UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

END OF SEMESTER 1, 2021/22 FINAL ASSESSMENT

BSc. Gen I, BSc.EDUC I, BSc. IT I & BSc.FM I, BSc. Econ & Stat I

Elements of Mathematics

MTC 1102

DATE : 09th March 2022

TIME : 9:30 AM - 12:30 PM

DURATION: 3 Hrs

Instructions

1. *Carefully read through ALL the questions before attempting.*
2. *ANSWER FOUR (4)Questions (All questions carry equal marks).*
3. *Ensure that your* ***Reg. number Name*** *and* ***Course*** *are indicated on all pages of your work.*
4. *Ensure that your work is* ***clear*** *and* ***readable****. Untidy work will be penalized.*
5. *Any type of examination Malpractice will lead to automatic disqualification.*

---

1. (a) Give the meaning of each of the following statements combined using given logical signs;

(i) $A \leftrightarrow B$ [2 Marks]

(ii) $A \vee B$. [2 Marks]

(iii) $A \rightarrow B$. 2 Marks]

(b) Explain, with the help of an example, the term "a formula F in proposition logic is":

(i) a tautology [3 Marks]

(ii) a contradiction. [3 Marks]

(iii) contingent. [3 Marks]

(c) Construct truth tables for

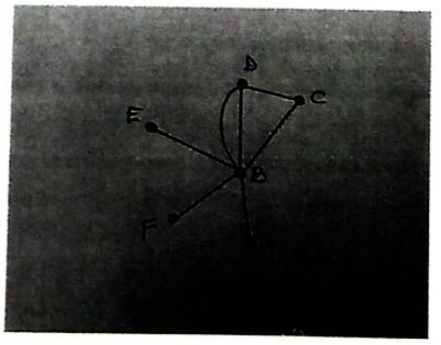
(i) $(\neg A \rightarrow \neg B) \rightarrow ((\neg A \rightarrow B) \rightarrow A)$. [3 Marks]

(ii) $(A \rightarrow B) \leftrightarrow (C \vee A)$. [5 Marks]

(iii) $A \rightarrow \neg B$. [2 Marks]

2. (a) With the help of a diagram, explain the difference between a *directed graph* and an *undirected graph.* [5 Marks]

(b) The figure below represents a graph G with vertices A, B, C, D, E, F;



(i) Find the cardinality of graph G above. [2 Marks]

(ii) Find the degree of vertex B. [2 Marks]

(iii) List the pending vertices of G. [3 Marks]

(iv) Is the graph above simple or multi? Give a reason for your answer. [3 Marks]

(v) List the isolated vertices of G. [2 Marks]

(vi) By means of a diagram, draw one example of a multi graph and one example of a simple graph other than G above. [3 Marks]

(c) (i) When do we say that a graph has a *circuit*? [3 Marks]

(ii) Give the difference between *indegree* and *outdegree* of a graph. [2 Marks]

3. (a) (i) Define a *relation* on a set. [1 Mark]

(ii) Differentiate between a *many to many* and a *one to many* relation with the help of an arrow diagram [4 Marks]

(iii) Given a set $A = \{1, 2, 3, 4, 5\}$. Compute the Cartesian product $A \times A$ [3 Marks]

(b) Given the pairs $(1, 1), (1, 2), (2, 1), (1, 1), (2, 2)$. Identify which of the relations below contains each of the given pairs, giving a reason in case.

(i) $R_1 = \{(a, b) : a \leq b\}$. [3 Marks]

(ii) $R_2 = \{(a, b) : a > b\}$. [1 Mark]

(iii) $R_3 = \{(a, b) : a = b\}$. [2 Marks]

(iv) $R_4 = \{(a, b) : a = b + 1\}$. [1 Mark]

(iv) $R_5 = \{(a, b) : a + b \leq 3\}$. [3 Marks]

(c) Given that set $A = \{a, b, c\}$ and set $B = \{1, 2, 3, 4, 5\}$.

(i) $A \times B$. [3 Marks]

(ii) $B \times A$. [2 Marks]

(iii) Comment on the results in c(i) and c(ii). [2 Marks]

4. (a) (i) Define a *set* and describe any two ways of representing elements of a set. [5 Marks]

(ii) Write the set of vowels used in the word *MISSISSIPOMUS*. [1 Mark]

(iii) If set $A = \{r, s, t, u\}$. Find all the proper subsets of A. [7 **Marks**]

(b) There is a group of 80 persons who can drive scooter or car or both. Out of these, 35 can drive scooter and 60 can drive car and 9 can drive neither scooter nor car.

(i) Represent the above data on a venn diagram. [5 **Marks**]

(ii) Find how many persons can drive both scooter and car. [3 **Marks**]

(iii) How many can drive scooter only? [2 **Marks**]

(iv) How many can drive car only? [2 **Marks**]

5. (a) (i) Define a *tree* as applied to graphs. [2 **Marks**]

(ii) Draw two trees, with 6 and 8 vertices respectively. [2 **Marks**]

(iii) Using a diagram, give one difference and one similarity between a *rooted tree* and a *binary tree.* [4 **Marks**]

(iv) Give any three applications of trees in mathematics and in real life. [3 **Marks**]

(b) Giving a precise and illustrative example in each case below, explain the conditions under which a relation said to be:

(i) Symmetric. [3 **Marks**]

(ii) Reflexive. [3 **Marks**]

(iii) Transitive. [3 **Marks**]

(c) Prove by mathematical induction that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n}{6}(n+1)(2n+1); n \in \mathbb{N}$$

[5 **Marks**]

End