1) Using laws of logic prove that

Q.B 1(i) $\underbrace{\neg(p \wedge q)}_{P} \rightarrow \underbrace{(\neg p \vee (\neg p \vee q))}_{Q}$

Using implication conversion law

$$p \rightarrow q \equiv \neg p \vee q$$

$\Leftrightarrow \neg(\neg(p \wedge q)) \vee (\neg p \vee (\neg p \vee q))$

$\Leftrightarrow (p \wedge q) \vee (\neg p \vee (\neg p \vee q))$ [Negation law]

$\Leftrightarrow (p \wedge q) \vee ((\neg p \vee \neg p) \vee q)$ [Associativity]

$\Leftrightarrow (p \wedge q) \vee (\neg p \vee q)$ [Idempotent law]

$\Leftrightarrow (\neg p \vee q) \vee (p \wedge q)$ [Commutative]

$\Leftrightarrow ((\neg p \vee q) \vee p) \wedge ((\neg p \vee q) \wedge q)$ [Distributive]

$\Leftrightarrow ((q \vee \neg p) \vee p) \wedge (\neg p \vee q) \wedge q)$ [Commutative]

$\Leftrightarrow (q \vee (\neg p \vee p)) \wedge (\neg p \vee (q \wedge q))$ [Associative]

$\Leftrightarrow (q \vee T) \wedge (\neg p \vee q)$ [Negation]

$\Leftrightarrow T \wedge (\neg p \vee q)$ [Dominant]

$= \neg p \vee q$

1) Apply PDNF for the following implications

1(ii) $(p \wedge q) \vee r \rightarrow \neg p$ and find PCNF.

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$	$\neg p$	$(p \wedge q) \vee r \rightarrow \neg p$	Min terms	Max terms
T	T	T	T	T	F	F		$p \vee q \vee r$
T	T	F	T	T	F	F		$p \vee q \vee \neg r$
T	F	T	F	T	F	F		$p \vee \neg q \vee r$
T	F	F	F	F	F	T	$p \wedge \neg q \wedge \neg r$	
F	T	T	F	T	T	T	$p \wedge \neg q \wedge r$	
F	T	F	F	F	T	T	$\neg p \wedge q \wedge \neg r$	
F	F	T	F	F	T	T	$\neg p \wedge \neg q \wedge r$	
F	F	F	F	F	T	T	$\neg p \wedge \neg q \wedge \neg r$	

PDNF $\Rightarrow (p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$ QB H(i)

$\vee (\neg p \wedge \neg q \wedge \neg r)$

PCNF $\Rightarrow (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg q \vee r)$

2) Prove the following using laws of logic

2(i) $P \to (Q \to R) \Rightarrow (P \to Q) \to (P \to R)$

It is enough to prove

$P \to (Q \to R) \to (P \to Q) \to (P \to R)$ is tautology

LHS

$$\left.\begin{aligned} P \to (Q \to R) &= P \to [\neg Q \vee R] \\ &= \neg P \vee (\neg Q \vee R) \end{aligned}\right\} \text{[Implication]}$$

$$= (\neg P \vee \neg Q) \vee R \quad \text{[Demorgan's]}$$

$$= \neg (P \wedge Q) \vee R \longrightarrow (1)$$

RHS

$$(P \to Q) \to (P \to R)$$

$$= (\neg P \vee Q) \to (\neg P \vee R)$$

$$= \neg (\neg P \vee Q) \vee (\neg P \vee R)$$

$$= (P \wedge \neg Q) \vee (\neg P \vee R)$$

$= (\neg Q \wedge P) \vee (\neg P \vee R)$

$= ((\neg Q \wedge P) \vee \neg P) \vee R$

$= ((\neg Q \vee \neg P) \wedge (P \vee \neg P)) \vee R$

$= ((\neg Q \vee \neg P) \wedge T) \vee R$

$= (\neg Q \vee \neg P) \vee R$

$= \neg(Q \wedge P) \vee R \longrightarrow ②$

$P \to (Q \to R) \to (P \to Q) \to (P \to R)$

$\equiv \neg(P \wedge Q) \vee R \longrightarrow \neg(Q \wedge P) \vee R$

$= \neg(\neg(P \wedge Q) \vee R)) \vee (\neg(Q \wedge P) \vee R)$

$= ((P \wedge Q) \vee R) \vee (\neg(P \wedge Q) \vee R) = T$

$\Rightarrow$ using Truth table

P	Q	R	$Q \to R$	$P \to (Q \to R)$	$P \to Q$	$P \to R$	$(P \to Q) \to (P \to R)$	$P \to (Q \to R) \to (P \to Q) \to (P \to R)$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

2(ii) Show that $J \wedge S$ logically follows from the premises $P \to Q$, $Q \to \neg R$, R, $P \vee (J \wedge S)$

Step No	Statement	Reason
1	$P \to Q$	Rule P
2	$Q \to \neg R$	Rule P
3.	$P \to \neg R$	1, 2, Hypothetical syllogism
4.	$R \to \neg P$	3, Contrapositive
5.	R	Rule P
6.	$\neg P$	4,5 Modus ponens
7.	$P \vee (J \wedge S)$	Rule P
8.	$J \wedge S$	6,7 Disjunction syllogism.

