KCG
COLLEGE OF TECHNOLOGY

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QUESTION BANK
23MA101 – MATRICES & CALCULUS
UNIT I
MATRICES
PART A

1. If λ is an eigen value of A, then prove that λ^2 is an eigen value of A^2. (AU'16)

   Ans: Let λ be an eigen value of A, then

   $AX = \lambda X$ ($\because X$ is an eigen vector and $X \neq 0$)

   Premultiplying both sides by A, we get

   $A(AX) = A(\lambda X)$

   $A^2 X = \lambda(AX)$

   $A^2 X = \lambda(\lambda X)$ $\quad (\because AX = \lambda X)$

   $A^2 X = \lambda^2 X$

   $\Rightarrow \lambda^2$ is an eigen value of A^2.

2. Two eigen values of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 3 and 5. What is the third eigen value? What is the product of eigen values of A?

   (AU'17)

   Ans: Given $\lambda_1 = 3,\ \lambda_2 = 5,\ \lambda_3 = ?$

   We know that

   Sum of the eigen values = Trace of A

   $\lambda_1 + \lambda_2 + \lambda_3 = 8 + 7 + 3$

   $3 + 5 + \lambda_3 = 18$

   $\lambda_3 = 10$

   $\therefore$ The third eigen value is $\lambda_3 = 10$

   Product of the eigen values $= |A| = \lambda_1 \lambda_2 \lambda_3 = 3 \times 5 \times 10 = 150$

3. Show that the eigen values of a null matrix are zero?

   (AU'17)

   Ans: Let A be a zero matrix of order 3 $= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

   Then the characteristic equation of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} 0-\lambda & 0 & 0 \\ 0 & 0-\lambda & 0 \\ 0 & 0 & 0-\lambda \end{vmatrix} = 0 \qquad \text{(ie)}\ (0-\lambda)\left[\lambda^2\right] = 0$$

$$\Rightarrow \lambda^3 = 0$$

Therefore the eigen values of A are 0, 0, 0.

4. If the sum of the eigen values and trace of 3×3 matrix A are equal, find the value of $|A|$.

Ans: Given

$$\lambda_1 + \lambda_2 = \text{Trace of } A$$

By Property $\lambda_1 + \lambda_2 + \lambda_3 = \text{Trace of } A$

$$= \lambda_1 + \lambda_2$$

$$\Rightarrow \lambda_3 = 0$$

$\therefore$ Product of the eigen values $= |A|$

$$= \lambda_1 \lambda_2 \lambda_3 = |A|$$

$$0 = |A|$$

$$\therefore |A| = 0$$

5. Show that the two matrices A and $P^{-1}AP$ have the same eigen values.

Ans: Let $B = P^{-1}AP$

$$B - \lambda I = P^{-1}AP - \lambda I$$

$$= P^{-1}[A - \lambda I]P$$

$$|B - \lambda I| = |P^{-1}||A - \lambda I||P|$$

$$|B - \lambda I| = |P^{-1}||A - \lambda I||P|$$

$$= |A - \lambda I||P^{-1}||P|$$

$$= |A - \lambda I||P^{-1}P|$$

$$= |A - \lambda I||I|$$

$$= |A - \lambda I|$$

$$|A - \lambda I| = 0 \Rightarrow |B - \lambda I| = 0$$

$\therefore$ The eigen values of A and $B = P^{-1}AP$ are the same.

6. If A and B are two non – singular matrices, Prove that AB and BA have the same eigen values.

Ans: $AB = I(AB)$

$$= (B^{-1}B)AB$$

$= B^{-1}(BA)B$

$\therefore$ AB and BA have same eigen values.

7. Prove that the eigen values of a orthogonal matrix are of unit modulus.

Ans: Let A be an orthogonal matrix, then

$$AA^T = A^T A = I$$

Let λ be an eigen value of A and X be the corresponding eigen vectors.

