TOC

PART - B

unit - 3

1. Find LMD & RMD, parse Tree for the following grammar $w = 00110101$

$S \to 0B \,/\, 1A$

$A \to 0 \,/\, 0S \,/\, 1AA$

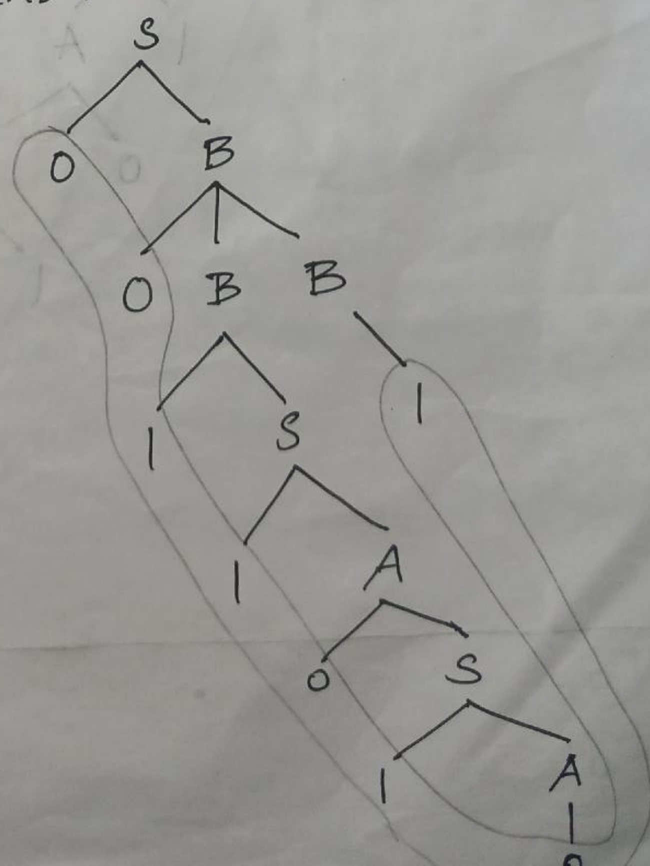
$B \to 1 \,/\, 1S \,/\, 0BB$

Sol:

LMD :

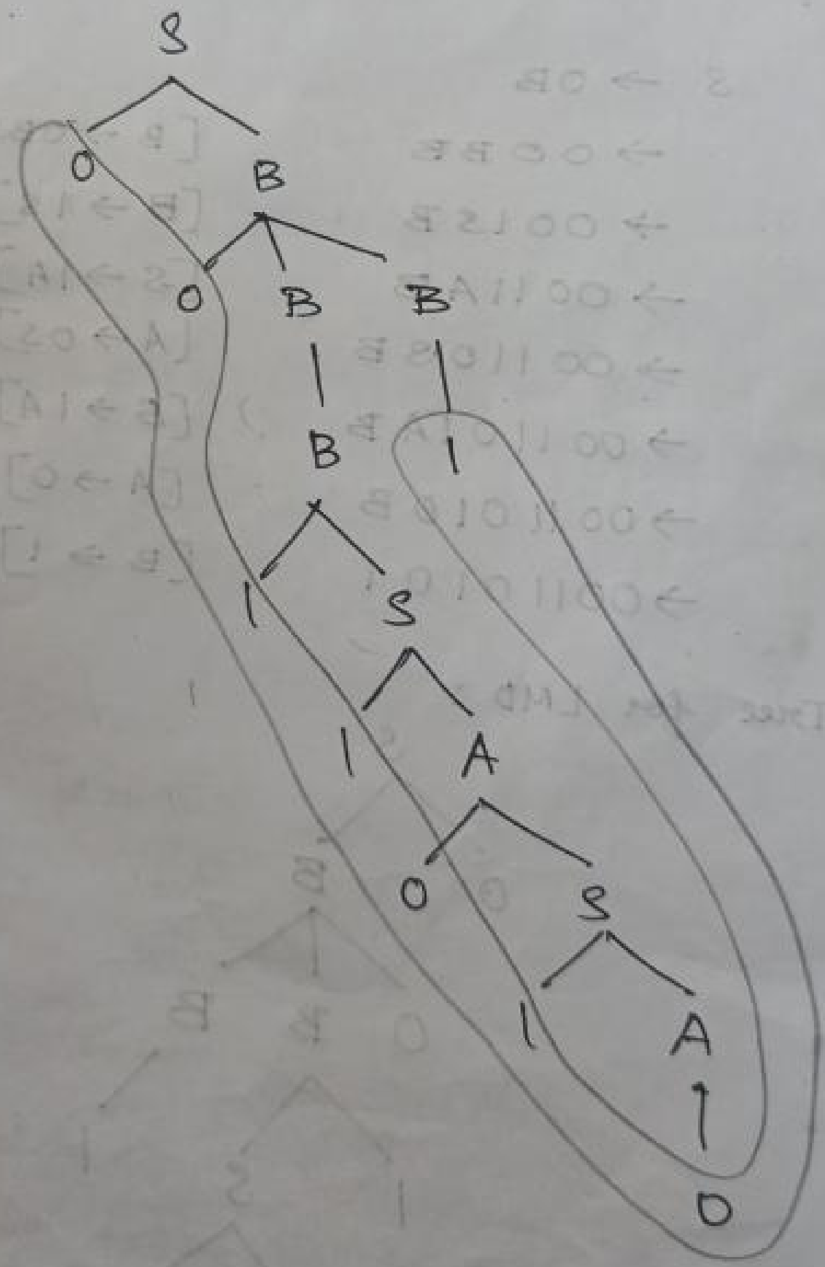
$S \to 0B$	
$\to 00BB$	$[B \to 0BB]$
$\to 001SB$	$[B \to 1S]$
$\to 0011AB$	$[S \to 1A]$
$\to 00110SB$	$[A \to 0S]$
$\to 001101AB$	$[S \to 1A]$
$\to 0011010B$	$[A \to 0]$
$\to 00110101$	$[B \to 1]$

