Foreword & Terminology

2024年12月22日 星期日 22:15

① Hurst index: a statistical measure used to analyze time series data.

- $H<0.5$. time series tends to revert to mean.
- $H=0.5$. the closer to $\frac{1}{2}$, the more the time series behaves like a random walk, i.e., Brownian motion.
- $H>0.5$. The larger the value, the more the time seires has a certain trend, ⇒ long-term positive autocorrelation in the time series.

$0\le H\le 1$ To reveal its correlation &

Method to calculate: $\{X_t\}$.

1. divide time series of length N, into A continuous subinterval, each length is n. $\{E_1, E_2, \cdots E_A\}$
2. In each interval, calculate $R=\max-\min$ (range), $S=\left(\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2\right)^{\frac{1}{2}}$ $\{R_A/S_A\}$.
3. for: $R/S\sim n^H \xrightarrow{\text{logarithm}} \ln R-\ln S=H\cdot\ln n$ do linear regression, then get H.

② take the case of Hurst index $\frac{2}{3}$ in particular.

In this case, the power law of the density decay in the same as that in KPZ phenomenon.

Need to show these two different settings are similar by numerical evidence.

Discuss theoretical argument in support of the conjecture that they're exactly same in the large-time limit.

⇒ may belong to same universality class.

③ Space-time white noise. (often used in SPDE).

Δ Def: finite interval in time & space. on $\mathbb{R}^+$ and $D\subset\mathbb{R}^d$. $\mathbb{R}^+\times\mathbb{R}^d$ let V be measurable on $\mathbb{R}^{d+1}$

$$V(dt,dx)=I_{\mathbb{R}^+}(t)\,I_D(x)\,dt\,dx$$

Δ Properties: Independence. Stationary. Gaussian distribution

often use a normal distribution with mean 0; a variance ensuring $\boxed{\text{power spectral}}$ is constant.

④ Feynman-Kac formula.

1° Purpose: used to solve parabolic PDEs (describe heat conduction, diffusion ...)

2° Formulation:

$$u(t,x)=E\left[e^{-\int_0^T V(X_s)ds}f(X_T)\mid X_t=x\right].$$

where: E denotes expectation operator

X_s is a stochastic process (often a Brownian motion)

$V(X_s)$ a potential function, a rate of decay/growth affects the value of $f(X_T)$.

$\int_0^T V(X_s)ds$ total cost accumulated over time on T.

$f(X_T)$ terminal condition, future time T

$X_t=x$ stochastic process X_s starts at the position x at time t. ⇒ initial position.

⑤ Hopf-Cole transform

1° Purpose: study nonlinear PDE, converting nonlinear Burger's equation into linear heat function

2° Form: $u=-2\nu\frac{w_x}{w}$ or $h=-2\nu\ln\phi$ $\left(h_x=\partial_x(-2\nu\ln\phi)=-2\nu\frac{\phi_x}{\phi}\right)$

1° Purpose: study

2° Form: $u = -2\nu \frac{w_x}{w}$ or $h = -2\nu \ln \phi$ $\left(h_x = \partial_x(-2\nu \ln \phi) = -2\nu \frac{\phi_x}{\phi}\right)$

where: ν is viscosity coefficient (粘度系数)

3° Remark: $u_t = -2\nu \left(\frac{w_t}{w}\right)_x$ $u_x = -2\nu \left(\frac{w_x}{w}\right)_x$ $u_{xx} = -2\nu \left(\frac{w_x}{w}\right)_{xx}$

Burger's equation: $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}$.

⑥ Comments on the formula about B.

$\phi(t,x) = \int e^{-\frac{1}{2\nu}[h(t,r_0) + \int_0^t F(s,r_s)ds]} W(dB)$ $r_s = x + \sqrt{2\nu}\, B_{t-s}$,

Always represents Brownian motion / Wiener process.

property: continuity. independent increment, Normality, start at $t=0$ with value 0

$\Downarrow$

增量在非重叠时间上是独立的.

⑦ standard Wiener measure:

1° Definition: The wiener measure is a probability measure on the space $C[0,\infty)$, consisting of all continuous functions starting at the origin.

2° Properties: continuity、Independent Increments、Normal Distribution

for any $t \geq 0$, increment $W(t) - W(s)$ is independent of the past $\sigma(W(u): u \leq s)$ and is normally distributed with mean 0 variance $t-s$.

