UGANDA MARTYRS UNIVERSITY
FACULTY OF SCIENCE
FINAL ASSESSMENT SEMESTER II 2006/2007
BSc I GEN, IT, B.ECON & FM DIFFERENRIAL EQUATIONS I

DATE: 10TH MAY 2007 **TIME 9:00-12:00 NOON**

Instructions

Attempt any **five** (05) questions

Read each question carefully before attempting
Questions carry equal marks

Q1. a) Find the general solutions to the following differential equations.

i) $y' = \dfrac{2 + \sin x}{3(y - 1)^2}$ ii) $\dfrac{dy}{dx} = \sqrt{(1 - y^2)(1 - x^2)}$.

b) Obtain a particular solution to the differential equation,
$x^2dy = (y^2 - xy + x^2)dx$; $y(1) = 2$.

Q2. a) Solve the following initial value problem,

$(3xy - y^2)dx + x(x - y)dy = 0$; $y(1) = 1$.

b) Two variables, x and y are connected by the equation, $\dfrac{dy}{dx} = 2y + x$.
Find an expression for y in terms of x such that when x = 0, y = 2.

Q3. According to Newton's law of cooling, the rate of fall of temperature of a body is directly proportional to the excess of its temperature over the surrounding, i.e.

$\dfrac{dT}{dt} \propto (T-\theta_R)$, where θ_R is room temperature.

Given that the temperature of an object falls from 200°C to 100°C in 40 minutes in a room temperature of 10°C. Show that T can be written as $T = 10 + 190e^{-kt}$; $k \in R$. Find the time taken for the temperature of the body to reach 50°C.

Q4. A second order differential equation is given as,

$xy'' + (1 - 2x)y' + (x - 1)y = 0$.

One of the linearly independent solution to the differential equation is $y = e^x$. Find a unique solution to the differential equation subject to the conditions,

$y(1) = 2e$; $y'(1) = -3e$.

Q5. A differential equation is related to an exponential function as given below.

$$\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 2y = 6xe^x$$

Obtain a particular solution to the equation above satisfying the conditions;

$y(0) = 1$ and $y'(0) = 0$.

Q6. a) Use the method of variation of parameters to solve the equation,

$$y'' - 2y' + y = \frac{e^x}{1 + x^2}.$$

b) Find a general solution to the equation,

$$\frac{d^3y}{dx^3} - y = xe^x.$$

Q7.a) Locate the ordinary, regular singular and irregular singular points (where applicable) of the differential equation,

$$x^3(1 - x^2)y'' + (2x - 3)y' + xy = 0.$$

b) Solve the differential equation,

$$y'' + x^2y = 0,$$

Using the power series method about $x = 0$.

END