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Math 20D C01

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Exercise 3.1
a)

Table for h = 0.2:

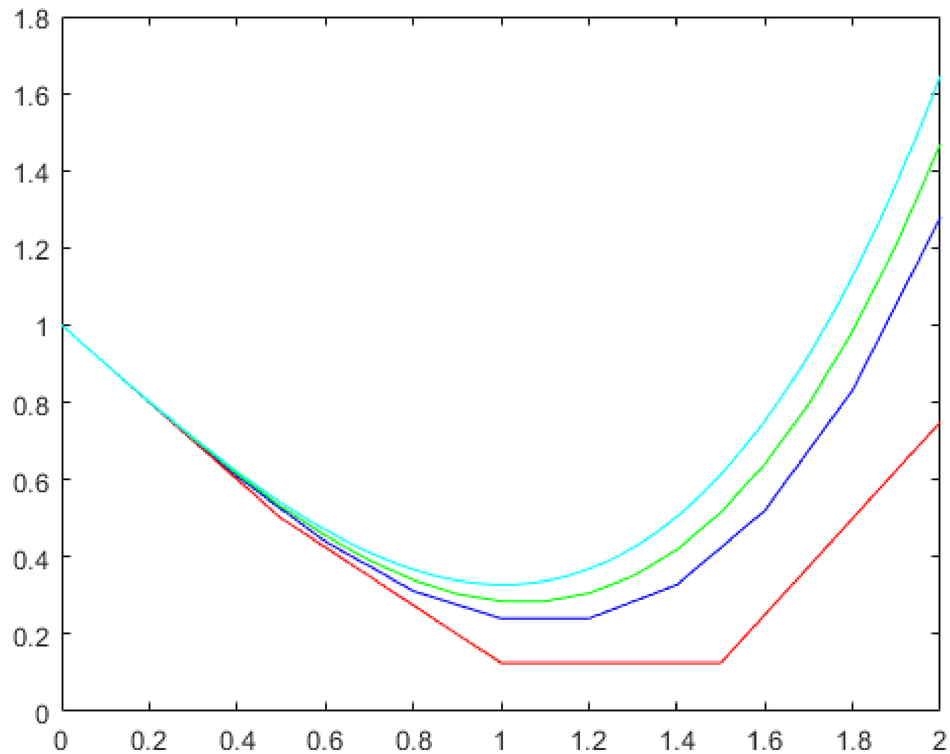
```
>> [x, y]

ans =

         0    1.0000
    0.2000    0.8000
    0.4000    0.6080
    0.6000    0.4400
    0.8000    0.3120
    1.0000    0.2400
    1.2000    0.2400
    1.4000    0.3280
    1.6000    0.5200
    1.8000    0.8320
    2.0000    1.2800
```

For h = 0.2 (b), the Euler function came up with y(1) = 0.2400, y(2) = 1.2800.

For h = 0.1 (g), the Euler function came up with y(1) = 0.2850, y(2) = 1.4700.

For h = 0.01 (c), the Euler function came up with y(1) = 0.3284, y(2) = 1.6467.

For the graph below, h = 0.5 (red), 0.2 (blue), 0.1 (green) and 0.01 (cyan).



b)

Hand worked solution:

dy/dx = (x^2-1)

dy = (x^2-1)dx

y = (x^3)/3 -x + C

Plugging in the initial condition, y(0) = 1, we get:

1 = C

So our specific solution is:

y = (x^3)/3 -x + 1

Matlab Solution:

```
>> dsolve('Dy=x^2-1', 'y(0)=1', 'x')

ans =

(x*(x^2 - 3))/3 + 1
```

Plugging in y(1) and y(2)

y(1) = 1/3 -1 + 1 = 1/3
y(2) = 8/3 - 2 + 1 = 5/3

c)

