(6) using Mathematical induction $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Q.B.

(1) (1)

$P(n) : \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

$P(n) = 1^2 + 2^2 + 3^2 + \cdots\cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

$P(n_0)$. $LHS = 1^2 = 1$.

$RHS = \frac{1(1+1)(2(1)+1)}{6}$

$= \frac{1(2)(3)}{6} = 1$.

$LHS = RHS$.

$\therefore P(n_0)$ is true

Assume the result for $n = K$,

$P(K)$ is true

$1^2 + 2^2 + 3^2 + \cdots K^2 = \frac{K(K+1)(2K+1)}{6}$.

To show that $P(K+1)$ is true

$1^2 + 2^2 + 3^2 + \cdots K^2 + (K+1)^2 = \frac{(K+1)((K+1)+1)(2(K+1)+1)}{6}$ is true

~~is true~~

$= \frac{(K+1)(K+2)(2K+3)}{6}$.

$1^2 + 2^2 + 3^2 + \cdots K^2 + (K+1)^2 = \frac{K(K+1)(2K+1)}{6} + (K+1)^2$

$= \frac{(K+1)}{6}(K(2K+1) + 6(K+1))$

$= \frac{K+1}{6}(2K^2 + K + 6K + 6)$

$\frac{12}{4} \Big| \frac{7}{3}$.

$= \frac{K+1}{6}(2K^2 + 7K + 6)$.

$= \frac{K+1}{6}(2K^2 + 4K + 3K + 6)$

$= \frac{K+1}{6}(2K(K+2) + 3(K+2))$

and $\tan\theta = \frac{1}{d}$.

3. Solve by recurrance relation of the fibonacci Sequence of the number $f_n = f_{n-1} + f_{n-2}$ Q.B (i) (ii)

$n \geq 2$, $f_1 = 1$ $f_2 = 1$

$f_n = f_{n-1} + f_{n-2} \Rightarrow f_n - f_{n-1} - f_{n-2} = 0$.

$n-(n-2) = 2$

Therefore it is second order recurrance relation.

∴ It is Characteristic equation is

$r^2 - r + 1 = 0$.

$r = \frac{1 \pm \sqrt{1+4}}{2}$ $\quad \frac{b \pm \sqrt{b^2-4ac}}{2a}$.

$r = \frac{1 \pm \sqrt{5}}{2}$

$r_1 = \frac{1+\sqrt{5}}{2}$, $r_2 = \frac{1-\sqrt{5}}{2}$

∴ since r_1 and r_2 are real & different

$a_n = A r_1^n + B r_2^n$.

$a_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n \rightarrow$ (1).

Where $n=1$

$a_1 = A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right)$

$1 = A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right)$

$\Rightarrow A(1-\sqrt{5}) + B(1-\sqrt{5}) = 2$

$A+B + \sqrt{5}(A-B) = 2 \rightarrow$ (2).

When $n=2$,

$$a_2 = A\left(\frac{1+\sqrt5}{2}\right)^2 + B\left(\frac{1-\sqrt5}{2}\right)^2$$

$$A\left(\frac{1+\sqrt5}{2}\right)^2 + B\left(\frac{1-\sqrt5}{2}\right)^2 = 1.$$

$$A\left[\frac{(1+\sqrt5)^2}{4}\right] + B\left[\frac{(1-\sqrt5)^2}{4}\right] = 1$$

$$A(1-\sqrt5)^2 + B(1-\sqrt5)^2 = 4$$

$$A(1+5+2\sqrt5) + B(1+5-2\sqrt5) = 4$$

$$6(A+B) + 2\sqrt5(A-B) = 4.$$

$$\cancel{2}\left[3(A+B)+\sqrt5(A-B)\right] = \cancel{4}^{2}$$

$$3(A+B)+\sqrt5(A-B) = 2 \rightarrow (2).$$

By solving (1) and (2)

$$\Rightarrow 3(A+B)+\sqrt5(A-B) = 2$$

$$(A+B)+\sqrt5(A-B) = 2$$

$$- \quad - \quad -$$

$$2(A+B) = 0.$$

$$\boxed{A = -B}$$

Sub $A = -B$ in equation (1)

