MAE 8 - Spring 2022
Final Exam
Form B
Time: 180 min

Instructions:

Open book, open note, open homework and midterm exam solution. No access to internet except for assessing and submitting exams in CANVAS.

Download **final_exam_template.zip** from CANVAS and use the files included in the folder for your final exam. Follow final exam template by filling in your name, id, hw_num = 'final' and form (e.g., form = 'B'). Put the code and answers in a MATLAB script named **final.m**. In problems 1, 3 and 4, you need to provide the code that produces the answers. Do not solve the problems by visual inspection.

Upon completion, create a zip archive named **final.zip** to include: **final.m**, **temperature.dat**, **survey.mat**, **SDweather.mat** and **spring_mass_damper.m**. Make sure all figures are plotted when your code is executed.

Submit **final.zip** in CANVAS before 6:00 PM on Monday June 6^{th}, 2022. Use double precision unless otherwise stated.

Problem 1 (20 points):

The data file **temperature.dat** contains temperature record (in Fahrenheit) in San Diego from 1852 to 2020. The data record has 13 columns: the first column indicates the year, and the next twelve columns are the monthly average temperature during that year. Load the file into MATLAB and perform the following exercises.

(a, b) In the year 1992, what was the warmest temperature? In what month(s) did it occur? Put the answers in **p1a** and **p1b**, respectively. Use 1 for Jan, 2 for Feb, 3 for Mar, etc.

(c, d) What is the warmest temperature throughout the record (all years)? In what year(s) did it occur? Put the answers in **p1c** and **p1d**, respectively.

(e) Between 1992 and 2012 (including these two years), in how many months was the temperature warmer than 71°F? Put the answer in **p1e**.

(f) Compute the seasonal average temperature during the autumn months (i.e. September, October and November) for each year. Put the answer in **p1f**.

(g) Create **figure 1** to include the following items:

- a solid black line to show how the autumn average temperature changes throughout the years.
- a red filled circle to indicate the year with the highest autumn temperature.
- a blue filled circle to indicate the year with the lowest autumn temperature.

Use marker size of 10. Remember to label the axes and include the legend and title. Set **p1g = 'See figure 1'**.

Problem 2 (20 points):

(a) Use **for** loop to evaluate the following series and put the answer in **p2a**.

$$\sum_{k=1}^{40}\sum_{l=1}^{50}\sum_{m=1}^{60}\frac{1}{2^k+2^l+2^m}$$

(b, c) The following series can be used to approximate the value of π:

$$\pi \approx 2\sum_{n=0}^{k}\frac{2^n n!^2}{(2n+1)!}$$

as the positive integer k approaches infinity. As k increases, the approximate value of π converges toward the true value. Find the smallest value of k such that the absolute error is less than 10^{-6}. Here, the absolute error is defined as the absolute difference between the approximate value and the MATLAB built-in value (i.e. **pi**). Put the approximate value of π in **p2b** and the value of k used to get the approximation in **p2c**.

(d) Consider the following two-dimensional function:

$$f(x,y) = exp\Big\{-\Big[cos(\frac{x}{2}) + sin(\frac{y}{3})\Big]^2\Big\}$$

where x = [-11:0.1:11] and y = [-12:0.1:12].

Create **figure 2** to include the following items:

- surface plot of f.
- blue filled circles to mark local maxima. A local maximum is defined as the point where f(x,y) is greater than or equal to the surrounding neighboring points. Ignore the local maxima at the edges of the x-y domain.

Use marker size of 5. Remember to label the axes and include the legend and title. Set **view(3)** and **p2d = 'See figure 2'**.

Problem 3 (20 points):

Load the data file **survey.mat** into MATLAB. The file contains data collected in the survey which you had taken during homework 5. The following 6 questions were asked:

1. What is your class level?
2. Do you have any MATLAB / coding experience prior to the course?
3. Which of the following lab sessions do you attend most frequently?

4. What grade do you expect for the course?

5. On average how many hours per week do you spend studying outside of class?

6. How difficult was the midterm?

The data file is a cell array with 155 elements. Each element corresponds one student's answers to the six questions. Note that, when students did not answer the question, the collected data is a string 'Null'. Use the cell array **survey** to perform the following exercises.