Parse Tree for LMD :



$S \rightarrow 0B$	
$\rightarrow 00BB$	$[B \rightarrow 0BB]$
$\rightarrow 00B1$	$[B \rightarrow 1]$
$\rightarrow 001S1$	$[B \rightarrow 1S]$
$\rightarrow 0011A1$	$[S \rightarrow 1A]$
$\rightarrow 00110S1$	$[A \rightarrow 0S]$
$\rightarrow 001101A1$	$[S \rightarrow 1A]$
$\rightarrow 00110101$	$[A \rightarrow 0]$

Parse Tree for RMD :



2.i) Construct a parse tree for the following grammar $G = (\{S, A\}, \{a, b\}, P, S)$ where P consists of $S \rightarrow aAS \mid b$, $A \rightarrow SbA \mid ba$. Draw the derivation tree for the string $w = abbbab$.

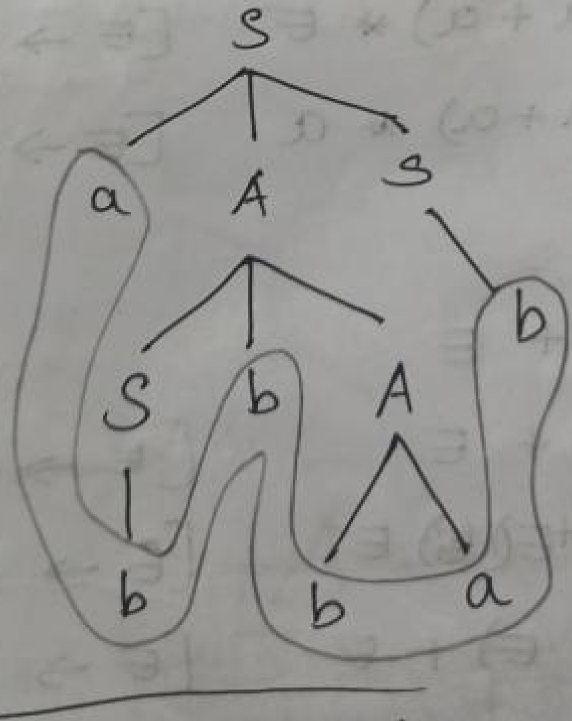
Sol:

Given Grammar:

$$S \rightarrow aAS \mid b$$
$$A \rightarrow SbA \mid ba$$

$$\begin{aligned} S &\rightarrow aAS \\ &\rightarrow aSbAS & [A \rightarrow SbA] \\ &\rightarrow abbAS & [S \rightarrow b] \\ &\rightarrow abbbaS & [A \rightarrow ba] \\ &\rightarrow abbbab & [S \rightarrow b] \end{aligned}$$

