UGANDA MARTYRS UNIVERSITY

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

University Examinations 2014/2015

Supplementary Assessment for BSc II FM

Monday August 3rd, 2015

MTF 2202 : INTRODUCTION TO FINANCIAL ENGINEERING

Time allowed: 3 hours

Instructions

(i) Answer **five** questions only.

(ii) Write both sides of the paper but begin a new question on a fresh page.

(iii) Only approved basic scientific calculators may be used in this examination.

(iv) A list of notation is given on the last printed page. Where necessary, assume that the currency of trading is Uganda Shilling.

(v) Show all your solutions clearly and neatly.

1 (a) State the following assumptions in the discrete time market model:

- Short selling,
- Divisibility,
- Discrete unit prices,
- Randomness.

(5 marks)

(b) Define the following:

(i) investment strategy,

(ii) self-financing investment strategy.

When is an investment strategy said to be predictable?

(5 marks)

(c) (i) Given the initial wealth $V(0)$ and a predictable sequence $(x_1(n), x_2(n), ..., x_m(n))$, $n = 1, 2, 3, ...$ of positions in risky assets, show that it is always possible to find a sequence $y(n)$ of risk-free positions such that $(x_1(n), x_2(n), ..., x_m(n), y(n))$ is a predictable self-financing investment strategy.

(4 marks)

(ii) Patricia is an investor with a portfolio consisting of two types of risky assets and risk-free assets. Let her position in risky assets held at time n of type one be $x_1(n)$, type two be $x_2(n)$ and her position in risk-free assets be $y(n)$ at time n. Consider a certain market scenario represented below.

$$\begin{array}{lll} S_1(0) = 75 & S_1(1) = 80 & S_1(2) = 90 \\ S_2(0) = 35 & S_2(1) = 30 & S_2(2) = 40 \\ A(0) = 115 & A(1) = 125 & A(2) = 136 \end{array}$$

Find the number of bonds $y(1)$ and $y(2)$ held by her during the first and second time steps of a predictable self-financing investment strategy with initial wealth value of $V(0) = 10965.2$ and the following risky assets positions if the asset prices follow the scenario above.

$$\begin{array}{ll} x_1(1) = 50.24 & x_1(2) = -55.50 \\ x_2(1) = 39.18 & x_2(2) = 25.13 \end{array}$$

Also find the time 1 value $V(1)$ and time 2 value $V(2)$ of this strategy.

(6 marks)

2 (a) What is the main difference between a Forward contract and a Futures contract.

(2 marks)

(b) What do you understand by the following:

- Short forward position,
- Long forward position.

Explain how an investor holding each of the above positions makes money.

(6 marks)

(c) (i) Let r be a constant risk-free interest rate, $S(0)$ be the price of a risky asset at time 0 and T be delivery time. Write an expression for forward price $F(0, T)$ in each of the following cases:

- stock paying no dividends,
- stock that will pay a dividend, *div* during the life time of the contract.

(3 marks)

(ii) Paula initiated a forward contract on 03/05/2010 for a stock worth 540/ = per share on this date. The risk-free interest rate then was 20%. Given that this stock doesn't pay dividend and that the contract has delivery date 03/05/2014. Calculate the forward price that was agreed if no arbitrage opportunity was created.

(2 marks)

(d) Suppose that the price of stock on 1st April 2010 turns out to be 10% lower that it was on 1st Jan. 2010. Assuming that the risk-free interest rate is constant at $r = 9\%$, what is the percentage drop of the forward price on 1st June 2010 as compared to that on 1st Jan. 2010 for a forward contract with delivery on 1st Jan. 2014.

(4 marks)

(e) Betty is a Ugandan importer of German cars who wants to arrange a forward contract to buy euros in a half a year. The interest rates for investments in Uganda shillings(UGx) and euros are 4% and 3%, respectively, the current exchange rate being 2499 UGx to one euro. Find her forward price of euros expressed in UGx?

(3 marks)

3 (a) With the aid of an example, explain what you understand by the No-Arbitrage principle.

(4 marks)

(b) Let $A(0) = 100$, $A(1) = 110$, $S(0) = 80$, and

$$S(1) = \begin{cases} 100 & \text{with prob. } \frac{4}{5} \\ 60 & \text{with prob. } \frac{1}{5} \end{cases}$$

Design a portfolio (x, y) (where x is the position in stock and y is the position in bonds) with the initial wealth of 10,000 split fifty-fifty between stock and bonds. Compute the expected return and risk as measured by standard deviation.

