Department of Science and Humanities
Second Year / Fourth Semester
23MA301- Linear Algebra
Question Bank
Unit – I- Matrices and System of Linear Equations

Q.No	Questions	CO's	Bloom's Level
1.	Find the rank of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 5 & 9 \end{bmatrix}$.	CO1	K3
2.	Prove that the system is inconsistent $x - 2y + 3z = 4;\ 3x + \ y = 3;$ $5x + 4y - 3z \ = 3.$	CO1	K3
3.	Solve $5x - 3y = 8; 3x + y = 2$ by Gauss Jordan method.	CO1	K3
4.	Define a direct and indirect methods of solving systems of simultaneous linear equations.	CO1	K1
5.	For solving a linear system of equations, compare Gauss Elimination method and Gauss Jordan method.	CO1	K2
6.	What is meant by Diagonally Dominant?	CO1	K1
7.	Write down the condition for the convergence and the iterative formula of Gauss Seidel technique.	CO1	K1
8.	Using Gauss elimination method solve $x + y = 2,\ 2x + 3y = 5$.	CO1	K3
9.	Compare between Gauss elimination and Gauss seidel methods.	CO1	K2
10.	Define homogeneous and non-homogeneous equations.	CO1	K1
11.	Define rank of a matrix.	CO1	K1
12.	Write the condition for system of equations to be consistent.	CO1	K1
13.	Write the condition for system of equations to be inconsistent.	CO1	K1
14.	Test the consistency of $x - y = 1; 2x + y = 6$.	CO1	K3
15.	Define linear and non-linear equations.	CO1	K1
16.	Give two direct method to solve a system of linear equations.	CO1	K1
17.	Using Gauss Jordan method solve $x - 4y = -2,\ 3x + y = 7$.	CO1	K3
18.	Explain briefly Gauss Seidel method of solving simultaneous linear equations.	CO1	K1
19.	Explain briefly Gauss Jordan method of solving simultaneous linear equations.	CO1	K1
20.	Explain briefly Gauss Elimination method of solving simultaneous linear equations.	CO1	K1
	Part – B		

1.	(i)Solve by Gauss–elimination method. $x_1 + 3x_2 - 2x_3 + 2x_5 = 0$; $2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1$; $5x_3 + 10x_4 + 15x_6 = 5$; $2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6$. (8 marks) (ii) Find the rank of A = $\begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$. (8 marks)	CO1	K3
2.	(i)Solve the following system of equations by Gauss Elimination method $3x + 4y + 5z = 18;\ 2x - y + 8z = 13;\ 5x - 2y + 7z = 20.$ (8 marks) (ii) Find K if the rank of A = $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & -1 \\ k & -1 & 0 \end{bmatrix}$ is 2? (8 marks)	CO1	K3
3.	(i)Solve the following system of equations by Gauss Elimination method $2x + y + 4z = 12;\ 8x - 3y + 2z = 20;\ 4x + 11y - z = 33.$ (8 marks) (ii) Find the rank of A = $\begin{pmatrix} 3 & 1 & 1 & 8 \\ -1 & 1 & -2 & -5 \\ 1 & 1 & 1 & 6 \\ -2 & 2 & -3 & -7 \end{pmatrix}$. (8 marks)		
4.	(i) Solve the system of equations by Gauss Seidal method $x - y + 4z = 4; x + 5y + 3z = 6; 5x - y - z = 1.$ (8 marks) (ii) Find K if the rank of A = $\begin{pmatrix} 1 & -2 & 3 & 4 \\ 3 & 1 & 0 & 3 \\ 5 & 4 & -3 & K \end{pmatrix}$ is 2? (8 marks)	CO1	K3
5.	(i) Solve the system of equations by Gauss Seidal method $8x - 3y + 2z = 20; 4x + 11y - z = 33; 6x + 3y + 12z = 35.$ (8 marks) (ii) Check whether the system is consistent and hence solve x+y+z =1; x-2y+z = 1; x+y-2z = 1. (8 marks)	CO1	K3
6.	(i) Solve the system of equations by Gauss Seidal method $27x + 6y - z = 85; 6x + 15y + 2z = 72; x + y + 54z = 110.$ (8 marks) (ii) Show that the equations x+y+z = 6; x-y+2z = 5; 3x+y+z = 8; 2x-2y+3z = 7 are consistent and solve them. (8 marks)	CO1	K3
7.	(i) Solve the following system of equations by Gauss Jordan method $x - y + z = 1; -3x + 2y - 3z = -6; 2x - 5y + 4z = 5.$ (8 marks) (ii) Examine if the following system of equations is consisitent and find the solution if it exists. x+y+z = 1; 2x-2y+3z = 1; x-y+2z = 5; and 3x+y+z = 2. (8 marks)	CO1	K3

