MAE 8 - Spring 2022 Project
The Dynamics of Bungee Jumping

The objective of the project is to simulate the trajectories of a person jumping off a cliff while connecting to a bungee cord. The dynamics of the jump is influenced by the tension of the elastic cord, the weight of the person and air frictional drag exerted on the body. You will perform various numerical experiments with different weight, spring constant and length of the chord to examine the safety of the jump.

The trajectory of the jump can be described by the following equations:

$$
\begin{aligned}
\frac{\partial U}{\partial t} &= -\frac{k}{m}\left(\frac{r-l}{r}\right)X - \frac{C_d \rho A V_{mag}}{2m}U, \\
\frac{\partial V}{\partial t} &= -\frac{k}{m}\left(\frac{r-l}{r}\right)Y - \frac{C_d \rho A V_{mag}}{2m}V, \\
\frac{\partial W}{\partial t} &= -\frac{k}{m}\left(\frac{r-l}{r}\right)Z - \frac{C_d \rho A V_{mag}}{2m}W - g, \\
\frac{\partial X}{\partial t} &= U, \\
\frac{\partial Y}{\partial t} &= V, \\
\frac{\partial Z}{\partial t} &= W,
\end{aligned}
\tag{1}
$$

where

$$
r = \sqrt{X^2 + Y^2 + Z^2},
$$
$$
V_{mag} = \sqrt{U^2 + V^2 + W^2}.
$$

Here, t is time (in second); X, Y, and Z are position (in meter) in Cartesian coordinate; and U, V, and W are the velocity components (in meter per second) in the x, y and z directions, respectively.

The parameters in the equations 1are defined as follows:

- m: Mass of the person in kilogram, to be varied in experiments
- k: Spring constant in Newton per meter, to be varied in experiments
- l: Length of the chord in meter, to be varied in experiments
- $C_d = 0.1$: Coefficient of drag due to air friction acted on the person
- $A = \pi$: Projected area of the person in flight in squared meter
- $\rho_a = 1.2$: Air density in kilogram per cubic meter

- $g = 9.81$: Gravity in meter per squared second

The following quantities will be of interest while analyzing the jump:

- $V_{mag} = \sqrt{U^2 + V^2 + W^2}$: Speed of the person in meter per second
- $Acc = dV_{mag}/dt$: Acceleration in meter per squared second
- $KE = 0.5mV_{mag}^2$: Kinetic energy in Joule

A total of eighteen experiments are to be performed to explore the effect of mass (Exp. # 1-6), spring constant (Exp. # 7-12), and cord length (Exp. # 13-18). In each experiment, the mass (m), spring constant (k) , cord length (l) as well as the initial position (Xo, Yo, Zo) and initial velocity component (Uo, Vo, Wo) of the person are stored in the data file: **bungee_data.txt**. The togography of the terrain (x_terrain, y_terrain, h_terrain) is stored in **terrain.mat**. A demo of how to plot the terrain is given in **terrain_demo.m**. Download the three files from CANVAS.

Using Euler-Cromer method, equation 1 can be transformed into the following algebraic form:

$$\begin{aligned}
r_n &= \sqrt{X_n^2 + Y_n^2 + Z_n^2}, \\
V_{mag,n} &= \sqrt{U_n^2 + V_n^2 + W_n^2}, \\
U_{n+1} &= U_n - \left[\frac{k}{m}\left(\frac{r_n - l}{r_n}\right) X_n + \frac{C_d \rho A V_{mag,n}}{2m} U_n\right] \Delta t, \\
V_{n+1} &= V_n - \left[\frac{k}{m}\left(\frac{r_n - l}{r_n}\right) Y_n + \frac{C_d \rho A V_{mag,n}}{2m} V_n\right] \Delta t, \\
W_{n+1} &= W_n - \left[\frac{k}{m}\left(\frac{r_n - l}{r_n}\right) Z_n + \frac{C_d \rho A V_{mag,n}}{2m} W_n + g\right] \Delta t, \\
X_{n+1} &= X_n + U_{n+1} \Delta t, \\
Y_{n+1} &= Y_n + V_{n+1} \Delta t, \\
Z_{n+1} &= Z_n + W_{n+1} \Delta t, \\
T_{n+1} &= T_n + \Delta t,
\end{aligned} \tag{2}$$

where subscript n denotes variables at current time, subscript n+1 denotes variables at time that is Δt ahead.

For this project, you are to write three MATLAB files: **bungee.m**, **read_input.m** and **project.m**. The description of these files are given below.

