

Program Structures and Algorithms  
Spring 2023(SEC –)

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**Task:**

Solve 3Sum problem and compare the time complexities between Quadratic, Quadrithmic, and Cubic solutions

**Relationship Conclusion:**

The 3Sum Problem is solved in four different ways namely, Quadratic, Quadratic with callipers, Quadrithmic and Cubic. Time complexity of Quadratic and Quadratic with callipers is  $N^2$  where as that of Quadrithmic is  $N^2 \log N$  and for Cubic it is  $N^3$

From the graph between log of N and log of time, we can verify this by calculating the slope of each line. The slope of Quadratic and Quadratic with callipers is 2.15 and 2.30 which is almost near to 2 which the ideal scenario. The slope for Quadrithmic solution is 2.41 which between 2 and 3 as expected (2 for Quadratic and 3 for Cubic). The slope for Cubic solution is 2.93 which is almost 3.

As the value of N increases, we can clearly see with increase in time the gap between Quadrithmic line and Cubic line from Quadratic line increases exponentially. This indicates that Cubic is worst solution compared to others.

**Evidence to support that conclusion:**

Observations of the 3Sum Problem are as follows:

		Quadratic		Quadratic with Calipers		Quadrithmic		Cubic	
N	Log N	Time in mSec	Log of time	Time in mSec	Log of time	Time in mSec	Log of time	Time in mSec	Log of time
250	7.96578428	0.37	1.434402824	0.19	2.395928676	0.22	2.184424571	3.19	1.673556424
500	8.96578428	1.04	0.056583528	0.24	2.058893689	2	1	21.92	4.454175893
1000	9.96578428	3.3	1.722466024	2.3	1.201633861	11.5	3.523561956	194.15	7.601027897
2000	10.9657843	18	4.169925001	11.6	3.5360529	61.7	5.947198584	1333.4	10.38089392
4000	11.9657843	80.4	6.329123596	69.4	6.116863758	274	8.098032083	10619.8	13.37446898
8000	12.9657843	508.67	8.9905862	360.67	8.494535617	1352.67	10.4015942		
16000	13.9657843	2329	11.18549492	2015	10.97656412	5876	12.52061868		

```
/Library/Java/JavaVirtualMachines/jdk-17.0.4.1.jdk/Contents/Home/bin/java ...
ThreeSumBenchmark: N=250
Run type: ThreeSumQuadratic
2023-01-28 17:50:10 INFO TimeLogger - Raw time per run (mSec): .37
2023-01-28 17:50:10 INFO TimeLogger - Normalized time per run (n^2): 5.92
Run type: ThreeSumQuadraticWithCalipers
2023-01-28 17:50:10 INFO TimeLogger - Raw time per run (mSec): .19
2023-01-28 17:50:10 INFO TimeLogger - Normalized time per run (n^2): 3.04
Run type: ThreeSumQuadrithmic
2023-01-28 17:50:10 INFO TimeLogger - Raw time per run (mSec): .22
2023-01-28 17:50:10 INFO TimeLogger - Normalized time per run (n^2 log n): .44
Run type: ThreeSumCubic
2023-01-28 17:50:11 INFO TimeLogger - Raw time per run (mSec): 3.19
2023-01-28 17:50:11 INFO TimeLogger - Normalized time per run (n^3): .20
ThreeSumBenchmark: N=500
Run type: ThreeSumQuadratic
2023-01-28 17:50:11 INFO TimeLogger - Raw time per run (mSec): 1.04
2023-01-28 17:50:11 INFO TimeLogger - Normalized time per run (n^2): 4.16
Run type: ThreeSumQuadraticWithCalipers
2023-01-28 17:50:11 INFO TimeLogger - Raw time per run (mSec): .24
2023-01-28 17:50:11 INFO TimeLogger - Normalized time per run (n^2): .96
Run type: ThreeSumQuadrithmic
2023-01-28 17:50:11 INFO TimeLogger - Raw time per run (mSec): 2.00
2023-01-28 17:50:11 INFO TimeLogger - Normalized time per run (n^2 log n): .89
Run type: ThreeSumCubic
2023-01-28 17:50:12 INFO TimeLogger - Raw time per run (mSec): 21.92
2023-01-28 17:50:12 INFO TimeLogger - Normalized time per run (n^3): .18
```

```
ThreeSumBenchmark: N=1000
Run type: ThreeSumQuadratic
2023-01-28 17:50:12 INFO TimeLogger - Raw time per run (mSec): 3.30
2023-01-28 17:50:12 INFO TimeLogger - Normalized time per run (n^2): 3.30
Run type: ThreeSumQuadraticWithCalipers
2023-01-28 17:50:12 INFO TimeLogger - Raw time per run (mSec): 2.30
2023-01-28 17:50:12 INFO TimeLogger - Normalized time per run (n^2): 2.30
Run type: ThreeSumQuadrithmic
2023-01-28 17:50:12 INFO TimeLogger - Raw time per run (mSec): 11.50
2023-01-28 17:50:12 INFO TimeLogger - Normalized time per run (n^2 log n): 1.15
Run type: ThreeSumCubic
2023-01-28 17:50:16 INFO TimeLogger - Raw time per run (mSec): 194.15
2023-01-28 17:50:16 INFO TimeLogger - Normalized time per run (n^3): .19
ThreeSumBenchmark: N=2000
Run type: ThreeSumQuadratic
2023-01-28 17:50:17 INFO TimeLogger - Raw time per run (mSec): 18.00
2023-01-28 17:50:17 INFO TimeLogger - Normalized time per run (n^2): 4.50
Run type: ThreeSumQuadraticWithCalipers
2023-01-28 17:50:17 INFO TimeLogger - Raw time per run (mSec): 11.60
2023-01-28 17:50:17 INFO TimeLogger - Normalized time per run (n^2): 2.90
Run type: ThreeSumQuadrithmic
2023-01-28 17:50:17 INFO TimeLogger - Raw time per run (mSec): 61.70
2023-01-28 17:50:17 INFO TimeLogger - Normalized time per run (n^2 log n): 1.41
Run type: ThreeSumCubic
2023-01-28 17:50:31 INFO TimeLogger - Raw time per run (mSec): 1333.40
2023-01-28 17:50:31 INFO TimeLogger - Normalized time per run (n^3): .17
```