$$A+B+\sqrt5(A-B) = 2$$

$$-B+B+\sqrt5(-B-B) = 2.$$

$$-\cancel{2}B = \frac{\cancel{2}}{\sqrt5} = \boxed{B = \frac{-1}{\sqrt5}}$$

$$= -2B\sqrt5 = 2$$

$$-2B = 2/\sqrt5.$$

$$B = \frac{\cancel{2}}{-\cancel{2}\sqrt5} = \frac{-1}{\sqrt5}$$

$$\Rightarrow A = -B = 1/\sqrt5$$

$$a_n = \frac{1}{\sqrt5}\left(\frac{1+\sqrt5}{2}\right)^n - \frac{1}{\sqrt5}\left(\frac{1-\sqrt5}{2}\right)^n \text{ //}$$

$$= 2^n - 2n + 1 //.$$

14/08/24

1.) Use the method of generation function solve the recurrence of $a_{n+1} - 8a_n + 16a_{n-1} = 4^n$. Q.B (2)(ii)

(*) .

$$a_{n+1} - 8a_n + 16a_{n-1} = 4^n$$

Multiply both side x^n.

$$\Rightarrow a_{n+1} x^n - 8a_n x^n + 16a_{n-1} x^n = 4^n x^n.$$

$$\sum_{n=1}^{\infty} a_{n+1} x^n - 8\sum_{n=1}^{\infty} a_n x^n + 16\sum_{n=1}^{\infty} a_{n-1} x^n = \sum_{n=1}^{\infty} (4x)^n.$$

$$x^{-1}\sum_{n=1}^{\infty} a_{n+1}x^{n+1} - 8\sum_{n=1}^{\infty} a_n x^n + 16x\sum_{n=1}^{\infty} a_{n-1}x^{n-1} =$$

$$4x + (4x)^2 + (4x)^3 + \dots$$

$$\frac{1}{x}\left[(a_2x^2 + a_3x^3 + \dots) - 8(a_1x + a_2x^2 + \dots) + 16x(a_0 + a_1x + a_2x^2 \dots\right.$$

$$= 4x(1 + 4x + (4x)^2 + \dots).$$

WKT,

$$G(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots$$

$$\Rightarrow \frac{1}{x}\left[G(x) - a_0 - a_1x\right] - 8\left[G(x) - a_0\right] + 16x(G(x)) = 4x(1-4x)^{-1}$$

$(a-b)^2$

$$\Rightarrow G(x)\left(\frac{1}{x} - 8 + 16x\right) - \frac{1}{x} - \cancel{8} + \cancel{8} = \frac{4x}{1-4x}.$$

$$\begin{array}{c|c} 16 & 8 \\ \hline 4 & 4 \end{array}$$

$$\Rightarrow G(x)\left(\frac{1 - 8x + 16x^2}{x}\right) = \frac{4x}{1-4x} + \frac{1}{x}.$$

$$\Rightarrow G(x)\left(\frac{(1-4x)^2}{x}\right) = \frac{4x^2 + 1 - 4x}{x(1-4x)}$$

$$\Rightarrow G(x) = \frac{1 - 4x + 4x^2}{\cancel{x}(1-4x)} \cdot \frac{\cancel{x}}{(1-4x)^2}$$

$$\Rightarrow G(x) = \frac{1 - 4x + 4x^2}{(1-4x)^3}.$$

$$\Rightarrow G(x) = (1 - 4x + 4x^2)(1-4x)^{-3}$$

$$\Rightarrow G(x) = (1 - 4x + 4x^2)\frac{1}{1\cdot 2}\left[1\cdot 2 + 2\cdot 3(4x) + 3\cdot 4(4x)^2 + \dots (n-1)n(4x)^{n-2} + n(n+1)(4x)^{n-1} + (n+1)(n+2)(4x)^n + \dots\right]$$

$$\Rightarrow a_n = \frac{1}{2}\left[(n+1)(n+2)4^n - 4n(n+1)4^{n-1} - 4n(n-1)\cdot 4^{n-2}\right]$$

$$\Rightarrow a_n = \frac{1}{2}\left[4^n(\cancel{n^2} + 3n + 2 - \cancel{n^2} - n) + 4^{n-1}(n^2 - n)\right]$$

$$\Rightarrow a_n = \frac{1}{2}\left[4^n(2n+2) + 4^{n-1}(n^2 - n)\right]$$

$$\Rightarrow a_n = \frac{4^{n-1}}{2}\left[4(2n+2) + n^2 - n\right]$$

$$\Rightarrow a_n = \frac{4^{n-1}}{2}\left[8n + 8 + n^2 - n\right)$$

$$\Rightarrow a_n = \frac{4^{n-1}}{2}(n^2+4n+8) //.$$

(5) Prove that $n^5 - n$ is divisible by 5 for all $n \geq 1$.

