Department of Science and Humanities – Mathematics Division

First Year / Second Semester

23MA204–Probability and Statistics

UNIT I PROBABILITY AND RANDOM VARIABLES

Q.No	Questions	CO's	Bloom's Level
1.	If P(A) = 18/36 , P(B)=11/36 & P(A∩B) = 6/36, Find P(A∪B^c) and P(A^c∪B)	CO1	K3
2.	State Bayes Theorem	CO1	K1
3	If a Random Variable X takes values 1, 2, 3, 4 such that 2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4). Determine the probability distribution of X	CO1	K1
4	If the probability mass function of a random variable is p(x) = kr^3, r = 1,2,3,4, find the value of k. Also find P($\frac{1}{2} < x < \frac{5}{2} / x > 1$)	CO1	K3
5	If the density function of a continuous random variable X is given by f(x) = $\begin{cases} ax, & 0 < x < 1 \\ a, & 1 < x < 2 \\ 3a - ax, & 2 < x < 3 \\ 0, & otherwise \end{cases}$. Find the value of a.	CO1	K3
6	A continuous random variable X has the density function $f(x) = k(1+x), 2 \le x \le 5$. Find P[X<4]	CO1	K3
7	State any two properties of probability mass function	CO1	K1
8	If a R.V X has the MGF $M_x(t) = \frac{2}{2-t}$, Determine the variance of X.	CO1	K3
9	A normal Distribution has mean 20 and standard deviation 10. Find P(15 ≤ X ≤ 40).	CO2	K3
10	State the limitations of Poisson Distribution.	CO2	K1
11	A binomial random variable has MGF = (0.25 + 0.75 e^t), Find E(X) and Var(X)	CO2	K3
12	If the probability that a target is destroyed on any one shot is 0.5, calculate the probability that it would be destroyed on 6^{th}attempt.	CO2	K3
13	The number of monthly breakdown of a computer is a R.V having Poisson distribution with mean equal to 1.8. Calculate the probability that this computer will function for a month with only one breakdown.	CO2	K3
14	The time required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. Calculate the conditional probability that a repair takes at 11h given that its direction exceeds 8h?	CO2	K3
15	If a R.V 'X' is uniformly distributed over (-3,3), then compute P (| X – 2 | < 2).	CO2	K3

<table>
<tr><td></td><td>Part - B</td><td></td><td></td></tr>
<tr><td>1</td><td>i)The members of a consulting firm rent cars from rental agencies. A,B and C as 60%,30% and 10% respectively, If 9%,20% and 6% of cars from A,B and C agencies need tune up
(a) If a rental car delivered to the firm does not need tune up what is the probability that it came from B agency
(b) If a rental car delivered to the firm need tune up what is the probability that came from B agency. (8Marks)
ii) Find the MGF, Mean and Variance of Binomial Distribution(8Marks)</td><td>CO1

CO2</td><td>K3

K3</td></tr>
<tr><td>2</td><td>i)There are 3 boxes containing respectively,1 white 2 red, 3 black balls; 2 white, 3 red and 1 black balls;3 white, 1 red and 2 black balls. A box is chosen at random and from it two balls are drawn at random. The two balls are 1 red and 1 white. What is the probability that they came from second box? (8Marks)
ii) Find the MGF, Mean and Variance of Exponential Distribution(8Marks)</td><td>CO1

CO2</td><td>K3

K3
</td></tr>
<tr><td>3</td><td>i) A discrete RV has the probability function
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>P(x)</td><td>a</td><td>3a</td><td>5a</td><td>7a</td><td>9a</td><td>11</td><td>13a</td><td>15a</td><td>17a</td></tr>
</table>
(i) Find the value of a
(ii) P(X<3), P(0<X<3), P(X≥3)
(iii) Find the distribution function of X
(iv) (iv) Mean of X (8Marks)
ii) Out of 800 families with 4 children each, how many families would be expected to have (i)2 boys and 2 girls (ii) at least 1 boy (iii) at most 2 girls and (iv) Children of both the genders. Assume equal probabilities for boys and girls. (8Marks)</td><td>CO1