8.	(i)Solve the following system of equations by Gauss Jordan method$10x+y+z=12; 2x+10y+z=13; x+y+5z=7$. (8 marks) (ii) Test the consistency and solve of x+2y-z-5w = 4; x+3y-2z-7w = 5 ; 2x-y+3z = 3. (8 marks)	CO1	K3
9.	(i) Solve the following system of equations by Gauss Jordan method 2x+2y-z+w = 4 ; 4x+3y-z+2w = 6; 8x+5y-3z+4w = 12 ; 3x+3y-2z+2w = 6. (8 marks) (ii) Test for consistency and solve x+2y+z+2w = 0; x+3y+2z+2w = 0; 2x+4y+3z+6w = 0; 3x+7y+4z+6w = 0. (8 marks)	CO1	K3
10.	(i) Investigate for what values of μ, λ the equations x+y+z = 6; x+2y+3z = 10; x+2y+ λz = μ have (a) no solution (b) unique solution (c) infinite no: of solutions. (10 marks) (ii) Solve the following system of equations by Gauss Elimination method $x-y+z=1; -3x+2y-3z=-6; 2x-5y+4z=5$. (6 marks)	CO1	K3

UNIT II- Vector Spaces

Q.No	Questions	CO's	Bloom's Level
1.	Define vector space.	CO2	K1
2.	Define subspace of a vector space?	CO2	K1
3.	State the necessary and sufficient condition for a subset W to be subspace of a vector space V over F.	CO2	K1
4.	Show that the vectors (1,2,3),(3,-2,1),(1,-6,-5) in R^3 are linearly dependent over R.	CO2	K3
5.	Define Linear span.	CO2	K3
6.	In R^3 over R test whether $(2, -5,4)$ is the linear combination of vectors$(1, -3,2)$ *and* $(2, -1,1)$.	CO2	K2
7.	What are the possible subspaces of R^2 ?	CO2	K1
8.	Define linear combination.	CO2	K1
9.	Define linearly dependent and linearly independent vectors.	CO2	K1
10.	For which values of k will the vector v = (1,-2, k) in R^3 be a linear combination of the vectors u = (3, 0,-2) and w = (2,-1,-5)?	CO2	K3
11.	Determine whether (-2,0,3) is a linear combination of (1,3,0) and (2,4,-1) in R^3 (R)?	CO2	K3
12.	Let $V = \{(a_1, a_2): a_1, a_2 \in R\}$ be a vector space over R. Test whether W = {(a ,0):a ∈ R} is a subspace over R.	CO2	K3
13.	Check whether the vectors (1,2,3),(2,3,1) in R^3(R) are linearly independent or not.	CO2	K2
14.	Determine $\{1+2x+x^2, 3+x^2, x+x^2\}$ is linearly independent or	CO2	K3

