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23MA203-PROBABILITY AND STATISTICS

Second Semester

(Regulation 2023)
CA1 -QUESTION BANK

Q.No	Part A- Questions	Marks	CO's	Bloom's Level
1.	A bag contains 3 red, 6 white and 7 black balls, Find out the probability that two balls drawn are red and white? $(3c_1 * 6c_1)/16c_2 = 3/20$	2	CO1	K2
2.	Derive the moment generating function of the distribution given by $$f(x) = \gamma e^{-\gamma x}, x > 0.$$ $E(e^{tx}) = \frac{\gamma}{\gamma - t}$	2	CO1	K2
3.	For a binomial distribution, mean is 2 and variance is 4/3.Find out the first term of the distribution : np=2 and npq=4/3 Solving p=1/3 and n=6 First term= $nc_x p^x q^{n-x} = 64/729 \ for \ x = 0.$	2	CO1	K2
4.	State any two properties of probability mass function $(i)\ p_i \geq 0$ $(ii) \sum_{1}^{n} p_i = 1$	2	CO1	K1
5.	Let X be a random variable with E(X)=1, E[X(X-1)]= 4. Find Var(X), Var(3-2X). E(X)=1, E[X(X-1]) = 4 $\Rightarrow$ E[X^2 – X] =4 $\Rightarrow$ E[X^2] – E[X] =4 $\Rightarrow$ E[X^2] – 1 =4 $\Rightarrow$ E[X^2] = 5 $Var(X) = E[X^2] - (E[X])^2 = 5 - 1 = 4$ $Var(2-3X) = (-3)^2 Var(X) = 9 \times 4 = 36 \qquad \because Var(aX+b) = (a)^2 Var(X)$	2	CO1	K3

6.	If the r.v has the mgf $M_x(t) = \dfrac{2}{2-t}$, determine the variance of X. $M_x(t) = \dfrac{2}{2-t} = \dfrac{2}{2\left(1-\frac{t}{2}\right)} = \left(1-\dfrac{t}{2}\right)^{-1} = 1+\left(\dfrac{t}{2}\right)+\left(\dfrac{t}{2}\right)^2+\left(\dfrac{t}{2}\right)^3+\cdots$ $= 1+\dfrac{1}{2}\left(\dfrac{t}{1!}\right)+\dfrac{1}{2}\left(\dfrac{t^2}{2!}\right)+\dfrac{3}{4}\left(\dfrac{t^3}{3!}\right)+\cdots$ $\mu_r' =$ coefficient of $\left(\dfrac{t^r}{r!}\right)$ in $M_X(t)$, $\quad \therefore \mu_1' = \dfrac{1}{2}, \mu_2' = \dfrac{1}{2}$ $\text{Var}(X) = \mu_2' - (\mu_1')^2 = \dfrac{1}{2} - \dfrac{1}{4} = \dfrac{1}{4}.$	2	CO1	K3
7.	If the probability that a target is destroyed on any one shot is 0.5, calculate the probability that it would be destroyed on 6th attempt Given p = 0.5 q = 0.5 By Geometric distribution $P[X = x] = q^x p, x = 0,1,2\ldots\ldots\ldots$ since the target is destroyed on 6th attempt x = 5 Required probability = $q^x p = (0.5)^6 = 0.0157$	2	CO2	K2
8.	A continuous random variable X has the density function $f(x) = 3x^2, 0 \le x \le 1,$ *find* α *such that* $P(X \le \alpha) = P(X > \alpha).$ $1 - P(x > \alpha) = P(x \le \alpha)$ *therefore* $2P(X > \alpha) = 1$ $\Rightarrow 2\int_{\alpha}^{1} 3x^2 dx = 1$ $\Rightarrow \alpha = 1$	2	CO1	K2
9.	One percent of jobs arriving at a computer system need to wait until weekends for scheduling, owing to core-size limitations. Calculate the probability that among a sample of 200 jobs there are no jobs that have to wait until weekends. p = 0.01, n = 200, $\lambda = np = 2$, X is the no. of jobs that have to wait $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!} = \dfrac{e^{-2}(2)^x}{x!} \Rightarrow P(X = 0) = \dfrac{e^{-2}(2)^0}{0!} = e^{-2} = 0.1353.$	2	CO1	K2

