1. prove that Inner product space $V(F)$

$\langle U, \alpha V+\beta W\rangle = \bar{\alpha}\langle U, V\rangle + \bar{\beta}\langle U, W\rangle$

Solution :

$$\langle U, \alpha V+\beta W\rangle = \overline{\langle \alpha V+\beta W, U\rangle}$$
$$= \overline{\langle \alpha V, U\rangle + \beta W, U\rangle}$$
$$= \bar{\alpha}\langle \overline{V, U}\rangle + \bar{\beta}\langle \overline{W, U}\rangle$$
$$= \bar{\alpha}\langle U, V\rangle + \beta\langle U, W\rangle$$

Hence proved

Q $\boxed{1(i)}$ Let $U=(a_1, a_2 \ldots a_n)$ $V=(b_1, b_2 \ldots b_n) \in F^n(C)$ Define $\langle U, V\rangle = a_1\bar{b_1} + a_2\bar{b_2} + \cdots + a_n\bar{b_n}$ verify is it an inner product of $F^n(C)$

Solution :

i) $$\langle \overline{U, V}\rangle = \overline{a_1\bar{b_1} + a_2\bar{b_2} \ldots + a_n\bar{b_n}}$$
$$= \bar{a_1}\,\bar{\bar{b_1}} + \bar{a_2}\,\bar{\bar{b_2}} \ldots + \bar{a_n}\,\bar{\bar{b_n}}$$
$$= \bar{a_1}b_1 + \bar{a_2}b_2 \ldots + \bar{a_n}b_n$$
$$= b_1\bar{a_1} + b_2\bar{a_2} + \ldots b_n\bar{a_n}$$
$$= \langle V, U\rangle$$

ii) $$\langle U, U\rangle = a_1\bar{a_1} + a_2\bar{a_2} + \ldots a_n\bar{a_n} \qquad \therefore\ a_1\bar{a_1} = |a_1|^2$$
$$= |a_1|^2 + |a_2|^2 + \ldots |a_n|^2 \geq 0$$
$$\langle U, U\rangle = 0 \text{ iff } |a_i|^2 = 0$$
$$|a_i| = 0$$
$$a_i = 0$$
$$\therefore U(0, 0 \ldots 0) = 0$$

iii) $$\langle \alpha U+\beta V, W\rangle = \langle \alpha(a_1, a_2 \ldots a_n) + \beta(b_1, b_2 \ldots b_n), (c_1, c_2 \ldots c_n)\rangle$$
$$\langle (\alpha a_1+\beta b_1)\bar{c_1}, (\alpha a_2+\beta b_2)\bar{c_2} \ldots (\alpha a_n+\beta b_n)\bar{c_n}$$
$$\langle (\alpha a_1\bar{c_1} + \alpha a_2\bar{c_2} \ldots \alpha a_n\bar{c_n}) + (\beta b_1\bar{c_1} + \beta b_2\bar{c_2} \ldots \beta b_n\bar{c_n})$$
$$\alpha\langle [a_1\bar{c_1} + a_2\bar{c_2} + \ldots a_n\bar{c_n}] + \beta[b_1\bar{c_1} + b_2\bar{c_2} + \ldots b_n\bar{c_n}]\rangle$$
$$= \alpha\langle U, W\rangle + \beta\langle V, W\rangle$$

From 1 & 2 & 3 it is an inner product in $F^n(C)$

Solution :

$\boxed{1(ii)}$

i) $\|u+v\|^2 + \|u-v\|^2 = 2\left[\|u\|^2 + \|v\|^2\right]$

$$\|u+v\|^2 + \|u-v\|^2 = \langle u+v, u+v\rangle + \langle u-v, u-v\rangle$$

$$= \left[\langle u,u\rangle + \cancel{\langle u,v\rangle} + \cancel{\langle v,u\rangle} + \langle v,v\rangle\right] +$$

$$\left[\langle u,u\rangle - \cancel{\langle u,v\rangle} - \cancel{\langle v,u\rangle} + \langle v,v\rangle\right]$$

$$= 2\langle u,u\rangle + 2\langle v,v\rangle$$

$$= 2\|u\|^2 + 2\|v\|^2$$

$\therefore$ Hence proved.