	not.		
15.	What is the dimension of a vector space (F), complex number over a field of real numbers?	CO2	K1
16.	Check whether $W = \{(a_1, a_2, a_3) \in R^3 : a_1 + 2a_2 - 3a_3 = 1\}$ is a subspace or not?	CO2	K2
17.	Let $V = \{(a_1, a_2): a_1, a_2 \in R\}$ be a vector space over R. Test whether the subset $W = \{(a_1, a_2) : 2a_1 + 3a_2 = 0; a_1, a_2 \in R\}$ is subspace or not.	CO2	K2
18.	Determine whether the vector $(2, -1,1)$ is in the span of $\{(1,0,2), (-1,1,1)\}$	CO2	K3
19.	Find the linear span of $S = \{(1,0,0), (2,0,0), (3,0,0)\} \subseteq R$ over R.	CO2	K3
20.	Test whether $v_1 = (1, -2,3)$, $v_2 = (5,6, -1)$ and $v_3 = (3,2,1)$ form a linear dependence or linear independence?	CO2	K2
	Part – B		
1.	(i)Prove that $P_n(R)$, the set of all polynomials of degree at most n with real coefficient is a vector space under usual addition and constant multiplication of polynomial. (10 marks) (ii) Check whether $2x^3 - 2x^2 + 12x - 6$ is a linear combination of $x^3 - 2x^2 - 5x - 3$ and $3x^3 - 5x^2 - 4x - 9$. (6 marks)	CO2	K3
2.	(i)Let W_1 and W_2 be subspaces of a vector space V. Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. (8 marks) (ii)Verify whether the set $S=\left\{\begin{pmatrix} 1 & -3 & 2 \\ -4 & 0 & 5 \end{pmatrix}, \begin{pmatrix} -3 & 7 & 4 \\ 6 & -2 & -7 \end{pmatrix}, \begin{pmatrix} -2 & 3 & 11 \\ -1 & -3 & 2 \end{pmatrix}\right\}$ in $M_{2x3}(R)$ is linearly dependent or not. (8 marks)	CO2	K3
3.	(i)Show that $F_n = \{(a_1, a_2, \dots , a_n): a_i \in F\}$ is a vector space over F with respect to addition and scalar multiplication defined component wise. (10 marks) (ii) Check if (2,-5,3) can be expressed as a linear combination of (1,-3,2), $(2, -4, -1)$, (1,-5,7). (6 marks)	CO2	K3
4.	(i)Prove that the set of all $(m \times n)$ matrices over F denoted by $M_{m\times n}(F)$ is a vector space over F with respect to matrix addition and scalar multiplication. (10 marks) (ii) Determine whether the set $W = \{(a_1, a_2): 2a_1 + 3a_2 = 0: a_1, a_2 \in R^2\}$ is subspace or not. (6 marks)	CO2	K3
5.	(i) The vectors $v_1 = (1,0,0)$; $v_2 = (0,1, 0)$; $v_3 = (1,1, 1)$; generate R^3 over R. Find the subset which is a basis of R^3 . (10 marks) (ii) Determine whether the set $W = \{(a_1, a_2, a_3) \in R^3 : a_1 - 3a_2 + a_3 = 3\}$ is subspace of R^3 under the operations of addition and scalar multiplication. (6 marks)	CO2	K3
6.	(i) Determine whether the set $W = \{(a, b, c) \in R^3 : a^2 + b^2 + c^2 = 5\}$ is subspace or not. (8 marks) (ii) Determine $\{1 + 2x + x^2, 3 + x^2, x + x^2\}$ is a basis for $P_2(R)$. (8 marks)	CO2	K3

7.	(i) Show that the matrices $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$ generate M_{2x2} (F). (8 marks) (ii)Give an example to show that the union of two subspaces need not be subspace. (8 marks)	CO2	K3
8.	(i)Check whether the set $S = \{v_1, v_2, v_3\}$ where $v_1 = (2, 1, 0)$, $v_2 = (-3, -3, 1)$, $v_3 = (-2, 1, -1)$ is a basis in the vector space $R^3(R)$. (8 marks) (ii) Let V be a vector space over F. If S = $\{v_1, v_2 \ldots \ldots v_n\}$is a basis for a vector space V, then prove that every vector v in V can be uniquely expressed in the form $v = c_1v_1 + c_2v_2 + \cdots . +c_nv_n$ in exactly one way. (8 marks)	CO2	K3
9.	(i)Write the vector v = (1,-2,5) as a linear combination of the vectors e_1 = (1,1,1) , e_2 = (1,2,3) and e_3 = (2,-1,1) in R^3. (8 marks) (ii)Let $R^3(R)$ be a Vector Space. Determine $\boldsymbol{x}$ so that the vectors $(1, -1, x - 1)$, $(2, x, -4)$, $(0, x + 2, -8)$ in R^3 are linearly dependent. (8 marks)	CO2	K3
10.	(i) Determine whether the polynomials $x^2 + 3x - 2$, $2x^2 + 5x - 3$, $-x^2 - 4x + 4$ generates the vector space of polynomials $P_2(R)$. (8 marks) (ii) Let S = {(1,3, -4,2), (2,2, -4,0), (1, -3,2, -4), (-1,0,1,0)} in R^4 . Show that S is linearly dependent and express one of the vectors in S as a linear combination of the other vectors in S. (8 marks)	CO2	K3