CO2</td><td>K3

K3</td></tr>
<tr><td>4</td><td>i) The density function of triangular distribution is given by $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$. Find the mean, variance and cumulative distribution of X. (8Marks)

ii) The temperature ‘T’ in Degrees Farenheit on a day is a Gaussian (85, 10) random variable. What is P(T>100), P(T<60), P(70 ≤ T ≤100)? (8Marks)</td><td>CO1

CO2</td><td>K3

K3</td></tr>
<tr><td></td><td>Part - C</td><td></td><td></td></tr>
<tr><td>5</td><td>Derive the MGF , Mean variance of Poisson distribution(8Marks)</td><td>CO2</td><td>K3</td></tr>
</table>

6	The number of accidents in a year attributed to taxi drivers in a locality follows Poisson distribution with an average 2. Out of 500 taxi drivers of that area, what is the number of drivers with at least 3 accidents in a year? (8Marks)	CO2	K3

UNIT II TWODIMENSIONAL RANDOM VARIABLES

Q.No	Questions	CO's	Bloom's Level
1.	Determine the value of c if the joint density function of two discrete random variables X and Y is given by p(x,y)= cxy, x=1,2,3 and y=1,2,3.	CO3	K3
2.	State the properties of the distribution function of a two dimensional random variable (X,Y).	CO3	K1
3	The Joint PMF of two random variables X and Y is given by $P_{XY}(x,y)=\begin{cases}\frac{1}{18}(2x+y) & x=1,2;\ y=1,2\\ 0 & otherwise\end{cases}$, What is the marginal PMF of X?	CO3	K1
4	If a two dimensional random variable (X,Y) has the Joint Probability distribution $f(x,y)=\begin{cases}\frac{kx}{y} & x=0,6,12\ and\ y=1,3,6\\ 0 & otherwise\end{cases}$, Find k.	CO3	K3
5	Find the value of k, if the joint density function of (X , Y) is given by $f(x,y)=\begin{cases}k(1-x)(1-y), 0<x<4, 1<y<5\\ 0 \quad ,otherwise\end{cases}$	CO3	K3
6	If the joint density function of (X , Y) is given by $f(x,y)=ke^{-(2x+y)}\quad x,y>0$. Find k.	CO3	K3
7	If the joint density function of (X , Y) is given by $f(x,y)=\begin{cases}\frac{1}{4} & 0\le x,y\le 2\\ 0 & otherwise\end{cases}$. Find P(X+Y<1).	CO3	K3
8	If X and Y are independent random variables with variance 2 and 3, find the variance of 3X+4Y.	CO3	K3
9	Define Covariance and coefficient of correlation between and two random variables X and Y.	CO3	K1
10	If X and Y are independent random variables, show that they are uncorrelated.	CO3	K1
11	State Central Limit theorem.	CO3	K1