A) Prove that $R^2(R)$ is an inner product space defined for $U=(a_1,a_2)$ $V=(b_1,b_2)$ by $\langle U,V\rangle =$

$\boxed{2(i)}$

$\langle U,V\rangle = a_1b_1 - a_2b_1 - a_1b_2 + 2a_2b_2$.

Solution :

i) $\overline{\langle U,V\rangle} = \overline{a_1b_1 - a_2b_1 - a_1b_2 + 2a_2b_2}$

$= a_1b_1 - a_2b_1 - a_1b_2 + 2a_2b_2$

$= b_1a_1 - b_1a_2 - b_2a_1 + 2b_2a_2$

$= \langle V,U\rangle$

ii) $\langle U,U\rangle = a_1a_1 - a_2a_1 - a_1a_2 + 2a_2a_2$

$= a_1^2 - 2a_1a_2 + 2a_2^2$

$= \underbrace{a_1^2 - 2a_1a_2 + a_2^2} + a_2^2$

$= (a_1-a_2)^2 + a_2^2 \geq 0$

$\langle U,U\rangle = 0$ iff $(a_1-a_2)^2 + a_2^2 = 0$

$(a_1-a_2)^2 = 0$, $a_2^2 = 0$

$a_1 = a_2$; $a_2 = 0$

$\boxed{a_1=0}$ $\boxed{a_2=0}$ $\Rightarrow \boxed{U=0}$

iii) $\langle \alpha U + \beta V, W\rangle = \alpha\langle U,W\rangle + \beta\langle V,W\rangle$

$= \langle \alpha(a_1,a_2) + \beta(b_1,b_2), (c_1,c_2)\rangle$

$= \langle (\underset{a_1}{\alpha a_1 + \beta b_1}, \underset{a_2}{\alpha a_2 + \beta b_2}), (\underset{b_1}{c_1}, \underset{b_2}{c_2})\rangle$

$= (\alpha a_1 + \beta b_1)c_1 - (\alpha a_2 + \beta b_2)c_1 - (\alpha a_1 + \beta b_1)c_2 + 2(\alpha a_2 + \beta_2)c_2$

$= \alpha a_1c_1 + \beta b_1c_1 - \alpha a_2c_1 - \beta b_2c_1 - \alpha a_1c_2 - \beta b_1c_2 + 2\alpha a_2c_2 + 2\beta b_2c_2$

$= \alpha(a_1c_1 - a_2c_1 - a_1c_2 + 2a_2c_2) + \beta(b_1c_1 - b_2c_2 - b_1c_2 + 2b_2c_2)$

$= \alpha\langle U,W\rangle + \beta\langle V,W\rangle$

From ① ② & ③ of $R^2(R)$ is an Inner product space.

...amental structure...
...ought of either as a generalization of a ...

3.

NORM (OR) LENGTH OF A VECTOR

Let V be a inner product space $v \in V$. The norm of v denoted by $\|v\| = \sqrt{\langle v, v \rangle}$.

The vector v is called a unit vector if $\|v\| = 1$

✓. Let $u = (2, 1+i, i)$ $v = (2-i, 2, 1+2i)$ be vectors in C^3 over C compute using standard inner product $\langle u, v \rangle$, $\|u\|$, $\|v\|$, $\|u+v\|$ $\boxed{3(i)}$

Solution:

$$u = (\underset{a_1}{2}, \underset{a_2}{1+i}, \underset{a_3}{i}) \qquad v = (\underset{b_1}{2-i}, \underset{b_2}{2}, \underset{b_3}{1+2i})$$

$$\begin{aligned}
\langle u, v \rangle &= a_1\overline{b_1} + a_2\overline{b_2} + a_3\overline{b_3} \\
&= 2(\overline{2-i}) + (1+i)(\overline{2}) + i(\overline{1+2i}) \\
&= 2(2+i) + (1+i)2 + i(1-2i) \\
&= 4 + 2i + 2 + 2i + i + 2 \\
&= 8 + 5i
\end{aligned}$$