Q.B 3 (i).

Let :-

$P(n)$: $n^5 - n$ is divisible by 5

$n = 1$

$P(n_0) = 1^5 - 1 = 1 - 1 = 0$ which is divisible by 5

$P(n_0)$ is true

Assume that the result is true for $n = k$.

$k^5 - k$ is divisible by 5.

$\Rightarrow k^5 - k = 5m$ where m is an integer

$k^5 = 5m + k \rightarrow (1)$.

To show that $P(k+1)$ is true

$(k+1)^5 - (k+1)$ is divisible by 5

$(k+1)^5 - (k+1) = {}^5C_0 k^5 + {}^5C_1 k^4 + {}^5C_2 k^3 + {}^5C_3 k^2 + {}^5C_4 k + {}^5C_5 (1 - k - 1)$

$= k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + \cancel{1} - k - \cancel{1}$

$= k^5 + 5k^4 + 10k^3 + 10k^2 + 4k$.

$= 5m + k + 5k^4 + 10k^3 + 10k^2 + 4k$

$= 5(m + k^4 + 2k^3 + 2k^2 + 1)$

$= 5(m + k^4 + 2k^3 + 2k^2 + 1$

$= 5n$ where $n = m + k^4 + 2k^3 + 2k^2 + 1$

Which is divisible by 5

$\Rightarrow P(k+1)$ is true.

hence by principle of Mathematical induction is true.

The no of Students ...

Find ~~the~~ The number of integer between 1 and 250 both inclusive that are divisible by 2, 3, 5 not divisible by 7. Q.B. 3(ii)

$|A| = \left\lfloor \frac{250}{2} \right\rfloor = 125$ $\quad |C| = \left[\frac{250}{5} \right] = 50.$

$|B| = \left[\frac{250}{3} \right] = 83$ $\quad D = \left\lfloor \frac{250}{7} \right\rfloor = 35.$

$|A \cap B| = \left[\frac{250}{2 \times 3} \right] = 41$ $\quad |B \cap C| = \left\lfloor \frac{250}{3 \times 5} \right\rfloor = 16$

$|A \cap C| = \left\lfloor \frac{250}{2 \times 5} \right\rfloor = 25$ $\quad |B \cap D| = \left\lfloor \frac{250}{3 \times 7} \right\rfloor = 11$

$|A \cap D| = \left\lfloor \frac{250}{2 \times 7} \right\rfloor = 17.$ $\quad |C \cap D| = \left\lfloor \frac{250}{5 \times 7} \right\rfloor = 7$

$|A \cap B \cap C| = \left\lfloor \frac{250}{2 \times 3 \times 5} \right\rfloor = 8$ $\quad |B \cap C \cap D| = \left\lfloor \frac{250}{3 \times 5 \times 7} \right\rfloor = 2$

$|A \cap B \cap D| \left\lfloor \frac{250}{2 \times 3 \times 7} \right\rfloor = 5$ $\quad |A \cap C \cap D| = \left\lfloor \frac{250}{2 \times 5 \times 7} \right\rfloor = 3$

$|A \cap B \cap C \cap D| = \left\lfloor \frac{250}{2 \times 3 \times 5 \times 7} \right\rfloor = 1$

$|A \cup B \cup C| = 125 + 83 + 50 - 41 - 25 - 16 + 8$

$= 184.$

$$|A \cup B \cup C \cup D| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |D|$$
$$|A \cap B \cap C| + |A \cap B \cap D| + |B \cap C \cap D| + |A \cap C \cap D|$$
$$- |A \cap B \cap C \cap D|.$$

$$= 125 + 83 + 50 + 35 - 41 - 25 - 17 - 16 -$$
$$11 - 7 + 8 + 5 + 2 + 3 - 1$$
$$= 194.$$

$\Rightarrow |A \cup B \cup C| - |A \cup B \cup C \cup D|$

$\Rightarrow$ $194 - 184 \Rightarrow 10.$

Total not divisible by 7 is 10%.

