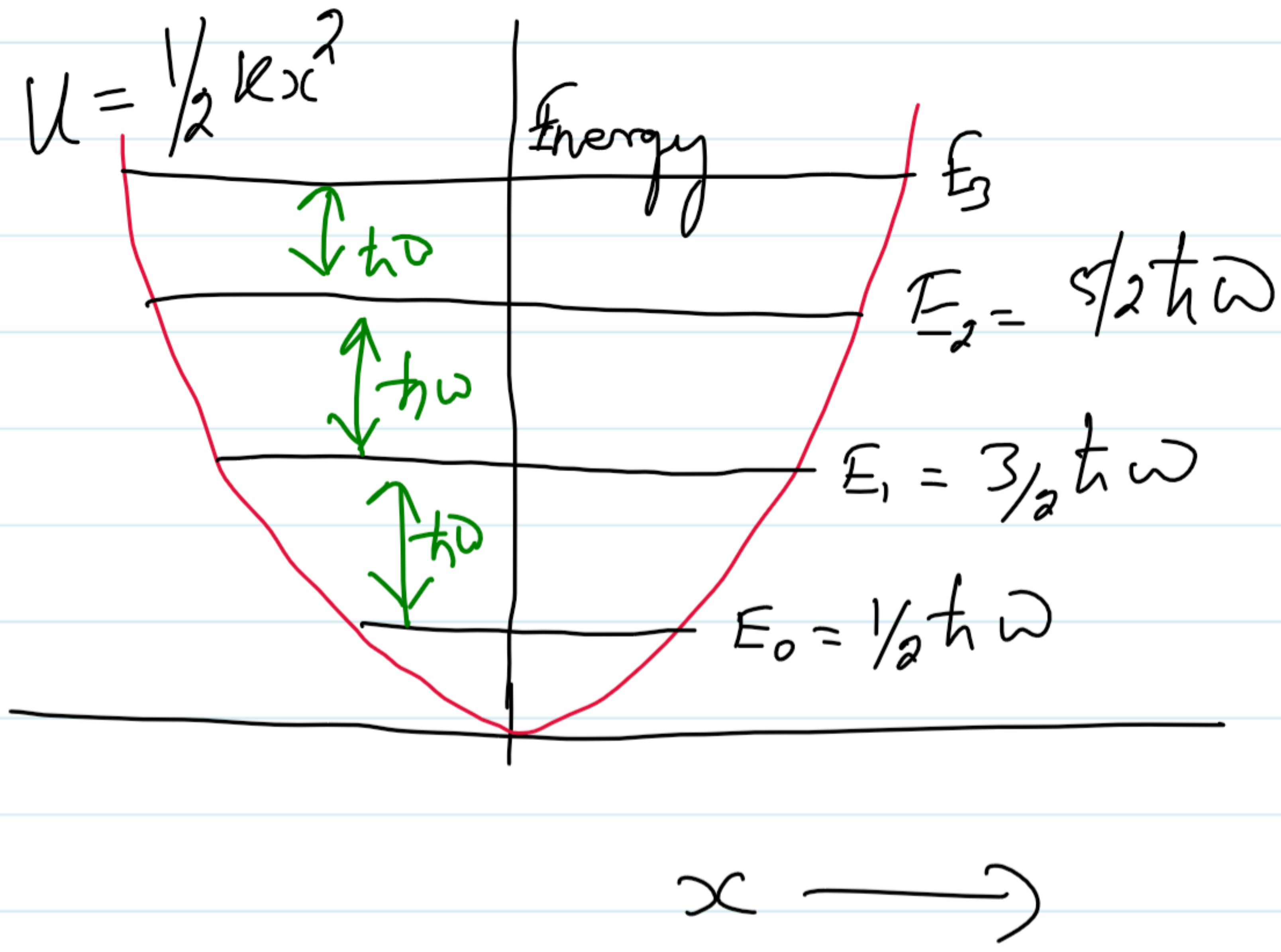
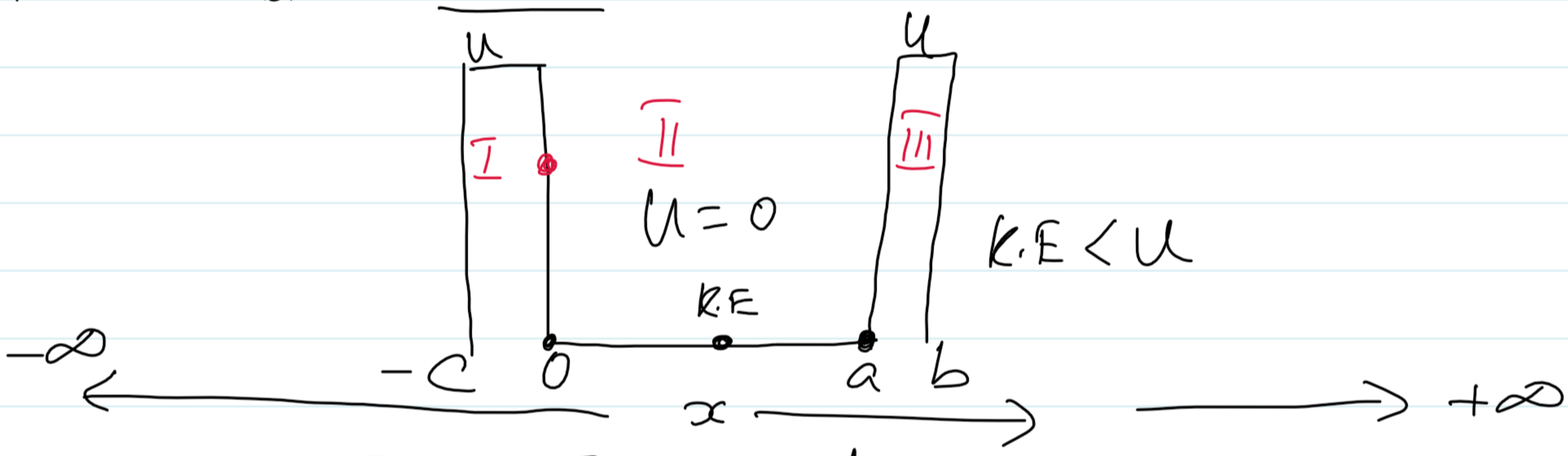