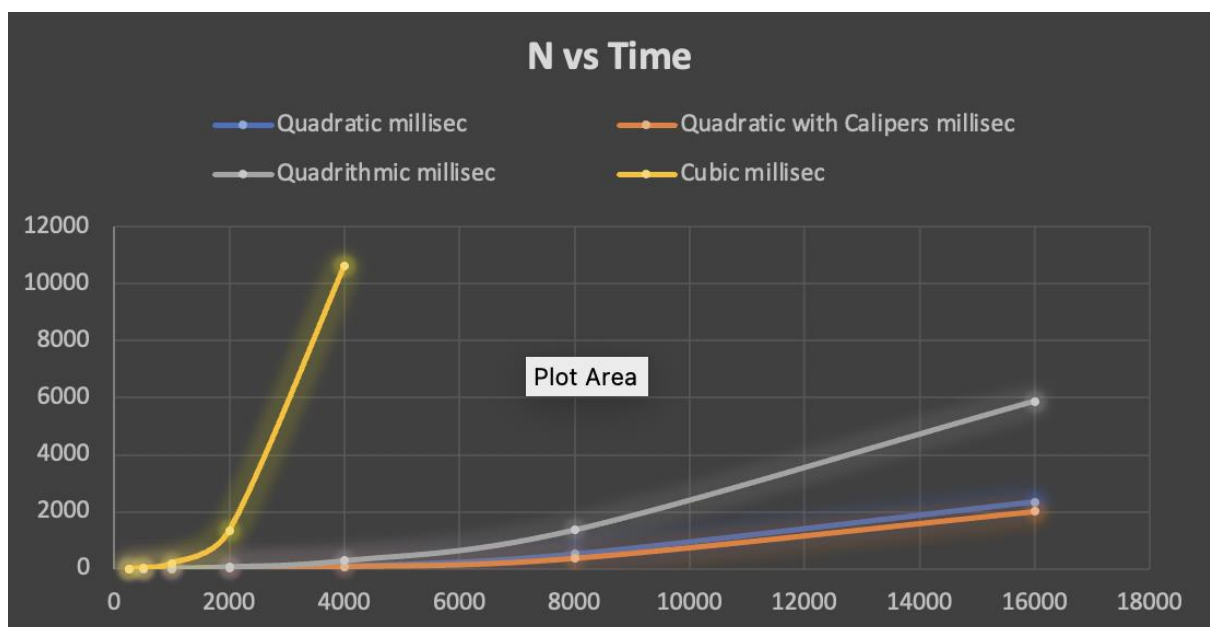
```

ThreeSumBenchmark: N=4000
Run type: ThreeSumQuadratic
2023-01-28 17:50:31 INFO  TimeLogger - Raw time per run (mSec): 80.40
2023-01-28 17:50:31 INFO  TimeLogger - Normalized time per run (n^2): 5.03
Run type: ThreeSumQuadraticWithCalipers
2023-01-28 17:50:31 INFO  TimeLogger - Raw time per run (mSec): 69.40
2023-01-28 17:50:31 INFO  TimeLogger - Normalized time per run (n^2): 4.34
Run type: ThreeSumQuadrithmic
2023-01-28 17:50:33 INFO  TimeLogger - Raw time per run (mSec): 274.00
2023-01-28 17:50:33 INFO  TimeLogger - Normalized time per run (n^2 log n): 1.43
Run type: ThreeSumCubic
2023-01-28 17:51:26 INFO  TimeLogger - Raw time per run (mSec): 10619.80
2023-01-28 17:51:26 INFO  TimeLogger - Normalized time per run (n^3): .17
ThreeSumBenchmark: N=8000
Run type: ThreeSumQuadratic
2023-01-28 17:51:27 INFO  TimeLogger - Raw time per run (mSec): 508.67
2023-01-28 17:51:27 INFO  TimeLogger - Normalized time per run (n^2): 7.95
