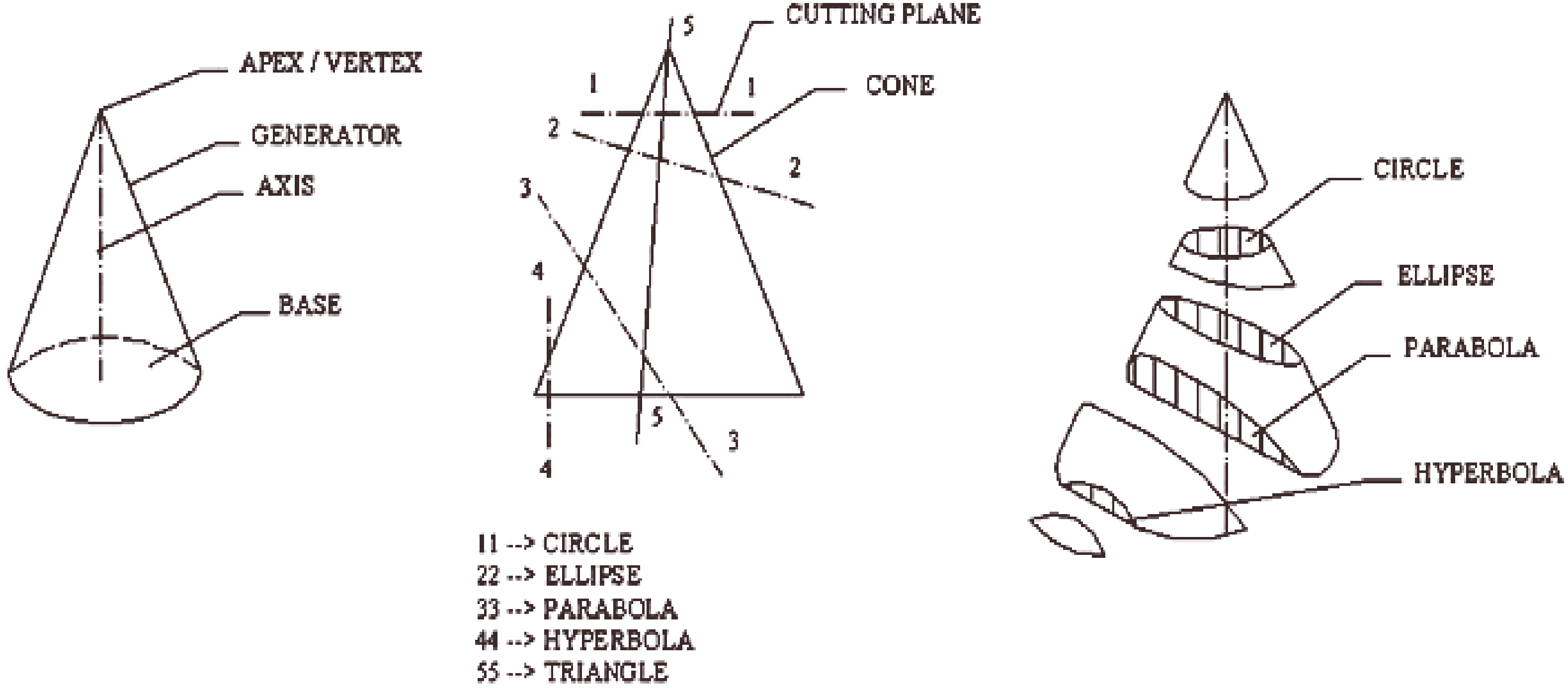
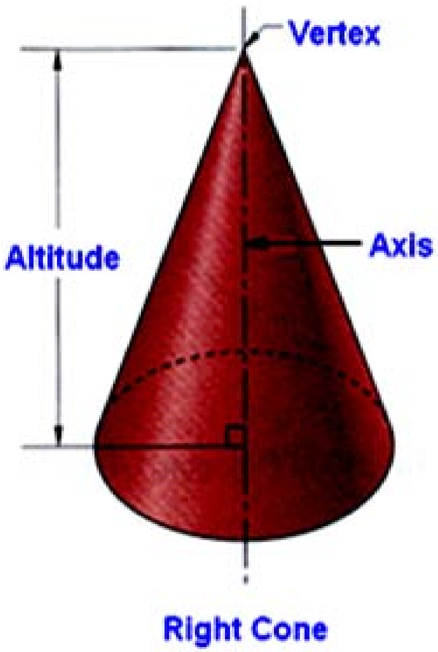
CONIC SECTIONS

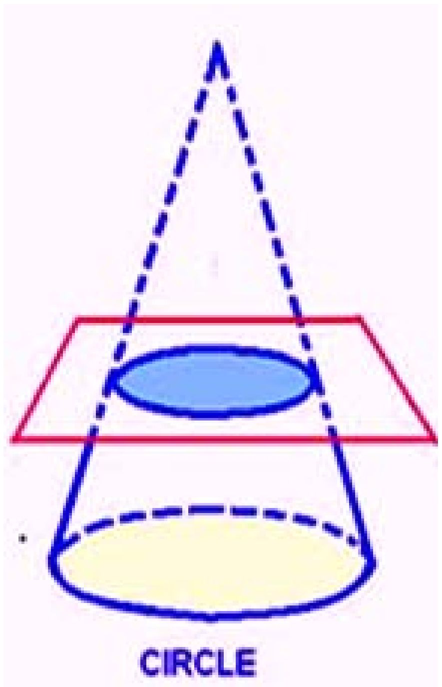
The sections obtained by the intersection of a right circular cone by a cutting plane in different positions are called conic sections or conics.



Circle:

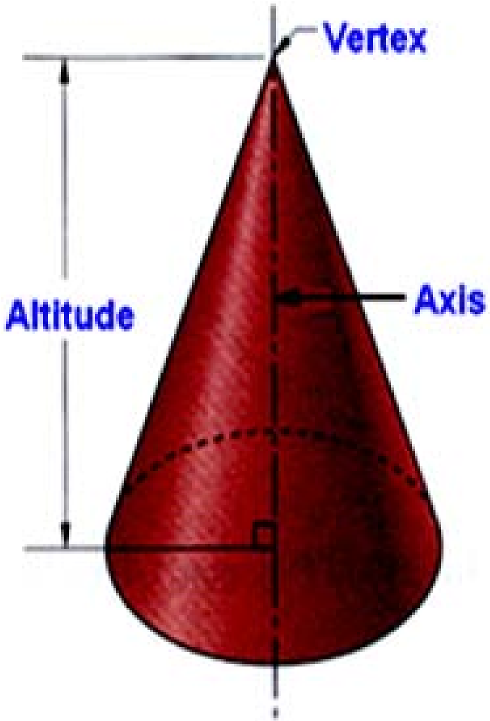
- When the cutting plane is parallel to the base or perpendicular to the axis, then the true shape of the section is circle.



