Department of Science and Humanities
Second Year / Third Semester
23MA202- Discrete Mathematics

Unit – I

Q.No	Questions	CO's	Bloom's Level
1.	Find $(\neg p \wedge (p \vee q)) \rightarrow q$ using truth table.	CO1	K1
2.	Find the contra positive of the statement "If it is raining then I get wet" Let p: it is raining and q: I get wet.	CO1	K1
3.	Tell is it true that the negation of a conditional statement is also a conditional statement?	CO1	K1
4.	Write the contra positive of the conditional statement: "If you obey traffic rules, then you will not be fined.	CO1	K1
5.	Show that the propositions $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.	CO1	K2
6.	Show that $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$ without using truth tables.	CO1	K2
7.	Show that $(\neg p) \rightarrow (p \rightarrow q)$ is a tautology.	CO1	K2
8.	Write the truth table for the formula $(p \wedge q) \vee (\neg p \wedge \neg q)$.	CO1	K1
9.	Show that $(p \vee q) \wedge (\neg p) \Rightarrow q$.	CO1	K2
10.	Write the negation of the statement "If there is a will, then there is a way".	CO1	K1
11.	Find an indirect proof of the theorem "If (3x + 2)) is odd then x is odd".	CO1	K1
12.	Show that $(p \leftrightarrow q) \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$.	CO1	K2
13.	Define Compound statement formula.	CO1	K1
14.	Define Universal quantification and Existential quantification.	CO1	K1
15.	Prove that $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction.	CO1	K2
16.	Define Predicate Calculus.	CO1	K1
	Part – B		
1.	(i). Using laws of logic, Prove that $\neg(p \wedge q) \rightarrow (\neg p \vee (\neg p \vee q)) \Leftrightarrow \neg p \vee q$ **[8 Marks]** (ii). Obtain PDNF of $(p \wedge q) \vee r \rightarrow \neg p$ and hence find its PCNF. **[8 Marks]**	CO1	K3
2.	(i). Prove the following: . $P \rightarrow (Q \rightarrow R) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$ **[8 Marks]** (ii). Show that $P \rightarrow Q, Q \rightarrow \neg R, R, P \vee (J \wedge S) \Rightarrow J \wedge S$. **[8 Marks]**	CO1	K3
3.	(i). Find the PCNF and PDNF of $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$.**[8 Marks]** (ii). Using indirect method of proof, derive $p \rightarrow \sim s$ from the premises $p \rightarrow (q \vee r)$, $q \rightarrow \sim p$, $s \rightarrow \sim r$ and p. **[8 Marks]**	CO1	K3