- File #1, **bungee.m**: This function solves the equation 2 for the jump trajectory from a given set of input parameters. Use the time step $\Delta t = 0.02\,s$. The function should have the following header: **function [T, X, Y, Z, U, V, W, safety] = rocket(m, k, l, Xo, Yo, Zo, Uo, Vo, Wo)** where the inputs are the mass, spring constant, cord length, initial position and velocity, respectively. The outputs are time, position and velocity of the person as well as the safety outcome of the experiment. All inputs are scalars while the outputs except **safety** are vectors. The output **safety** is either a logical true when the person is safe or a logical false when the person lands on the terrain. You need to simulate the trajectory for a duration of 120 s or until the person lands on the terrain. Consider MATLAB built-in function **interp2** to identify the landing location. The **terrain_demo.m** includes an example of how **interp2** works.

- File #2, **read_input.m**: This function reads the parameters in the file **bungee_data.txt** into MATLAB. The function should have the following declaration: **function [m, k, l, Xo, Yo, Zo, Uo, Vo, Wo]= read_input(input_filename, exp_num)** where **input_filename** is a string variable denoting the name of the file to be read and **exp_num** is an integer which denotes the experiment number. The outputs are the parameters needed to run the experiments. When the input **exp_num** is not available in the file, the function should set all outputs to **NaN** and display an error warning message to screen. Consider function **importdata** to read the parameters.

- File #3, **project.m**: the main script which includes the result of the tasks described below.

To run an experiment, call function **read_input** to get the input parameters and then call function **bungee** to obtain the trajectory. The project will be graded on the following 3 tasks.

Task 1: You will create 4 figures based on the experiment results. All figures should have title, axis labels with correct units and legends unless otherwise noted. If the figures are in 3-D, remember to include **view(3)**.

- Create **figure 1** to plot the six trajectories from Exp. # 1 through 6 in six panels. Use 2 row by 3 column arrangement for the panels. Each panel needs to include the trajectory in black solid line, final position in red filled circle marker and the terrain, and the terrain. No legend is required. Use function **plot3** and **surf**.

- Create **figure 2** to be similar to figure 1, but use Exp. # 7 through 12.

- Repeat the same process to create **figure 3**, but use Exp. # 13 through 18.

- Create **figure 4** to depict how the maximum distance varies among the experiments. Here, the maximum distance is defined as the furthest distance between the person and the coordinate origin. The figure should include 3 side-by-side panels. Plot maximum distance during the jump versus mass from Exp. # 1 through 6 on the left panel. Put maximum distance versus spring constant from Exp. # 7 through 12 on the middle panel. Finally, put maximum distance versus cord length from Exp. # 13 through 18 on the right panel. In all panels, put maximum distance on the vertical axis. Use black solid line and filled circle markers having different colors to indicate results from different experiments. Use marker size of 10. Remember to include the legend and set **axis tight**.

Task 2: Create a 18-element data structure named **exp_res** with the following fields:

- **number**: to include the experiment number.
- **max_speed**: to include the maximum speed.
- **max_acceleration**: to include the maximum acceleration.
- **integrated_KE**: to include the time integrated kinetic energy.
- **travel_distance**: to include the total travel distance along the trajectories.

Task 3: Use function **fprintf** to create a text file named **report.txt**. The text file should include the following lines:

- Your name on the first line.
- Your PID on the second line.
- A string '**exp number, max speed (m/s), max acc (m/s^2), int KE (J s), travel dist (m)**'on the third line.
- Corresponding values of Exp. number, maximum speed, maximum acceleration, time integrated kinetic energy and travel distance for each of the eighteen experiments from fourth to twenty first lines. Use a field with of 10 for the launch number and format %15.7*e* for others.

At the end of your **project.m** script, set the following:

```
p1a ='See figure 1';
p1b ='See figure 2';
p1c ='See figure 3';
p1d ='See figure 4';
p2a = exp_res(1);
p2b = [exp_res.max_speed];
p2c = [exp_res.max_acceleration];
p2d = [exp_res.integrated_KE];
p2e = [exp_res.travel_distance];
p3 = evalc('type report.txt');
```

Submission instructions: Follow the homework solution template. Remember to **clear all**, **close all**, **clc**, and fill in your name and PID. Set **hw_num = 'project'**. Create a zip archive named **project.zip**. The zip archive should include the following files: **project.m**, **bungee.m**, **read_input.m**, **bungee_data.txt**, **terrain.mat** and any other scripts / functions that you have written for the project. Make sure that you include all necessary files so that your **project.m** will run properly. Submit **project.zip** in CANVAS before 10 PM on Sunday 06/05/2022. Use double precision unless otherwise stated.