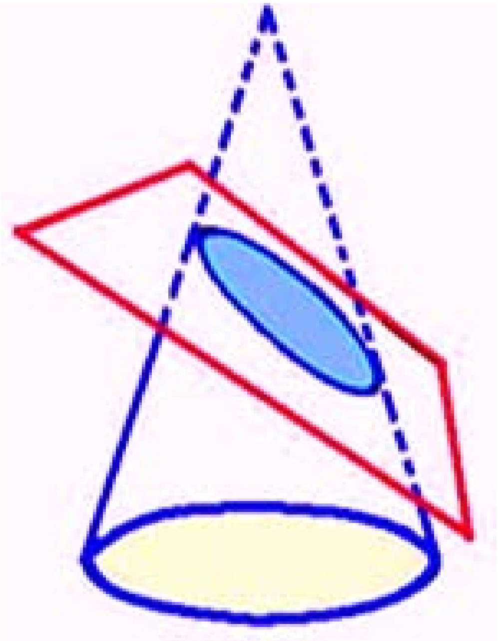


Ellipse:

- When the cutting plane is inclined to the horizontal plane and perpendicular to the vertical plane, then the true shape of the section is an ellipse.



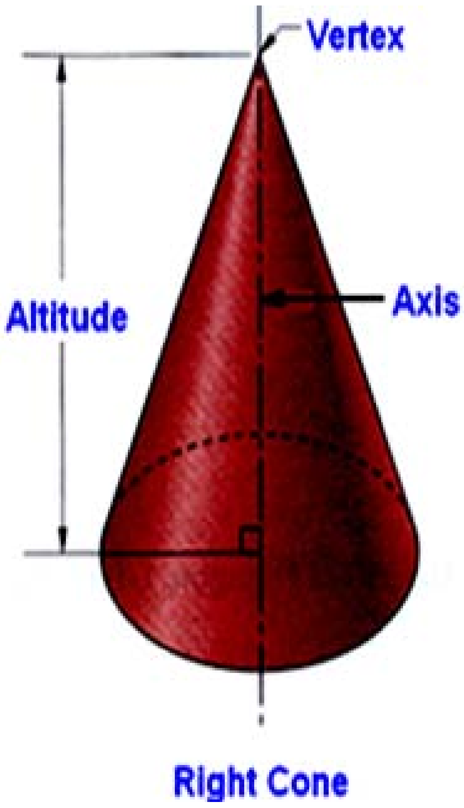
Right Cone

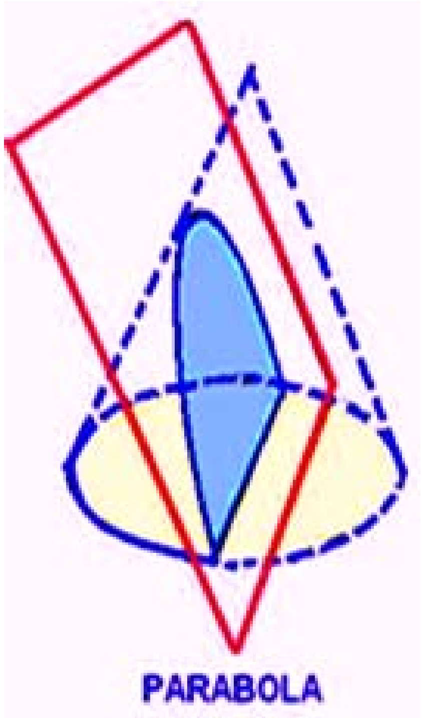


ELLIPSE

Parabola:

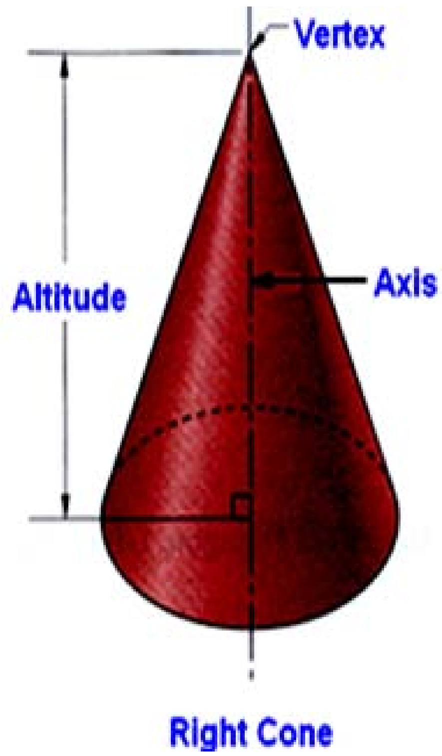
When the cutting plane is inclined to the axis and is parallel to one of the generators, then the true shape of the section is a parabola.

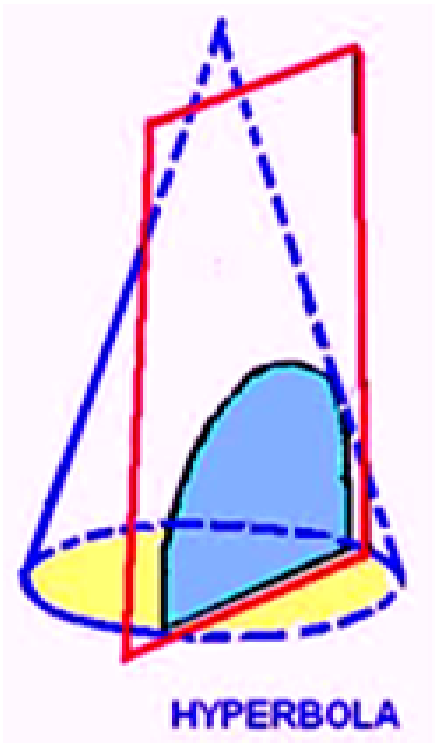




- **Hyperbola:**

When the cutting plane is parallel to the axis of the cone, then the true shape of the section is a rectangular hyperbola.





Eccentricity

Focus & Directrix

Conic may be defined as the locus of a point moving in a plane in such a way that the ratio of its distances from a fixed point, called focus and a fixed straight line called directrix.