$$\begin{aligned}
\|u\| = \sqrt{\langle u, u \rangle} &= \sqrt{a_1\overline{a_1} + a_2\overline{a_2} + a_3\overline{a_3}} \\
&= \sqrt{2(2) + 1+i(\overline{1+i}) + i(\overline{i})} \\
&= \sqrt{4 + (1+i)(1-i) + i(-i)} \\
&= \sqrt{4 + 1 \cancel{-i} \cancel{+i} - i^2 - i^2} \\
&= \sqrt{4+1+1+1} \\
&= \sqrt{7}
\end{aligned}$$

$$\begin{aligned}
\|v\| = \sqrt{\langle v, v \rangle} &= \sqrt{b_1\overline{b_1} + b_2\overline{b_2} + b_3\overline{b_3}} \\
&= \sqrt{(2-i)(\overline{2-i}) + 2(\overline{2}) + (1+2i)(\overline{1+2i})} \\
&= \sqrt{(2-i)(2+i) + 4 + (1+2i)(1-2i)} \\
&= \sqrt{4 - i^2 + 4 + 1 - 4i^2} \\
&= \sqrt{4+1+4+1+4} \\
&= \sqrt{14}
\end{aligned}$$

$\| u+v \| = \sqrt{\langle u+v, u+v \rangle}$

$u+v = (2, 1+i, i) + (2-i, 2, 1+2i)$

$= (2+2-i, 1+i+2, i+1+2i)$

$= (4-i, 3+i, 1+3i)$

$\| u+v \| = \sqrt{\langle u+v, u+v \rangle}$

$= \sqrt{(4-i)(4+i) + (3+i)(3-i) + (1+3i(1-3i)}$

$= \sqrt{16-i^2+9-i^2+1-9i^2}$

$= \sqrt{16+1+9+1+1+9}$

$= \sqrt{37}$

$\| f \| = \sqrt{}$

A linear operator T on $R^2(R)$ is defined by $T(x,y) = (2x+y, x-3y)$ with std IPS find $T^*(x,y)$ & $T^*(3,5)$.

4(i)

Solution:

The Basis of $R^2(R)$ $e_1 = (1,0)$ $e_2(0,1)$

$T(1,0) = 2(1)+0, 1-3(0) \Rightarrow (2,1)$

$T(0,1) = 2(0)+1, 0-3(1) \Rightarrow (1,-3)$

coefficient matrix $= \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}$

$$\text{Matrix}[T]_\beta = \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\text{Matrix } T^*_\beta = \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}$$

$$T^*(x,y) = \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x+y \\ x-3y \end{bmatrix}$$

$$= 2x+y, \quad x-3y$$

$$T^*(3,5) = 2(3)+5, \quad 3-3(5)$$

$$= (11, -12)$$

them $u, v \in V$

logram]

$\boxed{4(ii)}$

iii) $|\langle u, v\rangle| \leq \|u\| \, \|v\|$ $\boxed{4(ii)}$

case i)

If $u = 0$ (or) $v = 0$

$\Rightarrow \langle u, v\rangle = 0$

$\Rightarrow \|u\| = 0 \quad \|v\| = 0$

$\therefore |\langle u, v\rangle| = \|u\| \, \|v\|$

case ii)

Let $\langle u, v\rangle$ be real and $u \neq 0$ and λ is real

$0 \leq (\lambda u + v, \lambda u + v)$

$= \lambda\bar{\lambda}\langle u, u\rangle + \lambda\langle u, v\rangle + \bar{\lambda}\langle v, u\rangle + \langle v, v\rangle$ $\quad \therefore \bar{\lambda} = \lambda$

$= \lambda^2\langle u, u\rangle + \lambda\langle u, v\rangle + \lambda\langle u, v\rangle + \langle v, v\rangle$ $\quad \therefore \langle v, u\rangle = \langle u, v\rangle$

$= \lambda^2\langle u, u\rangle + 2\lambda\langle u, v\rangle + \langle v, v\rangle$

Let $a = \langle u, u\rangle$, $b = \langle u, v\rangle$, $c = \langle v, v\rangle$