Parse Tree:



ii) Prove that the expression grammar is ambiguous E+E, E*E, (E)

$E \to E + E$

$\to a + E$ [$E \to a$]

$\to a + E * E$ [$E \to E * E$]

$\to a + a * E$ [$E \to a$]

$\to a + a * a$ [$E \to a$]

$E \to E * E$

$\Rightarrow E + E * E$ [$E \to E + E$]

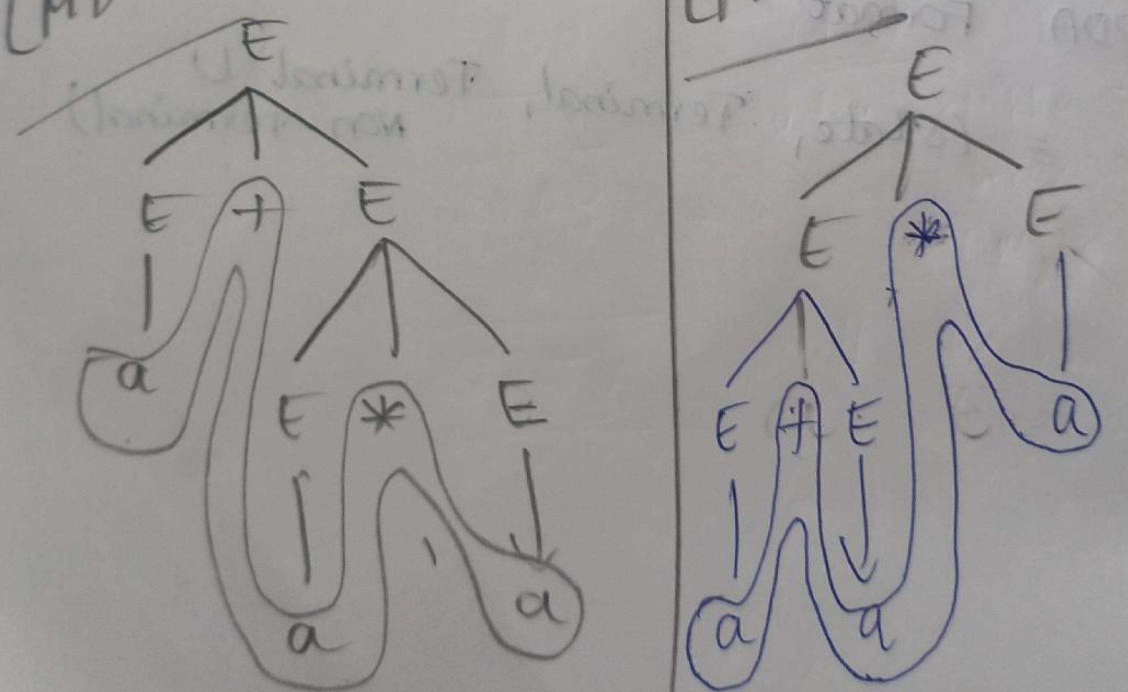
$\to a + E * E$ [$E \to a$]

$\to a + a * E$ [$E \to a$]

$\to a + a * a$ [$E \to a$]

Parse tree

LMD1: LMD2:



∴ The given grammar is ambiguous [because it contains more than 1 LMD].

3. Convert the PDA $P = \{(p,q), \{0,1\}, \{x, z_0\}, \delta, q, z_0\}$ to a CFG if δ is given by

$$\delta(q, 1, z_0) = (q, x z_0) \quad \text{Push}$$
$$\delta(q, 1, x) = (q, xx) \quad \text{Push}$$
$$\delta(q, 0, x) = (p, xx) \quad \text{Push}$$
$$\delta(q, \varepsilon, z_0) = (q, \varepsilon) \quad \text{Pop}$$
$$\delta(p, 1, x) = (p, \varepsilon) \quad \text{Pop}$$
$$\delta(p, 0, z_0) = (q, z_0) \quad \text{Read}$$

Sol:

Step 1 :

$G = (V, T, P, S)$

V → i) Start Symbol S

→ ii) Symbols of form $[p\,x\,q]$

$\boxed{V = Q^2 M + 1}$ where Q = No. of States, M = No. of stack symbol

$= (2)^2\, 2 + 1$

$= 4(2) + 1$

$= 8 + 1$

$= 9$

States ⇒ p, q

Stack Symbol ⇒ x, z_0

$$V = \{S, \underset{A}{p\,x\,p}, \underset{B}{p\,x\,q}, \underset{C}{q\,x\,p}, \underset{D}{q\,x\,q}, \underset{E}{p\,z_0\,p}, \underset{F}{p\,z_0\,q}, \underset{G}{q\,z_0\,p}, \underset{H}{q\,z_0\,q}\}$$