12	Let X and Y be two independent R.Vs with Var(X) = 9 and Var(Y)=3. Find Var(4X-2Y+6).	CO3	K3
13	If X has mean 4 and variance 9, while Y has mean –2 and variance 5 and the two are independent find (a) E[XY] (b) E[XY^2].	CO3	K3
14	The regression equations of X on Y and Y on X are respectively 5x – y = 22 and 64x – 45y = 24. Find the mean values of X and Y.	CO3	K3
15	The two lines of regression are 3x+2y-26=0 and 6x+y-31=0.Find the value of correlation coefficient.	CO3	K3
	Part - B		
1	X and Y are two random variables having the joint pmf f(x,y)=k(2x+3y), x=0,1,2; y=1,2,3.Find the marginal and conditional distributions. Also, find the probability distribution of (X+Y) and P(X+Y>3) (16 Marks)	CO3	K3
2	i)The joint PMF of X and Y is X/Y 0 1 2 0 0.1 .04 .02 1 .08 0.2 0.06 2 .06 0.14 0.30 Compute the marginal PMF of X and Y,P[X≤1,Y≤1] and check if X and Y are independent.? (8Marks) ii)The life time of a certain brand of an electric bulb may be considered as a RV with mean 1200h and standard deviation 250h. Find the probability using central limit theorem, that the average lifetime of 60 bulbs exceeds 1250h. (8Marks)	CO3 CO3	K3 K3
3	Let X and Y be discrete R.Vs with probability function f(x,y)=(x+y)/21, x=1,2,3, y=1,2 Find (i) Mean and Variance of X and Y (ii) COV(X,Y) (iii) Correlation of X and Y (16 Marks)	CO3	K3
4	The joint pdf of random variables X and Y is given by $f(x,y)=\begin{cases}2-x-y, 0<x<1, 0<y<1\\ 0 \quad ,elsewhere\end{cases}$. Find Cov (X , Y) and the correlation coefficient of X and Y. (16 Marks)	CO3	K3
5	In a partially destroyed laboratory record only the lines of regressions and variance of X are available. The regression equations are 8x – 10y + 66 = 0 and 40x – 18y = 214 and variance of X = 9. Find (a) the correlation coefficient between X and Y (b) Mean values of X and Y (c) variance of Y. (8Marks) ii) By the method of least squares find the straight line that best fits the following data X 1 2 3 4 5 Y 14 27 40 55 68 (8Marks)	CO3 CO3	K3 K3

	Part - C		
6	Fit a second degree polynomial to the following data by the method of least squares X 0 1 2 3 4 Y 1 1.8 1.3 2.5 6.3 (8Marks)	CO3	K3
7	Calculate the correlation coefficient for the following data. (8Marks) X 65 66 67 67 68 69 70 72 Y 67 68 65 68 72 72 69 71	CO3	K3

UNIT III ESTIMATION THEORY

Q.No	Questions	CO's	Bloom's Level
	Unit - IV		
1.	Write the difference between point estimator and interval estimator	**CO4**	K3
2.	What is an estimator?	**CO4**	K2
3	Define point estimator	**CO4**	K2
4	What are the characteristics of a good estimator?	**CO4**	K2
5	What are the commonly used methods for point estimation?	**CO4**	K2
6	Define Population estimation	**CO4**	K2
7	What is called interval estimation ?	**CO4**	K2
8	Define unbiased estimator	**CO4**	K2
9	What is robust statistic?	**CO4**	K2
10	what is an example of robust statistic?	**CO4**	K3
11	Define maximum likelihood estimator	**CO4**	K2
12	The time to failure of an electronic component follows an exponential distribution with parameter λ. Eight units are randomly selected and tested resulting in the following failure time 13.03, 6.07, 68.44, 17.11, 32.54, 8.77, 12.14, 23.42. Find the moment estimate of λ.	**CO4**	K3
13	The number of defective hard drives produced daily by a production line can be modelled as a Poisson distribution. The counts for 10 days are 7,3,1,2,4,1,2,3,1,2. Obtain the MLE of probability of 0.	**CO4**	K3
14	In one area along the inter-state , the number of dropped wireless phone tion per call follows a Poisson distribution. From four calls the number of dropped connection is 2, 0, 3, 1. Find the maximum likelihood estimate of λ.	**CO4**	K3
15	Define population moment estimator	**CO4**	K2
	Part - B		