$\therefore 0 \leq a\lambda^2 + 2b\lambda + c$

By theorem lemma $a>0$ and $a\lambda^2 + 2b\lambda + c \geq 0$ for all λ

then $b^2 \leq ac$ — ①

sub a, b, c in ①

$$\langle U, V\rangle^2 \leq \langle U, U\rangle \langle V, V\rangle$$

$$|\langle U, V\rangle|^2 \leq \|U\|^2 \|V\|^2 \qquad \therefore a^2 = |a|^2$$

Taking square root on both side

$$|\langle U, V\rangle| \leq \|U\| + \|V\|$$

case iii)

If $\alpha = \langle U, V\rangle$ is not real

$$\langle \frac{U}{\alpha}, V\rangle = \frac{1}{\alpha} \langle U, V\rangle$$

$$= \frac{1}{\cancel{\langle U, V\rangle}} \cancel{\langle U, V\rangle} = 1$$

$\therefore \langle \frac{U}{\alpha}, V\rangle = 1$ is a real

$$\therefore 1 = |\langle \frac{U}{\alpha}, V\rangle| \leq \|\frac{U}{\alpha}\| \|V\| \qquad \because [\text{By case(ii)}]$$

$$= |\frac{1}{\alpha}| \|U\| \|V\| \qquad \therefore \|\alpha U\| = |\alpha| \|U\|$$

$$1 \leq \frac{1}{|\alpha|} \|U\| \|V\|$$

$$|\alpha| \leq \|U\| \|V\| \qquad \alpha = U, V$$

$$|\langle U, V\rangle| \leq \|U\| \|V\|$$

$\therefore$ Hence proved.

iv) $\|U+V\| \leq \|U\| + \|V\|$

$$\|U+V\|^2 = \langle U+V, U+V\rangle$$

$$\langle U, U\rangle + \langle U, V\rangle + \langle V, U\rangle + \langle V, V\rangle$$

$$\langle \|U\|^2 + \langle U, V\rangle + \langle \overline{U, V}\rangle + \|V\|^2$$

$$\langle \|U\|^2 + 2|\langle U, V\rangle| + \|V\|^2 \qquad \therefore z + \bar{z} = 2x$$

$$\|U+V\|^2 = \langle \|U\|^2 + 2\|U\| \|V\| + \|V\|^2$$

Gram Schmidt orthogonalization process

working Rule:

1. $U_1 = V_1$

2. $U_2 = V_2 - \frac{\langle V_2, U_1 \rangle U_1}{\| U_1 \|^2}$

3. $U_3 = V_3 - \frac{\langle V_3, U_1 \rangle U_1}{\| U_1 \|^2} - \frac{\langle V_3, U_2 \rangle U_2}{\| U_2 \|^2}$

4. The orthonormal basis is $\{ w_1, w_2, w_3 \}$

$w_1 = \frac{U_1}{\| U_1 \|} \quad w_2 = \frac{U_2}{\| U_2 \|} \quad w_3 = \frac{U_3}{\| U_3 \|}$

In a IPS $R^3(R)$ with std inner product $B = \{(1,0,1), (1,0,-1) (0,3,4)\}$

$\boxed{5}$

Solution:

$B = \{(1,0,1), (1,0,-1) (0,3,4)\}$

$B = \{ v_1, v_2, v_3 \}$

$v_1 = (1,0,1) \quad v_2 = (1,0,-1) \quad v_3 = (0,3,4)$

i) $V_1 = U_1 = (1,0,1)$

ii) $U_2 = V_2 - \frac{\langle V_2, U_1 \rangle U_1}{\| U_1 \|^2}$

$\langle V_2, U_1 \rangle = ((1,0,-1)(1,0,1) \Rightarrow 1-1 \Rightarrow 0$

$\| U_1 \|^2 = \langle U_1, U_1 \rangle$

$= \langle (1,0,1)(1,0,1) \rangle \Rightarrow 1+1 \Rightarrow 2$

$U_2 = (1,0,-1) - \frac{0(1,0,1)}{2}$

$U_2 = (1,0,-1)$

iii) $U_3 = V_3 - \frac{\langle V_3, U_1 \rangle U_1}{\| U_1 \|^2} - \frac{\langle V_3, U_2 \rangle U_2}{\| U_2 \|^2}$

$\langle V_3, U_1 \rangle = \langle (0,3,4)(1,0,+1) \Rightarrow 0+0+4 \Rightarrow +4$

