UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

University Examinations 2015/2016

Final Assessment for BSc III GENERAL

Thursday May 5th, 2016

MTC 3102 : REAL ANALYSIS II

Time allowed: 3 hours

Instructions

(i) Answer **five** questions.

(ii) Write both sides of the paper but begin a new question on a fresh page.

(iii) Only approved basic scientific calculators may be used in this examination.

(iv) Show all your solutions clearly and neatly.

1. (a) Find the total work done in moving a particle in a force field given by $F = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.

(4 marks)

(b) What does a *line integral* mean to you?

(1 mark)

(c) If $A = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C A.dr$ from (0, 0, 0) to (1, 1, 1) a long the following paths C:

(i) $x = t, y = t^2, z = t^3$;

(4 marks)

(ii) the straight lines from $(0, 0, 0)$ to $(1, 0, 0)$, then to $(1, 1, 0)$, and then to $(1, 1, 1)$;

(7 marks)

(iii) the straight lines from $(0, 0, 0)$ to $(1, 1, 1)$.

(4 marks)

2. (a) (i) State Green's theorem in the plane.

(ii) Prove Green's theorem in the plane if C is a closed curve which has the property that any straight line parallel to the coordinate axes cuts C in at most two points.

(8 marks)

(b) Verify Green's theorem in the plane for $\oint(xy+y^2)dx+x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(7 marks)

(c) Evaluate $\int_{(0,0)}^{(2,1)}(10x^4 - 2xy^3)\, dx - 3x^2y^2dy$ along the path $x^4 - 6xy^3 = 4y^2$.

(5 marks)

3. (a) Define the following terms as applied to a point (x_0, y_0) of the function $f(x, y)$ of two variables:

(i) local minimum,

(ii) absolute maximum,

(iii) absolute minimum.

(5 marks)

(b) When is a pair of co-ordinates (m, n) said to be a *critical point* of a function $f(x, y)$? Define a *saddle point* of such a function.

(4 marks)

(c) State the second derivatives test that describes with appropriate equations the procedure you take to test the extrema points of a function $f(x, y)$ of two variables that has continuous second partial derivatives on some open disk.

(5 marks)

(d) Locate and classify all the critical points for $f(x, y) = 2x^2 - y^3 - 2xy$.

(6 marks)

4. (a) Let $u(x, y, z) = u_1 i + u_2 j + u_3 k$ be defined and differentiable at each point(x, y, z) in some region of space.

(i) State what you understand by the terms *divergence* and *curl* of u.

(2 marks)

(ii) Compute the divergence and the curl of $u(x, y, z) = xi + (y + \cos x)j + (z + e^{xy})k$.

(4 marks)

(b) Determine whether a vector field $F(x, y, z) = yi + (\cos y + x)j + (y \cos yz)k$ is conservative?

(3 marks)

(c) Evaluate the following integral:
$\int_0^{\frac{\pi}{6}} \int_0^{\frac{\pi}{2}} (x \cos y - y \cos x) dy dx$.

(5 marks)

(d) (i) What do you understand by the term *directional derivative*?

(2 marks)

(ii) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2i - j - 2k$.

(4 marks)

5. (a) (i) When is a function $f(x, y)$ said to be differentiable at a point (x_0, y_0)?

(2 marks)

(ii) If $V = \ln(x^2 + y^2)$, prove that V is a harmonic function.

(4 marks)

(b) (i) Given that $f(t)$ and $g(t)$ are differentiable vector valued functions, prove that

$$\frac{\partial}{\partial t}[f(t) \times g(t)] = f(t) \times g'(t) + f'(t) \times g(t)$$

for $f(t) = f_1 i + f_2 j + f_3(t)k$ and $g(t) = g_1 i + g_2 j + g_3(t)k$.

(5 marks)

(ii) Given that $f(t) = e^{2t}i + e^{-5t}j + tk$ and $g(t) = te^{2t}i + t^{4/3}j + (t^2+2)k$. Compute $\frac{\partial}{\partial t}[f(t) \cdot g(t)]$

(3 marks)

(c) Evaluate the following integral: $\int_{-1}^{1}\int_{0}^{4}\int_{x^2}^{2-x^2}(x+y)dzdydx$.

(6 marks)

6. (a) (i) Given that $f(x,y) = \frac{3x^3y+4x}{y^3}$, find f_{xy} and f_{yy}.

(5 marks)

(ii) By using the definition of partial derivative of a function $f(x,y)$ at a point (x_0, y_0), prove that if $f(x,y) = 5x^2 + 4y^3x + 2$, then $f_x(x,y) = 10x + 4y^3$.

(4 marks)

(b) State Stoke's theorem and Gauss's divergence theorem.

(4 marks)

(c) (i) $\mathbb{Z}_7$, the set of integers under addition modulo 7 is an abelian group.

	1	2	3	4	5	6
1	-	-	-	4	-	-
2	-	-	-	-	-	5
3	-	-	2	-	-	-
4	-	1	-	-	-	-
5	-	-	-	-	-	-
6	-	-	-	-	-	1

With the aid of an example, explain how the modular arithmetic above is used in Cryptography.

(ii) Workout the same table of $\mathbb{Z}_7$ as in $6(c)(i)$ but now under multiplication modulo 7.

(7 marks)

END