$\Leftrightarrow W(t) - W(s) \sim N(0, t-s)$

3° $C = \{w \in C_0([0,+\infty)): w(t_1) \in A_1, w(t_2) - w(t_1) \in A_2, \cdots w(t_n) - w(t_{n-1}) \in A_n\}$. $0 = t_0 < t_1 < \cdots < t_n$ are fixed time point. $A_1, \cdots A_n$ is Borel set in $\mathbb{R}$.

for every C, distribute a probability (i.d.d). $w(t_i) - w(t_{i-1}) \sim N(0, t_i - t_{i-1})$ $\Rightarrow$ Wiener measure of C is the product of the probabilities of these normal distributions restricted to the sets A_i

Introduction

2024年12月22日 星期日 23:25

KPZ equation: $\partial_t h + \frac{1}{2}(\partial_x h)^2 = \nu \partial_{xx} h + F$ (1), where F is space-time time noise.

describes the motion of growing surfaces that is subject to smoothing effects, slope-dependent growth speed, and space-time uncorrelated noise.

it is predicted that the $\boxed{\text{fluctuations}} \xrightarrow{} \text{of order } t^{\frac{1}{3}}$ of the height function, $h(t,x)$, $\boxed{\text{spatial correlation}}$ occurs at the scale of $t^{\frac{2}{3}}$

environment F, suppose F to be smooth (large-scale properties won't be affected by roughness of the environment F).

Besides the height function, there's also a geometrical approach to understand the KPZ scaling through the geometrical properties of optimal path: via Feynman-Kac formula after applying the Hopf-Cole transform $h(t,x) = -2\nu \cdot \ln\phi(t,x)$

$$\phi(t,x) = \int e^{-\frac{1}{2\nu}[h(t,\gamma_0)+\int_0^t F(s,\gamma_s)ds]} W(dB) \quad (2)$$

$\gamma_s = x + \sqrt{2\nu}\, B_{t-s}$, $W(\cdot)$ is the standard Wiener measure

Gibbs measure on paths: $P_0^{t,x}(d\gamma) = Z^{-1} e^{-\frac{1}{2\nu}[h(t,\gamma_0)+\int_0^t F(s,\gamma_s)ds]} W(dB)$.

Julia 自学笔记1. Strings & Data structures

2025年2月21日 星期五 19:29

0.

println(" ") 来输出

typeof( ) 来输出括号内的类型 如: Int64, Float64, String

基础运算: +, -, *, /, ^, % ↙取mod.

1. ① string: S_1 = "I'm your father"

out: I'm your father

S_2 = """ I'm your father """

out: "I'm your father"

S_3 = """ Look, no "error"! """

output: "Look, no \"errors\"!" ⇐ "\" 符号用来隔开输出的" "以避免出现string.

② Note that ' ' define a character, but NOT a string!

in: typeof('a')

out: Char ⇐ 单个字符.

③ 用 $ 符号将 Known-variables inserted into a string.

④ To concatenate strings 连接字符.

1° use string( )

2° use * 字符

3° use repeat(" ",) 数字

4° use ^ hi = "hi"^100 则输出100个 hi

example: S_3 = "How many cats";

S_4 = "is too many cats?";

string(S_3, S_4) 或者 S_3 * S_4

⇒ "How many cats is too many cats?"

2. Data structures { Tuples ①, Dictionaries ②, Arrays ③ }

①③ ordered 有序的.

②③ mutable 可变的.

① Tuples: Syntax (语法): use ( ) to enclose a collection of elements.

examples: mylove = ("Kasaki", "Kumiko")

mylove[1]

output: "Kasaki"

※ since it's immutable, we can't update it

如: mylove[1] = "Nozomi" 会报错!

② Named Tuples: (name1 = item1, name2 = item2, ---)

examples: MyLove = (Kasaki = "Nozomi", Oumae = "Kumiko")

input: MyLove[1] 或 MyLove.Kasaki

output: "Nozomi"

③ Dictionaries: Dict(key1 ⇒ value1, key2 ⇒ value2, ---) ⇐ not ordered, thus MyLove[1] is incorrect.

examples: MyLove = Dict("Kasaki" ⇒ "1203"; "Kumiko" = "0721")

input: MyLove["Kasaki"]

examples: MyLove = Dict(Kasaki => "1203" ...

input : MyLove ["Kasaki"]

output: "1203"

We can $\boxed{\text{add}}$ another entry to the dictionary as follows:

MyLove ["Leina"] = "1111" then we've add this into the dict.

we can $\boxed{\text{delete}}$ entry from the dict. by using $\boxed{\text{pop!}}$.