UNIT III- Linear Transformation

Q.No	Questions	CO's	Bloom's Level
1.	Define of linear transformation.	CO3	K1
2.	If T:R→R defined by T (x) = x + 1, ∀x ε R. Is T linear?	CO3	K2
3.	Define range and null spaces of a linear transformation	CO3	K1
4.	State Dimension theorem	CO3	K1
5.	Define nullity of a linear transformation	CO3	K1
6.	If $T: R^2 \rightarrow R^2$ is defined by $(a_1, a_2) = (2a_1 + a_2, a_1)$. Verify whether T is a linear transformation.	CO3	K3
7.	Define Kernel of T .	CO3	K1
8.	Is there a linear transformation T: $R^3 \rightarrow R^2$ such that T(1,0,3) =(1,1) and T(2,0, 6) = (2,1)? Justify.	CO3	K2
9.	Let T:$R^2 \rightarrow R^3$ be a linear transformation defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$. Write the matrix of the linear transformation.	CO3	K3
10.	Test the matrix $A = \begin{bmatrix}3 & 1 & 1\\0 & 4 & 2\\0 & 0 & -1\end{bmatrix} \in M_{3\times3}(R)$ for diagonalizability	CO3	K3
11.	Define Eigen space of linear operator T.	CO3	K1
12.	Let $T: R^2 \rightarrow R^2$ be defined by $(x, y) = (4x - 2y, 2x + y)$. Find the matrix of T relative to the standard basis.	CO3	K3
13.	Is T: $R^3(R) \rightarrow R^3$ $R)$ defined by $T(x, y, z) = (x, 0,0)$ is a linear transformation?	CO3	K3

14.	Test the map $T:R \to R$ defined by $(x) = x + 3\ \forall\ x \in R$ is a linear transformation.	CO3	K3
15.	Obtain the matrix representing the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ given by $(x, y, z) = (2x + 3y - z, x + z)$ with respect to the standard basis $\{e_{1,2},e_3\}$.	CO3	K3
16.	If Dim V= 5, rank(T) =3, find nullity(T)	CO3	K2
17.	If the Eigen values of a (3X3) matrix are 3,2 and trace(A)=1 then find the third eigen value?	CO3	K2
18.	Check whether the transformation is linear T: $R^2 \to R^2$ be defined by $T(a,b) = (a,b^2)$ is linear.	CO3	K2
19.	Check that T: $P_2(R) \to P_3(R)$ be defined as $T[f(x)] = x f(x) + f'(x)$ is linear or not.	CO3	K2
20.	Find the matrix representing the linear transformation T: $V_2(R) \to V_3(R)$ given by T (a, b) = (2a-b,3a+4b, a) with respect to the standard basis.	CO3	K3
	Part – B		
1.	(i)Let V and W be vector spaces and $T:V \to W$ be linear. Then prove that N(T) and R(T) are subspaces of V and W respectively. (8 marks) (ii) Test whether the matrix $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. is diagonalizable, if so find the Eigen Space. (8 marks)	CO3	K3
2.	(i) Let $T: R^3 \to R^2$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$ Find the basis for N(T) and compute the nullity of T. (8 marks) (ii) Let $T: R^2 \to R^3$ defined by $T(x, y) = (x - y, x, x + 2y)$, then B_1={(1,2), (2,3)} and B_2= {(1,1,0), (0,1,1), (2,2,3)} be a basis for R^3. Find the matrix [T]. (8 marks)	CO3	K3
3.	State & prove Dimension theorem. (16 marks)	CO3	K3
4.	Let $T: P_2(R) \to P_3(R)$ be defined by $T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt$. Find bases for N (T) and R(T) and hence verify dimension theorem. Is T one-to-one? Is T onto? Justify your answer. (16 marks)	CO3	K3
5.	Let $T: R^3 \to R^2$ be defined by $T(x, y, z) = (2x - y, 3z)$. Verify whether T is linear or not. Find N (T), R (T) and hence verify the dimension theorem. (16 marks)	CO3	K3
6.	Let $T : P_2(R) \to P_2(R)$ be defined as $T[(x)] = f(x) + (x + 1)f'(x)$. Find the eigen values and corresponding eigen vectors of T with respect to standard basis of $P_2(R)$. (16 marks)	CO3	K3
7.	If $T:R^2 \to R^3$ be defined by $T(x, y) = (2x - y, 3x + 4y, x)$ Compute the matrix of T in the standard basis of R^2 and R^3 . Is T one-to-one and onto? Find N(T) and R(T). (16 marks)	CO3	K3
8.	For the linear operator $T : P_2(R) \to P_2(R)$ be defined as $T[f(x)] = x f'(x) + x f(2) + f(3)$.Find the matrix T in an ordered basis B such that $[T]_B$ is diagonalizable. (16 marks)	CO3	K3
9.	(i)Find the linear transformation T: $R^3 \to R^3$ determined by	CO3	K3