Run type: ThreeSumQuadraticWithCalipers
2023-01-28 17:51:28 INFO  TimeLogger - Raw time per run (mSec): 360.67
2023-01-28 17:51:28 INFO  TimeLogger - Normalized time per run (n^2): 5.64
Run type: ThreeSumQuadrithmic
2023-01-28 17:51:33 INFO  TimeLogger - Raw time per run (mSec): 1352.67
2023-01-28 17:51:33 INFO  TimeLogger - Normalized time per run (n^2 log n): 1.63
ThreeSumBenchmark: N=16000
Run type: ThreeSumQuadratic
2023-01-28 17:51:37 INFO  TimeLogger - Raw time per run (mSec): 2329.00
2023-01-28 17:51:37 INFO  TimeLogger - Normalized time per run (n^2): 9.10
Run type: ThreeSumQuadraticWithCalipers
2023-01-28 17:51:41 INFO  TimeLogger - Raw time per run (mSec): 2015.00
2023-01-28 17:51:41 INFO  TimeLogger - Normalized time per run (n^2): 7.87
Run type: ThreeSumQuadrithmic
2023-01-28 17:51:53 INFO  TimeLogger - Raw time per run (mSec): 5876.00
2023-01-28 17:51:53 INFO  TimeLogger - Normalized time per run (n^2 log n): 1.64
Process finished with exit code 0

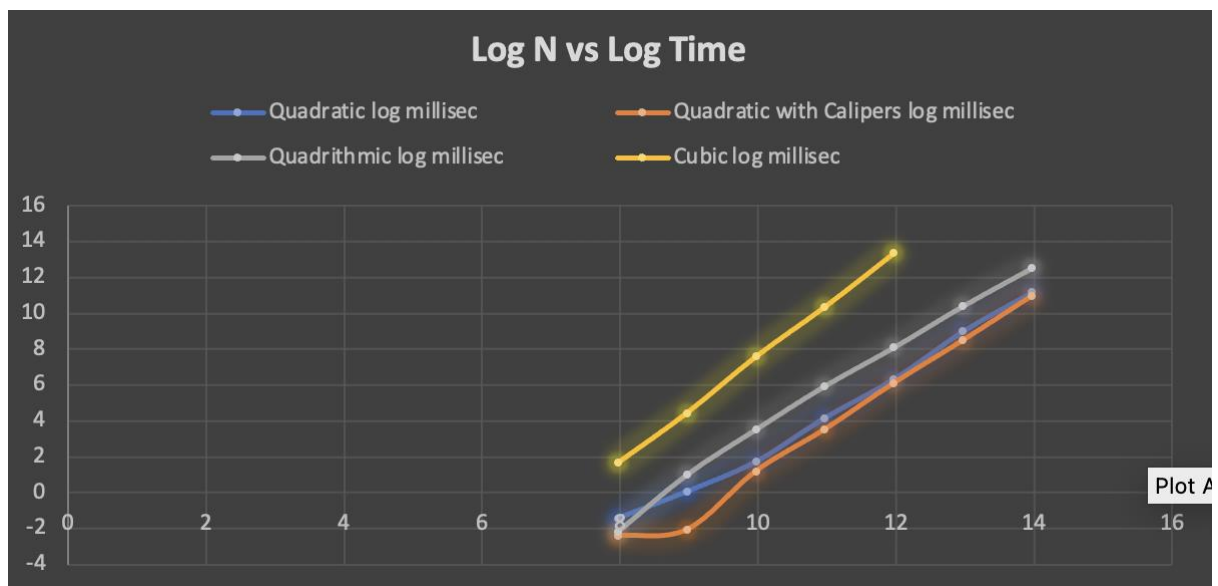
```

