MAE 107
Assignment 6
Due 10:00pm, Sunday, 4 June

Note: You must show all your work (including your codes) in order to get credit!

Problems to hand in (Not all problems may be graded.)

1. Write a matlab function which performs the fixed-point iteration for solution of n equations in n unknowns, where n may be greater than one. This function should call another function that evaluates the right-hand sides of the equations, which must be in the correct form for the fixed-point method. Apply your code to find a solution of the set of equations

$$\begin{aligned} x_1 &= \tfrac{1}{4}\sin(x_1 + x_2 + \arctan(x_3/2)), \\ x_2 &= \tfrac{1}{4}\cos(x_1 + x_2 + \arctan(x_3/2)), \\ x_3 &= 1 + \tfrac{1}{4}\cos(x_1 + \arctan(x_3/2)). \end{aligned}$$

You should do this for two different choices of initial conditions,

$$X^0 = \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad X^0 = \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix} = \begin{pmatrix} 300 \\ 200 \\ 100 \end{pmatrix}.$$

You may set up your function stop at a fixed number of iterations, rather than employing a more complex stopping condition. In each case, what are the resulting next three iterations, i.e., X^1, X^2 and X^3?. Also, after running the code for as many iterations as you like, provide an estimate of the actual solution to within 10^{-4}. You do not need to indicate all the iterations that led you to this estimate.

2. We have taken data on a system that we expect to behave according to

$$x(t) = a_0 + a_1 t + a_2 t^2,$$

where the coefficients a_0, a_1 and a_2 are unknown. The data is

$$\begin{aligned}
(t_0, x_0) &= (1.0, -0.9), \\
(t_1, x_1) &= (3.0, 1.7), \\
(t_2, x_2) &= (4.0, 2.1), \\
(t_3, x_3) &= (5.0, 0.2), \\
(t_4, x_4) &= (7.0, -0.7).
\end{aligned}$$

Use least-squares to estimate a_0, a_1 and a_2. Plot the data points and the resulting quadratic function, all on the same graph. You are free to solve the problem with either matlab or a calculator.

Problems 1 and 2 are worth 10 points each.

Study Problems (Will not be graded.)

- Suppose you are running the fixed-point method to solve $x = g(x)$. Starting from $x_0 = 0$, you obtained $x_1 = 100$. Suppose you know $|g'(x)| \leq 0.9$ for all x. What is the minimum n such that you can guarantee the error in x_n will be less than 10^{-8}?

- Consider the initial value problem

$$\begin{aligned}
\dot{y}(t) &= 2 + y(t), \\
y(0) &= 0,
\end{aligned}$$

  and suppose you wish to solve it up to time $T = 1$. Apply both the second-order Runge-Kutta method (RK2) with parameters $\beta_1 = \beta_2 = 1/2$, $\alpha = \nu = 1$, and the second-order Runge-Kutta method (RK2)with parameters $\beta_1 = 1/4$, $\beta_2 = 3/4$, $\alpha = \nu = 2/3$ to obtain approximate solutions. Use $n = 2$ steps, and obtain the resulting approximations to the solution at $T = 1$. What is the resulting error in each case?