(10 marks)

(c) (i) Give the differences between a call option and a long forward position.

(4 marks)

(ii) An investor has a portfolio $(x = 20, y = 32, z = 15)$ of x shares, y bonds, and z options. When time $t = 0$, $S(0) = 20, A(0) = 12$ and $C(0) = 38$ (where $C(0)$ is the strike price of the option). Find the wealth of this investor at $t = 0$.

(2 marks)

4 (a) (i) Explain the following terms as applied to Futures trading.

- Initial margin,
- Margin call.

(4 marks)

(ii) Suppose that the initial margin is set at 10% and the maintenance margin at 5% of the future price. The table below shows a scenario with futures prices $f(n, T)$. The columns labelled *margin 1* and *margin 2* show the deposit at the beginning and at the end of each day, respectively. The payment column contains the amounts paid to top up the deposit (negative numbers) or withdrawn(positive numbers). Find the numerical values of

a, b, c up to j.

n	$f(n,T)$	cash flow	margin 1	payment	margin 2
0	140	opening:	0	-14	14
1	138	-2	12	0	12
2	130	a	b	c	d
3	140	e	23	+9	f
4	150	g	h	i	j
		closing:	15	+15	0
			total:	10	

(5 marks)

(b) Given that the market presents a risk-neutral market conditions, and that $S(0) = 100$ and $r = 6\%$. Compute the expectation of $S(2)$ with respect to the risk-neutral probability.

(3 marks)

(c) Let return on time step t be denoted by $K(t)$ and expectation be denoted by $E(K(t))$. Let the expected stock price at time step n be denoted by $E(S(n))$. Show that $E(S(n)) = S(0)(1 + E(K(1)))^n$

(5 marks)

(d) Let the estimate probabilities of recession, stagnation and boom be $\frac{1}{3}, \frac{1}{5}, \frac{2}{3}$ respectively. If the predicted annual returns on some stock in these scenarios is -6%, 4%, 30% respectively, find the expected anual return on these stock.

(3 marks)

5 (a) (i) Write the expression for the rate of return $K(n, m)$ over a time interval $[n, m]$ in terms of $S(m)$ and $S(n)$.

(1 mark)

(ii) Show that the precise relationship between consecutive one-step returns and the return over the aggregate period is
$1+K(n,m) = (1+K(n+1))(1+K(n+2)) \cdots (1+K(m))$

(4 marks)

(b) The one-step returns on stock are identically distributed independent random variables such that

$$K(n) = \begin{cases} 0.9 & \text{with prob. } \frac{2}{5} \\ -0.3 & \text{with prob. } \frac{3}{5} \end{cases}$$

(i) Sketch a tree representing possible stock price movements over the next three days, given that the price today $S(0) = 140$.

(2 marks)

(ii) Compute the stock prices at all the nodes in the binomial 5(b)(i) above.

(5 marks)

(c) The one-step returns $K(n)$ on stock are identically distributed independent random variables such that

$$K(n) = \begin{cases} u & \text{with prob. } p \\ d & \text{with prob. } 1-p \end{cases}$$

at each time step n, where $-1 < d < u$ and $0 < p < 1$. This is one of the conditions of a binomial tree model of risky assets.

(i) Show that the stock price $S(n)$ can move up or down by a factor $1+u$ or $1+d$ at each step.

(3 marks)

(ii) Find d and u if $S(1)$ can take two values 87 or 76, and the top possible value of $S(2)$ is 92.

(5 marks)

6 (a) (i) What do you understand by the following terms:

- call option,
- put option.

(4 marks)

(ii) Explain the difference between the American call option and the European call option.

(2 marks)

(b) The stock prices in the scenarios $\omega_1, \omega_2, \omega_3$ are as follows:

scenario	$S(0)$	$S(1)$	$S(2)$
ω_1	100	110	120
ω_2	100	105	100
ω_3	100	90	100

with probabilities $\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$ respectively.

(i) Find the expected returns $E(K(1))$, $E(K(2))$ and $E(K(0,2))$

(ii) Compare $1+E(K(0,2))$ with $(1+E(K(1)))(1+E(K(2)))$

(8 marks)

(c) (i) State Martingale's property. Let r be the risk-free interest rate and suppose that the stock prices $S(n)$ has become known at time n, show that the risk-neutral conditional expectation of $S(n+1)$ is

$$E_*(S(n+1)/S(n)) = S(n)(1+r).$$

(4 marks)

(ii) If $r = 0.4$, find the risk-neutral conditional expectation of $S(3)$ given that $S(2) = 120$.

(2 marks)