1) $(\neg p \rightarrow r) \wedge (q \leftrightarrow p) \Rightarrow$ Find PCNF and PDNF

Q.B 3(i)

p	q	r	$\neg p$	$\neg p \wedge r$	$q \leftrightarrow p$	$(\neg p \wedge r) \wedge (q \leftrightarrow p)$	Min term	Max term
T	T	T	F	T	T	T	$p \wedge q \wedge r$	
T	T	F	F	T	T	T	$p \wedge q \wedge \neg r$	
T	F	T	F	T	F	F		$\neg p \vee \neg q \vee r$
T	F	F	F	T	F	F		$\neg p \vee \neg q \vee \neg r$
F	T	T	T	T	F	F		$p \vee q \vee r$
F	T	F	T	F	F	F		$p \vee q \vee \neg r$
F	F	T	T	T	T	T	$\neg p \wedge \neg q \wedge r$	
F	F	F	T	F	T	T		$p \vee \neg q \vee \neg r$

PDNF $\Rightarrow (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

PCNF $\Rightarrow (p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee r)$

$\wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee \neg r)$

$\wedge r)$ QB

3(ii) Prove that $P \rightarrow \neg S$ from the indirect premises

$P \rightarrow (q \vee r)$, $q \rightarrow \neg P$, $S \rightarrow \neg r$ and P

Assume the contradiction that $\neg(P \rightarrow \neg S)$

$$= \neg(\neg P \vee \neg S)$$

$$= P \wedge S$$

We use $P \wedge S$ are additional premises

Step NO	Statement	Reason
1	$P \rightarrow (q \vee r)$	Rule P
2	P	Rule P
3	$q \vee r$	1, 2, T Modus ponens
4	$P \wedge S$	Additional premise

5	s	Rule P
6	$s \rightarrow \neg r$	Rule P
7	$\neg r$	5, 6, Modous ponens
8	q	Rule P
9	$q \rightarrow \neg p$	Rule P
10	$\neg p$	8, 9 Modus ponens
11	$p \wedge \neg p = F$	2, 10 Conjunction.

QB

4(i)

Without constructing the truth table. Find the PDNF and PCNF of $P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$

let $A \equiv P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$

$\equiv P \vee (\neg(\neg P) \vee (Q \vee (\neg Q \rightarrow R)))$

$\equiv P \vee (P \vee (Q \vee (\neg Q \rightarrow R)))$

$= P \vee (P \vee (Q \vee (\neg(\neg Q) \vee R)))$

$= P \vee (P \vee (Q \vee (Q \vee R))$

$= P \vee (P \vee ((Q \vee Q) \vee R))$

$= P \vee (P \vee (Q \vee R))$

$= (P \vee P) \vee (Q \vee R)$

$= P \vee Q \vee R \longrightarrow$ Max term

$=$ PCNF

$$\neg A = (\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$$

$$\wedge (\neg P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)$$

$$\wedge (P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R)$$

$\neg(\neg A) =$ PDNF

$$= (P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$$

$$\vee (\neg P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$$

$$\vee (P \wedge Q \wedge R)$$

Q.13 #(ii) Show that the following premises are inconsistent $p \to q$, $q \to r$, $s \to \neg r$, $q \wedge s$

Step No	Statement	Reason
1	$p \to q$	Rule p.
2	$q \to r$	Rule p
3	$p \to r$	1, 2, Hypothetical Syllogism
4	$s \to \neg r$	Rule p
5	$r \to \neg s$	Contrapositive
6	$q \to \neg s$	2, 5, Hypothetical syllogism
7	$\neg q \vee \neg s$.	6, Conversion
8	$\neg (q \wedge s)$	7, DeMorgan's
9	$q \wedge s$	Rule p
10	$\neg (q \wedge s) \wedge (q \wedge s)$	9, Conjunction
11	F	Negation

Q.5(i) Prove that $\exists x [P(x) \wedge Q(x)] \rightarrow \exists x P(x) \wedge \exists x Q(x)$

Step No	Statement	Reason
1	$\exists x [P(x) \wedge Q(x)]$	Rule P
2	$P(a) \wedge Q(a)$	1, ES
3	$P(a)$	2, Simplification
4	$Q(a)$	2, Simplification
5	$\exists x P(x)$	3, EG
6	$\exists x Q(x)$	4, EG
7	$\exists x P(x) \wedge \exists x Q(x)$	5, 6, Conjunction

5(ii) 3) P.T the following constitute a valid argument

(i) If there was rain, then ~~travelling~~ travelling was diff

(ii) If they had an umbrella, there travelling was not difficult.

(iii) They had umbrella.

(iv) There was no rain

p: There was rain

q: Travelling was diff

r: They had umbrella.

$P \to Q$, $R \to \neg Q$, $R \Rightarrow \neg P$

Step NO	Statement	Reason
1	$P \to Q$	Rule p
2	$R \to \neg Q$	Rule p
3	$\neg Q \to R$	2, Contrapositive
4	$\neg P \to R$	3, 1, hypothetical syllogism
5	R	Rule p
6	$\neg P$	4, 5 Modus ponens.