3.) Solve $a_{n+2} - 2a_{n+1} + a_n = 2^n$, $a_0 = 2$, $a_1 = 1$ Q.B 4(i)

$a_{n+2} - 2a_{n+1} + a_n = 2^n$.

Multiply both sides by x^n.

$a_{n+2} x^n - 2a_{n+1} x^n + a_n x^n = 2^n x^n$.

$$\sum_{n=0}^{\infty} a_{n+2} x^n - 2\sum_{n=0}^{\infty} a_{n+1} x^n + \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} 2^n x^n.$$

$$x^{-2} \sum_{n=0}^{\infty} a_{n+2} x^{n+2} - 2x^{-1} \sum_{n=0}^{\infty} a_{n+1} x^{n+1} + \sum_{n=0}^{\infty} a_n x^n = \left[1 + 2x + (2x)^2 + \dots (2x)^n + \right]$$

$$\Rightarrow \frac{1}{x^2}\left(a_2 x^2 + a_3 x^3 + \dots + a_n x^n + \dots\right) - \frac{2}{x}\left(a_1 x + a_2 x^2 + \dots + \dots\right)$$

$$+ G(x) = (1-2x)^{-1}.$$

$$\Rightarrow \frac{1}{x^2}\left(G(x) - a_0 - a_1 x\right) - \frac{2}{x}\left[G(x) - a_0\right] + G(x) = \frac{1}{(1-2x)}$$

$$\Rightarrow G(x)\left(\frac{1}{x^2} + \frac{2}{x} + 1\right) - \frac{2}{x^2} - \frac{1}{x} + \frac{4}{x} = \frac{1}{1-2x}.$$

$$\Rightarrow G(x)\left[\frac{1+2x+x^2}{x^2}\right] = \frac{1}{1-2x} + \frac{2}{x^2} - \frac{3}{x}.$$

$$\Rightarrow G(x)\left[\frac{(1-x)^2}{x^2}\right] = \frac{x^2 + 2 - 4x - 3x + 6x^2}{x^2(1-2x)}$$

$$\Rightarrow G(x) = \frac{7x^2 - 7x + 2}{\cancel{x^2}(1-2x)} \cdot \frac{\cancel{x^2}}{(1-x)^2}$$

$$\Rightarrow G(x) = \frac{7x^2 - 7x + 2}{(1-2x)(1-x)^2}$$

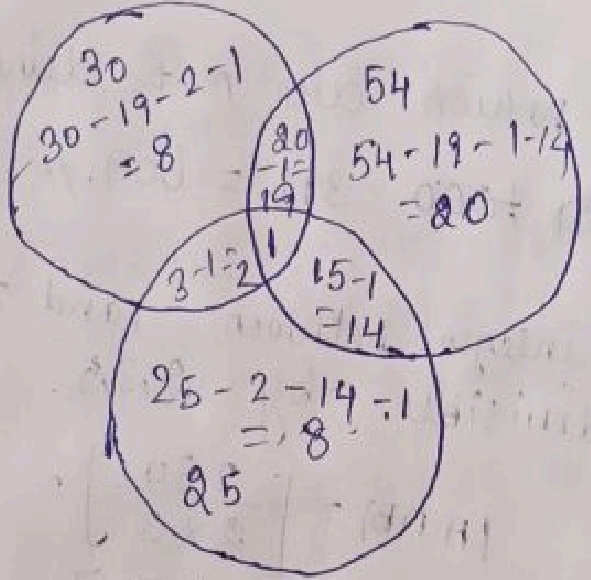
By Partial fraction

$$\frac{7x^2 - 7x + 2}{(1-2x)(1-x)^2} = \frac{A}{1-2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$$

A survey of 100 student it was found the 30. Studied Mathematics. 54 studied the Statistics 25 studied Operations Research. 1 ~~Statalls~~ 1 studied all 3 subjects. 20 studied Mathematics & Statistics 3 studieds Mathamatics & Operation research and 15 studieds Operation Research & Statistics.

find how many student studied non of the subject and how many student studieds

Q.B
4(ii)



Total

$M \cup S \cup O = 8 + 19 + 20 + 2 + 1 + 14 + 8$

$= 72$

Student studieds non of these subjects $= 100 - 72 =$ 28//.