Step 2 :

Start Symbol

$S \rightarrow [q_0\, z_0\, Q]$

$S \rightarrow [q\, z_0\, p]$ $\qquad S \rightarrow G$

$S \rightarrow [q\, z_0\, q]$ $\qquad S \rightarrow H$

Step 3 : $\therefore S \to G | H$

Push :

$\delta(q, 1, z_0) = (q, x z_0)$

$\boxed{[q_i, z_0, q_{i+k}] \to 1 [q_{i+1}, x, q_m][q_m, z_0, q_{i+k}]}$

$[q \ z_0 \ p] \to 1 [q \ x \ p][p \ z_0 \ p] \therefore G \to 1CE$

$[q \ z_0 \ p] \to 1 [q \ x \ q][q \ z_0 \ p] \therefore G \to 1DG$

$[q \ z_0 \ q] \to 1 [q \ x \ p][p \ z_0 \ q] \therefore H \to 1CF$

$[q \ z_0 \ q] \to 1 [q \ x \ q][q \ z_0 \ q] \therefore H \to 1DH$

$\therefore G \to 1CE | 1DG$

$H \to 1CF | 1DH$

Push :

$\delta(q, 1, x) = (q, xx)$

$[q \ x \ p] \to 1 [q \ x \ p][p \ x \ p] \therefore C \to 1CA$

$[q \ x \ p] \to 1 [q \ x \ q][q \ x \ p] \therefore C \to 1DC$

$[q \ x \ q] \to 1 [q \ x \ p][p \ x \ q] \therefore D \to 1CB$

$[q \ x \ q] \to 1 [q \ x \ q][q \ x \ q] \therefore D \to 1DD$

$\therefore C \to 1CA | 1DC$

$\therefore D \to 1CB | 1DD$

Push :

$\delta(q, 0, x) \to (p, xx)$

$[q \ x \ p] \to 0 [p \ x \ p][p \ x \ p] \therefore C \to 0AA$

$[q \ x \ p] \to 0 [p \ x \ q][q \ x \ p] \therefore C \to 0BC$

$[q \ x \ p]$

$[q\ x\ q] \rightarrow 0[p\ x\ p][p\ x\ q] \therefore D \rightarrow 0AB$

$[q\ x\ q] \rightarrow 0[p\ x\ q][q\ x\ q] \therefore D \rightarrow 0BD$

$$\therefore C \rightarrow 0AA \mid 0BC$$
$$D \rightarrow 0AB \mid 0BD$$

Pop :

$\delta(q, \epsilon, z_0) = (q, \epsilon)$

$\boxed{[q_i, z_0, q_{i+1}] \rightarrow \epsilon}$

$[q\ z_0\ q] \rightarrow \epsilon \quad \therefore H \rightarrow \epsilon$

$$\therefore H \rightarrow \epsilon$$

Pop :

$\delta(p, 1, x) = (p, \epsilon)$

$[p\ x\ p] \rightarrow 1 \quad \therefore A \rightarrow 1$

$$\therefore A \rightarrow 1$$

Read :

$\delta(p, 0, z_0) = (q, z_0)$

$\boxed{[q_i, z_0, q_{i+m}] \rightarrow 0[q\ z_0\ q_{i+m}]}$

$[p\ z_0\ p] \rightarrow 0[q\ z_0\ p] \quad \therefore E \rightarrow 0G$

$[p\ z_0\ q] \rightarrow 0[q\ z_0\ q] \quad \therefore F \rightarrow 0H$

$$\therefore E \rightarrow 0G$$
$$F \rightarrow 0H$$

Step 4:

Final productions:

$S \rightarrow G \mid H$

$G \rightarrow 1CE \mid 1DG$

$H \rightarrow 1CF \mid 1DH \mid \varepsilon$

$C \rightarrow 1CA \mid 1DC \mid 0AA \mid 0BC$

$D \rightarrow 1CB \mid 1DD \mid 0AB \mid 0BD$

$A \rightarrow 1$

$E \rightarrow 0G$

$F \rightarrow 0H$.