Eccentricity

The ratio of shortest distance from the focus to the shortest distance from the directrix is called eccentricity.

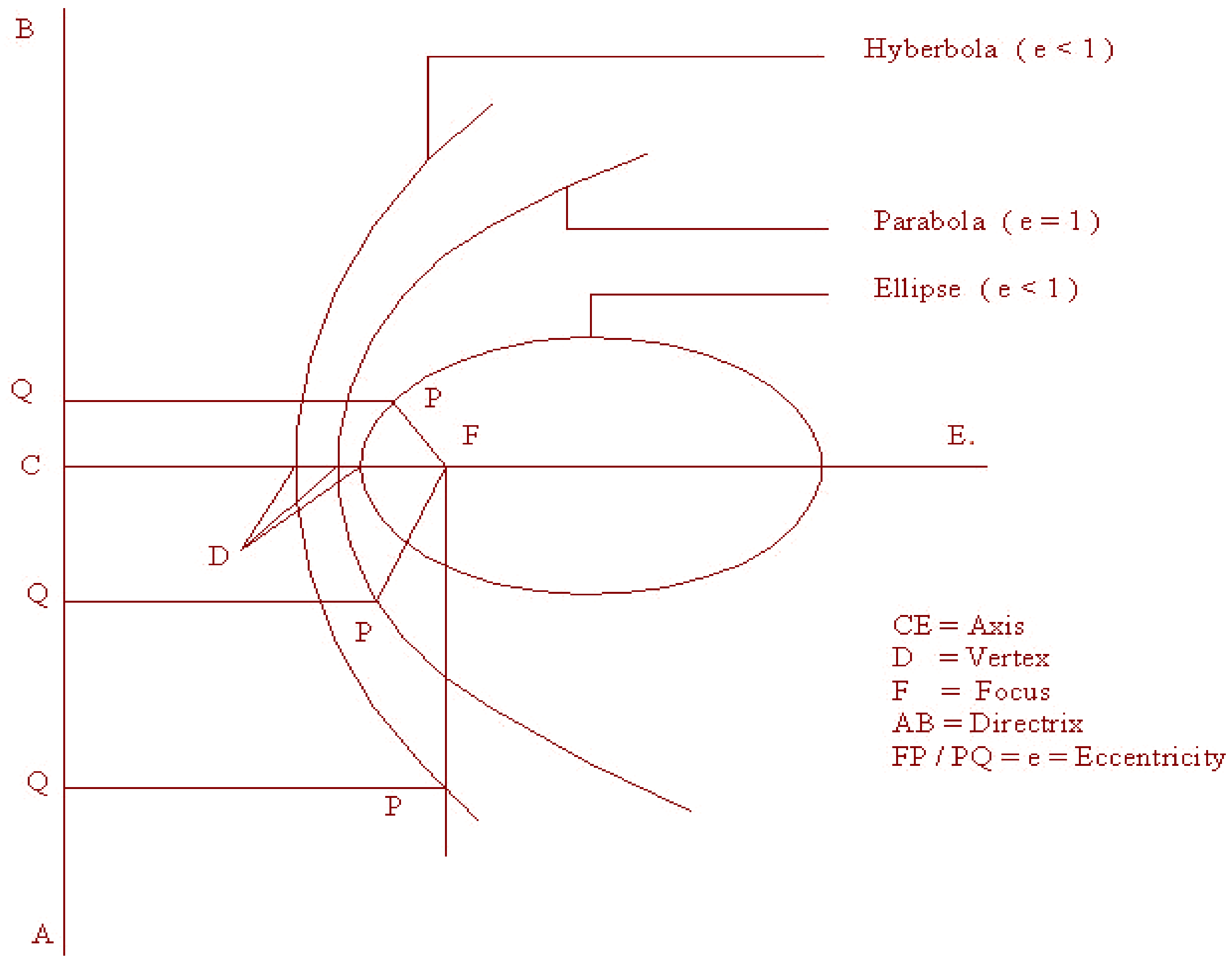
$$\textbf{Eccentricity} = \frac{\textbf{Distance of the point from the focus}}{\textbf{Distance of the point from the directric}}$$

When eccentricity

$< 1 \rightarrow$ **Ellipse**

$= 1 \rightarrow$ **Parabola**

$> 1 \rightarrow$ **Hyperbola**



B
Hyberbola (e < 1)
Parabola (e = 1)
Ellipse (e < 1)
Q
P
F
E.
C
D
Q
P
Q
P
A
CE = Axis
D = Vertex
F = Focus
AB = Directrix
FP / PQ = e = Eccentricity

Construct a parabola when the distance of the Focus from the directrix is 50 mm. **Eccentricity, e = 1**

1.Draw directrix AB and axis CC' as shown.

2.Mark F on CC' such that CF = 5 0 mm.