Particle inside a Finite Potential Well



Region I and III Finite Potential Barrier

Region II Zero potential

Schroedinger's equation Region I and Region III

$$\frac{d^2\psi_{0I}}{dx^2} + \frac{2m}{\hbar^2}(E-U)\psi_{0I} = 0$$

$$E - U < 0$$

$$\frac{d^2\psi_{0I}}{dx^2} - \left[\frac{2m}{\hbar^2}(U-E)\right]\psi_{0I} = 0$$

Let $\beta^2 = \frac{2m}{\hbar^2}(U-E)$

$$\frac{d^2\psi_{0I}}{dx^2} - \beta^2\psi_{0I} = 0 \quad \text{—(1)}$$

$$\frac{d^2\psi_{0III}}{dx^2} - \beta^2\psi_{0III} = 0 \quad \text{—(2)}$$

$$\psi_{0I} = Ae^{\beta x} + Be^{-\beta x} \quad - (3)$$

$$\psi_{0III} = Ce^{\beta x} + De^{-\beta x} \quad - (4)$$

A, B, C and D are integration constants

Schroedinger's equation for Region II $\quad 0 < x < a : U = 0$

$$\frac{d^2\psi_{0II}}{dx^2} + \frac{2m}{\hbar^2}(E - 0)\psi_{0II} = 0$$

$$\frac{d^2\psi_{0II}}{dx^2} + \frac{2mE}{\hbar^2}\psi_{0II} = 0$$

$$\frac{d^2\psi_{0II}}{dx^2} + k^2\psi_{0II} = 0 \quad - (5)$$

Let $k^2 = \dfrac{2mE}{\hbar^2}$

$$\psi_{0II} = F\sin kx + G\cos kx \quad - (6)$$

F and G are integration constants.

