MA3151-Matrices and Calculus

UNIT - II DIFFERENTIAL CALCULUS

Simha's Classes

S.Narasimhan

Differential Calculus

Representation of functions - Limit of a function - Continuity - Derivatives - Differentiation rules (sum, product, quotient, chain rules) - Implicit differentiation - Logarithmic differentiation - Applications : Maxima and Minima of functions of one variable.

Domain and range of functions

1. Find the domain of the function $f(x) = \dfrac{2x^3 - 5}{x^2 + x - 6}$.
Solution:
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2. Find the domain of the function $f(x) = \dfrac{2x^3 - 5}{x^2 + x + 6}$.
Solution:
For Video Explanation of this problem, Click Here

3. Find the domain of the function $f(x) = \dfrac{1}{x^2 - x}$
Solution:
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Limit of a function

1. Find $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + x + 1} - x\right)$
Solution:
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2. Evaluate $\lim\limits_{x \to 5} (2x^2 - 3x + 4)$.
Solution:
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3. Evaluate the limit $\lim\limits_{x \to 1} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$.
Solution:
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4. Prove that $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ does not exist.
Solution:
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Continuity

1. If $f(x) = \begin{cases} a^2x - 2a, & x \geq 2 \\ 12, & x < 2 \end{cases}$ is continuous for every real x, then find the value of a.

Solution:

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2.For what values of a and b, is $f(x) = \begin{cases} -2, & x \leq -1 \\ ax - b, & -1 < x < 1 \\ 3, & x \geq 1 \end{cases}$, continuous at every x?

Solution:

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3. Show that the function $f(x)$ is continuous on $(-\infty, \infty)$

$f(x) = \begin{cases} 1 - x^2, & x \leq 1 \\ \log x, & x \geq 1 \end{cases}$

Solution:

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4. For what values of the constant c is the function f continuous on $(-\infty, \infty)$?.

$f(x) = \begin{cases} cx^2 + 2x; & x < 2 \\ x^3 - cx; & x \geq 2 \end{cases}$

Solution:

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Differentiation rules

If $f(x)$ and $g(x)$ are functions of x and differentiable, then

1. $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
2. $\frac{d}{dx}(f(x)\,g(x)) = f(x)\frac{d}{dx}g(x) + f(x)\frac{d}{dx}g(x)$
3. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$

Using the first Principle of derivative, find the first derivative of the following function :

(1) x^2
Solution:
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(2) $\sqrt{x}$
Solution:
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(3) $\frac{1}{x}$
Solution:
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(4) $\cos x$
Solution:
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(5) $\sin x$
Solution:
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(6) $\log x$
Solution:
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(7) e^x
Solution:
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Differentiation product rule

1. Evaluate $\frac{d}{dx}\left(3x^5 \log x\right)$.
Solution:
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2.If $f(x) = xe^x$ then find $f'(x)$. Also find the n-th derivative $f^n(x)$.
Solution:
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3. Differentiate $y = (2x+1)^5 (x^3 - x + 1)^4$.
Solution:
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4. Find $\frac{dy}{dx}$, if $y = x^2 e^{2x} (x^2+1)^4$.
Solution:

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5.If $y = x \log\left(\frac{x-1}{x+1}\right)$, then find $\frac{dy}{dx}$.

Solution:

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Differentiation quotient rule

1. Find $\frac{d}{dx}\left(\frac{x^3}{3x-2}\right)$

Solution:

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2. Find $\frac{dy}{dx}$, if $y = \frac{(x^2+8)}{(2x+3)}$

Solution:

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Differentiation Parametric function

1. Find $\frac{dy}{dx}$, if $x = \cos t + t \sin t$ and $y = \sin t - t \cos t$

Solution:

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Logarithmic differentiation

1. Find the differential coefficients of $\frac{(a-x)^2 (b-x)^3}{(c-2x)^3}$.

Solution:

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2. If $y^x = x^{\sin y}$, Find y'.

Solution:

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3. If $y = (\sin x)^{\cos x}$, Find $\frac{dy}{dx}$.

Solution:

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4. If $x^y = e^{x-y}$, show that $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$.

Solution:

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Implicit differentiation

1.Find $\dfrac{dy}{dx}$, if $x^3 + y^3 = 3axy$.

Solution:

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2. Find y'' if $x^4 + y^4 = 16$.

Solution:

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3. Find y' for $\cos(xy) = 1 + \sin y$

Solution:

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Derivative of inverse Functions

1.Find the derivative of $f(x) = \cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$.

Solution:

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Applications of derivatives

Equation of tangent line

1.Find the slope of the circle $x^2 + y^2 = 25$ at $(3, -4)$.

Solution:

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1. If $x^2 + y^2 = 25$, then find $\dfrac{dy}{dx}$ and also find an equation of the tangent line to the curve $x^2 + y^2 = 25$ at the point (3,4) .

Solution:

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2. Differentiate the function $f(x) = \dfrac{\sec x}{1+\tan x}$. For what values of x, the graph of $f(x)$ has a horizontal tangent?
Solution:
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3. Find the equation of the tangent line to the curve $y = \dfrac{e^x}{(1+x^2)}$ at the point $\left(1, \frac{e}{2}\right)$
Solution:
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Applications of derivatives : Maxima and minima

1.Find the point of inflection of $f(x) = x^3 - 9x^2 + 7x - 6$.
Solution:
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1.For the function $f(x) = 2x^3 + 3x^2 - 36x$, find
(i) the intervals on which it is increasing or decreasing.
(ii) the local maximum and minimum value of f.
(iii) the intervals of concavity and the inflection points.
Solution:
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2.Find the maxima and minima of the function $x^5 - 5x^4 + 5x^3 + 10$.
Solution:
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3. Find the maximum and minimum values of the function $2x^3 - 3x^2 - 36x + 10$.
Solution:
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4. Find the absolute maximum and absolute minimum values of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ on the interval $[-2, 3]$.
Solution:
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5. Find the absolute maximum and minimum values of the function $f(x) = \log(x^2 + x + 1)$ in $[-1, 1]$.
Solution:
For Video Explanation of this problem, Click Here

6.Find the local maxima and minima for the function of the curve $y = x^4 - 4x^3$.

Solution:

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7. Find the intervals on which $f(x) = -x^3 + 12x + 5; -3 \leq x \leq 3$ is increasing and decreasing. Where does the function assume extreme values? What are the values?

Solution:

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