1	Below you are given the values obtained from a random sample of observations taken from an infinite population 32,34,35and 39. (i) Find a point estimator for μ. Is this an unbiased estimate of μ? $Explain$ (ii) Find a point estimator for σ^2. Is this an unbiased estimator of σ^2. (iii) Find a point estimator for σ **(iv)** What can be said about the sampling distribution of $\bar{x}$? Be sure to discuss the expected value , the standard deviation, and the shape of the sampling distribution of $\bar{x}$?(16 marks)	**CO4**	K3
2	i)If t_1 and t_2 are both most efficient estimators with equal variance V and if t_3 is an average of t_1 and t_2. Prove that Var(t_3)= $\frac{1}{2}V(1+\rho)$ where ρ is the correlation coefficient between t_1 and t_2. (8Marks) ii)Find the estimator of θ in the population with density f(x, θ)= θ $x^{\theta-1}$; 0<x<1; $\theta > 0$(8Marks)	**CO4** **CO4**	K3 K3
3	i)Let $x_1, x_2 \ldots x_n$ be a random sample from exponential distribution with probability density function of x as $f(x,\theta) = \theta e^{-\theta x}$, $0 < x < \infty, \theta > 0$. Estimate θ by the method of moments. (8Marks) ii)Derive the Maximum Likelihood estimator for Binomial Distribution(8Marks)	**CO4** **CO4**	K3 K3
4	i)Derive the Maximum Likelihood estimator for Geometric Distribution(8Marks) ii) In one area along the interstate the number of dropped wireless phone connection per call follows a Poisson distribution. From 4 calls the number of dropped connection is 2,0,3,1. Find the maximum likelihood estimator of λ. (8Marks)	**CO4** **CO4**	K3 **K3**
	Part - C		
5	A random sample of size 100 has a standard deviation of 5. What can you say about the maximum error with 95% confidence? (8Marks)	**CO4**	K3
6	Derive the Maximum Likelihood estimator for Poisson Distribution(8Marks)	**CO4**	K3

UNIT IV NON- PARAMETRIC TESTS

Q.No	Questions	CO's	Bloom's Level
	Unit - 4		
1.	Write difference between Mann whitney and wilcoxon tests	CO5	K2
2.	Define non parametric test	CO5	K1
3	What is the other name for non-parametric test?	CO5	K2
4	Name any 4 non parametric tests	CO5	K1
5	Write any two advantage of non-parametric method over parametric method	CO5	K2
6	What are the primary short comings of non-parametric test	CO5	K2
7	Define Sign test	CO5	K1
8	What are the types of sign test	CO5	K2
9	What is rank test?	CO5	K2
10	When will you use Mann Whitney U test?	CO5	K2
11	Give an example of run test	CO5	K1
12	Where do we use run test	CO5	K2
13	What are the uses of the run test	CO5	K2
14	What is the use of KW test	CO5	K2
15	What are the assumptions of Mann – whiteney?	CO5	K2

<table>
<tr><td></td><td>Part - B</td><td></td><td></td></tr>
<tr><td>1</td><td>i)In a factory 20 observations of the factors that could heat up a conveyor belt yielded the following results: 0.36,0.41,0.25,0.34,0.28,0.26,0.39,0.28,0.40,0.26,0.35,0.38,0.29,0.42,0.37,0.37,0.39,0.32,0.29 and 0.36. use the sign test at 0.01 level of significance to test the null hypothesis μ=0.34 and μ≠ 0.34(8Marks)
ii)The following are the average weekly losses of worker hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation
<table>
<tr><td>Before</td><td>45</td><td>73</td><td>46</td><td>124</td><td>33</td><td>57</td><td>83</td><td>34</td><td>26</td><td>17</td></tr>
<tr><td>After</td><td>36</td><td>60</td><td>44</td><td>119</td><td>35</td><td>51</td><td>77</td><td>29</td><td>24</td><td>11</td></tr>
</table>
Use the 0.05 level of significance to test whether the safety program is effective. (8Marks)</td><td>CO5

CO5</td><td>K4

K4</td></tr>
<tr><td>2</td><td>i)Two classes of students are tested using a certain competitive exam. The scores of a sample of a students from each class is given below: Use
<table>
<tr><td>Class A</td><td>45</td><td>44</td><td>47</td><td>48</td><td>55</td><td>53</td><td>55</td><td>63</td><td></td><td></td><td></td></tr>
<tr><td>Class B</td><td>65</td><td>67</td><td>77</td><td>65</td><td>56</td><td>67</td><td>78</td><td>55</td><td>66</td><td>65</td><td>58</td></tr>
</table>
Use Mann Whitney –U test to test whether both classes have similar scholastic. (8Marks)
ii) Checking on elm trees that were planted many years ago along a county road, a county official obtained the following arrangement of healthy, H, and diseased, D, trees: H H H H D D D H H H H H H H D D H H D D D D. Test at the α = .05 significance level whether this arrangement may be regarded as random (8Marks)</td><td>CO5

