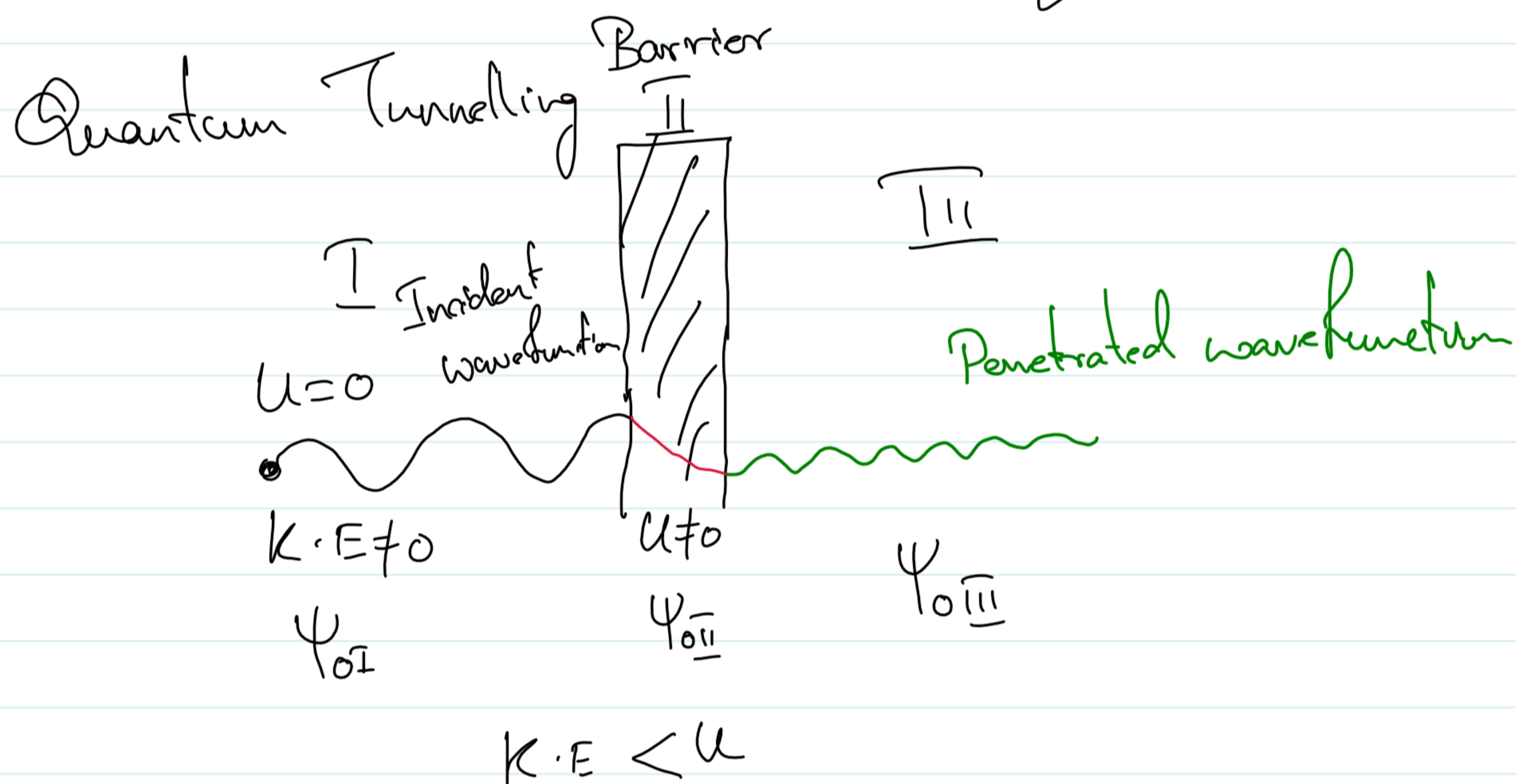
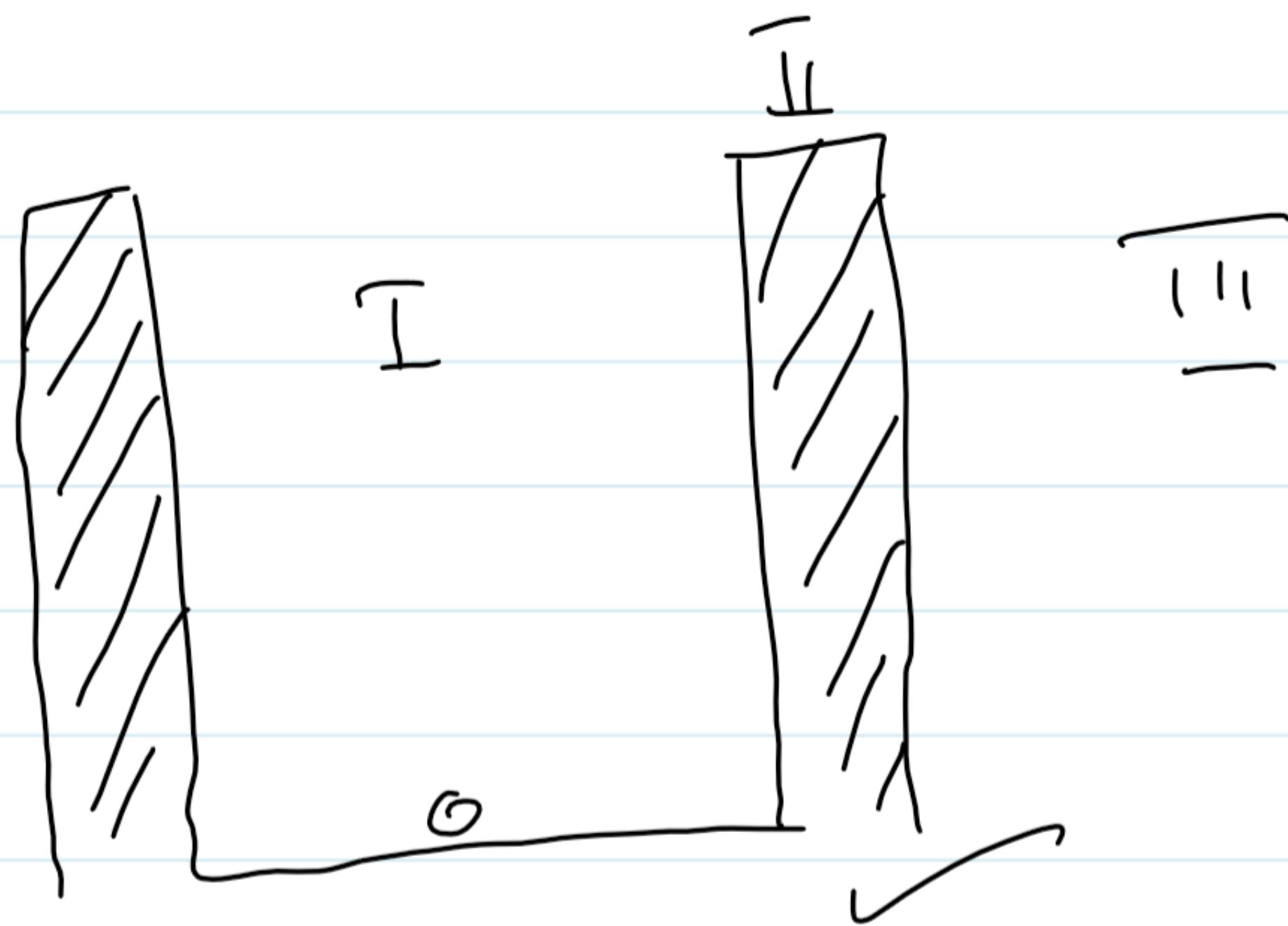


