Problem 1.

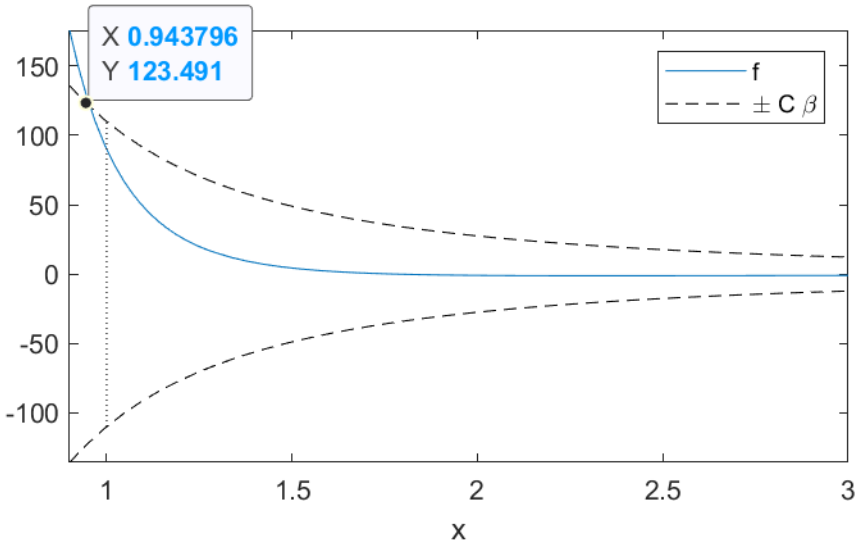
Solution. $f = \mathcal{O}(x^{-2})$.

$$|f(x)| = |\frac{100}{x^6} - \frac{10}{x^2}| \leq \frac{100}{x^6} + \frac{10}{x^2}.$$

Since $x^6 > x^2$ (equivalently $x^{-6} < x^{-2}$) for $x > 1$. Choosing $R = 1$, we obtain

$$|f(x)| \leq \frac{100}{x^2} + \frac{10}{x^2} = 110x^{-2}.$$

We see that $C = 110$ is a valid choice.



(Vertical line showing $R = 1$)

Problem 2.

Solution.

$$D_h(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} = \frac{2\sin(\frac{\pi}{3} + \frac{\pi}{6}) - 2\sin(\frac{\pi}{3})}{\frac{\pi}{6}} \approx 0.511\,745,$$

$$\hat{D}_h(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h} = \frac{2\sin(\frac{\pi}{3} + \frac{\pi}{6}) - 2\sin(\frac{\pi}{3} - \frac{\pi}{6})}{2 \times \frac{\pi}{6}} \approx 0.954\,930.$$

$$f'(x_0) = 2\cos\left(\frac{\pi}{3}\right) = 1, \quad f''(x_0) = -2\sin\left(\frac{\pi}{3}\right) = -\sqrt{3}, \quad f'''(x_0) = -2\cos\left(\frac{\pi}{3}\right) = -1.$$

$$e_h = |D_h(x_0) - f'(x_0)| \approx 0.488255 \quad > \frac{|f''(x_0)|}{2}h \approx 0.453\,450.$$

$$\hat{e}_h = |\hat{D}_h(x_0) - f'(x_0)| \approx 0.045070 \quad < \frac{|f'''(x_0)|}{6}h^2 \approx 0.045\,693.$$