Boundary condition

Left Boundary

$$\left.\begin{aligned} \psi_{0I}(0) &= \psi_{0II}(0) \\ \frac{d\psi_{0I}(0)}{dx} &= \frac{d\psi_{0II}(0)}{dx} \end{aligned}\right\} (7)$$

Continuity condition

Right Boundary

$$\left.\begin{aligned} \psi_{0II}(a) &= \psi_{0III}(a) \\ \frac{d\psi_{0II}(a)}{dx} &= \frac{d\psi_{0III}(a)}{dx} \end{aligned}\right\} (8)$$

Wave functions must be always finite for any value of 'x'

Boundary condition and finiteness of wave function is used to determine A, B, C, D, F, G

$$\psi_{0I} = Ae^{\beta x} + Be^{-\beta x} \quad -(3) \qquad x<0$$

$x \to -\infty$

$$\psi_{0I} = Ae^{\beta(-\infty)} + Be^{-\beta(-\infty)}$$

$$\psi_{0I} = Ae^{-\infty} + Be^{+\infty} \qquad e^{-\infty} = 0$$

$$\psi_{0I} = 0 + B \times \infty \qquad e^{+\infty} = \infty$$

$$\boxed{B = 0}$$

$$\psi_{0III} = Ce^{\beta x} + De^{-\beta x} \qquad x > a, x \text{ is positive}$$

$x \to \infty$

$$\psi_{0III} = Ce^{\beta\infty} + De^{-\beta\infty}$$

$$\psi_{0III} = C\underbrace{e^{\infty}}_{\cancel{\infty}} + D\underbrace{e^{-\infty}}_{0}$$

$$\boxed{C = 0}$$

$$\psi_{0I} = Ae^{\beta x} \quad -(9)$$

$$\psi_{0III} = De^{-\beta x} \quad -(10)$$

Left Boundary Condition

$$\psi_{0I}(0) = \psi_{0II}(0)$$

$$Ae^{\beta \times 0} = F\sin k0 + G\cos k0$$

$$Ae^{0} = 0 + G$$

$$\boxed{A = G} \quad - (11)$$

$$\frac{d\psi_{0I}(0)}{dx} = \frac{d\psi_{0II}(0)}{dx}$$

$$\psi_{0I} = Ae^{\beta x}$$

$$\frac{d\psi_{0I}}{dx} = A\beta e^{\beta x}$$

$$\psi_{0II} = F\sin kx + G\cos kx$$

$$\frac{d\psi_{0II}}{dx} = Fk\cos kx - Gk\sin kx$$

$$A\beta e^{\beta 0} = Fk\cos k0 - Gk\sin k0$$

$$A\beta = Fk$$

$$G\beta = Fk$$

$$\boxed{F = \frac{G\beta}{k}} \quad - (12)$$

Right Boundary

$$\psi_{0II}(a) = \psi_{0III}(a)$$

$$\psi_{0II} = F\sin kx + G\cos kx$$

$$\psi_{0III} = De^{-\beta x}$$

$$F\sin ka + G\cos ka = De^{-\beta a}$$

$$\frac{G\beta}{k}\sin ka + G\cos ka = De^{-\beta a}$$

$$G\left[\frac{\beta}{k}\sin ka + \cos ka\right] = De^{-\beta a}$$

$$\boxed{D = G\left[\frac{\beta}{k}\sin ka + \cos ka\right]e^{+\beta a}} \quad (13)$$

A, F, D interms of G

$$\frac{d\psi_{0II}(a)}{dx} = \frac{d\psi_{0III}(a)}{dx}$$

$$\psi_{0III} = De^{-\beta x}$$

$$\frac{d\psi_{0III}}{dx} = -D\beta e^{-\beta x}$$

$$Fk\cos ka - Gk\sin ka = -D\beta e^{-\beta a}$$

$$\frac{G\beta}{\cancel{k}} \times \cancel{k}\cos ka - Gk\sin ka = -D\beta e^{-\beta a}$$

$$\frac{G\beta}{\cancel{k}} \times \cancel{k} \cos ka - Gk \sin ka = -D\beta e^{-\beta a}$$

$$G\left[\beta \cos ka - k \sin ka\right] = -D\beta e^{-\beta a}$$

$$D = -\frac{G}{\beta} e^{+\beta a}\left[\beta \cos ka - k \sin ka\right] - (14)$$

$$D = G\left[\frac{\beta}{k} \sin ka + \cos ka\right] e^{+\beta a} \quad (13)$$