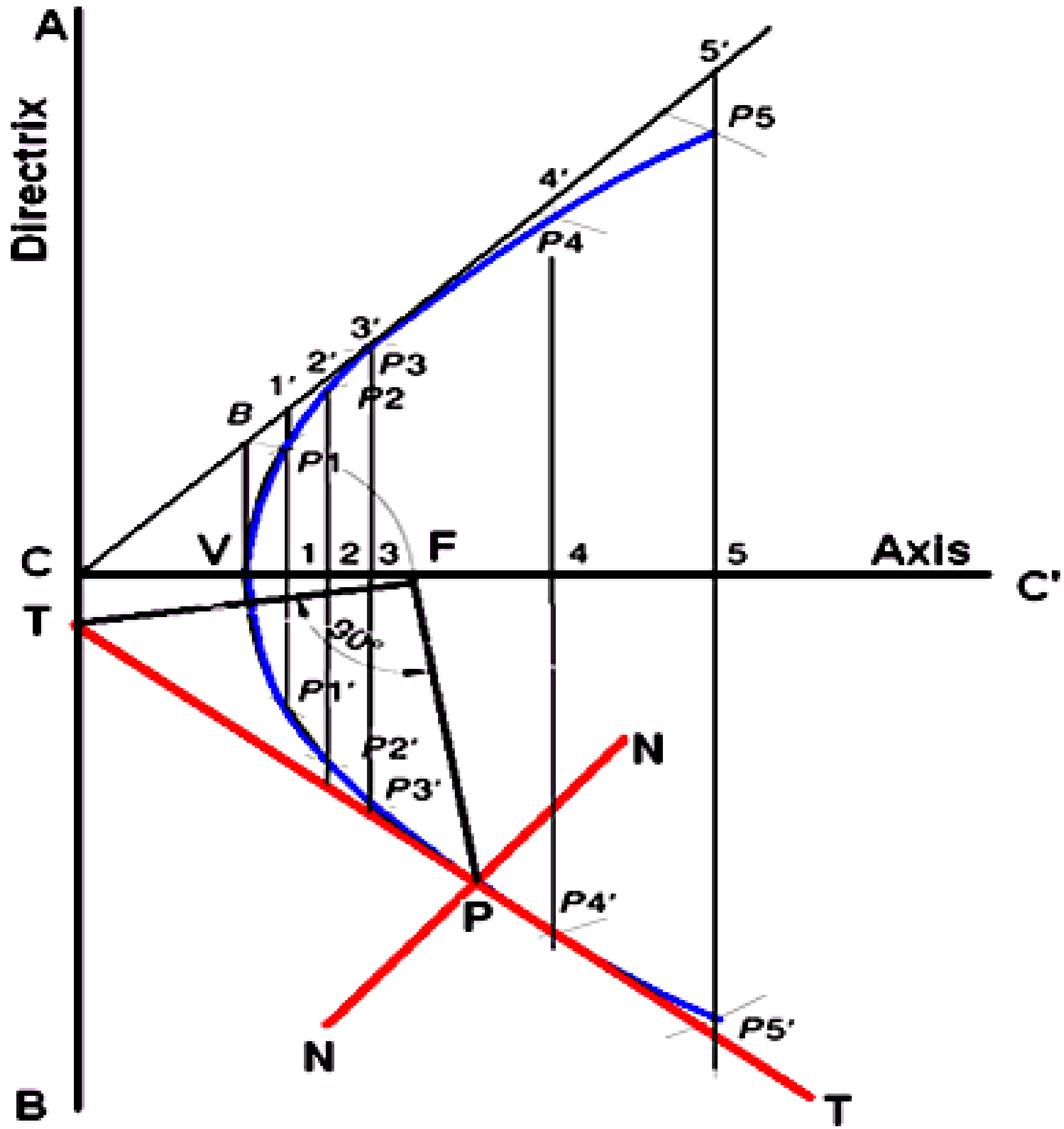
3.Mark V at the midpoint of CF. Therefore, e = VF/ VC = 1.

At V, erect a perpendicular VB = VF. Join CB.

4.Mark a few points, say, 1, 2, 3, … on VC' and erect perpendiculars through them meeting CB produced at 1',2', 3'…

5.With F as a centre and radius = 1–1', cut two arcs on the perpendicular through 1 to locate P1 and P1'. Similarly, with F as a centre and radii = 2–2', 3–3', etc., cut arcs on the corresponding perpendiculars to locate P2 and P2',P3 and P3' etc.

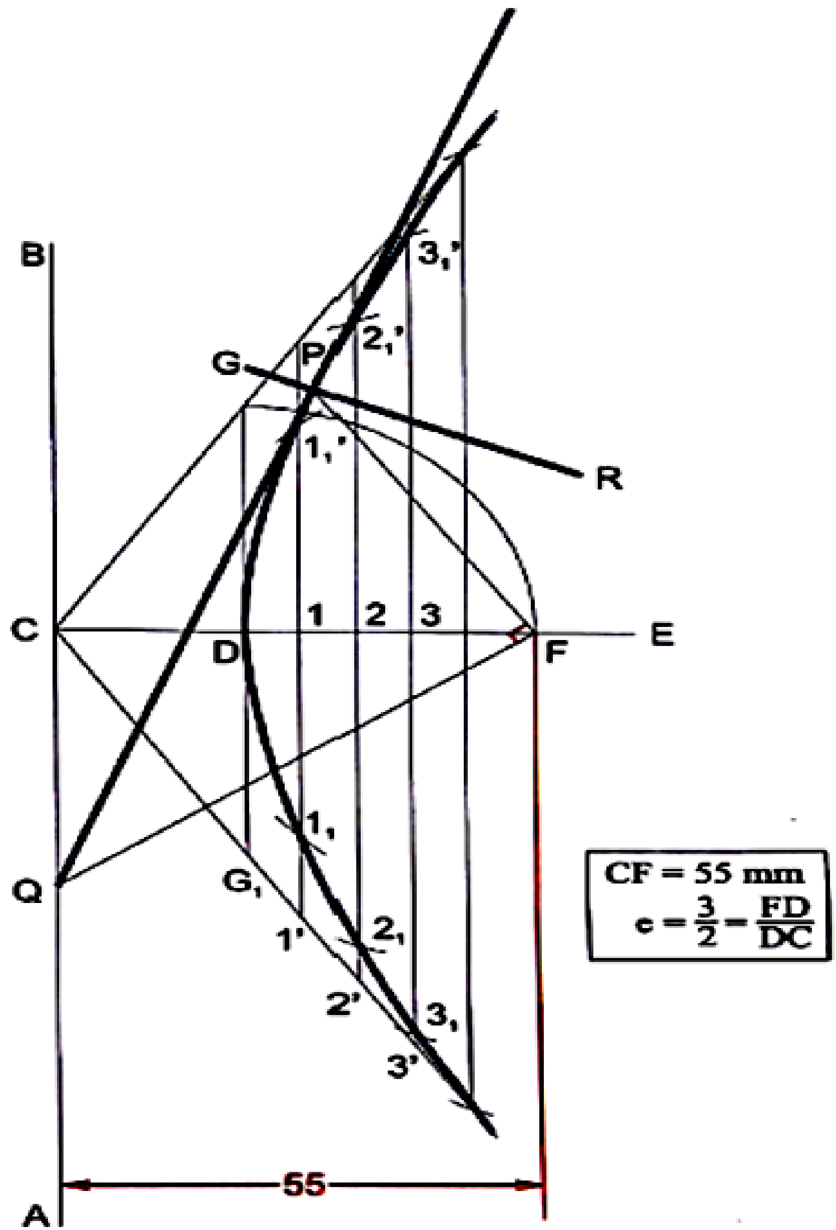
6.Draw a smooth curve passing through V, P1,P2,P3 …P3',P2',P1'.



A
Directrix
5'
P5
4'
P4
3'
P3
2'
P2
1'
B
P1
C
V
1
2
3
F
4
5
Axis
C'
T
30°
P1'
P2'
P3'
N
P
P4'
P5'
N
B
T

Draw a hyperbola when the distance of the focus from the directrix is 55 mm and eccentricity is 3/2.