10.	The number of monthly breakdown of a computer is a r.vhaving a Poisson distribution with mean equal to 1.8. calculatethe probability that this computer will function for a month with only one breakdown. Mean = λ = np = 1.8 $P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}=\frac{e^{-1.8}(1.8)^{x}}{x!}\Rightarrow P(X=1)=\frac{e^{-1.8}(1.8)^{1}}{1!}=0.2975.$	2	CO2	K2
11.	If X is a Uniformly distributed r.v with mean 1 and variance $\frac{4}{3}$, Determine P(X<0). Mean = $\frac{a+b}{2}=1\Rightarrow a+b=2$ and variance = $\frac{(b-a)^{2}}{12}=\frac{4}{3}\Rightarrow b-a=4$ By solving the above eqns. We get a = -1 and b = 3 $f(x)=\frac{1}{b-a}$ in $a<x<b=\frac{1}{6}, -3<x<3$ $P(X<0)=\int_{-1}^{0}f(x)dx=\int_{-1}^{0}\frac{1}{4}dx=\frac{1}{4}[x]_{-1}^{0}=\frac{1}{4}.$	2	CO2	K2
12.	The time required to repair a machine is exponentially distributed with parameter $\lambda=\frac{1}{2}$. Calculate the conditional probability that a repair takes at 11h given that its direction exceeds 8h? $P(X\geq 11 / X>8)=P(X>3)=$ $\int_{3}^{\infty}f(x)\,dx=\int_{3}^{\infty}\frac{e^{-\frac{x}{2}}}{2}\,dx=\frac{1}{2}\left[\frac{e^{-\frac{x}{2}}}{-1/2}\right]=e^{-1.5}=0.2231$	2	CO2	K2
13.	If X and Y are independent binomial variates following $B\left(5,\frac{1}{2}\right)$ and $B\left(7,\frac{1}{2}\right)$ respectively. Determine $P[X+Y=3]$. By additive property, $X+Y$ is also a binomial variate with parameters $n_1+n_2=12$ & $p=\frac{1}{2}$ $P(Z=z)=nC_z p^z q^{n-z}; z=0,1,2,3.....n$ $\therefore P[X+Y=3]=12C_3\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{9}=\frac{55}{2^{10}}$	2	CO2	K2

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<table>
<tr><td>14.</td><td>If a r.v 'X' is uniformly distributed over (-3,3), then compute P (| X – 2 | < 2).

$f(x) = \frac{1}{b-a}$ in $a < x < b = \frac{1}{6}, \; -3 < x < 3$
P (| X – 2 | < 2) = P (-2 < X – 2 < 2) = P (0 < X < 4)
$= \int_0^3 f(x)dx = \int_0^3 \frac{1}{6} dx = \frac{1}{6}[x]_0^3 = \frac{1}{2}.$</td><td>2</td><td>CO2</td><td>K2</td></tr>
<tr><td>15.</td><td>If a random variable X takes values 1, 2, 3, 4 such that 2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4). Determine the probability distribution of X

