dy = 0$
7. [6 marks] Choose (a) OR (b). Find the general solution of the differential equation.
- (a)  $y' + 2xy = 0$
- (b)  $(1 + x)y' = 4y$
8. [8 marks] Choose (a) OR (b). Find the explicit particular solution of the initial value problem
- (a)  $2y \frac{dy}{dx} = x(x^2 - 16)^{-1/2}$ ;  $y(5) = 2$
- (b)  $x \frac{dy}{dx} - y = 2x^2 y$ ;  $y(1) = 1$
9. [13 marks] Choose (a) OR (b). A third order homogeneous linear differential equation and three linearly independent solutions are given. Find a particular solution satisfying the given initial value problem
- (a)  $y''' - 3y'' + 3y' - y = 0$ ;  $y(0) = 2$ ,  $y'(0) = 0$ ,  $y''(0) = 0$ ;  $y_1 = e^x$ ,  $y_2 = xe^x$ ,  $y_3 = xe^{2x}$
- (b)  $y''' - 3y'' + 4y' - 2y = 0$ ;  $y(0) = 1$ ,  $y'(0) = 0$ ,  $y''(0) = 0$ ;  
 $y_1 = e^x$ ,  $y_2 = e^x \cos x$ ,  $y_3 = e^x \sin x$
10. [8 marks] Choose (a) OR (b) Find the explicit particular solution of the initial value problem
- (a)  $xy' + y = 3xy$ ;  $y(1) = 1$
- (b)  $xy' - 3y = x^3$ ;  $y(1) = 10$
11. [9 marks] Choose (a) OR (b). Find the general solution of the given differential equation
- (a)  $yy' + x = (x^2 + y^2)^{1/2}$
- (b)  $xy' = y + (x^2 + y^2)^{1/2}$
12. [8 marks] Choose (a) OR (b). Find the general solution of the given differential equations
- (a)  $2xy' - 3y = 9x^3$
- (b)  $3xy' + y = 12x$