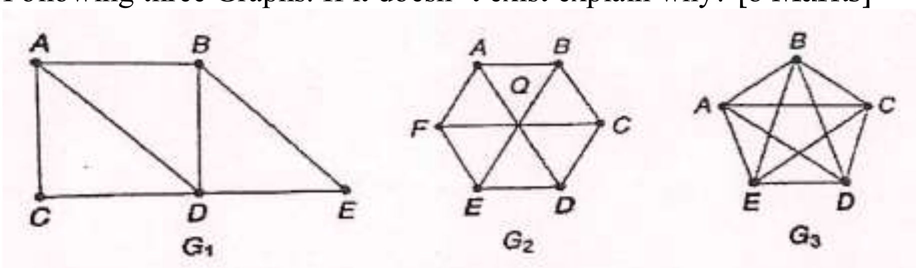
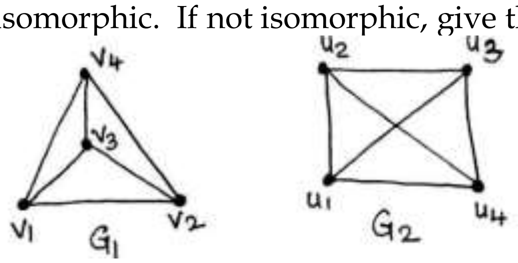
4.	(i). Without constructing the truth table find the PDNF and PCNF of $P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$. **[8 Marks]** (ii). Prove that the premises $p \rightarrow q, q \rightarrow r, s \rightarrow \sim r, q \wedge s$ are inconsistent**. [8 Marks]**	CO1	K3
5.	(i). Prove that $(\exists x)\ (P(x) \wedge Q(x)) \Rightarrow (\exists x)\ P(x) \wedge (\exists x)\ Q(x)$ **[8 Marks]** (ii). Prove that the following statements constitute a valid argument. If there was rain, then traveling was difficult. If they had umbrella, then traveling was not difficult. They had umbrella. Therefore there was no rain. **[8 Marks]**	CO1	K3
6.	(i)Using CP, obtain the following implication $\forall x(P(x) \to Q(x)); \forall x(R(x) \to \neg Q(x)) \Rightarrow \forall x(R(x) \to \neg P(x))$ **[8 Marks]** (ii) Prove that the following premises are inconsistent. (1) If Raja Kumar misses many classes through illness then he fails high school. (2) If Raja Kumar fails high school, then he is uneducated. (3) If Raja Kumar reads a lot of books then he is not uneducated. (4) Raja Kumar misses many classes through illness and reads a lot of books. **[8 Marks]**	CO1	K3
7.	(i) Verify the validity of the following argument. Every living thing is a plant or an animal. John's gold fish is alive and it is not a plant. All animals have hearts. Therefore John's gold fish has a heart. **[8Marks]** (ii) Verify the validity of the following argument. All integers are rational numbers. Some integers are powers of 2. Therefore, some rational numbers powers of 2. **[8 Marks]**	CO1	K3
8.	(i) Using rule CP, derive $P \to (Q \to S)$ from $P \to (Q \to R), Q \to (R \to S)$. **[8 Marks]** (ii) Show that the conclusion R follows from $P \to Q, Q \to R, P \vee R$ by using indirect method. **[8 Marks]**	CO1	K3

UNIT II

Q.No	Questions	CO's	Bloom's Level
1.	Using mathematical induction, show that $2+2^2+2^3+\dots\dots\dots+2^n=2^{n+1}-2$.	CO2	K2
2.	State pigeon hole principle.	CO2	K1
3.	Find the value of n if $np_3=5np_2$.	CO2	K1
4.	What is the value of 'r' if $5p_r=60$.	CO2	K1
5.	In how many ways can letters of the word "INDIA" be arranged?	CO2	K2
6.	Find the recurrence relation y(k) –8 (k–1)+16y(k–2) = 0, where k≥2, where y(2) = 16 and y(3) = 80.	CO2	K2
7.	Find the generating function for the sequence's' with terms 1, 2,3,4……..	CO2	K3
8.	Find $S(k)-7S(k-1)+10S(k-2)=0$.	CO2	K1
9.	How many ways are there to form a committee, if the committee consists of 3 educationalist and 4 socialist, if there are 9 educationalist and 11 socialist?	CO2	K2
10.	What is the number of arrangements of all the six letters in the word PEPPER?	CO2	K2
11.	Use mathematical induction to show that 1+2+3+……+n= $\frac{n(n+1)}{2}$.	CO2	K2
12.	Find the recurrence relation for the sequence $a_n=2n+9, n\geq 1$.	CO2	K3
13.	Show that, among 100 people, at least 9 of them were born in the same month.	CO2	K2
14.	There are 5 questions in a question paper in how many ways can a boy solve one or more questions?	CO2	K2
15.	If seven colours are used to paint 50 bicycles, then show that at least 8 bicycles will be the same colour.	CO2	K2
16.	Find the number of arrangements of the letters in MAPPANASSRR.	CO2	K2
	Part – B		
1.	(i). Using Mathematical induction prove that $\sum_{i=1}^{n} i^2=\frac{n\,(n+1)\,(2n+1)}{6}$. **[8 Marks]** (ii).Solve the recurrence relation of the Fibonacci sequence of numbers $f_n=f_{n-1}+\ f_{n-2}, n\ >2$ with initial conditions $f_1\ =\ 1$ and $f_2\ =\ 1$. **[8 Marks]**	CO2	K3
2.	(i).Using Mathematical Induction, prove that $1^2+3^2+5^2+\cdots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$. **[8 Marks]** (ii) Solve the recurrence relation $a_{n+1}-8a_n+16a_{n-1}=4^n$, $n\geq 1$ given that $a_0\ =1, a_1=8$. **[8 Marks]**	CO2	K3

