MAE 107
Assignment 1
Due 10:00pm, Friday, 14 April

Note: You must show all your work in order to get credit!

Problems to hand in (Not all problems may be graded.)

1. Construct the Taylor polynomials of order 0, 1 and 2 (i.e., $p_0(x)$, $p_1(x)$ and $p_2(x)$) for
$$f(x) = \frac{1}{\sin(x) + 2}$$
around $x_0 = \pi/6$. Sketch $f(x)$ and the three Taylor polynomials over $[0, \pi/3]$, all on the same graph. You can do this by hand or with the aid of a calculator or computer.

2. Construct the Taylor polynomials of order 3 for $f(x) = \exp(1 + x/2)$ around $x_0 = 0$. What is the actual error in the Taylor polynomial at $x = 1$ (i.e., $|f(1) - p_3(1)|$)? Obtain a bound on the error at $x = 1$ using the expression from class. Verify that the bound is greater than the actual error. Using the expression from class, obtain a single bound on the errors for all x in the interval from $[-2, 1]$, that is, the bound should be a single number that is greater than the actual error $|f(x) - p_3(x)|$ for any $x \in [-2, 1]$.

3. Suppose you will create a Taylor polynomial for $f(x) = \ln(4-x)$ around $x_0 = 1$ for use over the interval $[0, 2]$. Using the same error bounding method as in Problem 2, estimate the minimum order that is required such that the error in the Taylor polynomial is no greater than $\varepsilon = 0.005$ over $[0, 1]$. Similarly, what is the minimum order that is required such that the error in the Taylor polynomial is no greater than $\varepsilon = 0.005$ over $[1, 2]$? Lastly, what is the minimum required order in the case of the whole interval, $[0, 2]$? What is the corresponding Taylor polynomial for this last case, and what are the actual errors at $x = 0$, $x = 1$ and $x = 2$?

4. Using only what we have covered so far, estimate the number of FLOPS required to evaluate the final Taylor polynomial of Problem 3 at any

given $x \in [0, 2]$.

5. What is the asymptotic order of $f(x) = 5x^4 + \pi x^6 - 2x^3$ as $x \downarrow 0$? Show this using the definition given in class (or similarly equation (1.8) from the recommended book). What are C and δ? (Recall $f(x) = \mathcal{O}(x^n)$ as $x \downarrow 0$ if there exist $C < \infty$ and $\delta > 0$ such that $|f(x)| \leq Cx^n$ for all $x \in (0, \delta)$.) Draw a figure indicating the function and the asymptotic envelope $\pm Cx^n$, including δ.

Problems 1 and 4 are worth 5 points each. The other problems are worth 10 points each.

Study Problems (Will not be graded.)

- Consider the following variation on Problem 3,Suppose you will create a Taylor polynomial for $f(x) = \ln(4 - x)$ around $x_0 = 0$ for use over the interval $[0, 2]$. Estimate the minimum order that is required such that the error in the Taylor polynomial is no greater than $\varepsilon = 0.005$ over $[0, 2]$.