	$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ with respect to the standard bases. What is T(-2,2,3)? (8Marks) (ii) Let V and W be vector spaces of equal (finite) dimension and let $T{:}V \to W$ be linear. Then prove that the following are equivalent (i) T is one to one (ii) T is onto (iii) Rank(T) = dim(V) (8Marks)		
10.	(i) Let V and W be vector spaces, and let $T{:}V \to W$ be linear. Then prove that T is one-to-one function if and only if $N(T)$ = {0} (8Marks) (ii) Let T: $R^3 \to R^2$ over R defined on the basis T($\vec{\imath}$) = (0,0), T($\vec{\jmath}$) = (1,1), T($\vec{k}$) = (1,-1), Compute T($4\vec{\imath}$ -$\vec{\jmath}$ + $\vec{k}$) . Find N (T) and R (T). (8Marks)	CO3	K3

UNIT IV- Inner Product Spaces

Q.No	Questions	CO's	Bloom's Level
1.	Prove that in an inner product space V(F), < u, αv + βw >=$\bar{\alpha} < u, v >$ +$\bar{\beta} < u, w >$.	CO4	K2
2.	A linear operator on $R^2(R)$ is defined by $(x, y) = (x + 2y, x - y)$. Find T^*.	CO4	K2
3.	If u and v are any two vectors in an inner product space, then prove that $\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$.	CO4	K2
4.	State and prove Triangle inequality for Norm of a vectors.	CO4	K1
5.	Define Frobenius Inner product.	CO4	K1
6.	Define adjoint of linear operator.	CO4	K1
7.	Define Inner product space.	CO4	K1
8.	Define length of a vector.	CO4	K1
9.	Find the norm of (3, -4,0) in R^3 (R) with the standard inner product.	CO4	K2
10.	Let V be an inner product space over F. Then for all u, $v \in V$ and for all α, $\beta \in$, prove that $\|\alpha u\|$= $\|\alpha\|$ $\|u\|$.	CO4	K2
11.	Let V be an inner product space over F. Then for all u, $v \in V$ and for all α, $\beta \in F$, prove that $\|u + v\| \leq \|u\| + \|v\|$.	CO4	K3
12.	Define Orthonormal set.	CO4	K2
13.	In an inner product space V(F), if u and v are orthogonal vectors, then prove that$\|u + v\|^2 \leq \|u\|^2 + \|v\|^2$.	CO4	K2
14.	If V(F) is an inner product space and S, T are any linear operators on V. Then prove $(T^*)^* = T$.	CO4	K2
15.	If V(F) is an inner product space and S, T are any linear operators on V. Then prove $(\alpha\, T)^* = \bar{\alpha}\, T^*$ for all α ∈ F.	CO4	K2
16.	Find the distance between the vectors (7, 1), (3, -2) in R^2 with the standard inner product.	CO4	K3
17.	Test Cauchy-Schwarz inequality for x = (1, -1, 3) and y = (2, 0, -1).	CO4	K2