In: pop! (MyLove, "Leina")

Out: "1111"

then both elements are deleted.

④ Arrays : Syntax: [item1, item2, ---].

{ Array{String, 1} means 1 dimensional vector
{ Array{String, 2} means 2 - ___ matrix

括号内可以是 Int64, String、的混合.

example: mixture=[1,1,2,3, "Ted", "Robyn"]

use mixture[3]= 4

=> mixture=[1,1,4,3, "Ted", "Robyn"].

use push! (mixture, "Kasaki") ⇐ add it to the end of the array.

=> mixture = [1,1,4,3 ,"Ted", "Robyn", "Kasaki"]

use pop! (mixture) ⇐ remove the last element

=> mixture=[1,1,4,3, "Ted", "Robyn"]

use numbers= [[1,2,3],[4,5,6]] to represent a 2x3 matrix.

[1, 2, 3]
[4, 5, 6]

use rand(4,3) to get a random 4x3 matrix

rand(4,3,2) out: [:, :, 1] = [:, :, 2]=

4x3的矩阵 4x3 matrix

☆ fibonacci=[1,1,2,3,5,8,13]

sm= fibonacci

sm[1]= 44

input: fibonacci

output: 44,1,2,3,5,8,13

to avoid the updating of fibonacci,

we can use the function of copy.

smm = copy(fibonacci)

smm[1]= 44, fibonacci will not change

Julia 2 Loops & Conditionals.

2025年2月25日 星期二 16:47

Loops

1. "While" Fundamental syntax: While *condition*
   loop body
   end

Example:
```
n=0
while n<10
    n+=1
    println(n)
end
n
```
output: 1, 2, - — 10

my = [1,2,3,4]
length(my)=4.

2. "for" Fundamental Syntax: for *var* in *loop iterable*
   loop body
   end

Example:
```
for n in 1:10
    println(n)
end
```
output: 1, 2, 3, - - - 10

We can use loops in matrix, iterate over this array via column-major loops

Example: m,n=5,5   A=fill(0,(m,n)) ⟹

$$\begin{matrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{matrix}$$

```
for j in 1:n, i in 1:m
    A[i,j]= i+j
end
A
```

⟹

$$\begin{matrix} 2&3&4&5&6\\ 3&4&5&6&7\\ 4&5&6&7&8\\ 5&6&7&8&9\\ 6&7&8&9&10 \end{matrix}$$

⇐ A= [i+j for i in 1:m, j in 1:n]

Exercise: 生成一个横向的1x100数组

```
array= [ ]
for i in 1:100
    push!(array, i)
end
```

array= collect(1:100)

或：
```
transposed_array= transpose(array)
transposed_array = array'
```
} 以此获得其转置矩阵.

或者: array= [i^2 for i in 1:100].

Conditionals.

Syntax 1: if *condition 1*
    option 1
else if *condition 2*
    option 2
else
    option 3
end

Appendix: use A && B representing both A and B are true/false.

In Julia, && will be "short-circuit", only for boolean.
即: 在检测A,B是否为 boolean 值时, 若A为False 则不执行B,
若A为 True 再检查 B的运行.

Syntax 2: a ? b : c   equals to

```
if a
    b
else
    c
end.
```

Syntax 3: || operator, used as an "or" operation, also short-circuit.

A || B if A is true, then won't execute B

A is false, try to execute B.

Julia 3 Functions

2025年2月25日 星期二 21:28

1. Define a function

Syntax 1: use function and end keywords

Example 1:

```
function saying(name)
    println("$name, what can I say!")
end
```

Input: saying(Man)

Output: Man, what can I say!

Example 2:

```
function f(x)
    x^2
end
```

Input: f(25)

output: 625

Syntax 2: declare a function in a single line:

Example: `f2(x)= x^2`

In: f2(42)

Out: 1764

匿名的

Syntax 3: declare "anonymous" functions

Example: `sayhi3 = name -> println("Hi $name!")`

out: #1 ⇐ 只有序号,看不到函数名称

concatenation 连接.