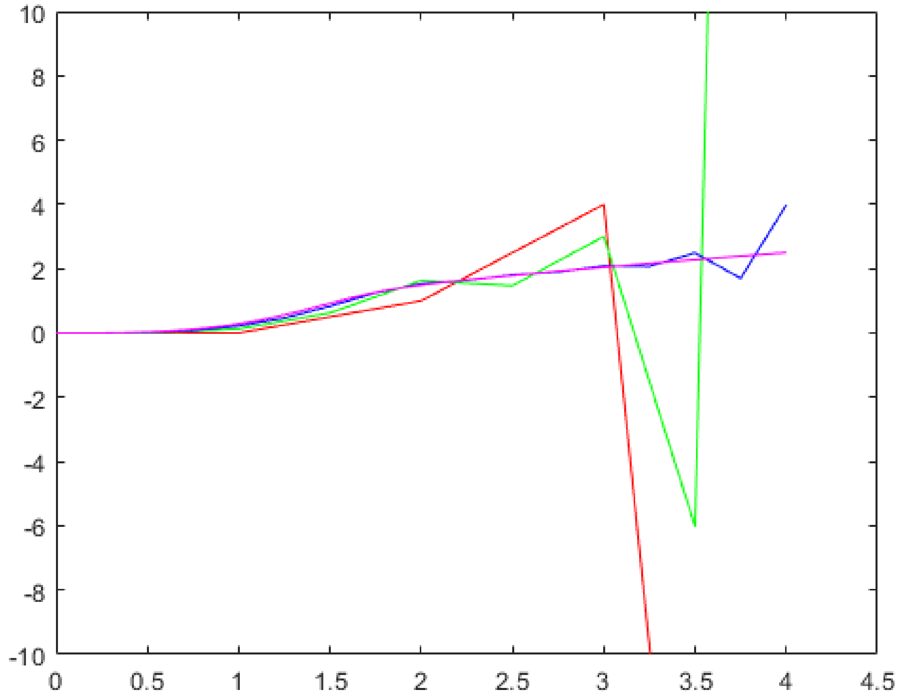
It appears that Euler's estimate gives better estimates as the value of h decreases. This is because, similar to an integral, lowering the step size makes the estimated slope closer to the true slope.

Exercise 3.2

Line (5): Initializes x with a starting point (x0).
Line (6): Initializes y with a starting point (y0).
Line (7): Begins a for loop with the interval i=2:nsteps (the amount of entries in the output)
Line (8): The next y value of the sequence is defined by adding the previous y value to the product of the slope of the derivative (at the previous x and y values) and the change in x (the step size).
Line (9): The next x value of the sequence is defined by the previous value of x plus the step size.
Line (10): Closes the for loop.

Exercise 3.3

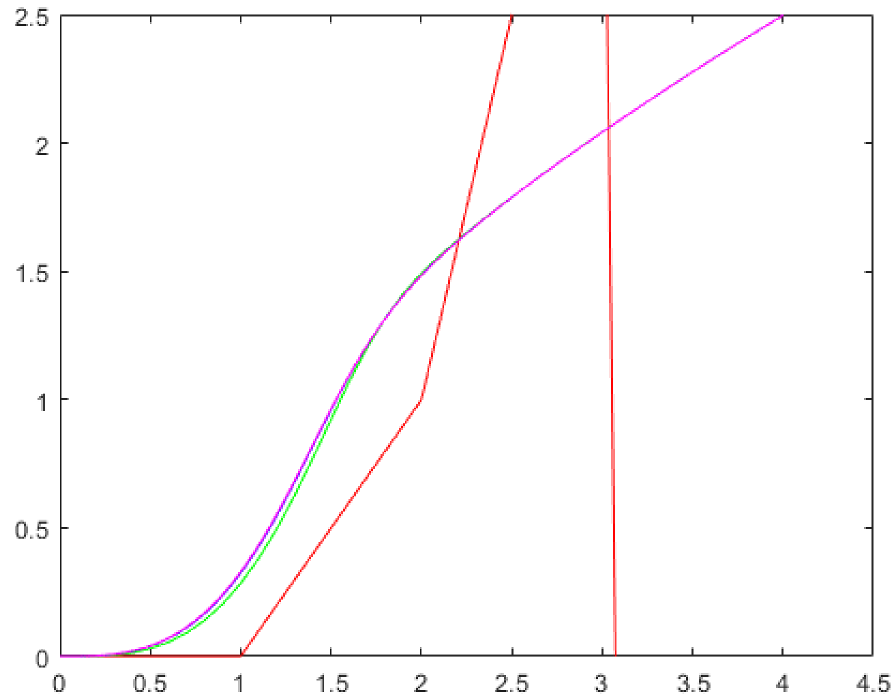
a)



Red: h =1, Green: h = 0.5, Blue: h = 0.25, Magenta: h = 0.1

Although it's likely not close to being the exact solution to the ODE, h = 0.1 is a fairly good representation of a solution to our function.

b)



Red: h =1, Green: h = 0.1, Blue: h = 0.01, Magenta: h = 0.001

The additional shrinking of h did not change the results significantly. This can be seen as the lines of h = 0.01 and 0.001 completely overlap. This would indicate that once we get to a step size of 0.01, our results become very accurate. Furthermore, the drawn solutions with h = 0.01 and h = 0.001 coincide with the direction field shown to us previously.