Then $AX = \lambda X$

$$(AX)^T AX = (\lambda X)^T \lambda X$$

$$X^T (A^T A)X = \lambda^2 X^T X$$

$$X^T (I)X = \lambda^2 X^T X$$

$$X^T X = \lambda^2 X^T X$$

$$\Rightarrow (1-\lambda^2)X^T X = 0$$

Since $X \neq 0, X^T \neq 0 \Rightarrow X^T X \neq 0$ and hence

$(1-\lambda^2) = 0.$

$\therefore \lambda = \pm 1 \, or \, |\lambda| = 1$

8. Find the characteristic roots of the orthogonal matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and verify that they are of unit modulus.

Ans: Let $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$|A - \lambda I| = \begin{vmatrix} \cos\theta - \lambda & -\sin\theta \\ \sin\theta & \cos\theta - \lambda \end{vmatrix}$$

$$\Rightarrow (\cos\theta - \lambda)^2 + \sin^2\theta = 0$$

$$\lambda^2 - 2\lambda\cos\theta + 1 = 0$$

$$\lambda = 2\cos\theta \pm \frac{\sqrt{4\cos^2\theta - 4}}{2}$$

$$\lambda = \cos\theta \pm i\sin\theta$$

$\therefore$ The characteristic roots are $\cos\theta \pm i\sin\theta$

$$\lambda = |\cos\theta \pm i\sin\theta|$$

$$= \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

9. Prove that the eigen values of unitary matrix are of unit modulus.

Ans: Let λ be an eigen value of A and X be the corresponding eigen vector, then $AX = \lambda X$.

Let A b an unitary matrix

Then $A^* A = I$

$$(AX)^* = (\lambda X)^*$$
$$X^* A^* = \bar{\lambda} X^*$$
$$(X^* A^*)(AX) = \bar{\lambda} X^* (\lambda X)$$
$$X^* (A^* A) X = \lambda \bar{\lambda} X^* X$$
$$X^* X = \lambda \bar{\lambda} X^* X$$
$$(1 - \lambda \bar{\lambda}) X^* X = 0$$
$$X \neq 0,\ X^* X = 0$$
$$(1 - \lambda \bar{\lambda}) = 0$$
$$\Rightarrow \lambda \bar{\lambda} = 1 \, or \, |\lambda^2| = 1, \ \therefore |\lambda| = 1$$

10. Show that 0 is a characteristic root of a matrix if and only if the matrix is singular.

Ans: Let A be square matrix and λ is a characteristic root. Then $|A - \lambda I| = 0$

If $\lambda = 0$ then $|A| = 0$.

If 0 is a characteristic root of A than A is singular, conversely A is singular, (ie) $|A| = 0$.

$|A - \lambda I| = 0 \Rightarrow |-\lambda I| = 0. \quad \therefore \lambda = 0$

$\therefore 0$ is a characteristic root

11. Prove that the eigen values of a triangular matrix are its diagonal elements.

Ans: Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdot & a_{1n} \\ 0 & a_{22} & a_{23} & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & a_{nn} \end{bmatrix}$ be an upper triangular matrix

The characteristic equation of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & a_{13} & \cdot & a_{1n} \\ 0 & a_{22} - \lambda & a_{23} & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & a_{nn} - \lambda \end{vmatrix} = 0$$

$(a_{11}-\lambda)(a_{22}-\lambda)\ldots(a_{nn}-\lambda)=0$

$(a_{11}-\lambda)=0,\ (a_{22}-\lambda)=0,\ldots(or)(a_{nn}-\lambda)=0$

$\therefore \lambda = a_{11}, a_{22}, \ldots (or) a_{nn}$

Therefore the eigenvalues A are its leading diagonal only

12. Show that the eigen values of a diagonal matrix are its leading diagonals.

Lets = $\begin{bmatrix} a_{11} & 0 & 0 & \cdot & 0 \\ 0 & a_{22} & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & a_{nn} \end{bmatrix}$ be a diagonal matrix.

The characteristic equation of A is $|A-\lambda I|=0$

(ie) $\begin{vmatrix} a_{11}-\lambda & 0 & 0 & \cdot & 0 \\ 0 & a_{22}-\lambda & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & a_{nn}-\lambda \end{vmatrix}$

$(a_{11}-\lambda)(a_{22}-\lambda)\ldots(a_{nn}-\lambda)=0$

$\therefore \lambda = a_{11}, a_{22}, \ldots (or) a_{nn}$

$\therefore$The eigen values of A are its leading diagonal only.