2. Duck-typing in Julia:

for $A = \text{rand}(3,3)$ which means a random 3x3 matrix, $f(A) = A^2 = A*A$

Also: f("hi") ⇒ hihi ⇐ Recall: $s_1 * s_2 = s_1s_2$ for s_1, s_2 are both strings

3. Mutating vs. non-mutating functions

Conventionally, functions followed by ! alter their contents

lacking ! do not

Example: v=[3,5,2].

Example: v=[3,5,2].

In:	sort(v)	sort!(v)
out:	2 3 5	2 3 5
In:	v	v
out:	3 5 2	2 3 5.

4. Some higher order functions.

☆ map use map to apply the function to every element of data structure such as map(f, [1,2,3]) ⇒ (f[1], f[2], f[3]).

or: map(x→x^3, [1,2,3]) ⇒ [1,8,27].

or: f.([1,2,3]) ⇒ (f[1], f[2], f[3]).

⇕ 等价于

broadcast(f, [1,2,3])

↳ 在处理多维数组时有用（自动扩展较小的数组以匹配较大数组的形状）

生成形如：$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ 的矩阵：A=[i+3*j for j in 0:2, i in 1:3]

$$f.(A) = \begin{bmatrix} 1 & 4 & 9 \\ 16 & 25 & 36 \\ 49 & 64 & 81 \end{bmatrix}$$

$$A.+2.*f.(A)./A = A + \begin{bmatrix} 2 & 8 & 18 \\ 32 & 50 & 72 \\ 98 & 128 & 162 \end{bmatrix} ./ \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

做除法时会转化为 float 以避免向下取整

$$= A + 2A = 3A = \begin{bmatrix} 3.0 & 6.0 & 9.0 \\ 12.0 & 15.0 & 18.0 \\ 21.0 & 24.0 & 27.0 \end{bmatrix}.$$

Julia 4 Packages & Plotting

2025年3月4日 星期二 14:26

1. Packages: we can use Pycall and Rcall to use foreign functions in Python or R.

use: https://julialang.org/packages/ to download.

We we first use a package, we can use:

```
using Pkg
Pkg.add("Example")
```

load the package: `using Example`

定义who为String类型

Appendix: `hello(who::String)="Hello, $who"`

2. Plotting

How to use Plots.jl which allows us to seamlessly change backends.

try gr( ) and plotyjs( ) backends.

Examples: In:

```
global=[14.4, 14.5, 15.8]
numpi=[4000, 2000, 1000]
gr( )
```

out: `Plots.GRBackend( )`

then use plot and scatter to generate plots.

plot: 创造线图

scatter: 创造散点图.

```
plot(numpi, global, label="line")
scatter!(numpi, global, label="points")
```

!符号,使函数 scatter 加入到 plot 图里,否则会生成两个图.

then add xlabel! ylabel! title! into the graph.

use xflip!( ) 来翻转x轴 (即左→右变为右→左 再改回坐标标注计).

we can also use the package UnicodePlots.

Examples: for $x = -10 \sim 10$, $y = x^2$, plot y vs. x. consider different backends.

In:
```
x= -10:10
y= x.^2
gr()
plot(x,y)
```
⇐ 可以生成一个连续图像，是后端的一种

若加用 gr()，则在Julia中生成一个点状图。

类似的有：GR、PyPlot，Plotly 等.

Examples: usage of subplot

```
p1=plot(x,x)
p2=plot(x,x.^2)
p3=plot(x,x.^3)
p4=plot(x,x.^4)
plot(p1,p2,p3,p4,layout=(2,2),legend=false)
plot(p1,p2,p3,p4,layout=(4,1),legend=false)
```

```
plot(layout=(2,2))
subplot(2,2,1)    plot(x,y,title="y=x")
    ⋮
```
代表如何

display(plot) 则需.

Julia 5 Mutiple dispatch

2025年3月5日 星期三 19:39

1. Specifying the types of our input arguments

In: foo(x::string, y::String) = println("My inputs x and y are both strings!")

foo("hello", "hi!")

out: My inputs x and y are both strings!

In: foo(3, 4)

out: 报错

In: foo(x::Int, y::Int) = — —

foo(3, 4)

out: — — —

Now, foo works both on integers and strings!

