Kasey Haman

A16978114

Math 18 B01

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Exercise 4.1:

a)

```
>> A = [8 11 2 8; 0 -7 2 -1; -3 -7 2 1; 1 1 2 4]

A =

     8    11     2     8
     0    -7     2    -1
    -3    -7     2     1
     1     1     2     4

>> B = [1 -2 0 5; 0 7 1 5; 0 4 4 0; 0 0 0 2]

B =

     1    -2     0     5
     0     7     1     5
     0     4     4     0
     0     0     0     2


>> det(A+B)

ans =

     0

>> det(A-B)

ans =

        1038

>> det(A*B)

ans =
```

```
   3.6480e+03

>> det(inv(A))

ans =

    0.0132

>> det(B')

ans =

    48
```

b) For a matrix to be invertible, its determinant must not be zero. As we can see, however, the determinant of A+B is zero. Therefore, it's not invertible.

c) If we knew the determinant of A and B, we can use determinant theorems to find det(B'), det(inv(A)), and det(A*B):

- det(B) = det(B')
- det(inv(A)) = 1/det(inv(A)).
- det(A)*det(B) = det(A*B)

Exercise 4.2:

```
>> N = [0.003 0.02 0; 0.1 1 0; 0 0 0.015]

N =

    0.0030    0.0200         0
    0.1000    1.0000         0
         0         0    0.0150

>> det(N^100)

ans =

 -1.7211e-201
```

When computing the determinant in Matlab, we find that the output is a value extremely close to, but not equal to zero. Because the value is not zero, the matrix should still have a determinant.

After computing the determinant by hand, we still are left with a nonzero value. Therefore, I would support my initial claim that the matrix should have a determinant.

Exercise 4.3:

a)

```
V = [-8 6 -6 -30; 3 9 12 -10; 3 -6 -1 18; 3 0 4 7]

V =

    -8     6    -6   -30
     3     9    12   -10
     3    -6    -1    18
     3     0     4     7

>> [P,D] = eig(V)

P =

    0.8660    0.8706    0.7612    0.6195
   -0.2887    0.0725    0.4087   -0.7088
   -0.2887   -0.4353   -0.5014    0.0383
   -0.2887   -0.2176   -0.0463   -0.3353


D =

    2.0000         0         0         0
         0    3.0000         0         0
         0         0    1.0000         0
         0         0         0    1.0000
```

b) The matrix V is invertible, as the matrix of eigenvalues, D, has a diagonal of non-zero elements.

c)

```
>> inv(P)*V*P

ans =

    2.0000   -0.0000   -0.0000   -0.0000
    0.0000    3.0000    0.0000    0.0000
   -0.0000   -0.0000    1.0000   -0.0000
```

```
   0.0000    0.0000    0.0000    1.0000
```

By computing P^-1*V*P, we are given our eigenvalue matrix (equivalent to our calculated matrix D).

Exercise 4.4:

a)

```
>> F = [0 1; 1 1]

F =

     0     1
     1     1

>> [P, D] = eig(F)

P =

  -0.8507    0.5257
   0.5257    0.8507


D =

  -0.6180         0
        0    1.6180
```

b)

```
>> F^10

ans =

    34    55
    55    89

>> P*D^10*(inv(P))

ans =

   34.0000   55.0000
   55.0000   89.0000
```

As we can see, they are the same matrix.

c)
```
>> f = [1 1]'

f =

     1
     1
```

After computing the values of F*f, F^2*f, F^3*f, F^4*f and F^5*f, I found that the vectors share a resemblance to the fibonacci sequence. The element in the second row of each vector is the addition of the elements in the previous vector. Furthermore, The element in the first row of each vector is equal to the element in the second row of the previous vector.

d)

```
>> F^29*f

ans =

      832040
     1346269
```

We find that our value for f(30) is equal to 832040.

Exercise 4.5:

```
>> P = [0.8100 0.0800 0.1600 0.1000;
0.0900 0.8400 0.0500 0.0800;
0.0600 0.0400 0.7400 0.0400;
0.0400 0.0400 0.0500 0.7800]

P =

    0.8100    0.0800    0.1600    0.1000
    0.0900    0.8400    0.0500    0.0800
    0.0600    0.0400    0.7400    0.0400
    0.0400    0.0400    0.0500    0.7800

>> x0 = [0.5106; 0.4720; 0.0075; 0.0099]

x0 =
```

```
    0.5106
    0.4720
    0.0075
    0.0099

>> [Q D] = eig(P)

Q =

    0.6656    0.7676    0.5432   -0.4641
    0.6165   -0.2841   -0.8148   -0.1254
    0.2946   -0.5682    0.1811   -0.2508
    0.3001    0.0848    0.0905    0.8402


D =

    1.0000         0         0         0
         0    0.6730         0         0
         0         0    0.7600         0
         0         0         0    0.7370
```

b) Because there is a 1 in the first column, and every other column has a value less than one, we know that the limit of D^n as n tends to infinity is a 1 in the first row of the first column with zeros in the rest of the matrix.

c)

```
>> L = D^1000

L =

    1.0000         0         0         0
         0    0.0000         0         0
         0         0    0.0000         0
         0         0         0    0.0000

>> Pinf = Q*L*inv(Q)

Pinf =

    0.3547    0.3547    0.3547    0.3547
    0.3285    0.3285    0.3285    0.3285
    0.1570    0.1570    0.1570    0.1570
```

```
    0.1599    0.1599    0.1599    0.1599
```

d)

```
>> Pinf*x0

ans =

    0.3547
    0.3285
    0.1570
    0.1599
```

When comparing this to our answer from last lab:

```
>> (P^100)*x0

ans =

    0.3547
    0.3285
    0.1570
    0.1599
```

They are the same.

e)

```
>> y = [0.2; 0.5; 0.1; 0.2]

y =

    0.2000
    0.5000
    0.1000
    0.2000

>> Pinf*y

ans =

    0.3547
    0.3285
    0.1570
```

0.1599

We can see that despite the change in our initial values, our final values stay the same. This makes mathematical sense, as each element in our rows of Pinf are the same. Because of this, when we multiply by any input vector that adds up to one, we will get the same number. This is because they are all being multiplied by the same proportion and then added up together.

Exercise 4.6

```
L = [0,0,0,0,1,0,0,0;
0,0,0,0,0,0,0,1;
0,1/2,0,0,0,0,1,0;
1/2,0,1/2,0,0,0,0,0;
0,0,1/2,0,0,1,0,0;
1/2,0,0,0,0,0,0,0;
0,1/2,0,0,0,0,0,0;
0,0,0,1,0,0,0,0;]

e0 = [1; 1; 1; 1; 1; 1; 1; 1]
```

a)

```
>> e10 = L^10*e0

e10 =

    1.0625
    1.1250
    1.1875
    1.1250
    1.1719
    0.5781
    0.5938
    1.1562

>> L^100*e0-L^99*e0

ans =

   1.0e-07 *

   -0.2287
   -0.1097
    0.0777
```

-0.0002
0.1844
-0.0117
-0.0185
0.1066

At a value of n=99, each progressive value will produce significantly small changes to e of n.

b)
We can see by the rows of L that C will point outward to websites B and G and by the columns of row L that C will be pointed to by websites D and E.