3.	(i).Prove that $n^5 - n$ is divisible by 5 for all $n \geq 1$. **[8 Marks]** (ii).Find the number of integers between 1 and 250 both inclusive that are divisible by 2,3,5 . **[8 Marks]**	CO2	K3
4.	(i). Solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n$, given that $a_0 = 2, a_1 = 1$. **[8 Marks]** (ii).A survey of 100 students, it was found that 30 studied Mathematics, 54 studied Statistics, 25 studied Operations Research, 1 studied all the three subjects, 20 studied Mathematics and Statistics, 3 studied Mathematics and Operation Research and 15 studied Statistics and Operation Research. Find how many students studied none of these subjects and how many students studied only Mathematics? **[8 Marks]**	CO2	K3
5.	(i). Prove by mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n. **[8 Marks]** (ii). Suppose there are six boys and five girls, (1) In how many ways can they sit in a row. (2) In how many ways can they sit in a row, if the boys sit together and the girls sit together. (3) In how many ways can they sit in a row, if the girls are to sit together (4) In how many ways can they sit in a row, if the boys are to sit together. **[8 Marks]**	CO2	K3
6.	(i)Find all the solution of the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$. **[8 Marks]** (ii) State the Inclusion and Exclusion Principle. Use it to find the number of students who play neither games, if in a class of 50 students, 20 students play football, 16 students play hockey with 10 students play both the games. **[8 Marks]**	CO2	K3
7.	(i) Find the number of integers between 1 and 1000 both inclusive that are not divisible by any of the integers 5, 7 &9. **[8 Marks]** (ii) Prove by mathematical induction that $(3^n + 7^n - 2)$ is divisible by 8 for $n \geq 1$ **[8 Marks]**	CO2	K3
8.	(i) Suppose a department consists of eight men and nine women. In how many ways can we select a committee of (1) Three men and four women? (2) Four persons that has at least one woman? (3) Four persons that at most one man? (4) Four persons that has persons of both gender? **[8 Marks]** (ii) Solve $a_n = 3a_{n-1} + 1;\ n \geq 1$ with $a_0 = 1$ **[8 Marks]**	CO2	K3

Unit – III

Q.No	Questions	CO's	Bloom's Level
1.	Define Graph.	CO3	K1
2.	Define Bipartite Graph.	CO3	K1
3.	Draw a complete bipartite graph of $K_{2,3}$ and $K_{3,3}$.	CO3	K1
4.	Define degree of a vertex in a graph.	CO3	K1
5.	Define isomorphism between graphs.	CO3	K1
6.	Define a complete graph.	CO3	K1
7.	Give an example of a graph which is Eulerian but not Hamiltonian.	CO3	K2
8.	Find the adjacency matrix of K_5	CO3	K2
9.	Define a Subgraph.	CO3	K1
10.	Show that C_6 is a bipartite graph?	CO3	K2
11.	Define Eulerian Circuit.	CO3	K1
12.	When a graph is called an Eulerian graph?	CO3	K1
13.	Define regular graph.	CO3	K1
14.	Define Connected graph.	CO3	K1
15.	When a graph is called a Hamiltonian graph?	CO3	K1
	Part – B		
1.	(i). State and prove the handshaking theorem in a graph. **[8 Marks]** (ii) Find an Euler path or an Euler Circuit, if it exists in each of the Following three Graphs. If it doesn"t exist explain why? [8 Marks]  **(8 Marks)**	CO3	K3
2.	(i) Prove that in a Simple Graph, the number vertices of odd degree is even. **[8 Marks]** (ii) Examine whether the following pair of graphs are isomorphic. If not isomorphic, give the reasons  **[8 Marks]**	CO3	K3
3.	(i) Prove that a simple graph with n vertices and k components	CO3	