Quantum Tunnelling - Barrier Penetration

Finite Potential well / barrier



Quantum tunnelling : The particle is able appear on the other side of the barrier even though the particle has a kinetic energy less than the Potential Barrier.

Schrodinger's equation in one dimension

$$\frac{d^2 \psi_0}{dx^2} + \frac{2m}{\hbar^2} (E - u) \psi_0 = 0$$

Region I and Region III $u = 0$

$$\frac{d^2 \psi_{0I}}{dx^2} + \left(\frac{2m}{\hbar^2} E \right) \psi_{0I} = 0$$

$$\frac{d^2 \psi_{0\text{II}}}{dx^2} + \left(\frac{2mE}{\hbar^2} \right) \psi_{0\text{II}} = 0$$

$$\text{Let } \alpha^2 = \frac{2mE}{\hbar^2}$$

$$\frac{d^2 \psi_{0\text{I}}}{dx^2} + \alpha^2 \psi_{0\text{I}} = 0$$

$$\frac{d^2 \psi_{0\text{II}}}{dx^2} + \alpha^2 \psi_{0\text{II}} = 0$$

$$\psi_{0\text{I}} = A e^{i\alpha x} + B e^{-i\alpha x}$$

$$\psi_{0\text{II}} = C e^{i\alpha x} + D e^{-i\alpha x}$$

Region III $u \neq 0, u > E$

$$\frac{d^2 \psi_{0\text{III}}}{dx^2} + \frac{2m}{\hbar^2} (E - u) \psi_{0\text{III}} = 0$$

$$\frac{d^2 \psi_{0\text{III}}}{dx^2} - \left[\frac{2m}{\hbar^2} (u - E) \right] \psi_{0\text{III}} = 0$$