Now we've added an additional method to the generic function called foo.

@which are used to see which method is being dispatched

add method like: foo(x::Number, y::Number) = println(" ") 则此时 foo 中有三个方法,

foo(x, y) = println(" 1 ") 没变改输出, 则此时四个方法

Julia 6

2025年3月5日 星期三 20:13

Consider sum(a) $= \sum_{i=1}^{n} a_i$, function name sum

用 """ 来引入C或 Python 语言, 如 C""" , Py"""

@simd 是一个宏，用于指示编译器尝试将循环自动向量化，以利用 CPU的向量化指令集来加速计算.

用 x=fill(1.0, (3,)) 来生成 3x1的向量： $\begin{matrix} 1.0 \\ 1.0 \\ 1.0 \end{matrix}$.

计算机基础知识

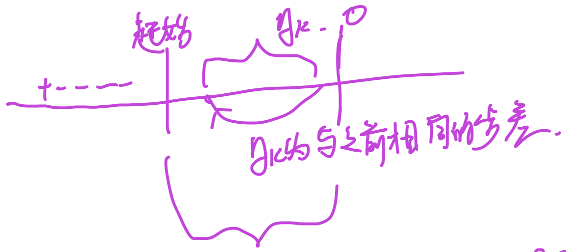
2025年2月21日 星期五 19:31

草稿：

2025年3月24日 星期一 20:17

$f(X\geq L)=$

X在$\{1,2,3,---\}$上取值.

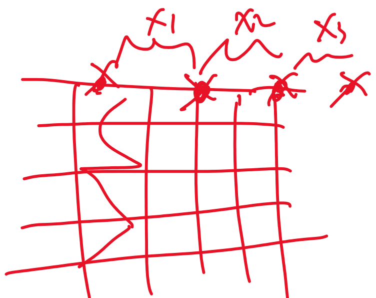


η_k与之前相同的步差.

这段区间中前面要步长，但是随机生成.

$\boxed{\text{去Markov性}}$：假果一步与前面的一步相关

最终探究：



--- $\{X_j\}$同分布，$X \sim N(0, t^{2H})$

3ε范围 $\Rightarrow$ 左右共$6\varepsilon = 6\sqrt{n}$，n为step数.

BM: $t\in[0,1]$ $\frac{S_{[nt]}}{\sqrt{n}} \to BM$. $(B_t)_{t\geq 0}$.

fractional BM $P(\eta_k \geq L) \sim CL^{-\alpha} \to$ 与H有关.