13. If the eigen values of a matrix A of order $3\times 3$are 1,2& 3, Then find the eigen values of adjoint of A.

(AU'16)

Ans: Given the eigen values of a matrix A are 1,2& 3

$\therefore$The eigen values of a matrix A^{-1} are $1, \frac{1}{2}, \frac{1}{3}$

$$A^{-1} = \frac{Adj\ A}{|A|} \Rightarrow Adj\ A = |A| A^{-1}$$

$$|A| = 6.$$

$\therefore$Eigen values of $adjA = 6, \frac{6}{2}, \frac{6}{3}$

$$adjA = 6, 3, 2$$

14. Find the eigen values of $3A+2I$ where $A=\begin{pmatrix}5 & 4\\ 0 & 2\end{pmatrix}$

(AU'16)

Ans: The eigen values of the given matrix A are 2 & 5

The eigen values of the given matrix $3A+2I = (3\times 2)+2, (3\times 5)+2 = 8, 17$

15. Write down the matrix corresponding to the quadratic form $2x_1^2+5x_2^2+4x_1x_2+2x_3x_1$

Ans: $A=\begin{pmatrix}2 & 2 & 1\\ 2 & 5 & 0\\ 1 & 0 & 0\end{pmatrix}$

(AU'15)

16. What is the nature of the quadratic form $x^2+y^2+z^2$ in four variables?

(AU'15)

Ans: The eigen values of the matrix corresponding to the quadratic form $x^2+y^2+z^2$ in four variables are 1,1,1,0

$\therefore$ The nature of the given quadratic form is Positive semi definite.

17. Find the sum and product of all the eigen values of $\begin{pmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{pmatrix}$.

Ans: Sum of the eigen values = Sum of the main diagonal elements

$= 8+7+3 = 18$

Product of the eigen values $= |A| = 45$

18. Give the nature of the quadratic form whose matrix is $\begin{pmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -2\end{pmatrix}$

The eigen values of the given matrix = -1,-1,-2

$\therefore$ The nature of the given quadratic form is negative definite.

19. Find the constants a & b such that the matrix $\begin{pmatrix}a & 4\\ 1 & b\end{pmatrix}$ has 3 & -2 as its eigen values.

Sum of the eigen values = Sum of the main diagonal elements

a+b = 3-2 = 1

Product of the eigen values $= |A| = -6$

$$ab-4 = -6$$

$$\therefore a = 2, -1, b = -2, 1$$

20. If $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, then find $2A^2 - 8A - 10I$ where I is the unit matrix.

The characteristic equation of A is $|A - \lambda I| = 0$

$$\lambda^2 - 4\lambda - 5 = 0$$

By CHT, $A^2 - 4A - 5I = 0 \quad \Rightarrow 2A^2 - 8A - 10I = 0$

21. Prove that any square matrix A and its transpose have the same eigen values.

$$|A - \lambda I| = \begin{vmatrix} a_{11} - \lambda & a_{12} & a_{13} & \cdot & a_{1n} \\ a_{21} & a_{22} - \lambda & a_{23} & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & a_{n3} & \cdot & a_{nn} - \lambda \end{vmatrix}$$

$$|A^T - \lambda I| = \begin{vmatrix} a_{11} - \lambda & a_{21} & \cdot & \cdot & a_{n1} \\ a_{12} & a_{22} - \lambda & \cdot & \cdot & a_{n2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{1n} & a_{2n} & \cdot & \cdot & a_{nn} - \lambda \end{vmatrix}$$

The value of the determinant is unaffected by interchanging of rows into columns and columns into rows $|A - \lambda I| = |A^T - \lambda I|$

$\therefore$ Any square matrix A and its transpose have the same eigen values

PART B

1. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$ (AU'16)

2. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 2 \end{pmatrix}$ (AU'16)

3. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ (AU'16)

4. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ (AU'15)

5. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$ (AU'15,AU'14)

6. Using Cayley – Hamilton theorem find A^{-1}, where $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$ (AU'16)

7. Using Cayley – Hamilton theorem find A^{-1} and A^4, where $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ (AU'17,AU'14)

8. Using Cayley – Hamilton theorem find A^{-1}, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$ (AU'16)

9. Using Cayley – Hamilton theorem find A^{-1}, where $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$ (AU'16)

10. Using Cayley – Hamilton theorem find A^{-1}, where $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ (AU'15)

11. Reduce the matrix $\begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{pmatrix}$ to diagonal form. (AU'17)

12. Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ to canonical form. (AU'16)

13. Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ to canonical form and hence find its rank. (AU'15)

14. Find the eigen values and eigen vectors of the matrix, where $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$

15. Using Cayley – Hamilton theorem find A^{-1}, where $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$

16. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

17. Using Cayley – Hamilton theorem find A^{-1}, where $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$

18. Reduce the quadratic form $2xy - 2yz + 2xz$ to canonical form by an orthogonal reduction.

19. Using Cayley – Hamilton theorem find A^{-1}, where $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$

20. Reduce the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to canonical form by an orthogonal reduction and find its nature.

UNIT-2 DIFFERENTIAL CALCULUS

PART A

1. Define a function.
   Ans: A function f from a set A to a set B is a rule that assigns to each element $x \in A$ a unique element $y \in B$.
2. Find the domain of the function $f(x) = \sqrt{x+2}$.
   Ans : Given $f(x) = \sqrt{x+2}$.
   f(x) is real ,if $x + 2 \geq 0 \Rightarrow x \geq -2$. $\therefore$ domain of $f(x) = D_f = [-2,\infty)$.
3. Find the domain and range of the function $f(x) = \sqrt{4-x^2}$.
   Ans : Given $f(x) = \sqrt{4-x^2}$
   f(x) is real ,if $4 - x^2 \geq 0 \Rightarrow x^2 - 4 \leq 0 \Rightarrow (x+2)(x-2) \leq 0 \Rightarrow -2 \leq x \leq 2$
   $\therefore$ domain of $f(x) = D_f = [-2,2]$.
   let $y = \sqrt{4-x^2} \Rightarrow y^2 = 4 - x^2 \Rightarrow x^2 + y^2 = 4$ which is a circle c(0,0) and R = 2.
   The graph is upper semicircle $y = \sqrt{4-x^2}$
   $\therefore$ y lies between 0 and 2
   $D_f = [-2,2]$, $R_f = [0,2]$.
4. Find the domain and range of the function $f(x) = \sqrt{x}$.
   Ans : Given $f(x) = \sqrt{x}$ So $x \geq 0$ & $y^2 = x$
   The graph is upper half parabola
   $D_f = [0,\infty)$, $R_f = [0,\infty)$.
5. Sketch the graph of the function $f(x) = |x|$
   Ans : We know that $f(x) = |x|$
   $$= \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$
   The graph of f coincides with the line y=x to the right of the y axis and coincides with the line y=-x to be left of the y axis.
6. Determine whether the given function $f(x)=x^5+x$ is an even or odd.
   Ans: $f(x)=x^5+x$
   $f(-x)=(-x^{5)}+(-x)$
   $=- x^5-x=-(x^5+x)= -f(x)$
   $\therefore$ f is an odd function.
7. Verify whether the given function : $f(x)=1-x^4$ is an even or odd.
   Ans: $f(x)=1-x^4$
   $f(-x)=1-(- x^4)= 1-x^4 =: f(x)$.
   $\therefore$ f is an even function.
8. Define increasing and Decreasing functions

Ans: A function f(x) is called an increasing on an interval I if $f(x_1)< f(x_2)$ whenever $x_1< x_2$ in I.

A function f(x) is called an decreasing on an interval I if $f(x_1)>f(x_2)$ whenever $x_1>x_2$ in I.