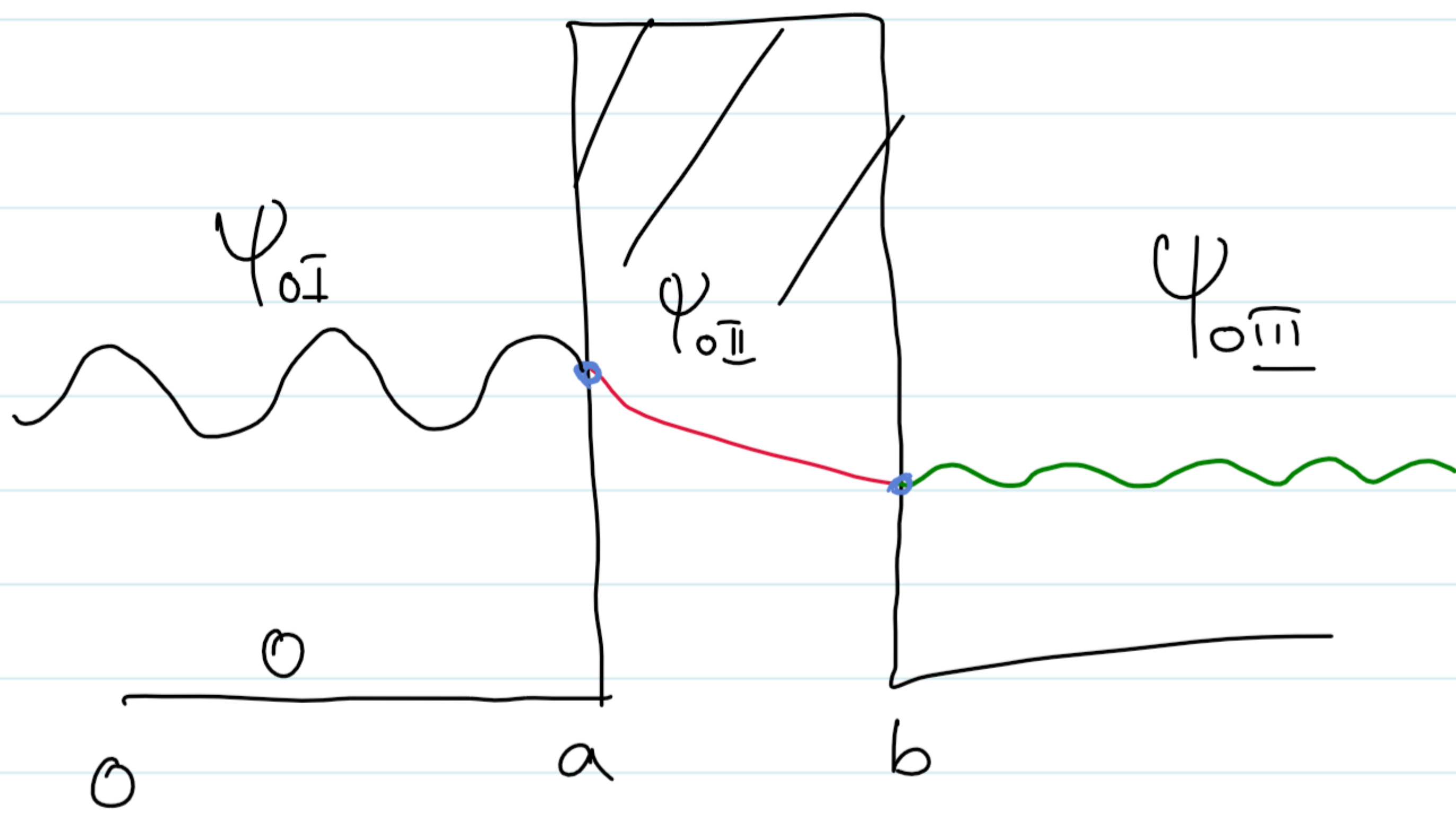
$$\text{Let } \beta^2 = \frac{2m}{\hbar^2} (u - E)$$

$$\frac{d^2 \psi_{0\text{III}}}{dx^2} - \beta^2 \psi_{0\text{III}} = 0$$

$$\psi_{0\text{III}} = F e^{\beta x} + G e^{-\beta x}$$

$$\psi_{0\text{I}} = A e^{i\alpha x} + B e^{-i\alpha x}$$

$$\psi_{0\text{III}} = C e^{i\alpha x} + D e^{-i\alpha x}$$



Boundary condition

$$\psi_{0I}(a) = \psi_{0II}(a)$$

Condition for continuity.

