MAE 8 - Fall 2021
Final Exam
Form A
Time: 180 min

Instructions: Open book, open note, open homework solution. Limited access to internet. Follow the homework solution template by filling in your name, id, hw_num = 'final' and form (e.g., form = 'A'). Put the code and answers in a MATLAB script named **final.m**. Create a zip archive named **final.zip** to include: **final.m** and **soccer.m**. Make sure all three figures are plot when your code is executed. Submit **final.zip** in CANVAS before 2:35 PM on Thursday December 9^{th}, 2021. Use double precision unless otherwise stated.

Problem 1 (20 points):

A three-dimensional floral pattern can be created using the following expressions:

$$
\begin{aligned}
x &= cos(4\theta)\,cos(\theta) \\
y &= cos(4\theta)\,sin(\theta) \\
z &= cos(8\theta)
\end{aligned}
$$

for $-10 \leq \theta \leq 350°$.

(a) Create a vector **theta** to include values from -10 to 350 with a consecutive difference of 0.1°. Use the expressions above to obtain the values for vectors **x**, **y** and **z**. Create **figure 1** to plot the pattern. The figure must include the following items:

- Use magenta solid line to mark the curve.
- Use green solid circle marker to mark the tips of the petals.
- Remember to provide axis labels, title and legend.
- Put the figure in 3-dimensional view by setting **view(45,60)**.

Set **p1a='See figure 1'**.

(b) Use forward finite-difference approximation to estimate the derivative dx/dθ at $\theta =$ 90°. Put the answer in **p1b**.

(c) Estimate the arc length of the three-dimensional curve. Approximate the arc length with straight lines between consecutive points. Put the answer in **p1c**.

Problem 2 (20 points):

Use the following code to generate **cellA** and **structA**:

```
rng(int8(form));
for n = 1:3
    for m = 1:3
        cellA{n,m} = randi(100,n,m);
        structA(n).field1 = char(80+[1:(3*n)]);
        structA(n).field2 = randi(100,n,m);
    end
end
```

(a) Consider the matrix on the 3^{rd} row and 2^{nd} column of **cellA**. Find the number on the 2^{nd} row and 2^{nd} column of that matrix. Put the answer in **p2a**.

(b) Consider the matrix on the 3^{rd} row and 3^{rd} column of **cellA**. Find the average value of that matrix. Put the answer in **p2b**.

(c) Consider all elements of **cellA**. Find the largest number and put it in **p2c**.

(d) What is the last character of **field1** in the second element of structure **structA**? Put the answer in **p2d**.

(e) Consider the matrix on the **field2** in the third element of structure **structA**. What is the number on the 2^{nd} row and 2^{nd} column? Put the answer in **p2e**.

(f) Copy **structA** into **p2f**. Modify all elements of **p2f** such that the 3^{rd} character on **field1** is lower case.

Problem 3 (20 points):

Consider the function f(x) in the interval $-20 \leq x \leq 20$:

$$f(x) = cos(2x)\, tanh(\frac{x}{10})$$

Use **for** loops and **if** statements to perform the following exercises.

(a, b) Compute f(x) for x = [-20:0.1:20]. Put **x** into **p3a** and **f** into **p3b**.

(c) Use trapezoid method to approximate the integral: $\int_{-20}^{20} f(x)\, dx$. Put the answer in **p3c**.

(d - f) How many local maxima does f(x) have (excluding the end values of f)? Put the answer in **p3d**. Find the **x** values and **f** values of the maxima and put the answers in **p3e** and **p3f**, respectively. **Do not use function islocalmax or islocalmin**.

(g - i) How many times does f(x) cross zero value? Put the answer in **p3g**. Find the **x** values and **f** values right before f(x) crosses zero value (or f(x) = 0) and put the answers in **p3h** and **p3i**, respectively.

(j) Make **figure 2** to include a solid black line for f(x) and different markers for the maxima and zero crossing. Label the axes and give title and legend. Set **p3j='See figure 2'**.

Problem 4 (15 points):

(a) Use **for** loop to compute the following series:

$$\sum_{m=1}^{100}\sum_{n=1}^{100}\frac{1}{2^{m\cdot n}}$$

Put the answer in **p4a**.

(b) On day 1, I put \$1 on my saving account. On day 2, I put \$3. On the next day, I continue to put three times the amount on the previous day. After how many days would I have at least \$10,000 on my account. Put the answer in **p4b**.

(c) Use **while** loop to find the smallest integer value for k that satisfies the following inequality equation:

$$\left|\pi - 4\sum_{n=0}^{k}\frac{(-1)^n}{2n+1}\right| \leq 10^{-3}$$

Put the answer in **p4c**.

Problem 5 (25 points):

Consider a soccer ball being kicked into the air with given initial velocity and rate of spinning. In this problem, you are to simulate the trajectory of the ball to examine how spinning affects the trajectory. The ball has a mass m = 0.4 kg, a radius r = 0.11 m. The simulation should produce the trajectory of the ball in Cartesian coordinates (X, Y, Z) and the corresponding velocity components (U, V, W) until the ball lands on the ground.

Using Euler method, the projectile of the ball can be computed using the following equations:

$$\begin{aligned}
U_{n+1} &= U_n + C_m\frac{\rho A r}{2m}\omega V_n \Delta t, \\
V_{n+1} &= V_n - C_m\frac{\rho A r}{2m}\omega U_n \Delta t, \\
W_{n+1} &= W_n - g\Delta t, \\
X_{n+1} &= X_n + U_n \Delta t, \\
Y_{n+1} &= Y_n + V_n \Delta t, \\
Z_{n+1} &= Z_n + W_n \Delta t.
\end{aligned} \tag{1}$$

where subscript n denotes variables at current time, subscript n+1 denotes variables at time that is Δt ahead. In these equations, ρ = 1.2 kg/m^3 is air density, A = πr^2 is the contact area of the ball, g = 9.81 m/s^2 is gravity , C$_m$ = 0.6 is the Magnus coefficient which controls the effect of spinning, and ω is the rate of spinning around z-axis in radians per second.

Write function **soccer** to simulate the motion of the ball. The function should have the following header: **[T, X, Y, Z, U, V, W] = soccer(omega)** where input **omega** is the rate of spinning. The outputs are time vectors **T**, the position vectors (**X**, **Y**, **Z**) and the velocity component vectors (**U**, **V**, **W**).

Use Δt = 0.001 s, **omega** = -10 rad/s and the following initial conditions to simulate the trajectory:

Xo = 0 m	Yo = 0 m	Zo = 0 m
Uo = 10 m/s	Vo = -20 m/s	Wo = 10 m/s

(a) Put the landing time in **p5a**.

(b) Put the landing position (X, Y, Z) of the ball in a 3-element vector **p5b**.

(c) Put the landing velocity components (U, V, W) of the ball in a 3-element vector **p5c**.

(d) Repeat the simulation for **omega** = -20, -30, -40, -50, -60 and -70 rad/s. Create **figure 3** to plot the trajectories from all simulations. The figure must include the following items:

- Use solid lines to show the seven trajectories. Use different colors to indicate the different simulations.
- Use solid circle markers to mark the landing locations. The marker should have the same color as the solid lines to denote results from the same simulation.

The figure must include axis labels, legend and title. The legend box doesn't need to include the landing the position. Set **p5d = 'See figure 3'**.