9. Define the Limit of a function.

Let f be a function defined in a neighbourhood of 'a' ,except possibly at the point 'a'. If f(x) is arbitrarily close to 'L' for all x sufficiently close to 'a' from either side then we say that f(x) approaches 'L' as x approaches 'a' and we write $\lim_{x\to a} f(x)=L$

10. Define left limit.

Ans: If the values of a function f approach arbitrarily close to 'l' for all x sufficiently close to 'a' from the left of 'a' (ie x<a) then the left hand limit exists and is written as $\lim_{x\to a^-} f(x)=l$.

11. Define right limit.

Ans: If the values of a function f approach arbitrarily close to 'l' for all x sufficiently close to 'a' from the right of 'a' (ie x>a) then the right hand limit exists and is written as $\lim_{x\to a^+} f(x)=l$.

12. Define the limit, $\lim_{x\to 3}\frac{x^2-9}{x-3}$

$$\lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3}\frac{(x+3)(x-3)}{x-3} = \lim_{x\to 3}(x+3) = 3+3=6.$$

13. Find the value of $\lim_{u\to -2}\sqrt{u^4+3u+6}$.

Ans: $\lim_{u\to -2}\sqrt{u^4+3u+6} = \sqrt{(-2)^4+3(-2)+6} = \sqrt{16-6+6} = \sqrt{16} = 4$

14. Evaluate $\lim_{x\to\frac{\pi}{2}}\left(\frac{1+\cos 2x}{(\pi-2x)^2}\right)$

Ans: $$\lim_{x\to\pi/2}\frac{1+\cos 2x}{(\pi-2x)^2} = \lim_{x\to\pi/2}\frac{2\cos^2 x}{4(\frac{\pi}{2}-x)^2} = \frac{1}{2}\lim_{x\to\pi/2}\left[\frac{\sin(\frac{\pi}{2}-x)}{(\frac{\pi}{2}-x)}\right]^2 = \frac{1}{2}$$

15. Show that $\lim_{x\to 0} x^2\sin\frac{1}{x} = 0$

Ans: We know that $-1\le \sin\frac{1}{x}\le 1 \Rightarrow -x^2\le x^2\sin\frac{1}{x}\le x^2$

$\lim_{x\to 0} x^2 = \lim_{x\to 0} -x^2 = 0$. ∴ By Sandwich Theorem $\lim_{x\to 0} x^2\sin\frac{1}{x} = 0$

16. Find the left limit of $f(x) = \begin{cases}\sqrt{x-4} & \text{if } x>4\\ 8-2x & \text{if } x<4\end{cases}$

Ans: $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}(8-2x) = 8-2(4) = 0$

17. Find the right limit of $f(x) = \begin{cases}4-x^2, & \text{if } x\le 2\\ x-1, & \text{if } x>2\end{cases}$

Ans: $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x-1) = 2-1 = 1$

18. Evalluate $\lim_{x\to\infty}\frac{5x^2+8x-3}{3x^2+2}$

Ans: $$\lim_{x\to\infty}\frac{5x^2+8x-3}{3x^2+2} = \lim_{x\to\infty}\frac{x^2(5+8/x-3/x^2)}{x^2(3+\frac{2}{x^2})} = \frac{5}{3}$$

19. Evalluate $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x}$

Ans: $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x} = \lim_{x\to 0}\frac{(e^{ax}-1)-(e^{bx}-1)}{x} = {}^*\lim_{x\to 0}\frac{(e^{ax}-1)}{x} - \frac{(e^{ax}-1)}{x} +$

$=\lim_{x\to 0}\frac{(e^{a})^{x}-1}{x} - \lim_{x\to 0}\frac{(e^{b})^{x}-1}{x} = bg_{e}(e^{a}) - bg_{e}(e^{b})$

$= a\log_e e - b\log_e e = a-b$

20. Define Continuity.

Ans: A function f is continuous at a point a if $\lim_{x\to a} f(x) = f(a)$

The definition implies the following conditions

(i) f is defined at a (ie) f(a) exists.