$$\psi_{0II}(b) = \psi_{0III}(b)$$

and the wave function must be finite

Using the Boundary conditions and the condition that wavefunction must be finite, we determine

$$A, B, C, D, E, G$$

$$B=0, D=0, F=0$$

$$\psi_{0II} = G e^{-\beta x}$$

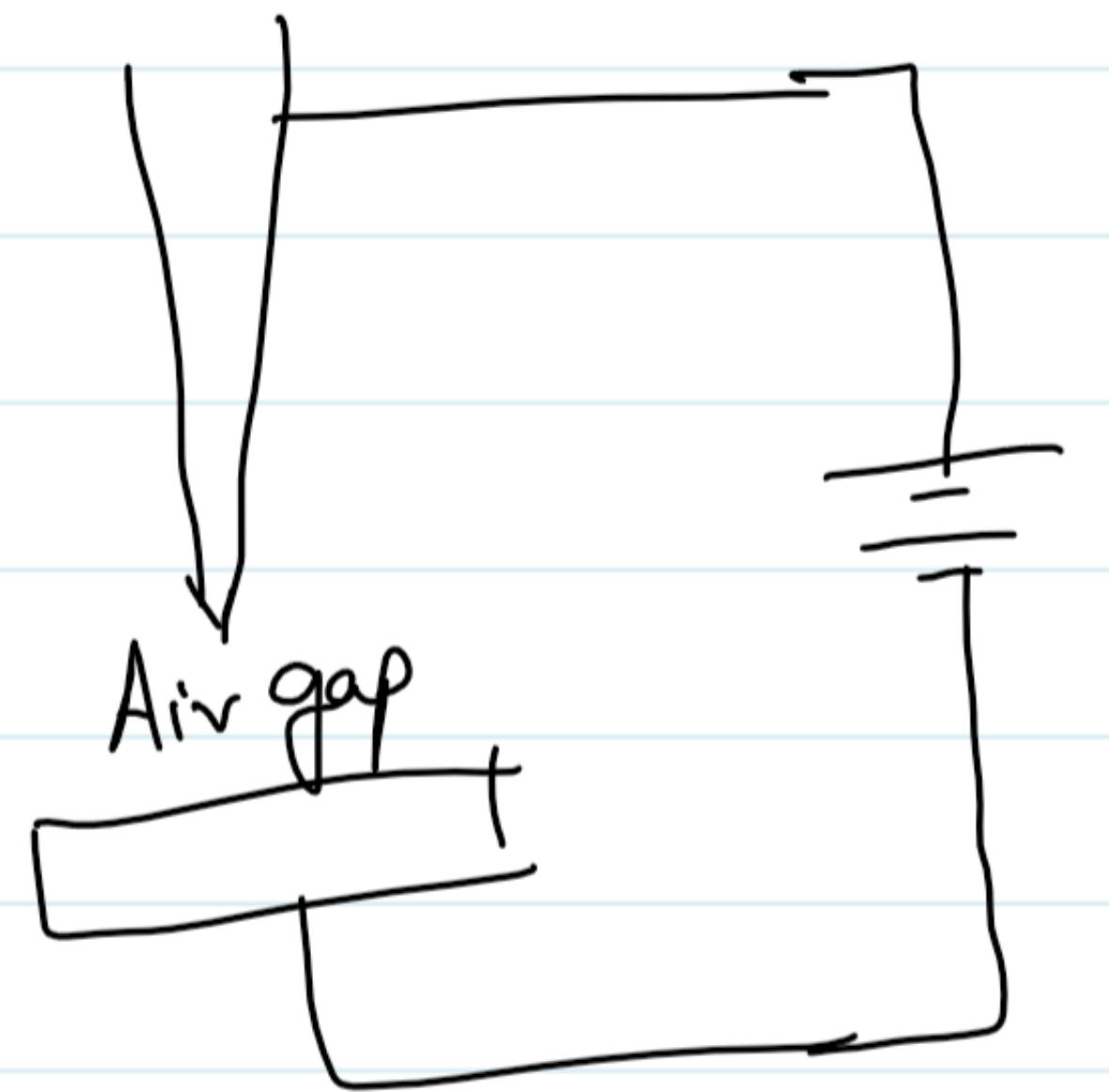
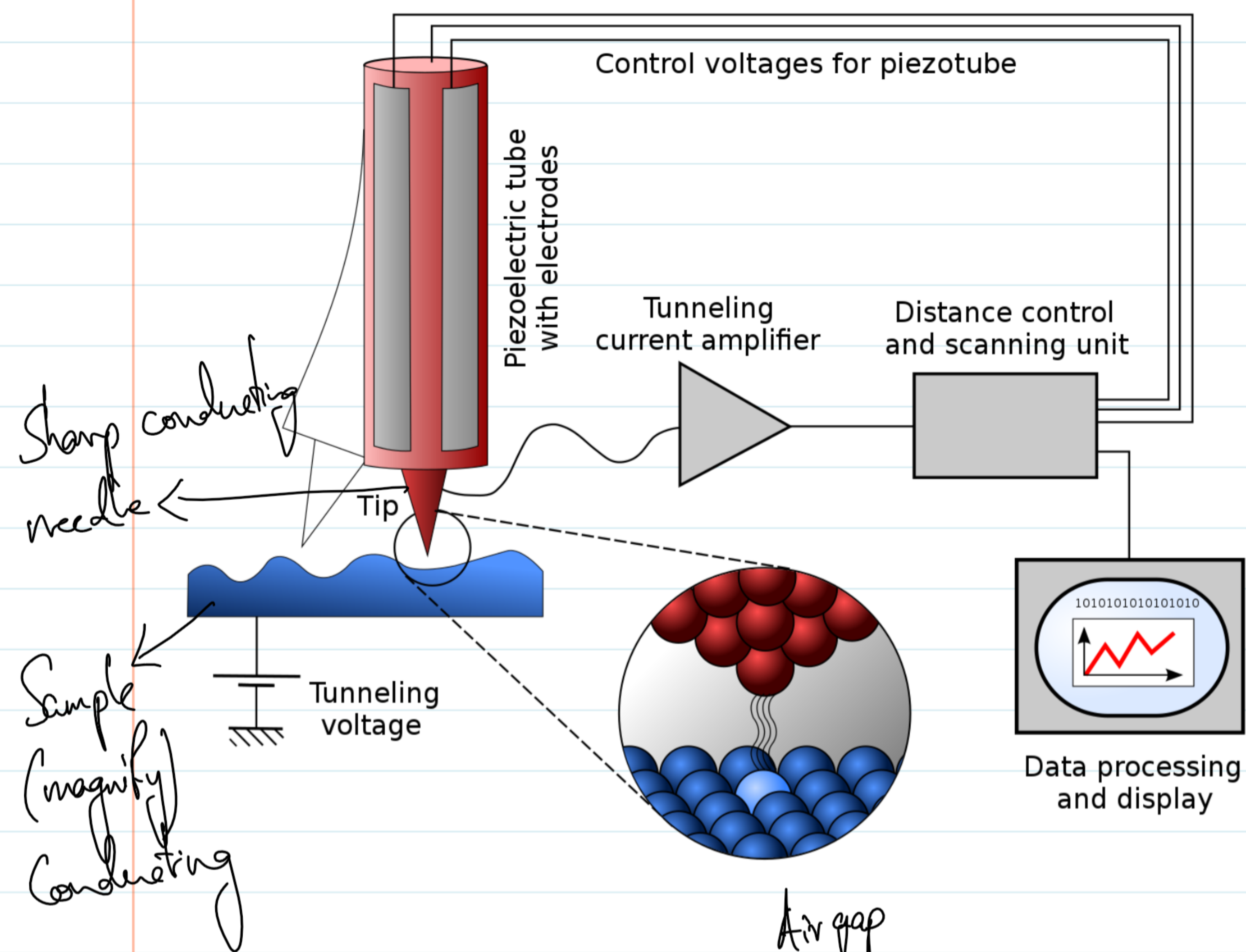
$$\psi_{0I} = A e^{i\alpha x}$$

$$\psi_{0III} = C e^{i\alpha x}$$

Scanning Tunneling Microscope

Wave nature of electron

Optical microscope : wave nature light.



Sample I Air gap Needle III
 Tunneling current flows between the sample and the needle
 Amount of tunneling current depends on the thickness of air gap

Electron Microscopes

- (i) Scanning Tunneling Microscope
- (ii) Scanning Transmission electron microscope
- (iii) Scanning electron microscope

Resonant Tunneling Diode

Special kind of PN junction diode.