18.	Let V = C(R), the vector space of polynomial over R with inner product defined by <f,g> $=\int_0^1 f(t)g(t)dt$ where f(t) = t+2 , g(t) = $t^2 - 2t - 3$, find <f,g>.	CO4	K2
19.	If W = span {i,j},subset of R^3 ,then find dimension of $W^{\perp}$.	CO4	K2
20.	Define Orthogonal vectors.	CO4	K1
	Part – B		
1.	(i) Let $u = (a_1, a_2, \dots , a_n)$, $v = (b_1, b_2, \dots , b_n) \in F^n(C)$. Define $< u, v > = a_1\overline{b1} + a_2\overline{b2} + \cdots + a_n\overline{bn}$ .Verify whether it is an inner product space of F^n. (10 marks) (ii) Let V be an inner product space over F. Then for all $u, v \in V$ and for all $\alpha, \beta \in F$, prove that $\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$. (6Marks)	CO4	K3
2.	(i) Prove that $R^2(R)$ is an inner product space defined for $u = (a_1, a_2)$ and $v = (b_1, b_2)$ by $< u, v > = a_1b_1 - a_2b_1 - a_1b_2 + 2a_2b_2$. (10 marks) (ii) Let V be an inner product space over F. Then for all $u, v \in V$ and for all $\alpha, \beta \in F$, prove that $\|u + v\|^2 - \|u - v\|^2 = 4< x, y >$ (6Marks)	CO4	K3
3.	(i) Let $u = (2, 1 + i, i)$, $v = (2 - i, 2, 1 + 2i)$ be vectors in $C^3(C)$. Compute using the standard inner product $< u, v >$,$\|u\|$, $\|v\|$, $\|u + v\|$. (8 marks) (ii) Let V be the set of all continuous real functions defined on the closed interval [0, 1]. The inner product on V be defined by $< f, g > = \int_0^1 f(t)g(t)dt$. Prove that (R) is an inner product space. (8 marks)	CO4	K3
4.	(i) Let V be an inner product space over F. Then for all $u, v \in V$ then prove that Cauchy-Schwartz inequality $\lvert< u, v >\rvert \leq \|u\| \, \|v\|$. (8 marks) (ii) If V(F) is an inner product space and S, T are any linear operators on V. Then prove a) $(ST)^* = T^*S^*$, b) $(S + T)^* = S^* + T^*$. (8 marks)	CO4	K3
5.	(i)A linear operator on $\mathbb{R}^2(\mathbb{R})$ with standard inner product is defined by $T(x, y) = (2x + y, x - 3y)$. Find $T^*(x, y)$ and $T^*(3, 5)$. (8 marks) (ii) State and prove Triangle inequality. (8 marks)	CO4	K3
6.	Let $V = P_2(R)$ be the inner product space with inner product defined by $< f, g > = \int_{-1}^1 f(t)g(t)dt$. Starting with the basis $B = \{1, x, x^2\}$, construct an orthonormal basis by Gram-Schmidt process. (16 marks)	CO4	K3
7.	In an inner product space $R^3(R)$ with the standard inner product, $B = \{(1,0,1), (1,0,-1), (0,3,4)\}$ is a basis. By Gram-Schmidt Orthogonalization process, find an orthogonal basis. Hence find an orthonormal basis. (16 marks)	CO4	K3
8.	In the inner product space R^3 (R) with the standard inner product, B={(1,1,0), (1,-1,1), (-1,1,2)} is a basis. By Gram-Schmidt orthogonalization process find an orthogonal basis. Hence find an orthonormal basis. Also find the Fourier coefficients of the vector (2,1,3) relative to orthonormal basis. (16 marks)	CO4	K3

9.	(i)Let V =P(R), the vector space of polynomials over R with inner product defined by $< f, g > = \int_0^1 f(t)g(t)dt$, where f (t) = t+ 2 and g(t) = t^2 –2t-3 Find $\|f\|$, $\|g\|$, $\|f + g\|$. (8 marks) (ii) If V(F) is an inner product space and S, T are any linear operators on V. Then prove a) $(T^*)^* = T$, b) $(\alpha T)^* = \bar{\alpha} T^*$ (8 marks)	CO4	K3
10.	Verify that the set {v_1, v_2, v_3} where v_1 =(0,1, -1), v_2 = (1+i ,1,1), v_3 =(1-i , 1,1) in C^3 is basis over C . Construct an orthogonal basis by Gram-Schmidt method. Hence find an orthonormal basis with the standard inner product. (16 marks)	CO4	K3