(4) Prove by Mathematical induction by $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n. Q.B 5 (i)

Let :-

$P(n) : 6^{n+2} + 7^{2n+1}$ is divisible by 43.

$n_0 = 1$:-

$P(1) : 6^3 + 7^3 = 216 + 343 = 559 = 43(13)$

$P(n_0)$ is true

assume that the result true for $n = k$.

$P(k)$ is true

$6^{k+2} + 7^{2k+1}$ is divisible by 43.

$\Rightarrow 6^{k+2} + 7^{2k+1} = 43m$ (where m is an integer)

$\Rightarrow 7^{2k+1} = 43m - 6^{k+2}$.

To show that $P(k+1)$ is true

$6^{(k+1)+2} + 7^{2(k+1)+1}$ is divisible by 43.

$6^{(k+1)+2} + 7^{2(k+1)+1} = 6^{k+3} + 7^{2k+3}$.

$= 6 \cdot 6^{k+2} + 7^2 \cdot 7^{2k+1}$.

$= 6\, 6^{k+2} + 49(43m - 6^{k+2}$

$= 43(49m) - 43\, 6^{k+2}$

$\Rightarrow 43(49m - 6^{k+2})$

$= 43n$ where $[n = 49m - 6^{k+2}]$

$\therefore P(k+1)$ is true

hence by principle of Mathematical induction true for $P(n)$ is true of $n \geq 1$

5.) find all the recurrence function where
$a_n = 5a_{n-1} - 6a_{n-2} + 7^n$.

Q.B 6(i).

A) $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

$a_n - 5a_{n-1} + 6a_{n-2} = 7^n \to (1)$.

$n - (n-2) = 2$.

It is second order recurrence relation

$$r^2 - 5r + 6 = 0$$

$$(r-2)(r-3) = 0.$$

$$r_1 = 2, \quad r_2 = 3$$

$$\begin{array}{c|c} 6 & -5 \\ \hline -2 & -3 \end{array}$$

$$Ar_1^n + Br_2^n.$$

$$a_n^{(c)} = A2^n + B3^n.$$

The Particular Solution is $C7^n$.

Substitute in (1) as $a_n = C7^n$.