(a, b) Are the second student and last student on the survey both **Freshman**? Put the answer in **p3a**. How about the 16^{th} and the 146^{th} students? Put the answer in **p3b**.

(c) How many students thought the midterm was **Moderate**? Put the answer in **p3c**.

(d) How many **Freshman** thought the midterm was **Moderate**? Put the answer in **p3d**.

(e) Create **figure 3**. In the figure, create a bar plot to show the numbers of **Freshman**, **Sophomore**, **Junior** and **Senior** in the class. Also include a column for the number of students who did not answer this questions (e.g. **Null**). Remember to label the axes and include the title. Set **p3e = 'See figure 3'**.

<u>Problem 4</u> (20 points):

Load the data file **SDweather.mat** into MATLAB. The file contain the rainfall (in inches) and temperature (in Fahrenheit) data in San Diego from 1852 to 2020. The data is organized into a structure with the following 5 fields:

- year: a single number to indicate the year
- rainfall: a 12-element vector to store monthly average rainfall
- temperature: a 12-element vector to store monthly average temperature
- annual_rainfall_avg: a single number to store annual average rainfall
- annual_temperature_avg: a single number to store annual average temperature

Use the structure **SDweather** to perform the following exercises.

(a, b) What was the coldest temperature in the year 1970? In what month(s) of that year did it occur? Put the answers in **p4a** and **p4b**, respectively.

(c, d) Throughout the record, what was the highest monthly average rainfall? In what year(s) did it occur? Put the answers in **p4c** and **p4d**, respectively.

(e) Create **figure 4** to include the following items:

- Use a bar plot to show how the annual average rainfall changes over the years.
- Use magenta and green filled diamonds to mark the years with the most and least rainfall, respectively.

Use marker size of 10. Remember to label the axes and include the legend and title. Set **p4e = 'See figure 4'**.

<u>Problem 5</u> (20 points):

Consider a spring-mass-damper system. The system has a mass (m = 0.1 kg), a spring with spring constant (k = 40 N/m) and a damper which acts to slow down the motion of the mass via frictional effect. In this problem, you will perform simulations with different values of frictional damping factor (c in N·s/m).

The governing equations for the system are as follows:

$$\begin{aligned} \frac{dV}{dt} &= -\frac{k}{m}X - \frac{c}{m}V, \\ \frac{dX}{dt} &= V, \end{aligned}$$

where X is the displacement of the mass from equilibrium position, V is the velocity of the mass and T is time.

Using Euler-Cromer method, the governing equations can be transformed into the following algebraic equations:

$$\begin{aligned} V_{n+1} &= V_n - (\frac{k}{m}X_n + \frac{c}{m}V_n)\Delta t, \\ X_{n+1} &= X_n + V_{n+1}\Delta t, \\ T_{n+1} &= T_n + \Delta t, \end{aligned}$$

where subscript n denotes variables at current time, subscript n+1 denotes variables at a time that is Δt ahead.

Write function **spring_mass_damper.m** to numerically solve for the motion of the system. The function should have the following header: **function [T, X, V] = spring_mass_damper(c)**. The input is the frictional damper factor c. The outputs vectors **T**, **X** and **V** are the time, displacement and velocity of the mass, respectively. Set initial displacement Xo = 0.1 m and initial velocity Vo = 10 m s^{-1}. Use $\Delta t = 0.001$ s and simulate the system for a period of 2 s. Give the function a description.

Use the function **spring_mass_damper** to perform six numerical experiments with c = [0, 0.6, 1.2, 1.8, 2.4, 3.2] N·s/m. Use the experiment results in the following exercises.

(a) Create **figure 5** to plot the six trajectories (i.e. displacement versus time) corresponding to the different values of c's. Use solid lines with different colors to mark the trajectories. Remember to label the axes and include the legend and title. Set **p5a = 'See figure 5'**.

(b) Compute the frequency ω (in rad s^{-1}) of the system in each of the experiments. Here, $\omega = 2\pi/t_p$ where t_p is the time period between the first two local maxima of the displacement **X**. Put the answer in a 6-element vector **p5b**.