Let 2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4) = k
Then P(X=1) =k/2 , P(X=2) =k/3, P(X=3) =k, P(X=4)=k/5
WKT ∑P(X)=1
k/2 + k/3 + k + k/5 =1⇒k=30/61
<table><tr><td>X=x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X=x)</td><td>15/6</td><td>10/61</td><td>30/61</td><td>6/31</td></tr></table></td><td>2</td><td>CO2</td><td>K2</td></tr>
<tr><td>Q.No</td><td>Part B- Questions</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>Find the mgf,mean and variance of Poisson distribution.</td><td>8</td><td>CO2</td><td>K3</td></tr>
<tr><td>2</td><td>If $P(X = x) = \frac{x}{15}$, x = 1,2,3,4,5
i) Find P(x=1 or x=2)
ii) P(1/2 < x < 5/2 / x>1)
iii) Distribution function of x
Find E(X), E(2x-2),Var(x)</td><td>8</td><td>CO1</td><td>K3</td></tr>
<tr><td>3</td><td>The number of accidents in a year attributed to taxi drivers in a locality follows Poisson distribution with an average 2. Out of 500 taxi drivers of that area, what is the number of drivers with at least 3 accidents in a year?</td><td>8</td><td>CO2</td><td>K3</td></tr>
<tr><td>4</td><td>The distribution function of a random variable X is given by $F(x) = 1-(1+x)e^{-x}$, $x \geq 0$.Find the density function, mean variance of X</td><td>8</td><td>CO1</td><td>K3</td></tr>
<tr><td>5</td><td>The length of the shower in a tropical island in a rainy season has an exponential distribution with parameter 2, time being measured in minutes. What is the probability that it will last for at least one more minute?</td><td>8</td><td>CO2</td><td>K3</td></tr>
</table>

6	Trains arrive at a station at 15 minutes interval starting at 4 a.m. If passengers arrive at a station at a time that is uniformly distributed between 9.00 a.m. and 9.30 a.m., find the probability that he has to wait for the train for (i) less than 6 minutes (ii) more than 10 minutes	**8**	CO2	K3
7	Out of 800 families with 4 children each, how many families would be expected to have (i)2 boys and 2 girls (ii) at least 1 boy (iii) at most 2 girls and (iv) Children of both the genders. Assume equal probabilities for boys and girls.	**8**	CO2	K3
8	State and prove the memoryless property of Geometric distribution	**8**	CO2	K3
9	6 dice are thrown 729 times how many times would you expect to have at least 3 dice should show a five or six.	**8**	CO2	K3
10	The probability that a razor blade manufactured by a firm is defective is 1/500. Blades are supplied in packets of 5 each. In a lot of 10,000 packets, how many packets would (i) be free defective blades? (ii) contains exactly one defective blade?($e^{-0.01}$=0.99) Let X be the number of defective blades in a packet of 5 blades. Then, X is B (n = 5, p = 1/500)	**8**	CO2	K3

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UNIT 5				
Q.No	**Part A- Questions**	**Marks**	**CO's**	**Bloom's Level**
1.	A garment was sampled on 10 consecutive hours of production. The number of defects found per garment is given below: Defects:5,1,7,0,2,3,4,0,3,2.Compute upper and lower control limits for monitoring number of defects.(Apr/May 2019) Solution: $\overline{C} = 2.7$,$UCL = 7.6295$ $LCL = -2.2295.$	2	6	K2
2.	When de we use X and R charts? In statistical quality control, the X **and R chart** is a type of control chart used to monitor variables data when samples are collected at regular intervals.	2	6	K1
3.	Define Tolerance limits "A (p, 1−α) upper tolerance limit (TL) is simply an 1−α upper confidence limit for the 100 p percentile of the population." A tolerance interval can be seen as a statistical version of a probability interval. "In the parameters-known case, a **95%** tolerance interval and a **95%** prediction interval are the same.	2	6	K1
4.	Define control chart A control chart provides a basis for deciding whether the variation in the output is due to random causes or due to assignable causes. It will assist us in making decisions whether to adjust the process or not	2	6	K1
5.	Define Statistical quality control: It is the procedure or method for the control of quality by the application of the theory of probability to the results of inspection samples of the population.	2	6	K1
6.	What are attributes? Attributes are the characteristics of products which are not measurable. Such characteristic can be felt by their presence or	2	6	K2

