

UGANDA MARTYRS UNIVERSITY

UNIVERSITY EXAMINATIONS

FACULTY OF SCIENCE

DEPARTMENT OF NATURAL SCIENCES

FINAL EXAMINATIONS FOR BACHELOR OF SCIENCE ECONOMICS AND
STATISTICS

SEMESTER II, 2022/2023

YEAR I [JANUARY INTAKE]

STA 1202: MATHEMATICAL STATISTICS I

DATE: MAY 23, 2023

TIME: 9:30 AM – 12:30 PM

DURATION: 3 hours

Instructions

1. Carefully read through ALL the questions before attempting
2. Attempt FOUR questions
3. Indicate the numbers you have done on the front page in their order
4. Ensure that ONLY your Registration number is indicated on the front page

Question 1

(a) Explain using relevant examples the practical application of the following concepts:-

(i) Probability tree diagram [3 Marks]

(ii) Baye's theorem [3 Marks]

(b) Box A contains 4 red sweets and 3 green sweets. Box B contains 5 red sweets and 6 green sweets. Box A is twice as likely to be picked as box B. If a box is chosen at random and two sweets are removed from it, one at a time without replacement;

(i) Construct a probability tree diagram for picking the sweets [6 Marks]

(ii) Determine the probability that the two sweets removed are of the same colour [3 Marks]

(iii) Construct a probability distribution table for the number of red sweets removed [3 Marks]

(c) In a factory, there are two machines making bolts. Machine A manufactures 75% of the bolts and the rest are manufactured by machine B. From machine A, 5% of the bolts are defective and from machine B, 2% are defective. A bolt is picked at random. Find the probability that:-

(i) It came from machine A [3 Marks]

(iv) It came from machine B given it is defective [4 Marks]

Question 2

(a) Outline one practical application of a Binomial distribution and a normal distribution

[3 Marks]

(b) A random variable X has a distribution probability function given by:-

$$f(x) = \begin{cases} kx; & 0 \leq x \leq 1; \\ k(4-x^2); & 1 \leq x \leq 2; \\ 0; & \text{Elsewhere} \end{cases}$$

(i) Find the value of the constant k

[3 Marks]

(ii) Determine the variance of the random variable

[4 Marks]

(iii) Find the median value of X

[3 Marks]

(iv) Find the cumulative distribution function of X

[6 Marks]

(c) The number of times a machine breaks down each month is modeled by a DRV X given by:-

$$f(x) = \begin{cases} k \left(\frac{1}{4} \right)^x; & x = 0, 1, 2, 3, 4. \\ 0, & \text{Otherwise} \end{cases}$$

Determine the following:-

(i) Value of k

[2 Marks]

(ii) Probability that the machine will break down more than two times in a month [3 Marks]

(iii) Expected number of times the machine will break down in a year

[3 Marks]

Question 3

(a) Two events M and N are such that; $P(M) = 0.7$, $P(M \cap N) = 0.45$ and $P(\overline{M} \cap \overline{N}) = 0.18$.

(i) Draw a contingency table for events M and N [4 Marks]

(ii) Find $P(\overline{N})$ [3 Marks]

(iii) Find $P(M \text{ or } N \text{ but not both } M \text{ and } N)$ [3 Marks]

(b) Using your own examples, explain the difference between a permutation and a combination

[3 Marks]

(c) Suppose you are a partner in a private equity firm. You want to invest \$5 million in two projects. Instead of equal allocation, you decided to invest \$3 million in the most promising project and \$2 million in the less promising project. After analysis, six projects were shortlisted for potential investment. Determine the number of possible arrangements available for your investment decision

[3 Marks]

(d) Using the knowledge of combinations or permutations to establish the following:-

(i) The number of greetings exchanged in a meeting of 10 staff members if every staff member greets each other once. [3 Marks]

(ii) The number of eight-digit numbers that can be formed with the numbers 2, 2, 2, 3, 3, 3, 4, 4.

[3 Marks]

(iii) The number of ways one can choose four pool balls from five different coloured balls in a pool. [3 Marks]

Question 4

(a) Identify the conditions under which a binomial distribution can be approximated by a normal distribution [2 Marks]

(b) In a school of 800 students, their average weight is 54.8 kg and standard deviation is 6.8 kg. If the weights assume a normal distribution, establish the:-

(i) Probability that a student picked at random weighs less than or equal to 58.2 kg [4 Marks]

(ii) Number of students who weigh more than 75 kg [3 Marks]

(iii) The weight range of the middle 56% of the students of the school [4 Marks]

(c) Twenty percent of the eggs supplied by UMU farm have cracks on them. Determine the probability that in a sample of 900 eggs supplied by UMU firm:-

(i) Exactly 200 eggs will have cracks [4 Marks]

(ii) More than 200 eggs will have cracks [4 Marks]

(iii) Between 200 and 400 eggs will have cracks [4 Marks]

Question 5

(a) Use the binomial formula to evaluate $(x + y)^n$ for $n = 6$

[3 Marks]

(b) The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is modeled by a continuous random variable X as follows:-

$$f(x) = \frac{3}{2}(1 - x^2); \quad 0 \leq x \leq 1$$

$$f(x) = 0; \quad \text{Otherwise}$$

Determine:-

(i) The distribution function of sales

[4 Marks]

(ii) The probability that X lies between 0.25 and 0.75 tons

[3 Marks]

(iii) The 20th percentile of the distribution

[3 Marks]

(c) Given the following distribution function

$$F(x) = \begin{cases} \frac{x^2 - 2}{2}; & 1 \leq x \leq 2; \\ 3x - \frac{x^2}{2}; & 2 \leq x \leq 3; \\ 1; & x \geq 3 \end{cases}$$

Determine the:-

(i) The probability density function of the random variable

[3 Marks]

(ii) The standard deviation of the random variable

[5 Marks]

(ii) The median of the random variable

[4 Marks]

END

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414