$\| U_2 \|^2 = \langle U_2, U_2 \rangle$

$\langle (1,0,-1)(1,0,-1) \Rightarrow 1+0+1 \Rightarrow 2$

~~$U_3 = (0,3,4) - \frac{0(1,0,1)}{2} - (+4)(1,0,+1)$~~

$\langle V_3, U_2 \rangle = \langle (0,3,4)(1,0,-1) \Rightarrow 0+0-4 \Rightarrow -4$

$U_3 = (0,3,4) - \frac{\overset{2}{\cancel{4}}(1,0,1)}{\cancel{2}} - \frac{(\overset{2}{\cancel{-4}})(1,0,-1)}{\cancel{2}}$

$= (0,3,4) - (2,0,2) - (-2,0,2) \Rightarrow (0-2+2), (3-0+0), (4-2-2)$

$U_3 = (0,3,0)$

The orthogonal basis are $\{U_1, U_2, U_3\}$

$\{(1,0,1), (1,0,-1), (0,3,0)\}$

iv) Let the orthogonal basis are $\{w_1, w_2, w_3\}$

$w_1 = \frac{U_1}{\|U_1\|} \Rightarrow \frac{(1,0,1)}{\sqrt{2}}$

$\therefore \|U_1\|^2 = 2$

$\|U_1\| = \sqrt{2}$

$w_2 = \frac{U_2}{\|U_2\|} \Rightarrow \frac{(1,0,-1)}{\sqrt{2}}$

$w_3 = \frac{U_3}{\|U_3\|} \Rightarrow \frac{(0,3,0)}{3}$

$\therefore \|U_3\|^2 = \langle U_3, U_3 \rangle$

$= \langle (0,3,0)(0,3,0) \rangle$

$= 0+9+0 \Rightarrow 9$

$\|U_3\| = 3$

$w_1 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

$w_2 = \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$

$w_3 = (0, 1, 0)$

3. In an IPS $R^3(R)$ with std inner product $B = \{(1,1,0)\ (1,-1,1)\ (-1,1,2)\}$ is a basis of G.S.O.P. find an fourier coefficients of $\boxed{6}$

Solution :

$$B = \{(1,1,0)\ (1,-1,1)\ (-1,1,2)\}$$

$$v_1 = (1,1,0) \quad v_2 = (1,-1,1) \quad v_3 = (-1,1,2)$$

i) $u_1 = v_1 = (1,1,0)$

ii) $u_2 = v_2 - \dfrac{\langle v_2, u_1\rangle u_1}{\|u_1\|^2}$

$$\langle v_2, u_1\rangle = \langle (1,-1,1)(1,1,0)\rangle \Rightarrow 1-1+0 \Rightarrow 0$$

$$\|u_1\|^2 = \langle u_1, u_1\rangle$$
$$= \langle (1,1,0)(1,1,0)\rangle$$
$$= 1+1+0 \Rightarrow 2$$

$$u_2 = (1,-1,1) - \frac{0\,\cancel{(1,1,0)}}{2}$$

$$u_2 = (1,-1,1)$$

$\|u_1\|$

$w_2 = \dfrac{u_2}{\|u_2\|} \Rightarrow ($

$w_3 = \dfrac{u_3}{\|u_3\|} \Rightarrow (-1,$

$w_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$w_2 = \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right.$

$w_3 = \left(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right.$

The Fourier coeffic

$v = \langle v, w_1\rangle w_1 + \langle$

$\langle v, w_1\rangle = \langle ($

$\langle v, w_2\rangle = \langle ($

$\langle v, w_3\rangle = \langle (2$

$\therefore$ The Fourier

iii) $U_3 = V_3 - \frac{<V_3, U_1> U_1}{\|U_1\|^2} - \frac{<V_3, U_2> U_2}{\|U_2\|^2}$