$\searrow C=1$,

$\Downarrow$

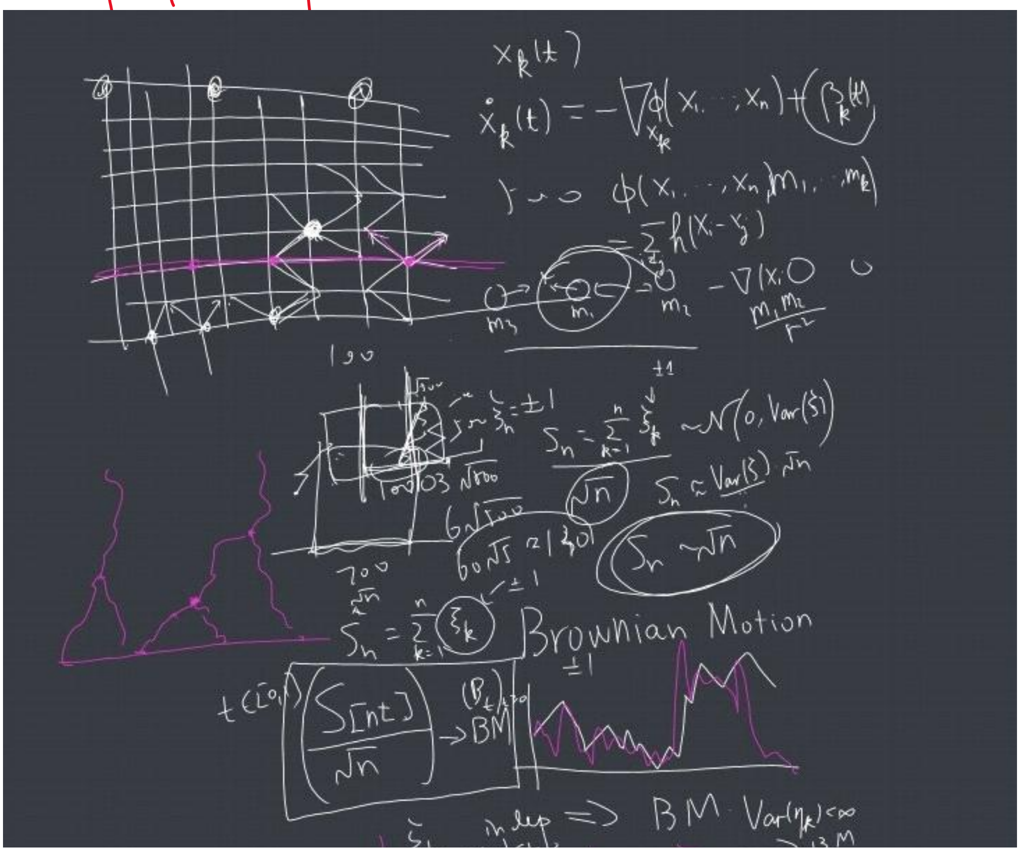
$B_t \sim N(0,t)$

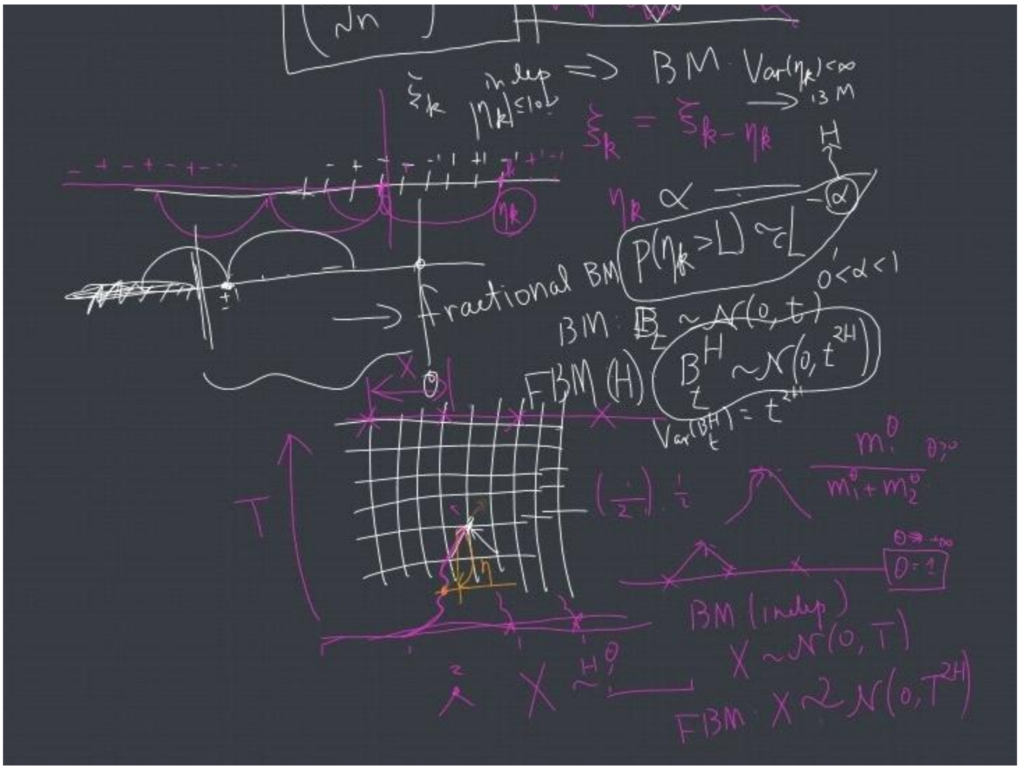
$B_t^H \sim N(0, t^{2H})$

other model：当 m_1, m_2 质量相碰时

以 $\frac{m_1^\theta}{m_1^\theta+m_2^\theta}$ 取前者

一方 取后者.




in dep ⟹ BM
Var(η_k) < ∞
ξ_k = ξ_k − η_k
η_k α
fractional BM
P(η_k > L) ~ c L^{-α}
0 < α < 1
BM: B_t ~ N(0, t)
FBM (H) B_t^H ~ N(0, t^{2H})
Var(B_t^H) = t^{2H}
T
BM (indep)
X ~ N(0, T)
FBM X ~ N(0, T^{2H})