$$a_n - 5a_{n-1} + 6a_{n-2} = 7^n.$$

$$C \cdot 7^n - 5C \cdot 7^{n-1} + 6C7^{n-2} = 7^n.$$

$$\cancel{7^n}(C - 5C7^{-1} + 6C7^{-2}) = \cancel{7^n}$$

$$C(1 - 5/7 + 6/49) = 1$$

$$C\left(\frac{49 - 35 + 6}{49}\right) = 1.$$

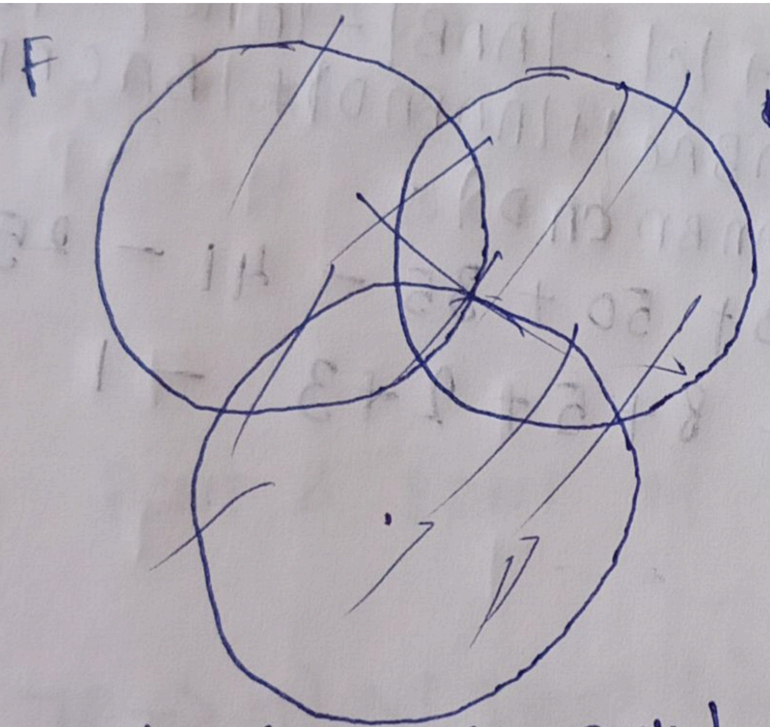
$$C = \frac{49}{20}.$$

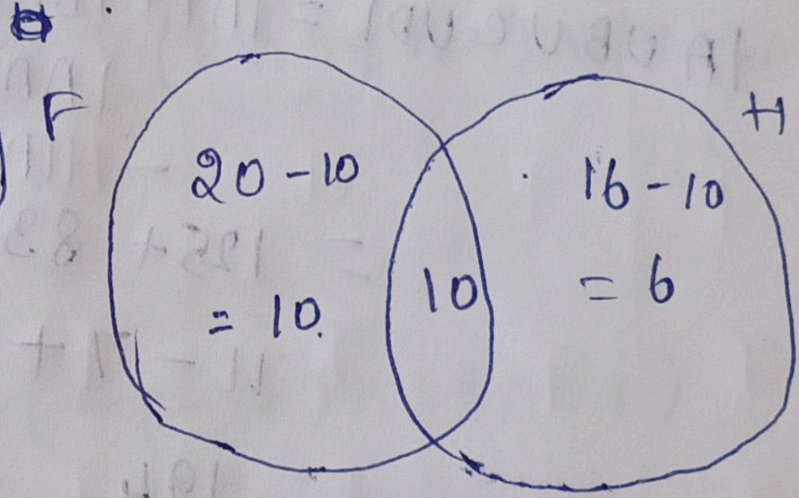
$$a_n = A2^n + B3^n + \frac{49}{20}7^n //.$$

State the inclusive and exclusive principle used to find the number of student who play neither games, if in a class of 50 student 20 student play football 15 student play Hockey and 10 student play both of the game.

if A and b are finite sub set of a finite universe sets.

Q.B
6 (ii)





$F \cup H = |F| + |H| - |F \cap H|$

$= 10 + 10 + 6 = 26$

The no of students who plays neither games $= 50 - 26 = 24$//.

Q.B/7(i)

find the no. of integers between 1 and 1000 both inclusive that are not divisible by any of the integer 5, 7, 9.

Let A, B, C be the set of integer between 1 and 1000 both inclusive that are divisible by 5, 7, 9.

$|A| = \left[\frac{1000}{5}\right] = 200$

$|C| = \left[\frac{1000}{9}\right] = 111$

$|B| = \left[\frac{1000}{7}\right] = 142$

$|A \cap B| = \left[\frac{1000}{5 \times 7}\right] = 28.$

$|A \cap C| = \left[\frac{1000}{5 \times 9}\right] = 22$

$[B \cap C] = \left[\frac{1000}{7 \times 9}\right] = 15.$

$[A \cap B \cap C] = \left[\frac{1000}{5 \times 7 \times 9}\right] = 3.$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$

$= 200 + 142 + 111 - 28 - 22 - 15 + 3.$

$= 391.$

The total number which are not divisible by 5, 7, 9. $\Rightarrow$ $1000 - 391 = 609.$ //.

(3) Prove by Mathematical induction.
$3^n + 7^n - 2$ is divisible by 8 $n \geq 1$.

Q.B 7(ii)

Let $P(n) = 3^n + 7^n - 2$ is divisible by 8.

$n = 1$ $P(n_0) = 3 + 7 - 2 = 8$ which is by 8.

$P(n_0)$ is true.

