UGANDA MARTYRS UNIVERSITY

NKOZI CAMPUS

UNIVERSITY EXAMINATIONS

FACULTY OF BUSINESS ADMINISTRATION AND MANAGEMENT

END OF SEMESTER TWO FINAL ASSESSMENT YEAR ONE

QUANTITATIVE METHODS II/TWO

DATE OF EXAMINATION: 19/07/2022

TIME ALLOCATED: 3 HOURS

INSTRUCTIONS

1. Carefully read through ALL the questions before attempting
2. **SECTION A IS COMPLUSORY AND ANSWER ANY TWO/2 QUESTIONS**
3. Ensure that your **Registration number** is indicated on all pages of your answer sheets.
4. All questions require clear workings and all numbers carry equal marks

SECTION A
QUESTION ONE

[30 marks]

The following game between the EU and UK begins in 2015 when the EU had to decide whether to change the terms of the UK's membership. If the EU says Yes to altered membership for the UK, the game ends and both players get a pay-off of 0. If the EU say No, the UK must decide whether to Remain or Leave. If the UK Remains, the game ends and the UK gets a pay-off of -5 while the EU gets a pay-off of 5. If the UK chooses Leave, the EU must choose either a Weak or Tough negotiating position. After this choice, the UK must decide whether to pursue a Hard or Soft Brexit. If the EU chooses Weak and the UK chooses Soft, the EU gets a pay-off of -10 while the UK gets a pay-off of 5. Any other combination of Weak/Tough and Hard/Soft will lead to a breakdown in negotiations with the EU getting a pay-off of -40 and the UK getting a pay-off of -80.

(i) (a) Draw the game tree of this game. **[3 Marks]**

(b) Represent this as a normal form game (i.e. a matrix) and find all the pure-strategy Nash equilibria. **[4 Marks]**

(c) Find all the Sub game Perfect Nash Equilibria in this game. How many are there? Why should we expect this to be the number of SPNE? **[4 Marks]**

(d) Is it a credible strategy for the EU to play Tough? Is it a credible strategy for the UK to play Hard? Discuss why or why not in each case. **[4 Marks]**

(ii) Suppose instead that if the UK chooses Leave then both players must choose their next moves simultaneously

(a) Draw the game tree of this new version of the game. **[3 Marks]**

(b) Represent this new game as a normal form game (i.e.a matrix) and find all the pure-strategy Nash equilibria. **[3 Marks]**

(c) Find all the Sub game Perfect Nash Equilibria in this new game. How many are there? Why should we expect this to be the number of SPNE? **[4 Marks]**

(iii) Suppose that a new report comes out showing that the pay-off for the UK to a from a Soft Brexit when the EU choose Weak is not 5, but rather -50. How does this change your answer to ii (c). **[5 Marks]**

QUESTION TWO

[30 marks]

For a small batch computing system the processing time per job is exponentially distributed with an average time of 3 minutes. Jobs arrive randomly at an average rate of one job every 4 minutes and are processed on a first-come-first-served basis. The manager of the installation has the following concerns.

(a) What is the probability that an arriving job will require more than 20 minutes to be processed (the job turn-around time exceeds 20 minutes)? **[6 Marks]**

(b) A queue of jobs waiting to be processed will form, occasionally. What is the average number of jobs waiting in this queue? **[8 Marks]**

marks]

(b). A company sells its product at $Shs.60$ per unit. Fixed cost for the company is $Shs.18000$ and the variable cost is estimated to be 25% of total revenue.

Determine:

(i) the total revenue function
(ii) the total cost function
(iii) the break even point.

Software Development Corporation (SDC) has developed a new encryption software to facilitate secure commercial transactions over the Internet. The feasibility of the product has been proven, but each sale will require significant customer support. SDC must make a decision regarding the level of sales and development resources that must be allocated to this product next year. The least expensive decision alternative (d1) is to start selling the new product through existing sales channels and provide customer support as needed. The next alternative (d2) is to assign one full-time sales person and one software specialist to focus on this product. The third alternative (d3) is to have a team of six people dedicated to this product. Finally, a complete division (d4) consisting of about twelve people may be created to fully automate the product and engage in an extensive marketing campaign. The potential profit from each decision alternative depends on the market acceptance or demand for this product which may be high, moderate or low. If market acceptance is high, each of the four decision alternatives, d1 through d4 will yield a profit of -200, 0, 300, and 900 thousand dollars respectively. If there is a moderate demand, the profits are likely to be 100, 100, 200, and -200 thousand dollars respectively. If the demand turns out to be low, then the profits will be 200, 150, -200, and -500 thousand dollars respectively. The industry experience with such products provides a probability estimate of demand to be high, moderate and low as .3, .5, and .2 respectively. Which of the four decision alternatives should be selected by SDC? What will be the expected profit from this decision? If a market research firm can provide perfect information about demand to SDC (i.e. whether it will be high, moderate, or low) before a product launching decision is made, how much is that information worth to SDC?

FORMULAE LIST

Portfolio Analysis Equations

1. Expected Portfolio Return $= \bar{r_p} = \sum_{i=1}^{n} x_i \bar{r_i}$

2. Covariance of two securities $= CoV_{xy} = \dfrac{\sum_{i=1}^{N} [R_x - \bar{R_x}][R_y - \bar{R_y}]}{N}$

3. Coefficient of correlation $= r_{xy} = \dfrac{CoV_{xy}}{\sigma_x \sigma_y}$

4. Variance of a portfolio $= \sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2(r_{12}\sigma_1\sigma_2)$

5. $\sigma_p^2 + (x_1\sigma_2 + x_2\sigma_2)^2$

6. Standard Deviation; $\sigma_p = \sqrt{x_1^2\sigma_1^2 + x_2^2\sigma_2^2}$

7. Portfolio Variance with more than one security; $\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j r_{ij} \sigma_i \sigma_j$

Transportation Equations

Total supply = Total demand

Total supply greater than Total demand

OR
Total Demand greater than Total Supply

Queuing Equations

Poisson's Formulae is as follows:

$P(n,t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$

1. Expected number of customers in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{p}{1-p}$$

2. Expected number of customers in the queue,

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{p^2}{1-p}$$

3. Average waiting time in the system,

$$W_s = \frac{1}{\mu - \lambda}$$

4. Average waiting time in the queue

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

5. Average waiting time for a customer

$$W(w/w > 0) = \frac{1}{\mu(1-p)} \text{ or } \frac{1}{\mu - \lambda}$$

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5. Average waiting time for a customer

$$W(w/w > 0) = \frac{1}{\mu(1-p)} \text{ or } \frac{1}{\mu - \lambda}$$

6. Expected length of non-empty queue

$$L(m/m>0) = \frac{\mu}{\mu - \lambda}$$

7. Probability that there are n customers in the system

$$P_n = \frac{\lambda}{\mu}^n$$

$$P_o = \frac{\lambda}{\mu}^n [1 - \frac{\lambda}{\mu}]$$

8. Probability that there is nobody in the system,

$$P_o = \frac{1 - \lambda}{\mu}$$

9. Probability that there is at least one customer or queue is busy

$$P_b = 1 - P_o$$

10. Traffic intensity

$$p = \frac{\lambda}{\mu}$$

GAME THEORY

$$p_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$p_2 = 1 - p_1$$

$$q_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$q_2 = 1 - q_1$$

Value of the game, $v = \frac{a_{11}a_{22} - a_{12} \cdot a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

GOOD LUCK