$P(a) \to Q(a)$ for some $a \Rightarrow \exists x\, [P(x) \to Q(x)]$

Q.B

Using CP (or) otherwise obtain the following implication

6(i)

$$\forall x\,[P(x) \to Q(x)],\ \forall x\,[R(x) \to \neg Q(x)] \Rightarrow \forall x\,[R(x) \to \neg P(x)]$$

Step No	Statement	Reason.
1	$\forall x\,[P(x) - Q(x)]$	Rule P
2	$P(a) \to Q(a)$	1 Rule US
3	$\forall x\,[R(x) - \neg Q(x)]$	Rule P
4	$R(a) \to \neg Q(a)$	3 Rule US
5	$Q(a) \to \neg R(a)$	4, Contrapositive
6	$P(a) \to \neg R(a)$	2,5 hypothetical syllogism

7	$R(a) \rightarrow \neg R(a)$	6, Contrapositive
8	$\forall x [R(x) \rightarrow \neg Q(x)]$	7, UG

Q.B 6(iii)

If Raja misses many classes through illness he fails in the highschool. If Raja fails highschool he is uneducated. If Raja reads a lot of books then he is not uneducated. Raja miss many classes through illness and reads lot of books

p: Raja misses many classes

q: Raja fails highschool.

r: He is uneducated

s: Raja reads a lot of books

$P \to Q$, $Q \to R$, $S \to \neg R$, $P \land S$

Step No	Statement	Reason
1	$P \to Q$	Rule P
2	$Q \to R$	Rule P
3	$P \to R$	1, 2 hypothetical syllogism
4	$S \to \neg R$	Rule P
5	$R \to \neg S$	Contrapositive
6	$P \to \neg S$	3, 5 hypothetical syllogism.
7	$\neg P \lor \neg S$	6, Conversion
8	$\neg (P \land S)$	7, Demorgan
9	$P \land S$	Rule P
10	$\neg (P \land S) \land (P \land S)$	8, 9 Conjunction
11	F	10, Negation

∴ The given premises are inconsistent

Q) Verify the validity of the statement

7(ii) All integers are rational numbers. Some integers are powers of 2. Therefore some rational numbers are powers of 2

$P(x)$: x is an integer

$Q(x)$: x is a rational number

$R(x)$: x is a power of 2

Implication $\rightarrow$ $\left.\begin{array}{l} \forall x\,[P(x) \rightarrow Q(x)] \\ \exists x\,[P(x) \rightarrow R(x)] \end{array}\right\}$ Premesis

Conclusion $\rightarrow \exists x\,[Q(x) - R(x)]$

$$\forall x\,[P(x) \rightarrow Q(x)],\ \exists x\,[P(x) \rightarrow R(x)]$$

$$\Rightarrow \exists x\,[Q(x) \rightarrow R(x)]$$

Step No	Statement	Reason
1	$\forall x\,[P(x) \rightarrow Q(x)]$	Rule P
2	$P(a) \rightarrow Q(a)$	1, US
3	$\exists x\,[P(x) \rightarrow R(x)]$	Rule P
4	$P(a) \rightarrow R(a)$	2, ES
5	$P(a)$	Rule P
6	$Q(a)$	2,5 Modus ponens ~~ponens~~
7	$R(a)$	4,5 Modus ponens
8	$Q(a) \rightarrow R(a)$	6, 7 Rule CP
9	$\exists x\,[Q(x) \rightarrow R(x)]$	8 Rule EG

Using CP rule derive $P \to (Q \to S)$ from

Q.B $P \to (Q \to R)$, $Q \to (R \to S)$

8(i)

Step NO	Statement	Reason
1	$P \to (Q \to R)$	Rule P
2	P	Rule P
3	$Q \to R$	1, 2 Modus ponens
4	$\neg Q \vee R$	3, Conversion
5	$Q \to (R \to S)$	Rule P
6	$\neg Q \vee (R \to S)$	5, Conversion
7	$(\neg Q \vee R) \wedge (\neg Q \vee (R \to S))$	4, 6, Conjunction
8	$\neg Q \vee S$	$(\neg Q \vee R) \wedge (\neg Q \vee (R \to S))$ $\neg Q \vee (R \wedge (R \to S))$ $\neg Q \vee ((R \wedge \neg R \vee S)$ $\neg Q \vee (F \vee S) \Rightarrow \neg Q \vee S$
9	$Q \to S$	Conversion
10	$P \to (Q \to S)$	2, 9 CP rule

Show that R follows from $P \to Q$, $Q \to R$
Q.B 8(ii)
$P \vee R$ by indirect method.

1.	$Q \to R$	Rule P
2.	$\neg R$	Additional Premises
3.	$\neg Q$	1, 2 Modulus Tollens
4	$P \to Q$	Rule P
5	$\neg P$	3, 4 Modulus Tollens
6	$P \vee R$	Rule P
7	R	5, 6, Disjunction syllogism
8	$R \wedge \neg R = F$	6, 7 Conjunction.