	absence.			
7.	What are the types of control charts? Type of control charts: i) Control charts for variables ii) Control charts for attributes.	2	6	K2
8.	What are the control charts for variables: The control charts for variables: i) The mean charge or X-chart, ii) The range chart or R-Chart, iii) σ – chart.	2	6	K2
9.	Name any two advantages of control charts? (i). In general it helps us to rectify the faults and errors during the process. (ii). By quality control methods, we contain and hold the variability in the production process due to change variation only, by maintaining the control specifications can be predicted.	2	6	K2
10.	What is the procedure for drawing X-R charts: The X and R charts are constructed usually in the same graph sheet on the basis of number of samples drawn from production process. These samples called rational sub groups or sub groups are random samples.	2	6	K1
11.	What are the control limits for mean? $\overline{\overline{X}} = \frac{\sum \overline{x}_i}{N}$, where N is the No of samples $UCL = \mu + \frac{3\sigma}{\sqrt{n}}$, and $LCL = \mu - \frac{3\sigma}{\sqrt{n}}$ $\mu = \overline{x}, \ \sigma = \frac{R}{d_2}$ where $\overline{R} = \frac{\sum R_i}{N}$ $UCL = \overline{\overline{X}} + A_2\overline{R}$ and $LCL = \overline{\overline{X}} - A_2\overline{R}$ where $A_2 = \frac{3}{d_2\sqrt{n}}$	2	6	K1
12.	What are the tools used in statistical quality control? 1) Descriptive Statistics 2) Statistical Process Control (SPC) 3) Acceptance Sampling	2	6	K2
13.	Define p-chart. Sol: Control chart for fraction defectives is called p-chart.	2	6	K1
14.	Define C-chart.	2	6	K1

