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Math 20D C01

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Exercise 4.1
a)

```
>> B = [1.2 2.5;4 0.7]

B =

    1.2000    2.5000
    4.0000    0.7000
```

b)

```
[eigvec, eigval] = eig(B)

eigvec =

    0.6501   -0.5899
    0.7599    0.8075

eigval =

    4.1221         0
         0   -2.2221
```

Exercise 4.2
a)

```
A =

     3     4
    -1    -2

>> [eigvec, eigval] = eig(A)
```

b)

```
eigvec =

    0.9701   -0.7071
```

-0.2425   0.7071

eigval =

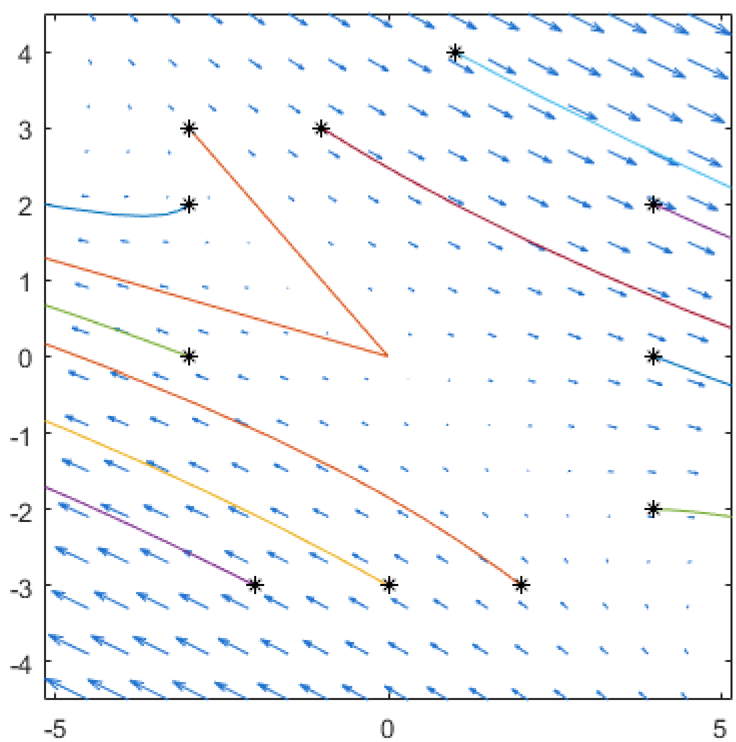
2   0
0   -1

c)

$$v(t) = c_1 e^{(2t)} \begin{pmatrix} 0.9701 \\ -0.2425 \end{pmatrix} + c_2 e^{(-t)} \begin{pmatrix} -0.7071 \\ 0.7071 \end{pmatrix} \quad : \quad v(t) = \begin{pmatrix} 0.9701 c_1 e^{2t} - 0.7071 c_2 e^{-t} \\ -0.2425 c_1 e^{2t} + 0.7071 c_2 e^{-t} \end{pmatrix}$$

As t gets large, the c1 component of the general solution will go towards infinity while the c2 portion will approach zero. When looking at the general solution, we can conclude that the solution set will approach negative infinity as x approaches infinity and positive infinity as x approaches negative infinity.

d)



The plot supports our previous conclusion that as t gets larger, the slope field is directed toward positive and negative infinity.

Exercise 4.3

a)

A =

```
    2.7000   -1.0000
    4.1000    3.7000
```

```
>> [B C] = eig(A)
```

B (eigenvectors) =

```
  -0.1093 + 0.4291i  -0.1093 - 0.4291i
   0.8966 + 0.0000i   0.8966 + 0.0000i
```

C (eigenvalues) =

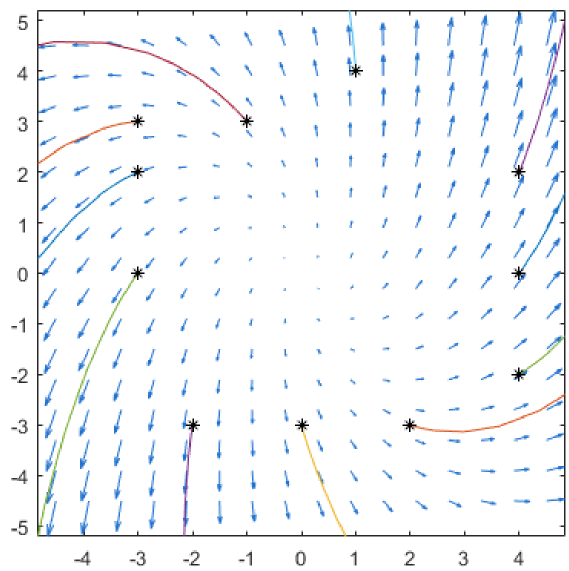
```
   3.2000 + 1.9621i   0.0000 + 0.0000i
   0.0000 + 0.0000i   3.2000 - 1.9621i
```

b)

$$v(t) = c_1 e^{(3.2+1.9621i)t}\begin{pmatrix} -0.1093 + 0.4291i \\ 0.8966 + 0 \cdot i \end{pmatrix} + c_2 e^{(3.2-1.9621i)t}\begin{pmatrix} -0.1093 - 0.4291i \\ 0.8966 + 0 \cdot i \end{pmatrix}$$

$$: \quad v(t) = \begin{pmatrix} c_1 e^{(3.2+1.9621i)t}(-0.1093 + 0.4291i) + c_2 e^{(3.2-1.9621i)t}(-0.1093 - 0.4291i) \\ 0.8966 c_1 e^{t(3.2+1.9621i)} + 0.8966 c_2 e^{t(3.2-1.9621i)} \end{pmatrix}$$

c)



When looking at the eigenvectors and eigenvalues, only the real numbers influence how the solution moves toward infinity. The eigenvector indicates the direction of the solution set while the eigenvalues determine the variance of the direction of the solution set.

