Unit 2 : Electromagnetic waves

Part A : Questions and Answers

1. Write Maxwell's equation 1 from Gauss's law in electrostatics

   Gauss's law in electrostatics states that the total electric flux through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the charge enclosed by it.

   Integral form of Gauss law

$$\oiint \vec{E}\,.\overrightarrow{ds} = \frac{1}{\varepsilon_0}\iiint \rho dV$$

   Differential form

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

2. Write Maxwell's equation 2 from Gauss's law in magnetostatics

   The total magnetic flux through any closed surface in a magnetic field is zero.

   Integral form

$$\oiint \vec{B}\,.\overrightarrow{ds} = 0$$

   Differential form

$$\vec{\nabla}.\vec{B} = 0$$

3. Write Maxwell's equation 3 from Faraday's law of electromagnetic induction

   Integral form

$$\oint \vec{E}.\overrightarrow{dl} = -\iint \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$$

   Differential form

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

4. Write Maxwell's equation 4 from Ampere's circuital law.

   Integral form of Ampere's circuital law

$$\oint \vec{H}.\overrightarrow{dl} = I$$

   Integral form of modified Maxwell-Ampere circuital law

$$\oint \vec{H}.\overrightarrow{dl} = \iint \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) ds$$

   Differential form of modified Maxwell-Ampere circuital law

$$\vec{\nabla} \times \vec{H} = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

5. Give the Maxwell's equations in differential form

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

6. Give the Maxwell's equations in integral form

$$\oiint \vec{E}\,.\overrightarrow{ds} = \frac{1}{\varepsilon_0}\iiint \rho dV$$

$$\oiint \vec{B}\,.\overrightarrow{ds} = 0$$

$$\oint \vec{E}.\overrightarrow{dl} = -\iint \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$$

$$\oint \vec{H}.\overrightarrow{dl} = \iint \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) ds$$

7. Write the Maxwell's equations for free space

$$\vec{\nabla}.\vec{E} = 0$$

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \left(\frac{\partial \vec{D}}{\partial t}\right)$$

8. Write the Maxwell's equations for conducting medium

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

9. What are the characteristics of Maxwell's first equation $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

- It is time independent or steady state equation
- Charge acts as a source or sink for the lines of electric force
- The flux of lines of electric force depends upon charge density

10. What are the characteristics of Maxwell's second equation $\vec{\nabla}.\vec{B} = 0$

- It states that the total magnetic flux entering and leaving a given volume is equal
- There is no source or sink for lines of magnetic force
- It states that magnetic monopoles do not exist.
- It is a time independent equation

11. What are the characteristics of Maxwell's third equation $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

- It relates the electric field vector E and magnetic flux density B.
- It states that the time varying magnetic flux density produces electric field.
- It is also known as Faraday-Lenz law of electromagnetic induction

12. What are the characteristics of Maxwell's fourth equation $\vec{\nabla} \times \vec{H} = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$

- It is also known as Maxwell-Ampere circuital law
- It relates the electric displacement vector D, current density J and magnetic field H.
- It states that a constant electric current and a time varying electric field produces magnetic field

13. Write down general electromagnetic wave equation in terms of electric field vector E for free space

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

μ_0 – permeability in free space
ε_0- permittivity in free space

14. Write down general electromagnetic wave equation in terms of magnetic field vector H for free space

$$\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

μ_0 – permeability in free space
ε_0- permittivity in free space

15. Write down the expression for velocity of EM wave in free space.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

For vacuum or free space we have $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$ and $\varepsilon_0 = 8.842 \times 10^{-12} Fm^{-1}$

16. Write down the general solution of wave equation for plane polarized EM wave

$$E_y = E_o \cos(\omega t - kx)$$
$$H_z = H_o \cos(\omega t - kx)$$

Where ω is the angular frequency and k is the wave vector

17. Write down a relation between the electric field vector E and magnetic field vector H.

$$\frac{\vec{E}}{\vec{H}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

18. What is intrinsic or characteristic impedance of free space?

- The ratio $\frac{\vec{E}}{\vec{H}}$ is having the unit of impedance (resistance) Ohm. Therefore, the quantity $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ has the dimensions of impedance.
- This ratio is known as intrinsic or characteristic impedance of free space, denoted by Z_0.
- It is constant quantity for free space and having value = 377Ω.

19. What is poynting vector?

The cross product of electric field vector E and the magnetic field vector H is called Poynting vector. It is denoted by

$$\vec{S} = \vec{E} \times \vec{H}$$

20. Write the general wave equation for the electric vector in an em wave in conducting medium.

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}}{\partial t} = 0$$

$\vec{E}$ – electric field vector
μ – permeability of medium
ε- permittivity of medium

21. Write the general wave equation for the magnetic vector in an em wave in conducting medium.

$$\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} - \mu\sigma \frac{\partial \vec{H}}{\partial t} = 0$$

$\vec{H}$ – magnetic field vector
μ – permeability of medium
ε- permittivity of medium

22. What is skin depth?

It is defined as the distance inside the conductor from the surface of the conductor at which the amplitude of the field vector is reduced to 1/e times its value at the surface.

23. Define intensity of EM wave.

The magnitude of the average value of $\vec{S}$ at a point is called the intensity of radiation at that point. The SI unit of intensity is W/m^2. It is given by

$$S_{av} = \frac{1}{2}\varepsilon_0 E_y^2$$

ε_0- permittivity in free space
c – velocity of light

24. Define radiation pressure.

The force per unit area on an object due to EM radiation is the radiation pressure P_r.

For total absorption of radiation

$$P_r = \frac{I}{c}$$

For total reflection back along the path

$$P_r = \frac{2I}{c}$$

25. Give the properties of electromagnetic waves

- Electromagnetic waves are produced by accelerated charges
- They do not require any material medium for propagation
- In an electromagnetic wave, the electric field vector E and the magnetic flux density vector B are at right angles to each other and to the direction of propagation. Hence, electromagnetic waves are transverse in nature.
- Variation of maxima and minima in both E and B occur simultaneously
- They travel in vacuum or free space with a speed 3×10^8 m/s given by the relation

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

- The energy in an electromagnetic wave is equally divided between electric and magnetic field vectors.
- The electromagnetic waves being chargeless, they are not deflected by electric and magnetic field.

UNIT- II
ELECTROMAGNETIC WAVES
PART-B

1. Derive Maxwell's third and fourth equation for electromagnetic waves by stating Faraday's law and ampere's circuital law.
2. Derive and explain Maxwell's equation for electromagnetic waves in integral and differential form.
3. Prove that the energy content due to electric and magnetic field in electromagnetic waves are equal.
4. Derive an expression for the momentum of an electromagnetic wave.
5. Derive the expression of plane electromagnetic wave equation in matter and explains electromagnetic wave properties.
6. Explain the properties of Electromagnetic waves.