Assume that $P(K)$ is true.
$P(K) = 3^K + 7^K - 2 = 8m$ where m is an integer.
$7^K = 8m - 3^K + 2 \rightarrow$ (1)

To show that $P(K+1)$ is true
$3^{K+1} + 7^{K+1} - 2$ is divisible by 8
$3^{K+1} + 7^{K+1} - 2 = 3 3^K + 7 7^K - 2$
$= 3 3^K + 7(8m - 3^K + 2) - 2$
$= 3 3^K + 7(8m) - 7(3^K) + 14 - 2$
$= 7(8m) - 4 3^K + 12$
$= 56m - 4 3^K + 12$
$= 56m - 4(3^K - 3)$

We know that, 3^K is odd no.
$\Rightarrow 3^K - 3$ is an even no.
$3^K - 3 = 2n$ where n is any integer
$3^{K+1} + 7^{K+1} - 2 = 56m - 4(2n)$
$= 8(7m - n)$ which is divisible by 8.

$P(K+1)$ true

Hence by the Principle of Mathematical induction
$P(n)$

Suppose a dept. consist of 8 men and 9 women. In h
many days can be select a commiting of.

Q.B 8(i)

(i) 3 men and 4 women.

(ii) 4 person at least 1 women.

(iii) 4 person atmost 1 man.

(iv) 4 person that has person's of both gender

(i) $n(m) = 8$

$n(W) = 9$.

3·M & 4 W.

$8C_3 \times 9C_4 = 7056$

(ii) 4 person at least 1 woment

$8C_3 \times 9C_1 + 8C_2 \times 9C_2 + 8C_1 \times 9C_3 + 9C_4$.

$= 2310$.

(iii) $8C_1 \times 9C_3 + 8C_0 \times 9C_4$

$= 798$

(iv) $8C_1 \times 9C_3 + 8C_2\, 9C_2 + 8C_3\, 9C_1$

2.) Solve $a_n - 3a_{n-1} = 2 \quad \forall\, n \geq 1,\ a_0 = 2.$

$$a_n - 3a_{n-1} = 2$$

Q.B
8 (ii).

Multiply both sides x^n;

$$a_n x^n - 3a_{n-1} x^n = 2x^n$$

$$\sum_{n=1}^{\infty} a_n x^n - 3\sum_{n=1}^{\infty} a_{n-1} x^n = \sum_{n\geq 1}^{\infty} 2x^n.$$

$(a_1x+a_2x^2+\cdots+a_nx^n+\cdots)-3x\sum_{n=1}^{\infty}a_{n-1}x^{n-1}=\sum_{n=1}^{\infty}2x^n$.

WKT,

$$G(x)=\sum_{n=0}^{\infty}a_nx^n$$

$$G(x)=a_0+a_1x+a_2x^2+a_3x^3+\cdots a_nx^n+\cdots$$

$$\Rightarrow G(x)-a_0-3x(a_0+a_1x+a_2x^2+\cdots+a_nx^n+\cdots)=2(x+x^2+x^3+\cdots x^n)$$

$$\Rightarrow G(x)-2-3x\,G(x)=2x\cdot(1+x+x^2+\cdots)$$

$$\Rightarrow G(x)\cdot(1-3x)=\frac{2x}{(1-x)}+2.$$

$$G(x)(1-3x)=\frac{\cancel{2x}+2-\cancel{2x}}{1-x}.$$

$$G(x)(1-3x)=2/_{1-x}.$$

$$G(x)=\frac{2}{(1-x)(1-3x)}$$

By partial fraction

$$\frac{2}{(1-x)(1-3x)}=\frac{A}{(1-3x)}+\frac{B}{1-x}.$$

$$2=A(1-x)+B(1-3x)$$

If $x=1/3$ $\quad 2=A(1-(1/3))+0\Rightarrow A(1-\frac{1}{3})\Rightarrow A\frac{3-1}{3}=A\,2/3$

$$=A\,2/3=2\Rightarrow A=\frac{2\times3}{2}=\frac{\cancel{6}^{3}}{\cancel{2}_{1}}=\boxed{A=3}$$

If $x=1$ $\quad 2=0+B(1-3(1))$

$$=0+B(1-3)=-2B=\boxed{B=-1}$$

$$G(x)=3(1-3x)^{-1}-1\cdot(1-x)^{-1}$$

$$=3[1+3x+(3x)^2+\cdots+(3x)^n+\cdots]-[1+x+x^2+\cdots x^n+\cdots].$$

$$a_n=3\cdot3^{n-1}$$ //.