$<V_3, U_1> = <(-1,1,2)(1,1,0)> \Rightarrow -1+1 \Rightarrow 0$

$<V_3, U_2> = <(-1,1,2)(1,-1,1)> \Rightarrow -1-1+2 \Rightarrow 0$

$\|U_2\|^2 = <U_2, U_2>$

$= <(1,-1,1)(1,-1,1)>$

$= 1+1+1 \Rightarrow 3$

$U_3 = (-1,1,2) - 0\frac{(1,1,0)}{2} - 0\frac{(1,-1,1)}{3}$

$U_3 = (-1,1,2)$

$\therefore$ The orthogonal basis are $\{U_1, U_2, U_3\}$

$U_1 = (1,1,0) \quad U_2 = (1,-1,1) \quad U_3 = (-1,1,2)$

iv) The orthogonal basis are $\{w_1, w_2, w_3\}$

$w_1 = \frac{U_1}{\|U_1\|} \Rightarrow \frac{(1,1,0)}{\sqrt{2}}$

$w_2 = \frac{U_2}{\|U_2\|} \Rightarrow \frac{(1,-1,1)}{\sqrt{3}}$

$w_3 = \frac{U_3}{\|U_3\|} \Rightarrow \frac{(-1,1,2)}{\sqrt{6}}$ $\quad \therefore \|U_3\|^2 = <(-1,1,2)(-1,1,2)>$ $= 1+1+4 \Rightarrow 6$

$w_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$w_2 = \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

$w_3 = \left(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

The Fourier coefficient are $<V, w_1>, <V, w_2>, <V, w_3>$ $\quad V = (2,1,3)$

$V = <V, w_1> w_1 + <V, w_2> w_2 + <V, w_3> w_3$

$<V, w_1> = <(2,1,3)\frac{(1,1,0)}{\sqrt{2}}> \Rightarrow \frac{2+1}{\sqrt{2}} \Rightarrow \frac{3}{\sqrt{2}}$

$<V, w_2> = <(2,1,3)\frac{(1,-1,1)}{\sqrt{3}}> \Rightarrow \frac{2-1+3}{\sqrt{3}} \Rightarrow \frac{4}{\sqrt{3}}$

$<V, w_3> = <(2,1,3)\frac{(-1,1,2)}{\sqrt{6}}> \Rightarrow \frac{-2+1+6}{\sqrt{6}} \Rightarrow \frac{5}{\sqrt{6}}$

$\therefore$ The Fourier coefficient are $\left(\frac{3}{\sqrt{2}}, \frac{4}{\sqrt{3}}, \frac{5}{\sqrt{6}}\right)$

2. Let $V = P(R)$ the vectorspace of product polynomials over R with (IP) defined by $\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$ where $f(t) = t+2$, $g(t) = t^2-2t+3$ find $\|f\|$, $\|g\|$, $\langle f,g\rangle$, $\|f+g\|$

$\boxed{7(i)}$

Solution :

Let V be an inner product space defined over P $t\in V$

$$\langle f, g\rangle = \int_0^1 f(t)\,g(t)\,dt$$

$$= \int_0^1 (t+2)(t^2-2t-3)\,dt$$

$$= \int_0^1 (t^3 - \cancel{2t^2} - 3t + \cancel{2t^2} - 4t - 6)\,dt$$

$$= \int_0^1 (t^3 - 7t - 6)\,dt$$

$$\left[\frac{t^4}{4} - \frac{7t^2}{2} - 6t\right]_0^1 \Rightarrow \frac{1}{4} - \frac{7}{2} - \frac{6}{1} \Rightarrow \frac{1-14-24}{4} \Rightarrow \frac{-37}{4}$$

L.c.m

$\|f\| = \sqrt{\frac{\ldots}{3}}$

$\|g\| = \sqrt{\langle g, g\rangle} = \int_0^1 g(t) \ldots$

$= \int_0^1 (t^2 - 2t \ldots$

$= \int_0^1 t^4 - 2t \ldots$

$= \int_0^1 (t^4 - 4t \ldots$

$= \int_0^1 (t^4 - 4t \ldots$

$= \left[\frac{t^5}{5} - \frac{4 \ldots}{\ldots}\right.$

$= \left[\frac{1}{5} - 1 - \frac{2}{3} \ldots\right.$

$= \left[\frac{210 + 3 \ldots}{15}\right.$

$= \sqrt{\frac{203}{15}}$

of either as a generalization of a Numpy arrays, or as a

nary.