- Draw a perpendicular line AB (directrix) and a horizontal line CC' (axis).
- Mark the focus point F on the axis line 55 mm from the directrix.
- Divide the CF in to 5 equal parts.
- As per the eccentricity mark the vertex V, in the third division of CF
- Draw a perpendicular line from vertex V, and mark the point B, with the distance VF.
- Join the points C& B and extend the line.
- Draw number of smooth vertical lines 1,2,3,4,5,6,etc., as shown in figure.
- Now mark the points 1′, 2′, 3′, 4′, 5′…
- Take the vertical distance of 11′ and with F as center draw an arc cutting the vertical line 11′ above and below the axis.
- Similarly draw the arcs in all the vertical lines (22′, 33′, 44′…)
- Draw a smooth curve through the cutting points to get the required hyperbola by free hand.


B
G
P
R
C
D
1
2
3
E
F
Q
A
55
CF = 55 mm
e = 3/2 = FD/DC

Parabola

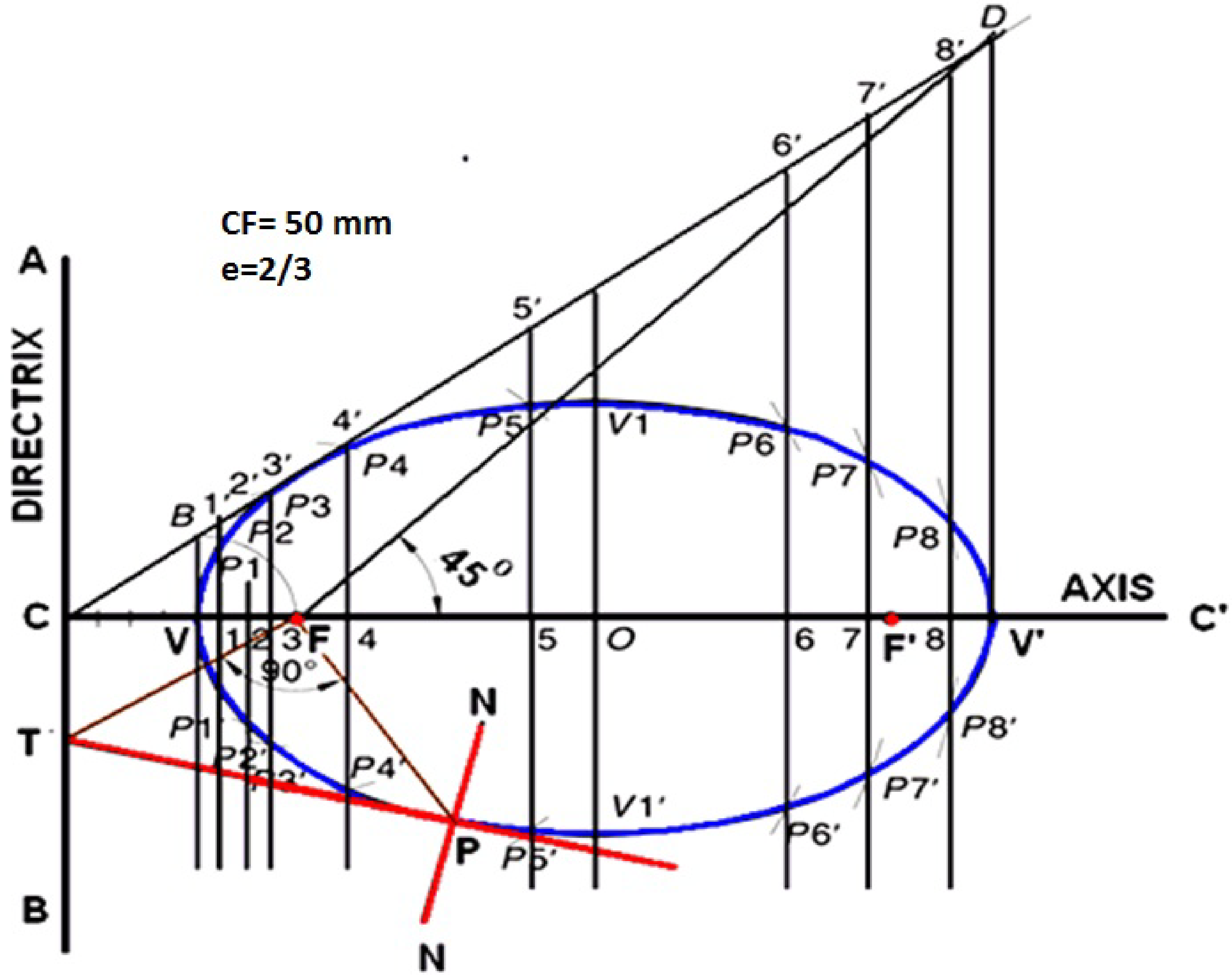
- https://youtu.be/D2MtgB1gXJg

Hyperbola

- https://youtu.be/M6t5HQpBAag

Draw an ellipse when the distance between the focus and directrix is 50mm and eccentricity is $\frac{2}{3}$.

1.Draw the directrix AB and axis CC'
2.Mark F on CC' such that CF = 50 mm.
3.Divide CF into 5 equal parts and mark V at the third division from C.
4.Now, e = FV/ CV = 2/3.
5.At V, erect a perpendicular VE = VF. Join CE. Through F, draw a line at 45° to meet CE produced at D. Through D, drop a perpendicular DV'on CC'. Mark O at the midpoint of V– V'.
6.With F as a centre and radius = 1–1', cut two arcs on the perpendicular through 1 to locate P1 and P1'. Similarly, with F as centre and radii =2-2', 3–3', etc., cut arcs on the corresponding perpendiculars to locate P2 and P2', P3 and P3', etc. Also, cut similar arcs on the perpendicular through O to locate V1 and V1'.
6. Draw a smooth closed curve passing through V, P1, P/2, P/3,., V1, …,V', …, V1', … P/3', P/2', P1'.
7. Mark F' on CC' such that V' F' = VF.



CF= 50 mm
e=2/3
DIRECTRIX
A
B
C
T
AXIS
C'
D
8'
7'
6'
5'
4'
B 1,2',3'
P1
P2
P3
P4
P5
V1
P6
P7
P8
45°
V
1 2 3 F
4
5
O
6 7 F' 8
V'
90°
N
P
N
P1'
P2'
P3'
P4'
P5'
V1'
P6'
P7'
P8'

Cycloidal curves

Cycloidal curves are generated by a fixed point on the circumference of a circle, which rolls without slipping along a fixed straight line or a circle.