CO5</td><td>K4

K4</td></tr>
<tr><td>3</td><td>The following are the number of misprints counted on pages select at random from a newspaper.
<table>
<tr><td>Day1</td><td>4</td><td>10</td><td>2</td><td>6</td><td>4</td><td>12</td></tr>
<tr><td>Day2</td><td>8</td><td>5</td><td>13</td><td>8</td><td>8</td><td>10</td></tr>
<tr><td>Day3</td><td>7</td><td>9</td><td>11</td><td>2</td><td>14</td><td>7</td></tr>
</table>
Use H-test to test whether the samples come from same population (16 Marks)</td><td>CO5</td><td>K4</td></tr>
<tr><td>4</td><td>The marks of statistics for selected 5 students from three colleges are given in the table. Test whether the performance of the students of the three college are same or not. (16 Marks)</td><td>CO5</td><td>K4</td></tr>
</table>

	A	50	62	75	48	65		
	B	80	95	98	87	90		
	C	60	45	30	58	57		
5	A new approach to prenatal care is proposed for pregnant women living in a rural community. The new program involves in home visits during the course of pregnancy in addition to the usual or regularly scheduled visits. A pilot randomized trial with 15 pregnant women is designed to evaluate whether women who participate in the program deliver healthier babies than women receiving usual care. The outcome is the APGAR scores measured 5 minutes after birth. The data are shown below. Usual Care 8 7 6 2 5 8 7 3 New Program 9 9 7 8 10 9 6 Is there statistical evidence of a difference in APGAR scores in women receiving the new and enhanced versus usual prenatal care?(16 marks)						CO5	**K4**
	Part - C							
6	The following are 15 measurements of the octane rating of a certain kind of gasoline: 97.5, 95.2, 97.3, 96.0, 96.8, 100.3, 97.4, 95.3, 93.2, 99.1, 96.1, 97.6, 98.2, 98.5, and 94.9. Use the Signed-Rank test with α = .05 to test whether the mean octane rating of the given kind of gasoline is 98.5. (8Marks)						CO5	**K4**
7	Test whether the following numbers 0.44, 0.81, 0.14, 0.05, 0.93 are uniformly distributed using Kolmogorov smirnov test(8Marks)						CO5	**K4**

UNIT V STATISTICAL QUALITY CONTROL

Q. No	Questions	CO's	Bloom's Level
1	A garment was sampled on 10 consecutive hours of production. The number of defects found per garment is given below: Defects: 5,1,7,0,2,3,4,0,3,2. Compute upper and lower control limits for monitoring number of defects.	CO6	K2
2.	When do we use X and R charts?	CO6	K1
3	Define Tolerance limits	CO6	K1
4	Define control chart	CO6	K1
5	Define Statistical quality control:	CO6	K1
6	What are attributes?	CO6	K2