$$\|f\| = \sqrt{\langle f, f \rangle} = \int_0^1 f(t)\, f(t)\, dt$$

$$= \int_0^1 (t+2)(t+2)\, dt$$

$$= \int_0^1 (t+2)^2\, dt$$

$$= \left[\frac{(t+2)^3}{3}\right]_0^1$$

$$= \frac{(3)^{\cancel{3}\,2}}{\cancel{3}} - \frac{(2)^3}{3} \Rightarrow 3^2 - \frac{2^3}{3}$$

$$= 9 - \frac{8}{3} \Rightarrow \frac{27-8}{3} \Rightarrow \frac{19}{3}$$

$$\|f\| = \sqrt{\frac{19}{3}}$$

$$\|g\| = \sqrt{\langle g, g \rangle} = \int_0^1 g(t)\, (g(t)\, dt$$

$$= \int_0^1 (t^2 - 2t - 3)(t^2 - 2t - 3)\, dt$$

$$= \int_0^1 t^4 - 2t^3 - 3t^2 - 2t^3 + 4t^2 + 6t - 3t^2 + 6t + 9)\, dt$$

$$= \int_0^1 (t^4 - 4t^3 + 4t^2 - 9)\, dt$$

$$= \int_0^1 (t^4 - 4t^3 - 2t^2 + 12t + 9)\, dt$$

$$= \left[\frac{t^5}{5} - \frac{4t^4}{4} - \frac{2t^3}{3} + \frac{12t^2}{2} + 9t\right]_0^1$$

$$= \left[\frac{1}{5} - 1 - \frac{2}{3} + 6 + 9\right] \Rightarrow \left[\frac{14}{1} + \frac{1}{5} - \frac{2}{3}\right]$$

$$= \left[\frac{210 + 3 - 10}{15}\right] \Rightarrow \left[\frac{213 - 10}{15}\right]$$

$$= \sqrt{\frac{203}{15}}$$

$f+g = (t+2+t^2-2t-3)$

$= t^2-t-1$

$\|f+g\| = \sqrt{<f+g, f+g>}$

$$<f+g, f+g> = \int_0^1 (f+g)(t)(f+g)t \, dt$$

$$= \int_0^1 (t^2-t-1)(t^2-t-1)dt$$

$$= \int_0^1 (t^4-t^3-\cancel{t^2}-t^3+\cancel{t^2}+t-t^2+t+1)dt$$

$$= \int_0^1 (t^4-2t^3-t^2+2t+1)dt$$

$$= \left[\frac{t^5}{5} - \frac{2t^4}{4} - \frac{t^3}{3} + \frac{2t^2}{2} + t\right]_0^1$$

$$= \left[\frac{1}{5} - \frac{1}{2} - \frac{1}{3} + 1 + 1\right] \Rightarrow \left[2 + \frac{1}{5} - \frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{60+6-15-10}{30} \Rightarrow \frac{66-25}{30} \Rightarrow \frac{41}{30}$$

$$\|f+g\| = \sqrt{\frac{41}{30}}$$

3\. Let V be inner product space over F then $u, v \in V$ & $x, y \in V$.

i) $\|u+v\|^2 + \|u-v\|^2 = 2[\|u\|^2 + \|v\|^2]$ [Parallelogram]

ii) $\|\alpha u\| = |\alpha| \, |u|$

iii) $|<u,v>| \leq \|u\| \, \|v\|$ [Schwarz inequality]

iv) $|u+v| \leq \|u\| + \|v\|$ Triangle inequality $\boxed{4(ii)}$

v) $|\|x\| - \|y\|| \leq \|x-y\|$

Adjoint of a matrix is $A^* = (\bar{A})^T$

Adjoint operator

Let $V(F)$ be finite dimensional IPS an T be a linear operator on V then the adjoint of T on V denoted by T^* is defined as $\langle T(U), V\rangle = \langle U, T^*(V)\rangle$ for all $U, V \in V$

THEOREM:

If $V(F)$ is an IPS and T is linear operator on V then i) $(T^*)^* = T$ ii) $(\alpha T)^* = \bar{\alpha} T^* \; \forall \alpha \in F$ $\boxed{7(ii)}$

i) $(T^*)^* = T$

consider $\langle U, (T^*)^*(V)\rangle$

$\langle T^*(U), V\rangle$

$\overline{\langle V, T^*(U)\rangle}$

$\overline{\langle T(V), U\rangle}$

$\langle U, T(V)\rangle$

$\therefore (T^*)^* = T$

$(T^*)^*(V) = T(V) \quad \forall V \in V$

$\therefore (T^*)^* = T$

ii) $(\alpha T)^* = \bar{\alpha} T^*$

$\langle U, (\alpha T)^* V\rangle$

$\langle \alpha T(U), V\rangle$

$\langle \alpha U, T^*(V)\rangle$

$\alpha \langle U, T^*(V)\rangle$

$\langle U, \bar{\alpha} T^*(V)\rangle \quad \forall V \in V$

$(\alpha T)^*(V) = \bar{\alpha} T^*(V)$

$(\alpha T)^* = \bar{\alpha} T^*$

The next fundamental structure in pandas is DataFrame. Like the Series object Frame can be thought of either as a generalization

7. verify the set $\{v_1, v_2, v_3\}$ $v_1 = (0, 1, -1)$ $v_2 = (1+i, 1, 1)$ $v_3 = (1-i, 1, 1)$ in C^3 is a basis over C construct an orthogonal basis by Gram Schmidt orthogonalization process Hence find the orthonormal basis with std inner product. $\boxed{8}$

Solution:

$v_1 = (0, 1, -1)$ $v_2 = (1+i, 1, 1)$ $v_3 = (1-i, 1, 1)$

$$A = \begin{vmatrix} 0 & 1 & -1 \\ 1+i & 1 & 1 \\ 1-i & 1 & 1 \end{vmatrix}$$

$$= -1(1+i-(1-i)) - 1(1+i-(1-i))$$

$$= -1(\cancel{1}+i-\cancel{1}+i) - 1(\cancel{1}+i-\cancel{1}+i)$$

$$= -1(2i) - 1(2i)$$

$$= -2i - 2i \neq 0$$

$\therefore$ The vectors are linearly independent $\dim(c^3) = 3$

$\Rightarrow \{v_1, v_2, v_3\}$ is a basis of $c^3(c)$

i) $u_1 = v_1 = (0, 1, -1)$

ii) $u_2 = v_2 - \dfrac{\langle v_2, u_1\rangle u_1}{\|u_1\|^2}$ $\because \|u_1\|^2 = \langle u_1, u_1\rangle \Rightarrow (0,1,-1)(0,1,-1) \Rightarrow 2$

$\langle v_2, u_1\rangle = (1+i, 1, 1) \cdot (0, 1, -1) \Rightarrow 1 - 1 = 0$

$u_2 = (1+i, 1, 1) - \dfrac{0(0,1,-1)}{2}$

$u_2 = (1+i, 1, 1)$

iii) $u_3 = v_3 - \dfrac{\langle v_3, u_1\rangle u_1}{\|u_1\|^2} - \dfrac{\langle v_3, u_2\rangle u_2}{\|u_2\|^2}$

$(1-i)(1+i) \Rightarrow 1 + \cancel{i} - \cancel{i} - i^2$

$1+1 = 2$

$\langle v_3, u_1\rangle = \langle (1-i, 1, 1), (0, 1, -1)\rangle \Rightarrow 0$

$\langle v_3, u_2\rangle = \langle (1-i, 1, 1), (1+i, 1, 1)\rangle \Rightarrow 2 + 1 + 1 \Rightarrow 4$

$\|u_2\|^2 = \langle u_2, u_2\rangle$

$\langle (1+i, 1, 1)(1+i, 1, 1)\rangle$

$(1+i)^2 + 1 + 1 \Rightarrow 1 + 2i + i^2 + 1 + 1$

$= \cancel{1} + 2i - \cancel{1} + 1 + 1$

$= 2i + 2$ $\therefore$ only take coefficient

$= 4$

$u_3 = (1-i, 1, 1) - \dfrac{0(0,1,-1)}{2} - \dfrac{\cancel{4}(1+i, 1, 1)}{\cancel{4}} \Rightarrow$ ~~xxxxx~~ 0

The orthogonal basis are $\{(0,1,-1), (1+i, 1, 1), (0, 0, 0)\}$