In engineering drawing some special curves(cycloidal curves) are used in the profile of teeth of gear wheels.

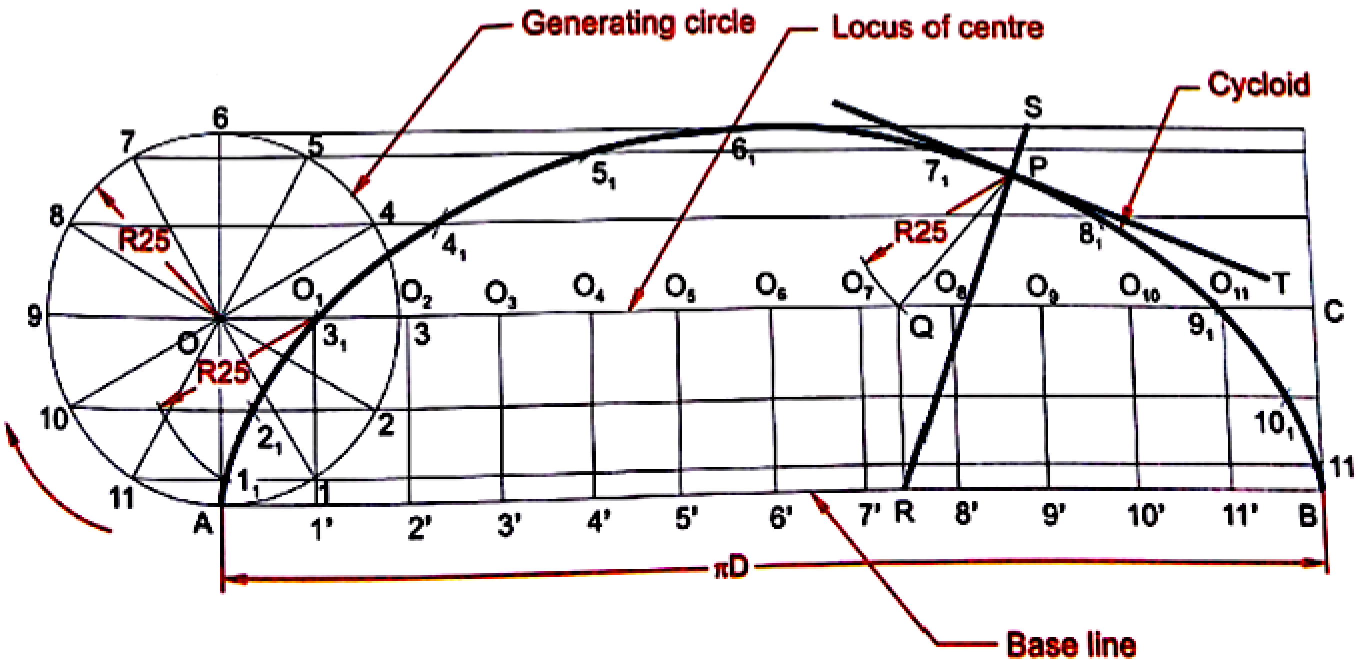
The rolling circle is called generating circle.

The fixed straight line or circle is called directing line or directing circle.

Draw ac cycloid given the diameter of the generating circle as 50mm. Also draw tangent and normal at a point on the curve 30mm above the directing line.

- Draw a circle with diameter 50mm and mark the center O.
- Divide the circle in to 12 equal parts as1,2,3…12.
- Draw horizontal line from the bottom points of the circle, with the distance equal to the circumference of the circle (ΠD) and mark the other end point B.
- Divide the line AB in to 12 equal parts. (1′, 2′, 3′…12′)
- Draw a horizontal line from O toand mark the equal distance point O_1, O_2,
- O_3…O_{12}.
- Draw smooth horizontal lines from the points 1,2,3…12.
- When the circle starts rolling towards right hand side, the point 1 coincides with 1′ at the same time the center O moves to O_1.

- Take OA as radius, O1 as center draw an arc to cut the horizontal line 1 to mark the point 11.
- Similarly O2 as center and with same radius OA draw an arc to cut the horizontal line 2 to mark the point 21.
- Similarly mark 31, 41…111.
- Draw a smooth curve through the points A,11, 21, 33,…. 111, B by free hand.
- The obtained curve is a cycloid.