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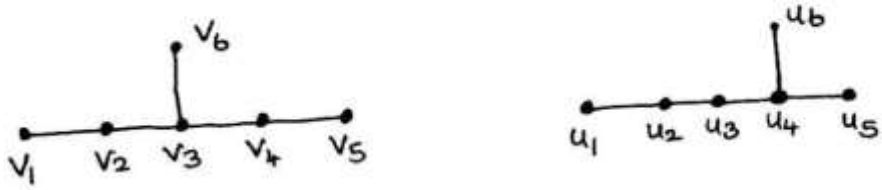
can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. [8 Marks]
(ii) Examine whether the following pair of graphs are isomorphic. If not isomorphic, give the reasons



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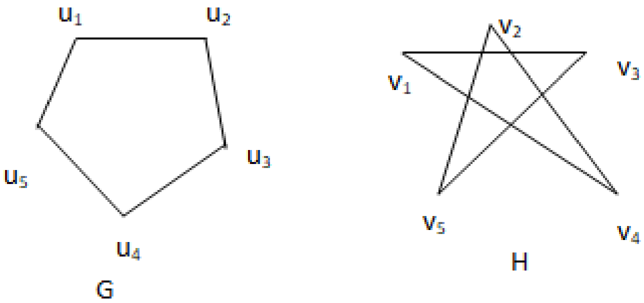
(i) Prove that a connected graph G is Eulerian if and only if every vertex of G is of even degree. [8 Marks]
(ii)When do we say that the two graphs are isomorphic? Examine whether the following graphs are isomorphic or not.
[8 Marks]



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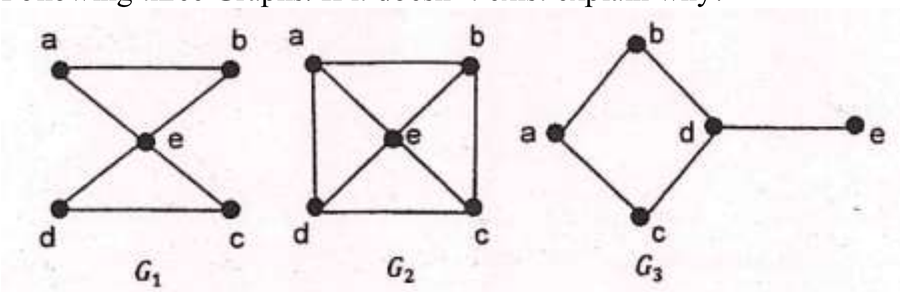
(i) Prove that the maximum number of edges in a simple graph with n vertices is $nC_2 = \frac{n(n-1)}{2}$ [8 Marks]
(ii). Find an Euler path or an Euler Circuit, if it exists in each of the Following three Graphs. If it doesn"t exist explain why?



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(i) If G is a self-complementary graph, then prove that G has n ≡0 (or) 1(mod 4) vertices [8 Marks]
(ii) Illustrate with an example for graphs which are a) Eulerian but not Hamiltonian b) Hamiltonian but not Eulerian c) Eulerian and Hamiltonian d) Neither Eulerian nor Hamiltonian.
[8 Marks]
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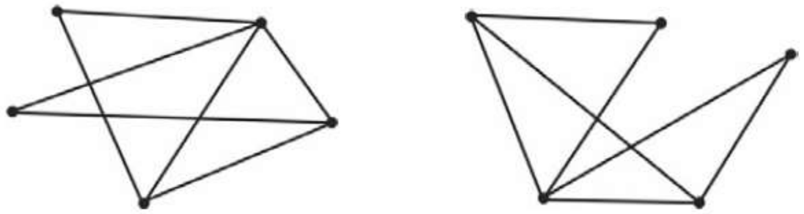
7.	(i) Examine whether the following pair of graphs are isomorphic. If isomorphic, label the vertices of the two graphs to show that their adjacency matrices are same **[8 Marks]**  (ii) Draw the complete graph K_5 with vertices A, B, C, D, E. Draw all complete subgraph of K_5 with 4 vertices. **[8 Marks]**	CO3	K3
8	(i) Represent each of the following Graphs with an adjacency matrix K_4, $K_{1,4}$, C_4, W_4 **[8 Marks]** (ii) Prove that the number of edges in a bipartite graph with n vertices is atmost $\frac{n^2}{4}$. **[8 Marks]**	CO3	K3