(ii) $\lim_{x\to a} f(x)$ exists (ie) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

(iii) $\lim_{x\to a} f(x) = f(a)$

21. Define right Continuity.

Ans: A function f is said to be continuous from the right at a if $\lim_{x\to a^+} f(x) = f(a)$

22. Define left Continuity.

Ans: A function f is said to be continuous from the left at a if $\lim_{x\to a^-} f(x) = f(a)$

23. Test the continuity of f(x) = $\begin{cases} \frac{2x^2-5x-3}{x-3}, & \text{if } x \text{ G } 3 \\ 6, & \text{if } x = 3 \end{cases}$

Ans: We have to test continuity at x = 3

$\lim_{x\to 3} f(x) = \lim_{x\to 3}\frac{2x^2-5x-3}{x-3} = \lim_{x\to 3}\frac{(x-3)(2x+1)}{x-3} = 7$

But f(3) = 6

$\therefore \lim_{x\to 3} f(x)$ G $f(3)$

$\therefore$ f is discontinuous at x = 3.

24. Test the continuity of f(x) = $\begin{cases} e^{x}, & \text{if } x < 0 \\ x^2, & \text{if } x \geq 0 \end{cases}$

Ans: We have to test continuity at x = 0. So we have to find the left and right limits.

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} = e^{x} = e^{0} = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0$

$\therefore \lim_{x\to 0^-} f(x)$ G $\lim_{x\to 0^+} f(x)$

$\therefore$ f is discontinuous at x = 0.

25. Define Derivative.

Ans : Let f be a function defined in a neighbourhood (c-δ , $c+\delta$) of a point c , where $\delta > 0$ and small . f is said TO be derivable or differentiable at c if $\lim\limits_{h\to 0}\frac{f(c+h)-f(c)}{h}$ exists. Then it is denoted by f'(c) and is called the derivative of f at c.

26. Define right Derivative.

Ans : Right derivative at c is defined as f'(c+) = $\lim\limits_{h\to 0+}\frac{f(c+h)-f(c)}{h}$ if the limit exists,h>0. f'(c+) is also denoted by f_+'(c).

27. Define left Derivative.

Ans : Left derivative at c is defined as f'(c-) = $\lim\limits_{h\to 0-}\frac{f(c+h)-f(c)}{h}$ if the limit exists,h<0. f'(c-) is also denoted by f_-'(c).

28. Find $\frac{d}{dx}[\sin x]^{\cos x}$

Ans : $\frac{d}{dx}[\sin x]^{\cos x} = \frac{d}{dx}(e^{\log(\sin x^{\cos x})}) = \frac{d}{dx}(e^{\cos x\log(\sin x)})$

$=(e^{\cos x\log(\sin x)})[\frac{\cos x}{\sin x}\cos x + \log(\sin x)(-\sin x)]$

$=\sin x^{\cos x}(\frac{\cos^2 x}{\sin x} - \sin x \log\sin x)$

29. If f(1)=10 , f'(x)≥ 2 for $1\le x\le 4$, how small can f(4) possibly be ?

Ans : By Mean Value Theorem $\frac{f(4)-f(1)}{4-1}$ = f'(c)

$\frac{f(4)-10)}{3} \ge 2$

f(4)-10≥6

f(4) ≥ 16. ∴ The minimum value is 16.

30. The motion of a particle is given by S = $2t^3-5t^2+3t+4$. Find the acceleration , and what is the acceleration after 2 seconds.

Ans : S = $2t^3-5t^2+3t+4$

$\frac{ds}{dt} = 6t^2-10t+3$ ∴ Velocity = = $6t^2-10t+3$

Acceleration = $\frac{d^2s}{dt^2}$ = 12t-10

Acceleration at t = 2 is $\frac{d^2s}{dt^2}$ = 12(2)-10 =14

31. Find the domain of f(x) = $\sqrt{3-x}$ - $\sqrt{2+x}$

Ans: Given f(x) = $\sqrt{3-x}$ - $\sqrt{2+x}$

f(x) is real ,if $x+2 \ge 0 \Rightarrow x \ge -2$ and $3-x\ge 0 \Rightarrow x \le 3$

∴ domain of f(x) = D_f = (-∞.3]∩[-2,∞) .

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32. Evaluate $\lim\limits_{t\to 1}\frac{t^4-1}{t^3-1}$.