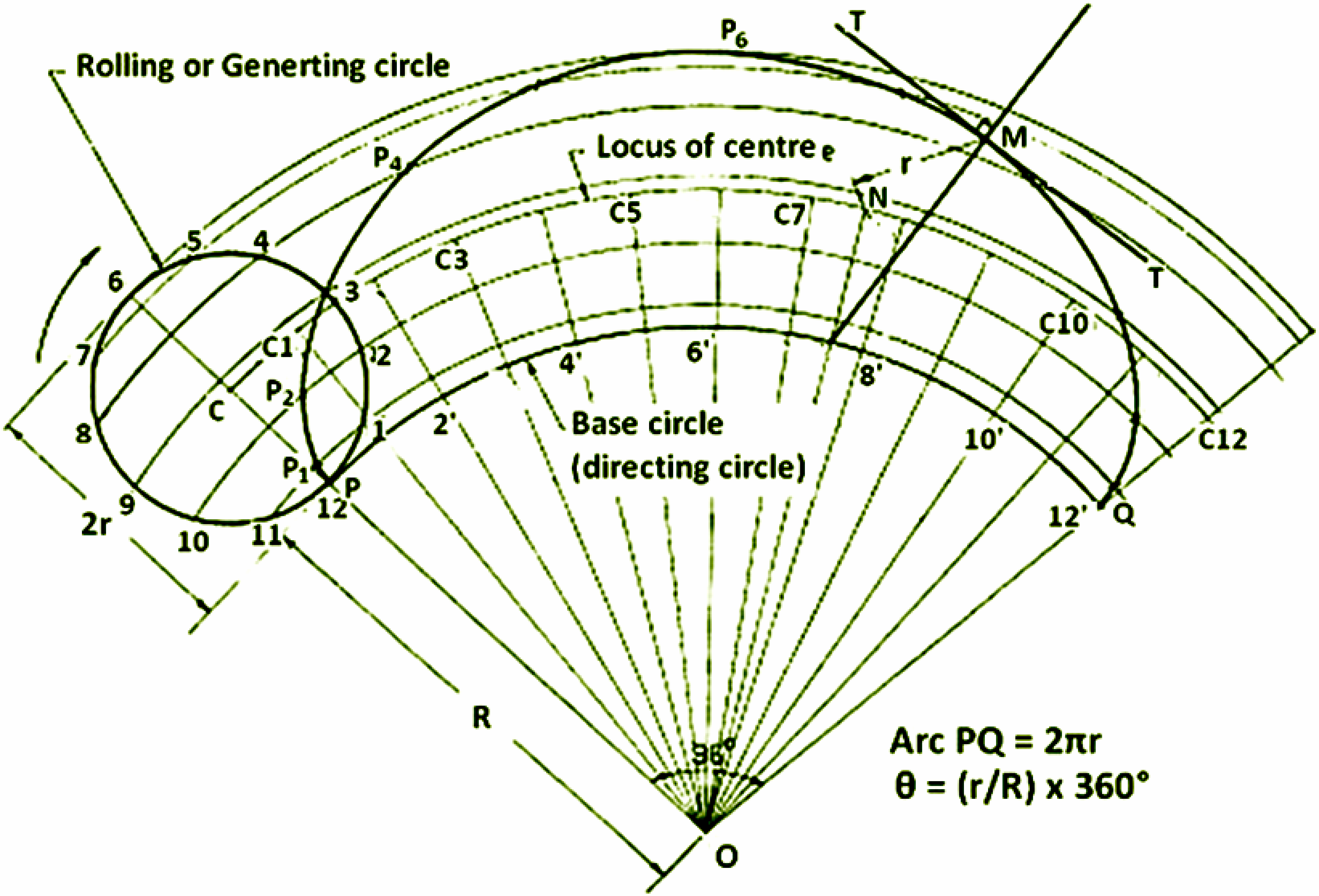
Epicycloids

Problem : 5

A circle of diameter 50mm rolls on the outside of another circle of diameter 180mm without slipping. Draw the path traced by a point on the smaller circle.

- With O as centre and radius OP (base circle radius), draw an arc PQ.
- The included angle $\theta = (r/R) \times 360°$.
- With O as centre and OC as radius, draw an arc to represent locus of centre.
- Divide arc PQ in to 12 equal parts and name them as 1’, 2’, …., 12’.
- Join O1’, O2’, … and produce them to cut the locus of centres at C1, C2, ….C12. Taking C1 as centre, and radius equal to *r*, draw an arc cutting the arc through 1 at P1.
- Taking C2 as centre and with the same radius, draw an arc cutting the arc through 2 at P2Similarly obtain points P3, P3, …., P12.
- Draw a smooth curve passing through P1, P2….. , P12, which is the required epiclycloid.

Epicycloids



Problem 6

A circle of diameter 50mm rolls along the inside of another circle of diameter 200mm without slipping. Draw the path traced by a point on the smaller circle.

1.With O as centre and radius OB (base circle radius), draw an arc BA.

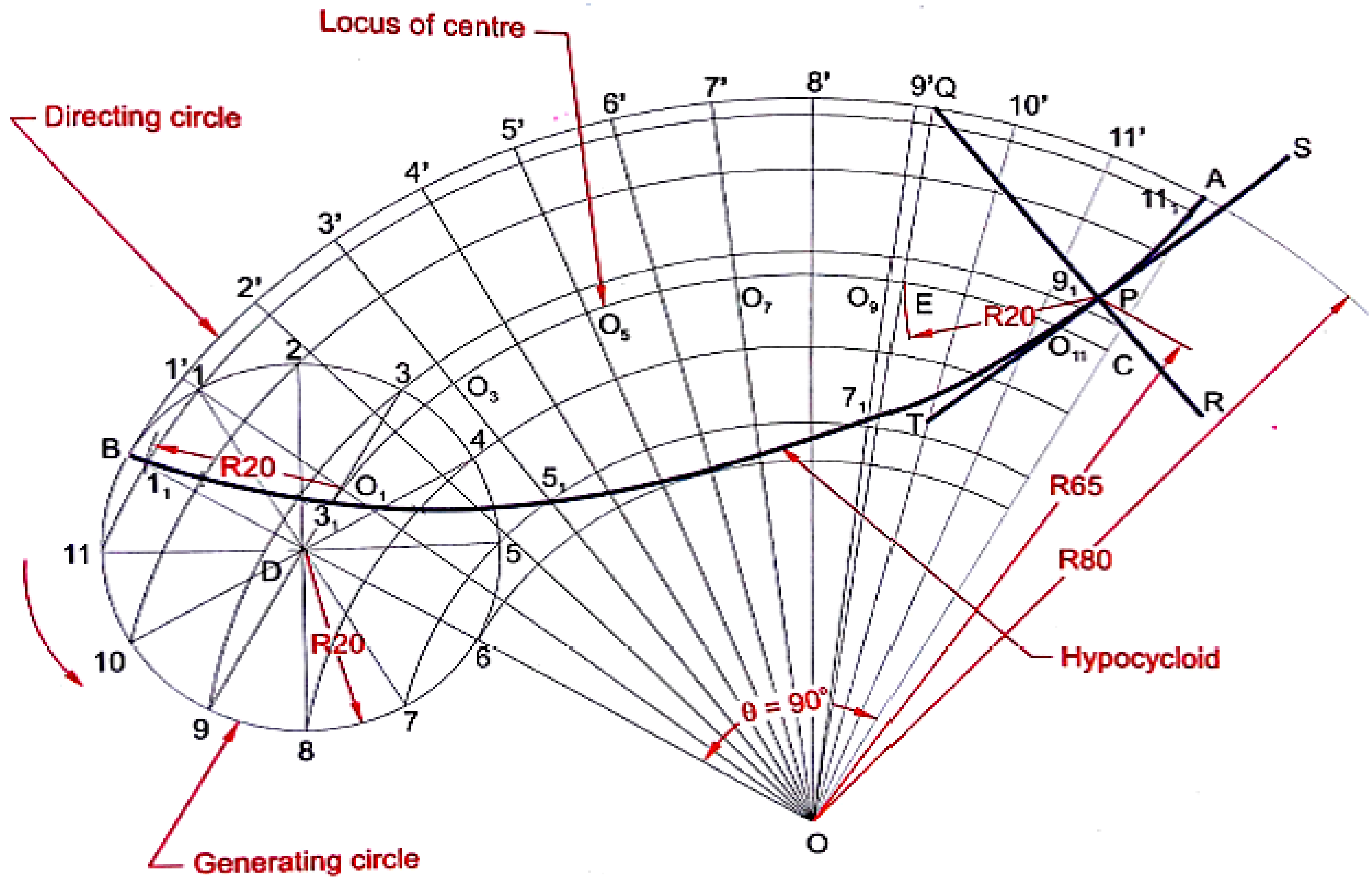
2.The included angle θ = (r/R) x 360°. With O as centre and OB as radius, draw an arc to represent locus of centre.