---

4. Convert the following CFG into PDA using empty Stack.

$S \rightarrow aB \mid bA$

$A \rightarrow bAA \mid aS \mid a$

$B \rightarrow aBB \mid bS \mid b$

aB
aaBB
aabSB
aabbAB
aabbaB
aabbab

Sol:

Step 1:

$G = (V, T, P, S)$

$G = (\{S, A, B\}, \{a, b\}, \{aB, bA, bAA, aS, aBB, bS, a, b\}, S)$

Step 2: state T N U T stc

$PDA = (\{q\}, \{a, b\}, \{a, b, B, A, S\}, \delta, q, S)$

Step 3:

Transition Function:

For each Non-Terminal,

$\boxed{\delta(q, \varepsilon, A) = (q, B) \quad \text{when } A \rightarrow B}$

$\delta(q, \varepsilon, S) = \{(q, aB), (q, bA)\}$

RMD .

$\delta(q, \varepsilon, A) = \{(q, bAA), (q, aS), (q, a)\}$

$\delta(q, \varepsilon, B) = \{(q, aBB), (q, bS), (q, b)\}$

For each Terminal,

$\boxed{\delta(q, a, a) = (q, \varepsilon)}$

$\delta(q, b, b) = (q, \varepsilon)$

Step 4:

Instantaneous Description for the string aabbab

$(q, aabbab, S) \vdash_P^* (q, \cancel{a}abbab, \cancel{a}B)$

$\vdash_P^* (q, abbab, B)$

$\vdash_P^* (q, \cancel{a}bbab, \cancel{a}BB)$

$\vdash_P^* (q, bbab, BB)$

$\vdash_P^* (q, \cancel{b}bab, \cancel{b}SB)$

$\vdash_P^* (q, bab, SB)$

$\vdash_P^* (q, \cancel{b}ab, \cancel{b}AB)$

$\vdash_P^* (q, ab, AB)$

$\vdash_P^* (q, \cancel{a}b, \cancel{a}B)$

$\vdash_P^* (q, b, B)$

$\vdash_P^* (q, \cancel{b}, \cancel{b})$

$\vdash_P^* (q, \varepsilon, \varepsilon)$

$\therefore$ The string $w = aabbab$ is accepted by empty Stack.

5. i) Construct PDA for the Language $\{wcw^R \mid w \in \{0,1\}^*\}$

Sol:

$L = \{c, 0c0, 1c1, 01c10, 10c01, 00c00, 11c11,$
$001c100, 110c011, \ldots\ldots\}$

Transition Function:

$\delta(q_0, 0, z_0) = (q_0, 0z_0)$

$\delta(q_0, 1, z_0) = (q_0, 1z_0)$

$\delta(q_0, 0, 0) = (q_0, 00)$

$\delta(q_0, 1, 0) = (q_0, 10)$

$\delta(q_0, 0, 1) = (q_0, 01)$

$\delta(q_0, 1, 1) = (q_0, 11)$

$\delta(q_0, c, z_0) = (q_1, z_0)$

$\delta(q_0, c, 0) = (q_1, 0)$

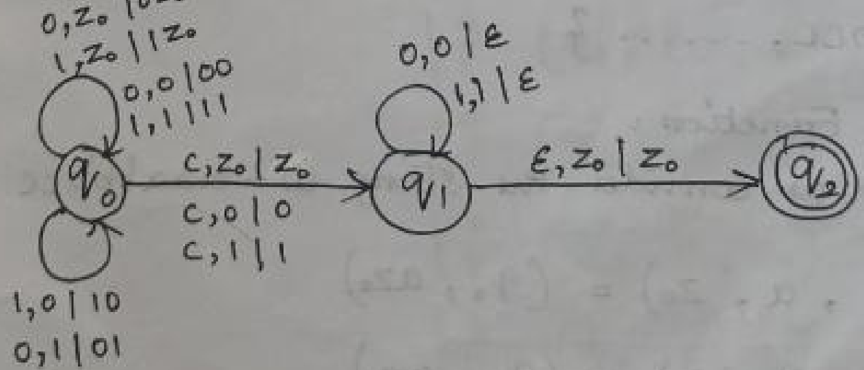
$\delta(q_0, c, 1) = (q_1, 1)$

$\delta(q_1, 0, 0) = (q_1, \epsilon)$

$\delta(q_1, 1, 1) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

Transition Diagram:



The string is accepted by final state.