2025年3月16日 星期日 23:08

Coalsecing Brownian motion: Two particle combine, one of it vanish

Question: whether if the mass increase, the probability of other particles that moving towards it (tendency of approaching is increasing? (like $p \to 8$)

how to quantify these

Smoluchowski: $\mathbb{R}^3$, $m, n \to m+n$, with prob. $\frac{1}{2}$ to get one of their situation.

in a fluid, viscosity (黏度系数 η), radius a → sphere. diffusion speed is $\frac{KT}{6\pi a\eta}$

The system of coalescing Brownian motions on the line can be formulated:

denote $C_x(t) = C_x(t, w)$, the place w at time t of the particle.

System: $C = C(w)$: a real valued stochastic process indexed by $(x,t) \in \mathbb{R} \times [0, \infty)$

$C(w)$ has the following properties: $x, y \in \mathbb{R}$, $t \geq 0$;

① $C_x(0) = x$

② $C_x(\cdot) \in C_{\mathbb{R}}[0, +\infty)$ continuous path.

③ $C_x(t) = C_y(t)$ implies that $C_x(s) = C_y(s)$ for all $s > t$.

④ $C_x(\cdot)$ must be a family of coalescing Brownian motions.

第2种: a projection of the system C: for $t \geq 0$, $X_t = \{C_x(t) : x \in \mathbb{R}\}$.

$(X_t, t \geq 0)$ is a Markov process. analogous to the system $\xi_t^{\mathbb{Z}}$ of coalescing random walks.

distribution of X_t is invariant, i.e. $X_t \overset{d}{=} \sqrt{t} X_1$: for $A \subset \mathbb{R}$, rA means $\{rx : x \in A\}$.

The Percolation Substructure:

A graphical representation due to Harris: 渗透

Define: for an arbitrary subset A, $X_t^A = \{C_x(t) : x \in A\}$. $X_t^{A \cup B} = X_t^A \cup X_t^B$

$C = (C_x(s,t) : x \in \mathbb{R}, t \geq s \geq 0)$ $C_x(s,t)$ give a time t of a particle, which is "born" at time s at location x

加点进去 (是否有随机产生点, 或有规律的产生点, 有可能吗?)

$C_x(u,t) = C_y(r,t) \Leftrightarrow C_x(u,s) = C_y(r,s)$ for all $s > t$.

To specify distribution of C, define $(C^u)_x(t) = C_x(u, u+t)$ for each $u \geq 0$

for $\forall t$, part of c before t: $C_x(r,s) : 0 \leq r \leq s \leq t$, $x \in \mathbb{R}$.

after t: $C_x(u,v) : t \leq u \leq r < \infty$, $x \in \mathbb{R}$.

对于 random walk, take each step $\frac{1}{\sqrt{s}}$, jumping interval $\frac{1}{s}$, then as $s \uparrow$,

this is almost Brownian Motion.

A stationary "Gaussian" Process

π^c be the distribution of the random set X_1.

for $\forall t > 0$, distribution of the rescaled set $\frac{1}{\sqrt{t}} X_t$ is π^c

write $(X_t^{\pi^c}, t \geq 0)$ for the mixture of the processes X_t^A, for $t > 0$, $\frac{1}{\sqrt{t}} X_t^{\pi^c} \overset{d}{=} X_0^{\pi^c}$.

均衡状态, 在时间上不会改变

考虑一个时间变换: 令 $Y_t = e^{-\frac{t}{2}} X_{e^t}$

for $\forall t \in \mathbb{R}$, distribution of Y_t is π^c.

for diffusion coefficient 1 for every particle, if it is at x, its drift 位移 is $-\frac{x}{2}$.

⇒ 粒子从无穷远向原点移动, 平衡态为 π^c

参考应用随机过程中的 Markov Chain.

$\boxed{\text{Duality}}$:

Methods involving $\boxed{\text{time reversal}}$ are reffered to as "duality methods".

便是.

