2.14 PROPERTIES (SPEED, ORIENTATION, AMPLITUDE AND PHASE) OF ELECTROMAGNETIC WAVES IN MATTER - NON-CONDUCTING (DIELECTRIC) MEDIUM

(i) Speed of electromagnetic waves in dielectric medium

We know the most general (vector) wave equation in terms of electric and magnetic fields in a dielectric medium are

(i) **For electric field :** $\nabla^2\vec{E} - \varepsilon\mu\frac{\partial^2\vec{E}}{\partial t^2} = 0$ .... (1)

(ii) **For magnetic field :** $\nabla^2\vec{B} - \varepsilon\mu\frac{\partial^2\vec{B}}{\partial t^2} = 0$ .... (2)

Therefore, we can write the compact single wave equation for both electric and magnetic fields as,

$$\nabla^2 f - \varepsilon\mu\frac{\partial^2 f}{\partial t^2} = 0 \qquad \text{........ (3)}$$

Where f is a scalar wave function, which represents the electric and magnetic field components $(i.e., E_x, E_y, E_z \text{ for } \vec{E} \ \& \ B_x, B_y, B_z \text{ for } \vec{B})$.

We know that the standard form of wave equation which travel at a velocity 'v',can be written as

$$\nabla^2\psi - \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = 0 \qquad \text{........ (4)}$$

Where ψ is a scalar wave function.

Comparing equations (3) and (4), we get

$$\frac{1}{v^2} = \varepsilon\mu$$

(or) $$\boxed{v = \frac{1}{\sqrt{\varepsilon\mu}}} \qquad \text{........ (5)}$$

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Since, $\varepsilon = \varepsilon_0\varepsilon_r$ and $\mu = \mu_0\mu_r$, we can write eqn (5) as

$$v = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}} = \frac{1}{\sqrt{\varepsilon_0\mu_0}\sqrt{\varepsilon_r\mu_r}}$$

Since, $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ we can write the above equation as

$$\boxed{v = \frac{c}{\sqrt{\varepsilon_r\mu_r}}} \quad \text{........ (6)}$$

Thus, from equation (6), we can see that the speed of electromagnetic wave in dielectric medium is less than the speed of light (c).

Comparing eqn (5) and (6) we can write

$$\frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}}$$

Squaring on both side, we get

$$\frac{1}{\varepsilon\mu} = \frac{c^2}{\varepsilon_r\mu_r}$$

(or) $$\boxed{\varepsilon\mu = \frac{\varepsilon_r\mu_r}{c^2}} \quad \text{........ (7)}$$

substituting eqn (7) in eqn (1) and (2) we get

$$\boxed{\nabla^2\vec{E} - \frac{\varepsilon_r\mu_r}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0} \quad \text{........ (8)}$$

and $$\boxed{\nabla^2\vec{B} - \frac{\varepsilon_r\mu_r}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} = 0} \quad \text{........ (9)}$$

Equations (8) and (9) represents the wave equations of electric and magnetic field, respectively in terms of ***speed in dielectrics.***

(ii) Orientation of electromagnetic wave in dielectric medium

The plane wave solution for electric field wave equation (8) and magnetic field wave equation (9) can be written as

$$\vec{E}(r,t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \quad \text{........ (10)}$$

$$\vec{B}(r,t) = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \quad \text{........ (11)}$$

Where, $\vec{E}_0$ and $\vec{B}_0$ are the complex amplitude of electric and magnetic field and $\vec{k}$ is the wave vector of electromagnetic wave given by $\vec{k} = k\hat{n}$.