<table>
<tr><td></td><td>Sol: Control chart for number of defects is called c-chart.</td><td></td><td></td><td></td></tr>
<tr><td>15.</td><td>The total number of defects in 20 pieces of cloth is 220. What are UCL and LCL?
UCL = 20.95 and LCL = 1.05</td><td>2</td><td>6</td><td>K2</td></tr>
<tr><td colspan="5">PART-B</td></tr>
<tr><td>1.</td><td>The following are the sample means and ranges for 10 samples each of size 5 . Construct the control chart for the mean and range and comment on the nature of the control.
<table>
<tr><th>Sample no</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr>
<tr><td>Mean $\bar{x}$</td><td>12.8</td><td>13.1</td><td>13.5</td><td>12.9</td><td>13.2</td><td>14.1</td><td>12.1</td><td>15.5</td><td>13.9</td><td>14.2</td></tr>
<tr><td>Range R</td><td>2.1</td><td>3.1</td><td>3.9</td><td>2.1</td><td>1.9</td><td>3.0</td><td>2.5</td><td>2.8</td><td>2.5</td><td>2.0</td></tr>
</table></td><td>8</td><td>6</td><td>K4</td></tr>
<tr><td>2</td><td>Write a short note on Statistical Quality Control</td><td>8</td><td>6</td><td>K3</td></tr>
<tr><td>3</td><td>The following data gives readings of 10 samples of size 5 each in the production of a certain product. Draw control chart for mean and range with its control limits.
<table>
<tr><th>Sample no</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><td>$\bar{X}$</td><td>43</td><td>49</td><td>37</td><td>44</td><td>45</td></tr>
<tr><td>R</td><td>5</td><td>6</td><td>5</td><td>7</td><td>7</td></tr>
</table>
<table>
<tr><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr>
<tr><td>37</td><td>51</td><td>46</td><td>43</td><td>47</td></tr>
<tr><td>4</td><td>8</td><td>6</td><td>4</td><td>6</td></tr>
</table></td><td>8</td><td>6</td><td>K4</td></tr>
<tr><td>4</td><td>A plant produces paper for newsprint and rolls of paper are inspected for defects. The result of inspection of 20 rolls of papers are given draw the control chart for the given data:
9,10,8,12,15,22,7,13,18,13,16,14,8,7,6,4,5,6,8,9.</td><td>8</td><td>6</td><td>K3</td></tr>
<tr><td>5</td><td>The number of scratch marks on a particular piece of furniture is recorded. The data for 20 samples are given below:
Draw the appropriate control chart and write the comments about the state of the process when:
i)the management sets a goal of 5 scratch marks on an average per piece.
ii) the management does not set the average number of marks per piece.</td><td>8</td><td>6</td><td>K4</td></tr>
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<table>
<tr><td>Sample Number</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Scratch Mark</td><td>6</td><td>3</td><td>14</td><td>7</td><td>2</td><td>5</td><td>12</td><td>4</td><td>7</td><td>3</td></tr>
<tr><td>Sample Number</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr>
<tr><td>Scratch Mark</td><td>2</td><td>7</td><td>6</td><td>8</td><td>4</td><td>10</td><td>5</td><td>4</td><td>13</td><td>9</td></tr>
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<tr>
<td>6</td>
<td>To monitor the manufacturing process of laptops, a quality control engineer randomly selects 50 laptops from the production line, each day over a period of 20 days. The laptops are inspected for certain defects and the number of defective laptops found each day is recorded in the following table:
Construct NP chart and state whether the process is in control.
<table>
<tr><th>Day</th><th>Number of Laptops Inspected</th><th>Number of Defective Laptops</th><th>Day</th><th>Number of Laptops Inspected</th><th>Number of Defective Laptops</th></tr>
<tr><td>1</td><td>50</td><td>4</td><td>11</td><td>50</td><td>6</td></tr>
<tr><td>2</td><td>50</td><td>8</td><td>12</td><td>50</td><td>1</td></tr>
<tr><td>3</td><td>50</td><td>6</td><td>13</td><td>50</td><td>5</td></tr>
<tr><td>4</td><td>50</td><td>10</td><td>14</td><td>50</td><td>3</td></tr>
<tr><td>5</td><td>50</td><td>4</td><td>15</td><td>50</td><td>2</td></tr>
<tr><td>6</td><td>50</td><td>3</td><td>16</td><td>50</td><td>3</td></tr>
<tr><td>7</td><td>50</td><td>4</td><td>17</td><td>50</td><td>7</td></tr>
<tr><td>8</td><td>50</td><td>7</td><td>18</td><td>50</td><td>9</td></tr>
<tr><td>9</td><td>50</td><td>8</td><td>19</td><td>50</td><td>2</td></tr>
<tr><td>10</td><td>50</td><td>4</td><td>20</td><td>50</td><td>4</td></tr>
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<td>8</td><td>6</td><td>K4</td>
</tr>
<tr>
<td>7</td>
<td>Mobile manufacturer inspects 30 mobiles at the end of the day of production and notes the number of defective mobiles. This procedure is continued up to 12 days and 2, 1, 3, 0, 2, 1, 0, 5, 2, 0, 3, 1 defective mobile are found. Is the production process under control with respect to the proportion defective?</td>
<td>4</td><td>6</td><td>K3</td>
</tr>
<tr>
<td>8</td>
<td>A garment was sampled on 10 consecutive hours of production. The number of defective per garment is given below :5,1,7,0,2,3,4,0,3,2. Compute the upper and lower control limits for monitoring number of defects.</td>
<td>4</td><td>6</td><td>K3</td>
</tr>
<tr>
<td>9</td>
<td>Construct a control chart for defectives for the following data
<table>
<tr><th>Sample no</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr>
<tr><th>No. Inspect ed</th><td>90</td><td>65</td><td>85</td><td>70</td><td>80</td><td>80</td><td>70</td><td>95</td><td>90</td><td>75</td></tr>
</table>
</td>
<td>8</td><td>6</td><td>K4</td>
</tr>
</table>

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	No. of defectives | 9 | 7 | 3 | 2 | 9 | 5 | 3 | 9 | 6 | 7			
10	As part of an overall quality improvement programme, a textile manufacturer decides to monitor the number of defects found in each inspected bolt (large bundle) of cloth. The data from 20 inspections are recorded in the table given below:	8	6	K4

Bolt of Cloth	1	2	3	4	5	6	7	8	9	10
Number of Defects	10	19	5	9	2	8	7	13	3	2
Bolt of Cloth	11	12	13	14	15	16	17	18	19	20
Number of Defects	22	4	6	9	7	2	5	12	4	2