Unit – IV

Q.No	Questions	CO's	Bloom's Level
1.	Is it true $\left(Z_5^{\bullet}, \times_5\right)$ a cyclic group? Justify your answer.	CO4	K1
2.	Define Group Homomorphism	CO4	K3
3.	Prove that in any group, identity element is the only idempotent element.	CO4	K3
4.	Prove that every cyclic group is abelian.	CO4	K3
5.	Define semi Group.	CO4	K1
6.	If Z be the group of integers with the binary operations $*$ defined by $a*b = a+b-2$, for all $a, b \in Z$. Find the identity element the group $\langle z, *\rangle$.	CO4	K3
7.	Give an example of an integral domain which is not in Field.	CO4	K2
8.	Let $f:(G,*) \rightarrow (G', \Delta)$ be a group homomorphism. Then prove that $[f(a)]^{-1} = f(a^{-1}), \forall a \in G$.	CO4	K3
9.	Define "kernel of homomorphism" in a group.	CO4	K1
10.	State any two properties of a group.	CO4	K1
11.	If $P(S)$ is the power set of a non-empty set S, prove that $(P(S), \cap)$ is a monoid.	CO4	K3
12.	Define Monoid.	CO4	K1
13.	Define a Field	CO4	K1
14.	Define a Commutative ring.	CO4	K1
15.	Define cyclic group.	CO4	K1
16.	Define a Normal Subgroup.	CO4	K1
	Part – B		
1.	(i) let G = $\left\{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}\right\}$,Show that G is a group under Matrix Multiplication. **[8 Marks]** **(ii).** Prove that if every element in a group G is its own inverse, then G is abelian. Justify that the converse is not true. **[8 Marks]**	CO4	K3
2.	(i).Prove that (Z, *) is an abelian group where * is defined by a*b = a+b+1. **[8 Marks]** (ii) In any group $(G,*)$, show that $(a*b)^{-1} = b^{-1} * a^{-1}$, for all $a, b \in G$. **[8 Marks]**	CO4	K3
3.	(i) If a and b are any two elements of a group (G, *), then G is	CO4	K3

	abelian iff $(a*b)^2 = a^2*b^2$ **[10 Marks]** **(ii)** Show that the Group G = {1, -1, i, -i} is cyclic and find its generators **[6 Marks]**		
4.	State and prove Lagrange's theorem on groups. **[16 Marks]**	CO4	K3
5.	**(i).** Show that $(Q^+,*)$ is an abelian group, where $*$ is defined by $a*b=\frac{ab}{2}, \forall a,b \in Q^+$. **[8 Marks]** (ii). Find all the non-trivial subgroups of $(Z_6,+_6)$. **[8 Marks]**	CO4	K3
6.	(i) Show that the Group Homomorphism preserves the properties of Identity element and Invertibility **[8 Marks]** (ii) Show that the set Z_6 = {0, 1, 2, 3, 4, 5} is a Monoid under the operation multiplication modulo 6 **[8 Marks]**	CO4	K3
7.	Show that the set Z_5 = {0, 1, 2, 3, 4} is a Field under the operations addition modulo 5 and multiplication modulo 5. **[16 Marks]**	CO4	K3
8.	Show that (Z, *, #) is a commutative ring with identity, where the binary operations are defined by a*b = a+b-1 and a#b = a+b-ab for all a,b in Z. **[16 Marks]**	CO4	K3