3.Divide arc AB in to 12 equal parts and name them as 1’, 2’, …., 8’. Join O1’, O2’, …, O12’ so as to cut the locus of centres at C1, C2,….C8.

4.Taking C1 as centre, and radius equal to *r*, draw an arc cutting the arc through 1 at P1. Taking C2 as centre and with the same radius, draw an arc cutting the arc through 2 at P2.

5.Similarly obtain points P3, P3, …., P8.

6.Draw a smooth curve passing through P1, P2….. , P8, which is the required hypocycloid.



Locus of centre
Directing circle
Generating circle
Hypocycloid
R20
R20
R20
R65
R80
θ = 90°
O
B
A
S
Q
P
R
T
E
C
D
O_1
O_3
O_5
O_7
O_9
O_{11}
1' 2' 3' 4' 5' 6' 7' 8' 9' 10' 11'
1 2 3 4 5 6 7 8 9 10 11
1_1 3_1 5_1 7_1 9_1 11_1

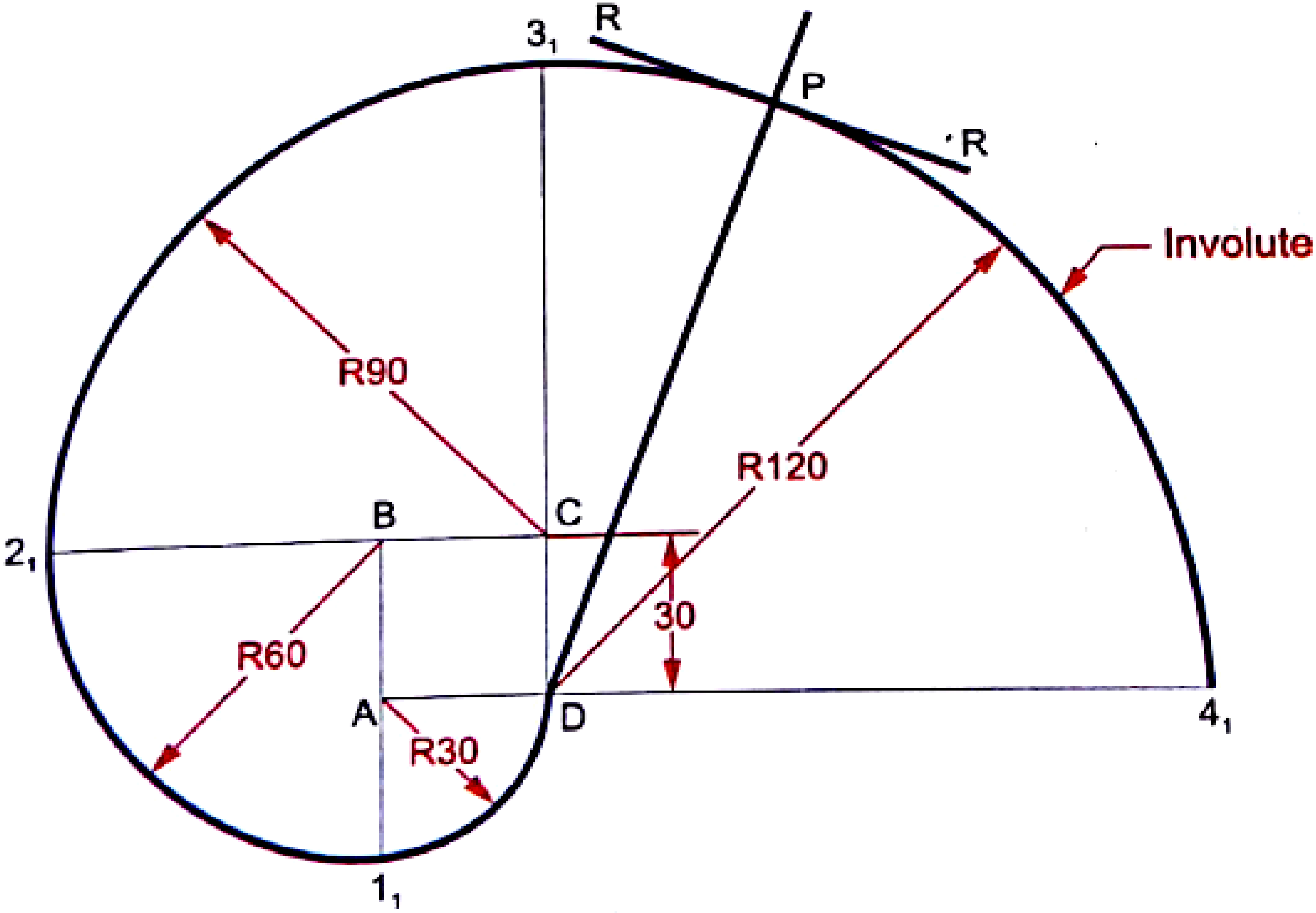
- An involutes is a curve traced by a point on a perfectly flexible string, while unwinding from around a circle or polygon the string being kept taut (tight).

Problem 7

Draw an involute of a given square of size 30mm

1. Draw the given square ABCD of side a.
2. Taking D as the starting point, with centre A and radius DA=a, draw an arc to intersect the line BA produced at P1.
3. With Centre B and radius BP1 = 2 a, draw on arc to intersect the line CB produced at P2.
4. Similarly, locate the points P3 and P4.
5. The curve through D, PI' P2, P3 and P4 is the required involutes.
6. DP 4 is equal to the perimeter of the square.

involutes of a square



Problem 8

Construction of Involutes of circle diameter 40mm

1. Draw the circle with O as centre and OA as radius.
2. Draw line P-P12 = 2π D, tangent to the circle at P
3. Divide the circle into 12 equal parts. Number them as 1, 2…
4. Divide the line PQ into 12 equal parts and number as 1´, 2´…..
5. Draw tangents to the circle at 1, 2,3….
6. Locate points P1, P2 such that 1-P1 = P1´, 2-P2 = P2´
7. Join P, P1,P2...
8. The tangent to the circle at any point on it is always normal to the its involutes.

Involutes of circle