Ans: $\lim_{t\to1}\frac{t^4-1}{t^3-1}=\lim_{t\to1}\frac{(t^2+1)(t+1)\cancel{(t-1)}}{\cancel{(t-1)}(t^2+t+1)} = \frac{4}{3}$

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33. Sketch the graph of the function $f(x)=\begin{cases} 1+x\,, \text{if } x<-1 \\ x^2, \text{if } -1\le x\le 1 \\ 2-x\,,\ \text{if } x\ge 1\end{cases}$ and use it to determine the value of 'a' for which $\lim_{x\to a} f(x)$ exists ?

Ans: $\lim_{x\to a} f(x)$) exists when a = 1

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34. Does the curve $y=x^4-2x^2+2$ have any horizontal tangents ? If so where ?

$\frac{dy}{dx} = 4x^3-4x$

Horizontal tangents occur when slope = 0

$\frac{dy}{dx} = 4x^3-4x = 0$

X = 0,1,-1

Y= 2, 1,1

The **horizontal tangent** lines are at the **points** (1,1) (0 , 2) and (-1,1) Jan 2018

PART B

1. Find the value of $\lim_{x\to0}\frac{\sqrt{1+x+x^2}-1}{x}$
2. Evaluate $\lim_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}$
3. Find the value of $\lim_{x\to2}\frac{x^2-4}{\sqrt{3x-2}-\sqrt{x+2}}$
4. Find the equation of tangent and normal line to the curve $y=\frac{x-1}{x-2}$ at the point(3,2).
5. Find the equation of tangent and normal line to the curve $y=\frac{5x}{1+x^2}$ at the point(2,2).
6. If $f(x)=x^4-2x^2+3$ then (i)What are the critical points of f ? (ii)On what interval is f increasing or decreasing ? (iii)At what points,if any,does f assume local maximum and local minimum values?
7. If $f(x)= 3x^4-4x^3-12x^2+5$ then (i)What are the critical points of f ? (ii)On what interval is f increasing or decreasing ? (iii)At what points,if any,does f assume local maximum and local minimum values? Find the intervals of concavity and the inflection points
8. Find the values of a,b,c if $f(x)=\begin{cases}\sin(a+1)\,x+\sin x, \text{if } x<0 \\ C \qquad , \text{if } x=0 \\ \frac{(x+bx^2)^{1/2}-x^{1/2}}{bx^{1/2}}, \text{if } x>0\end{cases}$
9. For what value of the constant c is the function f continuous on (-∞,∞) , $f(x)=\begin{cases} cx^2+2x\,, \text{if } x<2 \\ x^3-cx\,, \text{if } x\ge 2\end{cases}$
10. Find the local maximum and local minimum values of the function $f(x)=\sqrt{x}-\sqrt[4]{x}$

using both the first and second derivative tests.

11. Find y" if $x^4+y^4 = 16$
12. Find the equation of tangent line $x^3+y^3 = 6xy$ at the point (3,3) and at what point the tangent line horizontal in the first quadrant.
13. Find the maxima and minima of the function $x^3-5x^4+5x^3+10$
14. Find the maxima and minima of the function $10x^6-24x^5+15x^4-40x^3-108$
15. Guess the value of the limit (if it exists) for the function $\lim_{x\to 0}\frac{e^{5x}-1}{x}$ by evaluating the function at the given numbers $x = \pm 0.5, \pm 0.1, \pm .01, \pm 0.001, \pm 0.0001$ (correct to six decimal places)
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16. For the function $f(x) = 2+2x^2-x^4$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity and the inflection points.

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17. Find the values of a and b that make f continuous on $(-\infty,\infty)$.

$$f(x)=\begin{cases}\frac{x^3-8}{x-2}, & if\ x<2\\ ax^2-bx+3, & if\ 2\le x<3\\ 2x-a+b, & if\ x\ge 3\end{cases}$$

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18. Find the derivative of $f(x)=\cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$ Nov/Dec 2018

19. Find y' for $\cos(xy) = 1+\sin y$ Nov/Dec 2018