

MAE 107
Assignment 2
Due 10:00pm, Saturday, 22 April

Note: You must show all your work in order to get credit!

Problems to hand in (Not all problems may be graded.)

1. What is the order as $x \rightarrow \infty$ of $f(x) = \frac{100}{x^6} - \frac{10}{x^2}$? Show this using the definition given in class. What are C and R ? (Recall $f(x) = \mathcal{O}(x^{-n})$ as $x \rightarrow \infty$ if there exist $C < \infty$ and $R < \infty$ such that $|f(x)| \leq Cx^{-n}$ for all $x \in (R, \infty)$.) By hand, draw a figure indicating the function and the asymptotic envelope $\pm Cx^{-n}$, including R .
2. With the aid of a calculator, compute $D_h(x_0)$ and $\hat{D}_h(x_0)$ for $f(x) = 2\sin(x)$ at $x_0 = \pi/3$, with $h = \pi/6$, where

$$D_h(x_0) = \frac{f(x_0 + h) - f(x_0)}{h}, \quad \hat{D}_h(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}.$$

In each case, what is the resulting error? Compare these errors with the *approximate* error bounds obtained in class.

Problem 1 is worth 5 points. Problem 2 is worth 10 points.

Study Problems (Will not be graded.)

- What is the order as $x \rightarrow \infty$ of $f(x) = \frac{7}{x^5 + \pi x^4} + \frac{7}{x^2 + \pi x^3}$?
- Rewrite the polynomial

$$f(x) = 3/7 + \sqrt{5}x + 9/7x^2 + 2\sqrt{5}x^3 + 15/7x^4 + 6\sqrt{5}x^5$$

using the Horner's rule form as given in class. Do you need to perform a square-root operation in the code? What is the minimal required number of FLOPS? For concreteness, use a massively simplified model of the machine in which it stores floating point numbers with exactly 7 decimal digits.

- Suppose you would like to compute $e^{(2+x)}$ for any $x \in [-1, 1]$ so that the error is no greater than 0.005. Using a Taylor polynomial and Horner's method, try to compose some pseudo code that would achieve this with the minimal, or at least close to the minimal, number of FLOPS.