<table>
<tr><td>7</td><td>What are the types of control charts?</td><td>CO6</td><td>K2</td></tr>
<tr><td>8</td><td>What are the control charts for variables: The control charts for variables:</td><td>CO6</td><td>K2</td></tr>
<tr><td>9</td><td>Name any two advantages of control charts?</td><td>CO6</td><td>K2</td></tr>
<tr><td>10</td><td>What is the procedure for drawing X-R charts:</td><td>CO6</td><td>K1</td></tr>
<tr><td>11</td><td>What are the control limits for mean?</td><td>CO6</td><td>K1</td></tr>
<tr><td>12</td><td>What are the tools used in statistical quality control?</td><td>CO6</td><td>K2</td></tr>
<tr><td>13</td><td>Define p-chart.</td><td>CO6</td><td>K1</td></tr>
<tr><td>14</td><td>Define C-chart.</td><td>CO6</td><td>K1</td></tr>
<tr><td>15</td><td>The total number of defects in 20 pieces of cloth is 220. What are UCL and LCL?</td><td>CO6</td><td>K2</td></tr>
<tr><td></td><td colspan="3">Part - B</td></tr>
<tr><td>1</td><td>The following are the sample means and ranges for 10 samples each of size 5 . Construct the control chart for the mean and range and comment on the nature of the control. (16 marks)
<table>
<tr><td>Sample no</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Mean $\bar{x}$</td><td>12.8</td><td>13.1</td><td>13.5</td><td>12.9</td><td>13.2</td><td>14.1</td><td>12.1</td><td>15.5</td><td>13.9</td><td>14.2</td></tr>
<tr><td>Range R</td><td>2.1</td><td>3.1</td><td>3.9</td><td>2.1</td><td>1.9</td><td>3.0</td><td>2.5</td><td>2.8</td><td>2.5</td><td>2.0</td></tr>
</table></td><td>CO6</td><td>K4</td></tr>
<tr><td>2</td><td>A factory manufacturing small bolts. To check the quality of the bolts, the manufacturer selected 20 samples of same size 100 from the manufacturing process time to time. He visually inspected each selected bolt for certain defects. After the inspection, he obtained the following data: (16 marks)
<table>
<tr><td>Sample no</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Proportion defective</td><td>0.10</td><td>0.04</td><td>0.08</td><td>0.15</td><td>0.08</td><td>0</td><td>0.01</td><td>0.05</td><td>0.05</td><td>0.08</td></tr>
<tr><td>Sample no</td><td>10</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr>
<tr><td>Proportion defective</td><td>0.10</td><td>0</td><td>0.06</td><td>0.05</td><td>0.03</td><td>0.20</td><td>0.05</td><td>0.07</td><td>0.01</td><td>0.08</td></tr>
</table></td><td>CO6</td><td>K4</td></tr>
<tr><td>3</td><td>A plant produces paper for newsprint and rolls of paper are inspected for defects. The result of inspection of 20 rolls of papers are given draw the control chart for the given data.19,10,8,12,15,22,7,13,18,13,16,14,8,7,6,4,5,6,8,9.(16 Marks)</td><td>CO6</td><td>K4</td></tr>
<tr><td>4</td><td>The number of scratch marks on a particular piece of furniture is recorded. The data for 20 samples are given below:
Draw the appropriate control chart and write the comments about the state of the process .(16 Marks)</td><td>CO6</td><td>K4</td></tr>
</table>

	Sample No	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
	Scratch Mark	6	3	14	7	2	5	12	4	7	3	
	Sample No	**10**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
	Scratch Mark	2	7	6	8	4	10	5	4	13	9	

5	As part of an overall quality improvement programme, a textile manufacturer decides to monitor the number of defects found in each inspected bolt (large bundle) of cloth. The data from 20 inspections are recorded in the table given below:**(16 Marks)**	**CO6**	**K4**

Bolt of cloth	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
No. Of defects	10	19	5	9	2	8	7	13	3	2
Bolt of cloth	**10**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
No. Of defects	22	4	6	9	7	2	5	12	4	2

	Part - C		
6	The following data are found during the inspection of the first 15 samples of size 100 each from a lot of two-wheelers manufactured by an automobile company. Draw the P-chart and comment on the state of control. (8Marks)	**CO6**	**K4**

Sample No	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
No of defective	3	4	6	2	12	5	3	6	3	5
Sample No	**10**	**12**	**13**	**14**	**15**					
No of defective	4	15	5	2	3					

7	The following data gives the readings for 8 samples of size 6 each in the production of a certain product. Find the control limits using mean and range chart. (8Marks)	**CO6**	**K4**

Sample no	**1**	**2**	**3**	**4**	**5**	**6**
Mean $\bar{x}$	300	342	351	319	326	333
Range R	25	37	20	28	30	22