Exercise 4.4

a)

A =

1.2500	-0.9700	4.6000
-2.6000	-5.2000	-0.3100
1.1800	-10.3000	1.1200

```
>> [B C] = eig(A)
```

B (eigenvectors) =

0.7351 + 0.0000i	0.4490 + 0.2591i	0.4490 - 0.2591i
-0.1961 + 0.0000i	-0.3375 + 0.2242i	-0.3375 - 0.2242i
0.6490 + 0.0000i	-0.7530 + 0.0000i	-0.7530 + 0.0000i

C (eigenvalues) =

```
   5.5698 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -4.1999 + 2.6606i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -4.1999 - 2.6606i
```

b) As we can see from the data set, the first eigenvalue is positive while the others are negative. Because the theorem states that our system is stable if and only if all eigenvalues are negative, our system will tend to infinity and will not be stable.

Exercise 4.5

a)

```
A =

   -0.0558   -0.9968    0.0802    0.0415
    0.5980   -0.1150   -0.0318         0
   -3.0500    0.3880   -0.4650         0
         0    0.0805    1.0000         0

>> B = [0.01; -0.175; 0.153; 0]

B =

    0.0100
   -0.1750
    0.1530
         0

>> [C D] = eig(A)

C (eigenvectors) =

   0.1994 - 0.1063i   0.1994 + 0.1063i  -0.0172 + 0.0000i   0.0067 + 0.0000i
  -0.0780 - 0.1333i  -0.0780 + 0.1333i  -0.0118 + 0.0000i   0.0404 + 0.0000i
  -0.0165 + 0.6668i  -0.0165 - 0.6668i  -0.4895 + 0.0000i  -0.0105 + 0.0000i
   0.6930 + 0.0000i   0.6930 + 0.0000i   0.8717 + 0.0000i   0.9991 + 0.0000i


D (eigenvalues) =

  -0.0329 + 0.9467i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0329 - 0.9467i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.5627 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0073 + 0.0000i
```

b)
The system is asymptotically stable, as every real eigenvalue is < 0.

c) The bottom component is the biggest at 0.9991. Because this value is in x4, this type of rotation in this eigenvector is most closely associated with the pitch rate.

Exercise 4.6

a)

```
>> F = [ 0 7 0 -1]

F =

     0     7     0    -1

>> B*F

ans =

         0    0.0700         0   -0.0100
         0   -1.2250         0    0.1750
         0    1.0710         0   -0.1530
         0         0         0         0
```

b)

```
>> F = [0 5 0 -0.1]

F =

         0    5.0000         0   -0.1000

>> B*F

ans =

         0    0.0500         0   -0.0010
         0   -0.8750         0    0.0175
         0    0.7650         0   -0.0153
         0         0         0         0
```

c)

F = [0 5 0 -0.1]

```
>> A + (B*F)

ans =

   -0.0558   -0.9468    0.0802    0.0405
    0.5980   -0.9900   -0.0318    0.0175
   -3.0500    1.1530   -0.4650   -0.0153
         0    0.0805    1.0000         0

[F G] = eig(A +(B*F))

F =

  -0.1906 + 0.0001i  -0.1906 - 0.0001i  -0.0197 + 0.0000i  -0.0873 + 0.0000i
  -0.0668 + 0.1126i  -0.0668 - 0.1126i  -0.0356 + 0.0000i  -0.0789 + 0.0000i
   0.2521 - 0.5971i   0.2521 + 0.5971i   0.0908 + 0.0000i  -0.5879 + 0.0000i
  -0.7256 + 0.0000i  -0.7256 + 0.0000i  -0.9950 + 0.0000i   0.8003 + 0.0000i


G =

  -0.3400 + 0.8104i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.3400 - 0.8104i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0883 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.7425 + 0.0000i
```

The eigenvalues are slightly different from A.

d)
Our yaw is in x2, we want the complex eigenvalues to have real part less than -0.2, and we want there to be a real eigenvalue within 0.02 of zero.
F = [0 2.5 0 -0.09],
eig(A+B*F) =

```
EigenVectors
   0.2072 - 0.0503i   0.2072 + 0.0503i  -0.0084 + 0.0000i   0.0451 + 0.0000i
   0.0015 - 0.1506i   0.0015 + 0.1506i  -0.0398 + 0.0000i   0.0368 + 0.0000i
  -0.1432 + 0.6539i  -0.1432 - 0.6539i   0.0192 + 0.0000i   0.5395 + 0.0000i
   0.6955 + 0.0000i   0.6955 + 0.0000i  -0.9990 + 0.0000i  -0.8400 + 0.0000i

EigenValues
  -0.2057 + 0.9227i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.2057 - 0.9227i   0.0000 + 0.0000i   0.0000 + 0.0000i
```

```
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0160 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.6458 + 0.0000i
```

A value that dampens yaw oscillation is approximately F = [0 2.5 0 -0.09]