We know the Maxwells equations for dielectric medium are

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{......... (12)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{......... (13)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{......... (14)}$$

$$\vec{\nabla}\times\vec{B} = \varepsilon\mu\frac{\partial\vec{E}}{dt} \quad \text{......... (15)}$$

The field vector of wave equations for dielectric medium suggest that, the ***del operator*** $\vec{\nabla}$ ***is equivalent to*** $i\vec{k}$ ***and*** $\frac{\partial}{\partial t}$ ***is equivalent to*** - $i\omega$.

i.e., $\vec{\nabla} = i\vec{k}$ *and* $\frac{\partial}{\partial t} = -i\omega$

Therefore, equation (12) becomes,

$$\vec{\nabla}\cdot\vec{E}(r,t) = 0$$

(or) $\quad i\vec{k}\cdot\vec{E} = 0$

(or) $\quad \boxed{\vec{k}\cdot\vec{E} = 0} \quad$ .... (16)

Similary, Equation (13) becomes,

$$\vec{\nabla}\cdot\vec{B}(r,t) = 0$$

(or) $\quad i\vec{k}\cdot\vec{B} = 0$

(or) $\quad \boxed{\vec{k}\cdot\vec{B} = 0} \quad$ .... (17)

Similarly, Equation (14) becomes,

$$\vec{\nabla} \times \vec{E}(r,t) = -\frac{\partial \vec{B}}{\partial t}$$

(or) $i\vec{k} \times \vec{E} = -(-i\omega \vec{B})$

(or) $i\vec{k} \times \vec{E} = i\omega \vec{B}$

(or) $\boxed{\vec{k} \times \vec{E} = \omega \vec{B}}$ .... (18)

Similarly, Equation (15) becomes,

$$\boxed{\vec{\nabla} \times \vec{B}(r,t)} = \varepsilon\mu \frac{\partial \vec{E}}{\partial t}$$

(or) $i\vec{k} \times \vec{B} = \varepsilon\mu\left[-i\omega \vec{E}\right]$

(or) $i\vec{k} \times \vec{B} = -i(\omega\varepsilon\mu)\vec{E}$

(or) $\boxed{\vec{k} \times \vec{B} = -(\omega\mu\varepsilon)\vec{E}}$ .... (19)

Conclusions

From equations (16), (17), (18) and 19), we can conclude the following points, *viz.*,

(i) From equations (16) and (17) we can say that the wave vector $(\vec{k})$ is perpendicular to Electric $(\vec{E})$ and magnetic $(\vec{B})$ field vectors, respectively

(ii) From equation (18) we can say that the magnetic field vector $(\vec{B})$ is perpendicular to wave vector and electric field vector.

(iii) From equation (19) we can say that the electric field vector $(\vec{E})$ is perpendicular to magnetic field vector and wave vector.

From the above points, we can conclude that the electric and magnetic field vectors are transverse in nature and $\vec{E}$, $\vec{B}$ & $\vec{k}$ are mutually perpendicular to each other as shown in Fig. 2.8.

Therefore, equation (18) shall be written as,

$$\boxed{\vec{k}_Z \times \vec{E}_X = \omega \vec{B}_Y} \quad \text{........ (20)}$$

Similarly Equation (19) shall be written as,

$$\vec{k}_z \times \vec{B}_y = -(\omega\mu\varepsilon)\vec{E}_x$$

(or) $$\boxed{\vec{B}_Y \times \vec{k}_Z = \omega\mu\varepsilon \vec{E}_X} \quad \text{........ (21)}$$

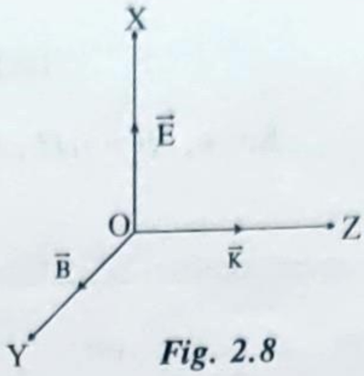

Fig. 2.8

Therefore, $\{\vec{E}, \vec{B}, \vec{k}\} \rightarrow \{X, Y, Z\}$ are right handed triad of vector.