UNIT V- Eigenvalue Problems and Matrix Decomposition

Q.No	Questions	CO's	Bloom's Level
1.	Explain power method to find the dominant eigen value of a matrix.	CO5	K1
2.	Write the possible initial vectors in power method.	CO5	K1
3.	Determine the largest eigen value and vector of A=$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	CO5	K3
4.	What are the methods used to find eigen values and eigen vectors?	CO5	K2
5.	Define eigen value and eigen vector.	CO5	K1
6.	When can we use Jacobi method to find the eigen value and vector?	CO5	K1
7.	How will you find all eigen values using power method?	CO5	K1
8.	How will you find all eigen values using Jacobi method?	CO5	K1
9.	State the fundamental principles of Jacobi method?	CO5	K1
10.	Find all the eigen values of A=$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ by power method. Use initial vector as $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.	CO5	K3
11.	Define singular value decomposition.	CO6	K1
12.	Write any two properties of singular value decomposition.	CO6	K1
13.	Define QR factorization.	CO6	K1
14.	Write down iterative algorithm for QR decomposition of a matrix	CO6	K1
15.	Define Least-Square solution and write the objectives of least square method	CO6	K1
16.	Determine a canonical basis for A -$\begin{bmatrix} 3 & 5 \\ -2 & -4 \end{bmatrix}$	CO6	K3
17.	Define pair wise orthogonal vectors.	CO6	K1
18.	Verify whether the following vectors are pairwise orthogonal $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	CO6	K3

19.	Find the norm of $\begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$	CO6	K3
20.	Find the inner prodect of $\begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$	CO6	K3
	Part – B		
1.	(i)Using Power method find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix}1 & 6 & 1\\1 & 2 & 0\\0 & 0 & 3\end{bmatrix}$. (8 marks) (ii) Using Jacobi's method find the eigen values and the corresponding eigen vectors of $\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$. (8 marks)	CO5	K3
2.	(i)Find the dominant Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix}1 & 3 & -1\\3 & 2 & 4\\-1 & 4 & 10\end{bmatrix}$ by Power method, correct to two decimal places. Choose the initial vector as $\begin{pmatrix}0\\0\\1\end{pmatrix}$. (8 marks) (ii) Solve the system of equations in the least square sense x + 2y + z = 1, 3x – y = 2, 2x + y -z = 2, x + 2y + 2z = 1. (8 marks)	CO5	K3
3.	Using Power method, find all the Eigen values of $\begin{bmatrix}5 & 0 & 1\\0 & -2 & 0\\1 & 0 & 5\end{bmatrix}$. (16 marks)	CO5	K3
4.	Determine all the Eigen values and the corresponding Eigen vector of the matrix $A = \begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 2\end{bmatrix}$ by Jacobi method. (16 marks)	CO5	K3
5.	Find the least square line fitted to the data (1,1), (2,2), (3,2), (4,3). Also find the least square error. (16 marks)	CO5	K3
6.	Find the singular value decomposition of A = $\begin{bmatrix}5 & 5\\-1 & 7\end{bmatrix}$. (16 marks)	CO6	K3
7.	Find the singular value decomposition of A = $\begin{bmatrix}3 & 0\\4 & 5\end{bmatrix}$. (16 marks)	CO6	K3
8.	Find the QR factorization of $\begin{bmatrix}1 & 1 & 1\\1 & 0 & 1\\0 & 0 & 1\end{bmatrix}$. (16 marks)	CO6	K3
9.	Find the QR factorization of $\begin{bmatrix}1 & -1 & 1\\1 & 0 & 1\\-1 & 1 & 1\end{bmatrix}$. (16 marks)	CO6	K3
10.	Find the QR factorization of $\begin{bmatrix}1 & 2 & 0\\0 & 1 & 1\\1 & 0 & 1\end{bmatrix}$. (16 marks)	CO6	K3