Instantaneous Description :

$$(q_0, w, z_0) \vdash_P^* (q_0, 110c011, z_0)$$

$$\vdash_P^* (q_0, 10c011, 1z_0)$$

$$\vdash_P^* (q_0, 0c011, 11z_0)$$

$$\vdash_P^* (q_0, c011, 011z_0)$$

$$\vdash_P^* (q_1, \cancel{0}11, \cancel{0}11z_0)$$

$$\vdash_P^* (q_1, \cancel{1}1, \cancel{1}1z_0)$$

$$\vdash_P^* (q_1, \cancel{1}, \cancel{1}z_0)$$

$$\vdash_P^* (q_1, \varepsilon, z_0)$$

$$\vdash_P^* (q_2, \varepsilon, z_0)$$

$\therefore$ The string $w = 110c011$ is accepted by PDA.

---

ii) Construct a PDA for empty stack for the language $\{a^m b^m c^n \mid m, n \geq 1\}$

Sol:

$L = \{$ abc, aabbc, aaabbbcc, aaabbbc, aabbccc abcc, ... $\}$

Transition Function :

Let us consider the string $w = aaabbbcc$

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, a) = (q_1, \varepsilon)$$

$$\delta(q_1, b, a) = (q_1, \varepsilon)$$

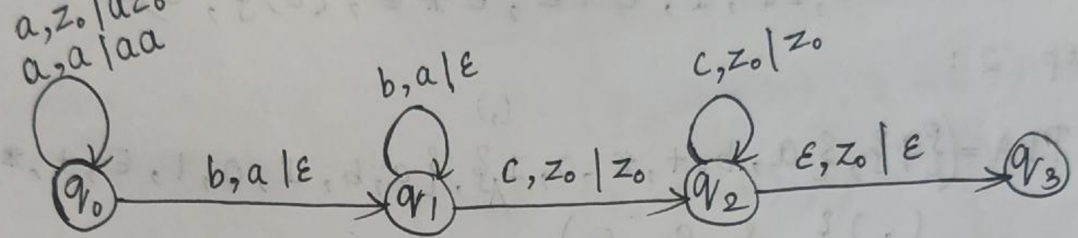
$$\delta(q_1, c, Z_0) = (q_2, Z_0)$$

$$\delta(q_2, c, Z_0) = (q_2, Z_0)$$

$$\delta(q_2, \varepsilon, Z_0) = (q_3, \varepsilon)$$

$\therefore$ The string is accepted by empty stack.

Transition Diagram :



Instantaneous Description :

$$(q_0, w, Z_0) \overset{*}{\underset{P}{\vdash}} (q_0, aaabbbcc, Z_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_0, aabbbcc, aZ_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_0, abbbcc, aaZ_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_0, \cancel{b}bbcc, \cancel{a}aaZ_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_1, \cancel{b}bcc, \cancel{a}aZ_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_1, \cancel{b}cc, \cancel{a}Z_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_1, cc, Z_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_2, c, Z_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_2, \varepsilon, Z_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_3, \varepsilon, \varepsilon)$$

$\therefore$ The string $w = aaabbbcc$ is accepted by PDA.

6. i) Convert the following CFG to PDA and analyse the answer (a+b) and a++.

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

$E \rightarrow I \mid E+E \mid E*E \mid (E)$

Sol:

Step 1:

$G = (V, T, P, S)$

$= (\{I, E\}, \{a, b, +, *, 0, 1, (,)\}, \{a, b, Ia, Ib, I0, I1, I, E+E, E*E, (E)\}, E)$

Step 2:

$PDA = (\{q\}, \{a, b, +, *, 0, 1, (,)\}, \{a, b, I, 0, 1, E, +, *, (,)\}, \delta, q, E)$

Step 3:

Transition Function:

For each Non-Terminal,

$\delta(q, \varepsilon, I) = \{(q, a), (q, b), (q, Ia), (q, Ib), (q, I0), (q, I1)\}$

$\delta(q, \varepsilon, E) = \{(q, I), (q, E+E), (q, E*E), (q, (E))\}$

For each Terminal,

$\delta(q, a, a) = (q, \varepsilon)$

$\delta(q, b, b) = (q, \varepsilon)$

$\delta(q, +, +) = (q, \varepsilon)$

$\delta(q, *, *) = (q, \varepsilon)$

$\delta(q, 0, 0) = (q, \varepsilon)$

$\delta(q, 1, 1) = (q, \varepsilon)$

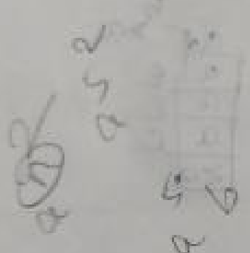
$\delta(q, (, () = (q, \varepsilon)$

$\delta(q,),)) = (q, \varepsilon)$

Step 4 :