[Duality]:

Methods involving $\boxed{\text{time reversal}}$ are reffered to as "duality methods".

in the lattice 晶格, 格子), coalescing and annihilating (湮灭) random walks, voter model (投票模型)

形成两对偶运动, 一个向前一个向后

⇓

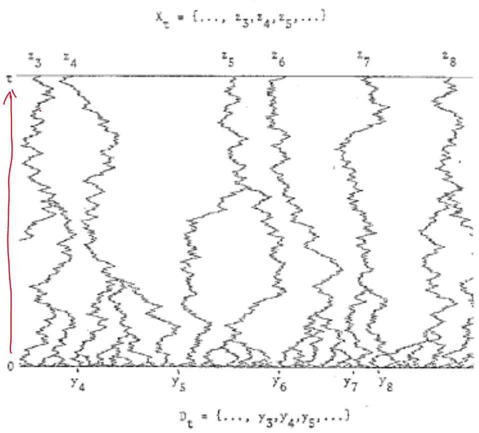
两个 system 不交叉.

a joint realization of c and c', two systems of coalescing Brownian motions as specified by ↙.

s.t. for all $w \in \Omega$, $x, y \in \mathbb{R}$, $0 \le s, t \le T$, $(C_x(s) - C_y'(T-s))(C_x(t) - C_y'(T-t)) \geq 0$

对偶关系由 使CBM当作 CRW(random walks) 给出.

考虑逆向, 编程时由 $y \to z$, 对每个 z_i, 有区间 $[y_i, y_{i+1})$ 使其初始点倒回去.



$D_t = \{y \in \mathbb{R} : \lim_{x \to y^-} C_x(t) \neq \lim_{x \to y^+} C_x(t)\}$.

points of D_t do not move:

for $s \geq t > 0$, $D_s \subset D_t$.

Consider the distribution of D_t?

构造对偶的 c 与 c' 于 $[0,t]$ 区间上.

记 $X_t' = \{C_x'(t) : x \in \mathbb{R}\}$. Now $D_t = X_t' \overset{d}{=} X_t$. ☆.

对 Duality 的第二种用法, a formula for π^c, the distribution of X_1.

X_t is determined by its zero function, $\phi_t(B) = P(X_t \cap B = \phi)$ for $B \subset \mathbb{R}$.

$I_0 = \{B \subset \mathbb{R} : B = [a_1, b_1] \cup \dots \cup [a_n, b_n]$ for some $n, a_i, b_i\}$.

system c and c' in duality on $[0,t]$ ⇒ $\phi_t(B) = P(X_t \cap B = \phi) = P(C_{a_i}'(t) = C_{b_i}'(t)$ for $i = 1, 2, \dots K\}$..

for a finite $F \subset \mathbb{R}$, A^F be a system of annihilating Brownian motions starting with particles on F: two particles collide then they'll both vanish.

⇓

duality equation: $B \in I_0$, as $\phi_t(B) = P(X_t \cap B = \phi) = P(A_t^{\partial B} = \phi)$, ∂B is the boundary of B.

Consider Poisson process with intensity λ: $\phi(B) = e^{-\lambda |B|}$. $|B|$ is Lebegue measure of B, Note of the first version of Codes.

2025年3月20日 星期四 22:57

函数/命令	意义	函数/命令	意义
plot	基本二维图形	ylabel	y轴说明
fplot	一元函数图像	zlabel	z轴说明
plot3	空间曲线	clabel	等高线高度标志
meshgrid	网格数据生成	grid	格栅
mesh	网面图	hold	图形保持
surf	曲面图	axis	定制坐标轴
contour	等高线图	view	改变视点
comet	二维动画	subplot	子图
contour3	三维等高线图	figure	新图形窗口
comet3	三维动画	clf	清除图形
title	标题	close	关闭图形窗口
xlabel	x轴说明	ndgrid	n维网格数据生成

名称	说明	名称	说明
area	区域填充图	quiver(quiver3)	二维矢量图（三维）
bar(bar3)	条形图（三维）	stairs	阶梯图
barh(bar3h)	水平条形图（三维）	meshc	带等高线网格图
comet(comet3)	彗星图（三维）	meshz	带垂帘线网格图
errorbar	误差带图	surfc	带等高线着色图
feather	箭号图	trimesh	三角形网格图
fill	多边形填充图	trisurf	三角形表面图
hist	统计直方图	waterfall	瀑布图
pie(pie3)	饼图（三维）	cylinder	柱面图
stem(stem3)	火柴杆图（三维）	sphere	球面图