## Graphical Representation:

### 1. Time Vs N



## 2. Log T vs Log N



### Unit Test Screenshots:

```
Tests passed: 11 of 11 tests - 803 ms
ThreeSumTest (edu.neu.coe.inf 803 ms)
  testGetTriples0 13 ms
  testGetTriples1 5 ms
  testGetTriples2 0 ms
  testGetTriplesC0 0 ms
  testGetTriplesC1 2 ms
  testGetTriplesC2 0 ms
  testGetTriplesC3 225 ms
  testGetTriplesC4 557 ms
  testGetTriplesJ0 0 ms
  testGetTriplesJ1 0 ms
  testGetTriplesJ2 1 ms

/Library/Java/JavaVirtualMachines/jdk-17.0.4.1.jdk/Contents/Home/bin/java ...
ints: [-40, -20, -10, 0, 5, 10, 30, 40]
triples: [Triple{x=-40, y=0, z=40}, Triple{x=-40, y=10, z=30}, Triple{x=-20, y=-10, z=30}, Triple{x=-10, y=0, z=10}]
[Triples{x=-51, y=2, z=49}, Triple{x=-51, y=9, z=42}, Triple{x=-44, y=2, z=42}, Triple{x=-11, y=2, z=9}]
[Triples{x=-51, y=2, z=49}, Triple{x=-51, y=9, z=42}, Triple{x=-44, y=2, z=42}, Triple{x=-11, y=2, z=9}]
[-72, -50, -43, -29, -14, 5, 12, 24, 39, 54]
[Triples{x=-29, y=5, z=24}]
ints: [-40, -20, -10, 0, 5, 10, 30, 40]
triples: [Triple{x=-40, y=0, z=40}, Triple{x=-40, y=10, z=30}, Triple{x=-20, y=-10, z=30}, Triple{x=-10, y=0, z=10}]
[Triples{x=-51, y=2, z=49}, Triple{x=-51, y=9, z=42}, Triple{x=-44, y=2, z=42}, Triple{x=-11, y=2, z=9}]
[Triples{x=-51, y=2, z=49}, Triple{x=-51, y=9, z=42}, Triple{x=-44, y=2, z=42}, Triple{x=-11, y=2, z=9}]
[-72, -50, -43, -29, -14, 5, 12, 24, 39, 54]
[Triples{x=-29, y=5, z=24}]

Process finished with exit code 0
```

### Explanation of why quadratics work:

As the value of N increases, we can clearly see with increase in time the gap between Quadrithmic line and Cubic line from Quadratic line increase exponentially.

We can see that the time for Quadrithmic in case of N=250 and 500 is between the two Quadratic solutions but as the value of N increases, the Quadratic solutions are more efficient as we only run the loop twice.

The best solution is quadratic since it can solve the problem in  $N^2$  time.

We want  $x + y + z = 0$  to be true, then  $x + z = -y$  is also true. Since we need to traverse through the loop for each y, which takes N time complexity and finding complementary x and z would take another traversal of the whole loop. So, the least possible time to solve the problem would require a minimum of  $N^2$  time complexity.