Instantaneous Description for the string $(a+b)$ and $a++$.

$$
\begin{aligned}
(q, (a+b), E) &\vdash_P^* (q, \cancel{(}a+b), \cancel{(}E)) \\
&\vdash_P^* (q, a+b), E)) \\
&\vdash_P^* (q, a+b), E+E)) \\
&\vdash_P^* (q, a+b), I+E)) \\
&\vdash_P^* (q, \cancel{a}+b), \cancel{a}+E)) \\
&\vdash_P^* (q, \cancel{+}b), \cancel{+}E)) \\
&\vdash_P^* (q, b), E)) \\
&\vdash_P^* (q, b), I)) \\
&\vdash_P^* (q, \cancel{b}), \cancel{b})) \\
&\vdash_P^* (q, \cancel{)}, \cancel{)}) \\
&\vdash_P^* (q, \varepsilon, \varepsilon)
\end{aligned}
$$



$\therefore$ The string $w = (a+b)$ is accepted by empty stack PDA .

$$
\begin{aligned}
(q, a++, E) &\vdash_P^* (q, a++, E+E) \\
&\vdash_P^* (q, a++, I+E) \\
&\vdash_P^* (q, \cancel{a}++, \cancel{a}+E) \\
&\vdash_P^* (q, \cancel{+}+, \cancel{+}E) \\
&\vdash_P^* (q, +, E)
\end{aligned}
$$

∴ The string $w = a{+}{+}$ is not accepted by PDA.

∴ The PDA accepts only $(a+b)$ and not accepts string $a{+}{+}$.

---

ii) Construct a PDA by empty stack, $L = \{a^m b^n \mid n <$

Sol:

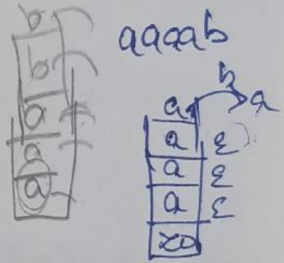
$L = \{aab, aaab, aaabb, aaaabbb, \ldots\}$

Transition Function:

Let us consider the string $w = aaabb\varepsilon$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$



$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \varepsilon)$

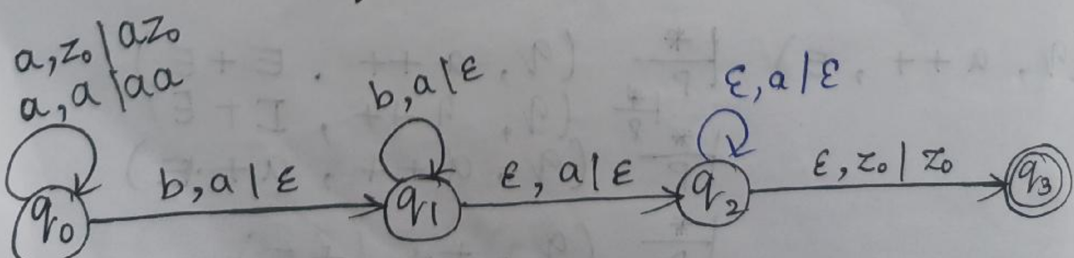
$\delta(q_1, b, a) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, a) = (q_2, \varepsilon) \rightarrow \delta(q_2, \varepsilon, a) = (q_2, \varepsilon)$

$\delta(q_2, \varepsilon, z_0) = (q_3, z_0)$

The string is accepted by final state.

Transition Diagram:



Instantaneous Description :

$$(q_0, w, z_0) \overset{*}{\underset{P}{\vdash}} (q_0, aaabb, z_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_0, aabb, az_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_0, abb, aaz_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_0, \cancel{b}b, \cancel{a}aaz_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q, \cancel{b}, \cancel{a}az_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_1, \cancel{\varepsilon}, \cancel{a}z_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_2, \varepsilon, z_0)$$

$$\overset{*}{\underset{P}{\vdash}} (q_3, \varepsilon, z_0)$$

$\therefore$ The string $w = aaabb$ is accepted by PDA .