UNIT- IV
BASIC QUANTUM MECHANICS
Part-A

1. **Explain Compton effect?**

   When a monochromatic beam of high energy photons is scattered by a substance of low atomic number, the scattered radiation consists of two components, one has the same wavelength λ as the incident ray and the other has a slightly longer wavelength λ' . This phenomenon of change in wavelength of scattered high energy photons is known as Compton effect. The shift in wavelength is known as Compton shift.

2. **Write an expression for Compton Shift.**

$$d\lambda = \frac{h}{m_0 c}\,(1 - \cos\theta)$$

h= Planck's constant,

c=speed of light,

λ- wavelength,

θ- Angle through which photons are scattered.

3. **What are matter waves or de-Broglie waves?**

   The waves associated with moving particles are known as matter waves or de-Broglie waves. The wavelength of matter waves is given by

$$\lambda = \frac{h}{mv}$$

h - Planck's constant,

v-velocity of the particle,

m- mass of the particle.

4. **Write an expression for the de-Broglie wavelength in terms of energy of the particle?**

$$\lambda = \frac{h}{\sqrt{2mE}}$$

h - Planck's constant,

E – Kinetic Energy of the particle,

m- mass of the particle.

5. **Write an expression for the de-Broglie wavelength associated with an electron accelerated by a potential V.**

$$\lambda = \frac{h}{\sqrt{2meV}}$$

h- Planck's constant,

e - charge of an electron,

m- mass of an electron,

V-accelerating voltage.

6. **State the properties of matter waves?**

   i. Lighter the particle, greater is the wavelength associated with it.
   ii. Smaller the velocity of the particle, greater is the wavelength.
   iii. Matter waves are associated only with moving particles.
   iv. Matter waves are not electromagnetic waves. The wave velocity is not constant.

7. Show that the phase velocity of matter waves is greater than the velocity of light

We know that,

$E = mc^2$ and $E = h\gamma$

$mc^2 = h\gamma$

$\gamma = \frac{mc^2}{h}$

Wavelength of mater waves $\lambda = \frac{h}{mv}$

Velocity of the wave = $\lambda\,\gamma$ = $\frac{h}{mv}\,\frac{mc^2}{h}$ = $\frac{c^2}{v}$ which is greater than c. Since no particle can travel with a velocity greater than velocity of light. So, $\frac{c^2}{v} > c$.

8. What is Schrödinger wave equation?

The equation that describes the wave nature of a particle in mathematical form is Schrödinger wave equation. The general form of it is,

$$\left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\Psi = = i\,\hbar\frac{\partial\Psi}{\partial t}$$

9. What is a wave function?

A complex function which characterizes de-Broglie wave is known as wave function and is denoted by Ψ.

10. Mention the physical significance of wave function?

i. Ψ relates the particle and wave nature of matter statistically

ii. It is a complex quantity, and we cannot measure it.

iii. $\Psi^*\Psi$ is a real positive number and it is the probability density of finding a particle.

iii. If the particle is found somewhere in space, then the probability of finding the particle is equal to 1, P = $\iiint \boldsymbol{\Psi}^2\, dx\, dy\, dz = 1$.

11. What are Eigen values and Eigen functions ?

The following is the eigen value equation

$$AX = \lambda X;$$

X is the eigen function.

A is a linear operator

λ is the eigen value.

The time independent Schroedinger equation can be represented in the form of eigen value equation $H\Psi_o = E\Psi_o$,

where Ψ_o is the eigen function, E is the eigen value, H is the Hamiltonian operator.

$$H = \left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)$$

12. Briefly explain the basis of normalization of wave function.

Normalization is the process of making the total area under modulus square of the wave function to be equal to one. The process involves fixing the arbitrary constant appearing in the wave function so that the total probability of finding a particle within the boundary is one.

$$\int_{-\infty}^{\infty} \Psi^*\Psi = 1$$

If normalization is not done, then the total probability of finding the particle can exceed one and this is absurd.

13. For a free particle moving within a 1D box the ground state energy cannot be zero. Why? (Jan 2018)

The energy of a particle in a 1D box is

$$E = \frac{n^2h^2}{8ma^2}$$

$$E = 0 \Rightarrow n = 0$$

The wave function of a particle inside a 1D box is given as

$\Psi = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a} e^{-i\omega t}$. If n=0, then Ψ= 0 for all values of x and t which implies the total probability of finding the particle inside the box is $\int \Psi^*\Psi = \int 0.\,dv = 0$. This means that the particle is not inside the box. But the we know that the particle is inside the box. Therefore, the condition $n = 0 \Rightarrow E = 0$ is physically unacceptable.

14. What is Compton wavelength?

The shift in the wavelength corresponding to the scattering angle 90^0 is known as Compton wavelength.

$$d\lambda = \frac{h}{m_oc}(1 - \cos\theta), \text{ when } \theta = 90^\circ$$

$$d\lambda = \frac{h}{m_oc}(1 - \cos 90),$$

$$= \frac{6.635 \text{ x } 10-34}{9.1 \text{ x } 10^{-31} \text{ x} 3 \text{ x } 10^8}(1\text{-}0) = 0.0243 \text{ x } 10^{-10} \text{ m}$$

Part-B

1. Derive Schrödinger's time independent and time dependent equation for matter waves?
2. Solve the Schrodinger wave equation for particle in a one-dimensional box and obtain the energy eigen values and eigen functions?
3. What is Compton effect? Give the theory of Compton effect and show that the Compton shift. $\lambda' - \lambda = \frac{h}{m_0c}(1 - cos\emptyset)$ **(Jan 2018)**

Problems

1. Calculate the de-Broglie wavelength of an electron which has been accelerated from by the application of potential 400 V.
2. Calculate the minimum energy that an electron can possess in an infinitely deep potential well of width 4nm.
3. An electron is confined to a one-dimensional box of side 10^{-10} m. Find the first four eigen values of the electron?
4. Calculate the energy in eV of a photon of wavelength 1.2 A°