Exercise 3.4

a)

```
>> h = @(x) -cos(x) + 2;
[x, y, h(x), abs(y - h(x))]

ans =

        0    1.0000    1.0000         0
   0.2500    1.0311    1.0311    0.0000
   0.5000    1.1224    1.1224    0.0000
   0.7500    1.2683    1.2683    0.0000
   1.0000    1.4597    1.4597    0.0000
   1.2500    1.6848    1.6847    0.0001
   1.5000    1.9293    1.9293    0.0000
   1.7500    2.1782    2.1782    0.0001
   2.0000    2.4161    2.4161    0.0000
```

```
    2.2500    2.6282    2.6282    0.0001
    2.5000    2.8012    2.8011    0.0000
    2.7500    2.9243    2.9243    0.0000
    3.0000    2.9900    2.9900    0.0000
    3.2500    2.9941    2.9941    0.0000
    3.5000    2.9365    2.9365    0.0000
    3.7500    2.8206    2.8206    0.0000
    4.0000    2.6536    2.6536    0.0000
    4.2500    2.4460    2.4461    0.0001
    4.5000    2.2108    2.2108    0.0000
    4.7500    1.9625    1.9624    0.0001
    5.0000    1.7163    1.7163    0.0000
    5.2500    1.4879    1.4879    0.0001
    5.5000    1.2913    1.2913    0.0000
    5.7500    1.1388    1.1388    0.0000
    6.0000    1.0398    1.0398    0.0000
    6.2500    1.0006    1.0006    0.0000
    6.5000    1.0234    1.0234    0.0000
    6.7500    1.1070    1.1070    0.0000
    7.0000    1.2461    1.2461    0.0000
    7.2500    1.4321    1.4321    0.0001
    7.5000    1.6534    1.6534    0.0000
    7.7500    1.8961    1.8962    0.0001
    8.0000    2.1455    2.1455    0.0000
    8.2500    2.3858    2.3857    0.0001
    8.5000    2.6020    2.6020    0.0000
    8.7500    2.7808    2.7808    0.0000
    9.0000    2.9111    2.9111    0.0000
    9.2500    2.9848    2.9848    0.0000
    9.5000    2.9972    2.9972    0.0000
    9.7500    2.9476    2.9476    0.0000
   10.0000    2.8391    2.8391    0.0000
```

The functions are nearly identical to each other. We find that we have at most 0.0001 unit error between our data at any given point. Or in other words, the difference between the two is negligible using the ode45 function.

b)

```
>> [x, z] = Euler(0.25, 0, 1, 10, g);
[y, z, h(x), abs(y - h(x)), abs(z - h(x))]

ans =

    1.0000    1.0000    1.0000         0         0
```

1.0311	1.0000	1.0311	0.0000	0.0311
1.1224	1.0619	1.1224	0.0000	0.0606
1.2683	1.1817	1.2683	0.0000	0.0866
1.4597	1.3521	1.4597	0.0000	0.1076
1.6848	1.5625	1.6847	0.0001	0.1222
1.9293	1.7997	1.9293	0.0000	0.1295
2.1782	2.0491	2.1782	0.0001	0.1291
2.4161	2.2951	2.4161	0.0000	0.1210
2.6282	2.5224	2.6282	0.0001	0.1057
2.8012	2.7169	2.8011	0.0000	0.0842
2.9243	2.8666	2.9243	0.0000	0.0577
2.9900	2.9620	2.9900	0.0000	0.0280
2.9941	2.9973	2.9941	0.0000	0.0031
2.9365	2.9702	2.9365	0.0000	0.0338
2.8206	2.8825	2.8206	0.0000	0.0620
2.6536	2.7396	2.6536	0.0000	0.0860
2.4460	2.5504	2.4461	0.0001	0.1043
2.2108	2.3267	2.2108	0.0000	0.1159
1.9625	2.0823	1.9624	0.0001	0.1199
1.7163	1.8325	1.7163	0.0000	0.1161
1.4879	1.5927	1.4879	0.0001	0.1048
1.2913	1.3780	1.2913	0.0000	0.0867
1.1388	1.2016	1.1388	0.0000	0.0628
1.0398	1.0745	1.0398	0.0000	0.0347
1.0006	1.0047	1.0006	0.0000	0.0041
1.0234	0.9964	1.0234	0.0000	0.0270
1.1070	1.0502	1.1070	0.0000	0.0568
1.2461	1.1627	1.2461	0.0000	0.0834
1.4321	1.3269	1.4321	0.0001	0.1051
1.6534	1.5327	1.6534	0.0000	0.1207
1.8961	1.7672	1.8962	0.0001	0.1290
2.1455	2.0159	2.1455	0.0000	0.1296
2.3858	2.2632	2.3857	0.0001	0.1226
2.6020	2.4938	2.6020	0.0000	0.1082
2.7808	2.6935	2.7808	0.0000	0.0874
2.9111	2.8497	2.9111	0.0000	0.0615
2.9848	2.9527	2.9848	0.0000	0.0321
2.9972	2.9962	2.9972	0.0000	0.0010
2.9476	2.9774	2.9476	0.0000	0.0298
2.8391	2.8975	2.8391	0.0000	0.0584

When comparing Euler's method to the real solution, we can see that there is significant error in our data set. And so, because the ode45 function is more accurate, we can conclude that ode45 is a more reliable method of getting solutions to an ODE.