(iii) Amplitude and phase of electromagnetic wave in dielectric

From equation (18) we know that the vector product of wave vector & electric field vector is

$$\vec{k} \times \vec{E} = \omega \vec{B}$$

Since, $\vec{k} = k\hat{n}$, we can write the above equation as

$$k\hat{n} \times \vec{E} = \omega \vec{B}$$

(or) $$\hat{n} \times \vec{E} = \frac{\omega}{k}\vec{B}$$

For dielectrics, velocity $\text{v} = \frac{\omega}{k}$ $\therefore$ We can write the above equation as

$$\hat{n} \times \vec{E} = \text{v}\vec{B}$$

(or) $$\boxed{\vec{B} = \frac{1}{\text{v}}\hat{n} \times \vec{E}} \quad \text{........ (22)}$$

The magnitude of magnetic field vector can be written as,

$$|\vec{B}| = \frac{1}{v}|\vec{E}|$$

$$\text{(or)} \quad \boxed{B = \frac{1}{v} E} \qquad \cdots\cdots\cdots (23)$$

Since, $B = \mu H$, equation (23) becomes

$$\mu H = \frac{1}{v} E$$

$$\text{(or)} \quad \boxed{\frac{E}{H} = v\mu} \qquad \cdots\cdots\cdots (24)$$

We know the velocity in dielectric medium $v = \frac{1}{\sqrt{\varepsilon\mu}}$ .... (25)

Substituting eqn (25) in eqn (24), we get

$$\frac{E}{H} = \frac{\mu}{\sqrt{\varepsilon\mu}}$$

$$\text{(or)} \quad \frac{E}{H} = \frac{\sqrt{\mu}\sqrt{\mu}}{\sqrt{\varepsilon}\sqrt{\mu}}$$

$$\text{(or)} \quad \boxed{\frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}}} \qquad \cdots\cdots\cdots (26)$$

Intrinsic Impedance

We know the relative permittivity $\varepsilon = \varepsilon_0\varepsilon_r$ and the relative permeability $\mu = \mu_0\mu_r$

$\therefore$ We can write eqn (26) as

$$\frac{E}{H} = \sqrt{\frac{\mu_r\mu_0}{\varepsilon_r\varepsilon_0}}$$

Since, the intrinsic impedence of vacuum $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$, we can write

$$\frac{E}{H} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

(or) $$\boxed{\eta = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}} \quad \text{........ (27)}$$

Where η is intrinsic impedence (real quantity) of dielectric medium given by

$$\boxed{\eta = \frac{E}{H}} \quad \text{........ (28)}$$

Equation (27) represents that the ***electric and magnetic field vectors are in same phase*** as shown in ***Fig. 2.9.***

Amplitude

From the equation (28) we can write

(or) $$\boxed{E = \eta H} \quad \text{........ (29)}$$

Thus, The amplitude of electric field vector is η times that of the magnetic field vector.

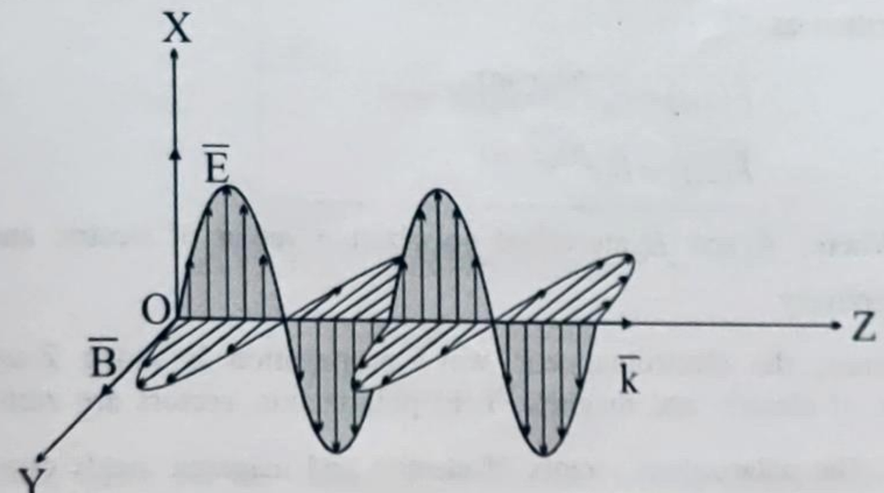

Fig. 2.9 Electromagnetic wave in dielectric

From ***Fig. 2.9,*** we can observe that the electric and magnetic field vectors have same relative magnitude at every plane which is perpendicular to the direction of wave propagation at any instant of time.