MA3151- MATRICES AND CALCULUS

Simha's Classes
S.Narasimhan

Partial derivatives

If $u = \log\left(x^2 + y^2 + z^2 - 3xyz\right)$, then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = \dfrac{3}{x+y+z}$.

Solution:

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If $u = x^3 + y^3$ and where $x = a\cos t$ and $y = b\sin t$ then find $\dfrac{du}{dt}$

Solution:

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Given the transformations $u = e^x \cos y$ and $v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = \left(u^2 + v^2\right)\left(\dfrac{\partial^2 f}{\partial u^2} + \dfrac{\partial^2 f}{\partial v^2}\right)$

Solution:

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Euler's Theorem

If $u\left(x, y\right)$ is homogeneous function of degree n in x and y with all first and second derivatives continuous, then

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n\left(n-1\right)u$$

Solution:

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Jacobians

If $u = e^x \sin y$ and $v = x + \log \sin y$ then find the Jacobian of u, v with respect to x, y.

Solution:

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If $u = \dfrac{2x - y}{2}$ and $v = \dfrac{y}{2}$ then find $\dfrac{\partial (u, v)}{\partial (x, y)}$

Solution:

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Taylor's Expansion for function of 2 variables

Let $f(x, y)$ be a function of 2 variables x, y defined in a region R of the xy-plane and let (a, b) be a point in R. Suppose $f(x, y)$ has all its partial derivatives in a nieghbourhood of (a, b) then

$$\begin{aligned} f(x,y) = f(a,b) &+ \frac{1}{1!}\left[(x-a) f_x(a,b) + (y-b) f_y(a,b)\right] \\ &+ \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b) f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right] \\ &+ \frac{1}{3!}\Big[(x-a)^3 f_{xxx}(a,b) + 3(x-a)^2 (y-b) f_{xxy}(a,b) \\ &\quad + 3(x-a)(y-b)^2 f_{xyy}(a,b) + (y-b)^3 f_{yyy}(a,b)\Big] + \cdots \end{aligned}$$

Problem 1

Expand $e^x \sin y$ in powers of x and y upto three degree by using Taylor's series.

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Problem 2

Obtain the Taylor's series expansion of $e^x \log(1+y)$ at the origin.

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Problem 3

Find the Taylor series expansion of $e^x \sin y$ at the point $\left[-1, \frac{\pi}{4}\right]$ upto 3rd degree terms.

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Problem 4

Expand $x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's expansion.

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Problem 5

Expand $f(x,y) = 4x^2+xy+6y^2+x-20y+21$ in Taylor's series about $(-1,1)$.

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Problem 6

Expand $f(x,y) = \sin(xy)$ in powers of $(x-1)$ and $\left(y-\frac{\pi}{2}\right)$ upto second degree terms.

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Problem 7

Find the Taylor's series expansion of the function $f(x,y) = \sin x \sin y$ near the origin.

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Lagrange's Constraints

working rule

Step 1: Maximise or minimise $f(x, y, z)$ subject to $g(x, y, z)$
Write $F = f + \lambda g$ Find F_x, F_y, F_z, F_λ
Step 2: Find $F_x = 0 \ldots (1), F_y = 0 \ldots (2), F_z = 0 \ldots (3), F_\lambda = 0 \ldots (4)$.
Step 3: $(1) \div (2) \Rightarrow y$ in terms of $x \ldots (5)$.
Step 4: $(1) \div (3) \Rightarrow z$ in terms of $x \ldots (6)$
Step 5: Substitute (5) and (6) in (4) we get x, so that we can find y and z also.

Problem 1

Find the stationary value of $x^2 + y^2 + z^2$ subject to the condition
$ax + by + cz = p$.

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Problem 2

A rectangular box open at the top is to have a volume of 32 C.C. Find the dimensions of the box requiring the least amount sheets for its construction.

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Problem 3

Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

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Problem 4

Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = a$.

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Problem 5

Find the maximum and minimum values of the the function $f(x, y) = 3x+4y$ on the circle $x^2 + y^2 = 1$.

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Problem 6

The temperature T in space is $T = 400xyz^2$. Find the highest temperature on the surface on the unit sphere $x^2 + y^2 + z^2 = 1$.

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Problem 7

Find the maximum and minimum distances of the point (3,4,12) from the unit sphere $x^2 + y^2 + z^2 = 1$.

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Problem 8

Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

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Problem for practice

1.Find the maximum and minimum distances from the origin to the curve $3x^2 + 4xy + 6y^2 = 140$

2.Find the maximum and minimum values of $x^2 + y^2$ subject to the condition $3x^2 + 4xy + 6y^2 = 140$