Unit – V

Q.No	Questions	CO's	Bloom's Level
1.	Define Lattice.	CO5	K1
2.	Define Lattice homomorphism.	CO5	K1
3.	Show that in a lattice if $a \le b$ and $c \le d$ then $a * c \le b * d$.	CO5	K2
4.	State Isotonic property of lattice.	CO5	K1
5.	Draw the Hasse diagram of $(D_{20}, /)$ where D_{20} denotes the set of positive divisors of 20 and $/$ is the relation "division".	CO5	K2
6.	In a lattice prove that $a \le b \Rightarrow a * b = a$.	CO5	K3
7.	Prove that in a lattice $(L, \le)$, $a * (a \oplus b) = a$ where $*$ and $\oplus$ denote the meet and join.	CO5	K3
8.	State the De Morgan's laws of Boolean algebra.	CO6	K1
9.	Prove that a lattice with 5 elements is not a Boolean algebra.	CO6	K3
10.	Show that the absorption laws are valid in Boolean algebra.	CO6	K2
11.	When a lattice is said to be Boolean algebra?	CO6	K1
12.	Validate the elements 0 and 1 of a Boolean algebra B are unique?	CO6	K2
13.	Define sub lattice with example.	CO5	K1
14.	Reduce the expression $a.\bar{a}b$.	CO6	K2
15.	Reduce the expression a(a+c).	CO6	K2
16.	Define Boolean algebra.	CO6	K1
	Part – B		
1.	(i) State and Prove De Morgan's laws in complemented and distributive lattice. **[8 Marks]** (ii) In a Boolean algebra show that $ab' + a'b = 0$ iff $a = b$. **[8 Marks]**	CO5 CO6	K3 K3
2.	(i) In a lattice, show that the isotonicity property is true **[8 Marks]** (ii) Show that in any Boolean algebra, $a\bar{b} + b\bar{c} + c\bar{a} = \bar{a}b + \bar{b}c + \bar{c}a$ **[8 Marks]**	CO5 CO6	K3 K3
3.	**(i)** Draw the Hasse diagram of $(D_{100}, /)$ where D_{100} denotes the set of positive divisors of 100 and $/$ is the relation "division". Find (a)glb$\{10,20\}$, (b)lub$\{10,20\}$, (c) glb$\{5,10,20,25\}$,(d)lub$\{5,10,20,25\}$. **[8 Marks]** (ii) State and Prove Involution laws, Absorption laws in Boolean Algebra. **[8 Marks]**	CO5 CO6	K3 K3
4.	(i) State and Prove the Modular Inequality for a Lattice **[8 Marks]** **(ii)** In a Boolean algebra, show that the following statements are equivalent (i) a+b = b, (ii) a.b = a, (iii) a'+b = 1, (iv) a.b' = 0 **[8 Marks]**	CO5 CO6	K3 K3

5.	(i). Prove that every chain is a distributive lattice. **[8 Marks]** **(ii).** State and Prove Idempotent laws, Boundedness laws in Boolean Algebra. **[8 Marks]**	CO5 CO6	K3 K3
6.	(i) Show that Every distributive lattice is modular**. [8 Marks]** (ii) Show that in a Boolean algebra $a \le b \Leftrightarrow a \wedge \overline{b} = 0 \Leftrightarrow \overline{a} \vee b = 1 \Leftrightarrow \overline{b} \le \overline{a}$	CO5 CO6	K3 K3
7.	(i) Show that every chain is modular. **[8 Marks]** (ii) Show that in a Boolean algebra B $a+b' = 1$ iff $a+b = a$ for any a, b in B **[8 Marks]**	CO5 CO6	K3 K3
8.	**(i)** State and prove the Joint Cancellation law in a distributive lattice [**8 Marks]** (ii) In any Boolean algebra, prove that $(a+b).(a'+c)= ac+a'b+bc$ [**8 Marks]**	CO5 CO6	K3 K3