$$AD^{-1}V = \lambda V$$

$$D = \begin{bmatrix} k_1 & 0 & 0 \cdots \\ 0 & k_2 & 0 \cdots \\ \vdots & 0 & k_3 \end{bmatrix} \Rightarrow D^{-1} = \begin{bmatrix} 1/k_1 & \cdots & 0 & 0 \\ 0 & 1/k_2 & \cdots & 0 \\ 0 & \cdots & 1/k_3 & \end{bmatrix}$$

$$A = \begin{bmatrix} 1/k_1 & \cdots & \\ \vdots & 1/k_2 & \\ \vdots & & 1/k_3 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$$

$$= \begin{bmatrix} k_1/k_1 & 0 & 0 & 0 \\ 0 & k_2/k_2 & 0 & 0 \\ 0 & & & k_3/k_3 \end{bmatrix} \begin{bmatrix} k_1/k_1 \\ k_2/k_2 \\ k_3/k_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \quad = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$A \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & \alpha_{ij} & \alpha_{33} & \alpha_{14} & \cdots \\ \alpha_{21} & & & & \\ \alpha_{31} & & & & \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

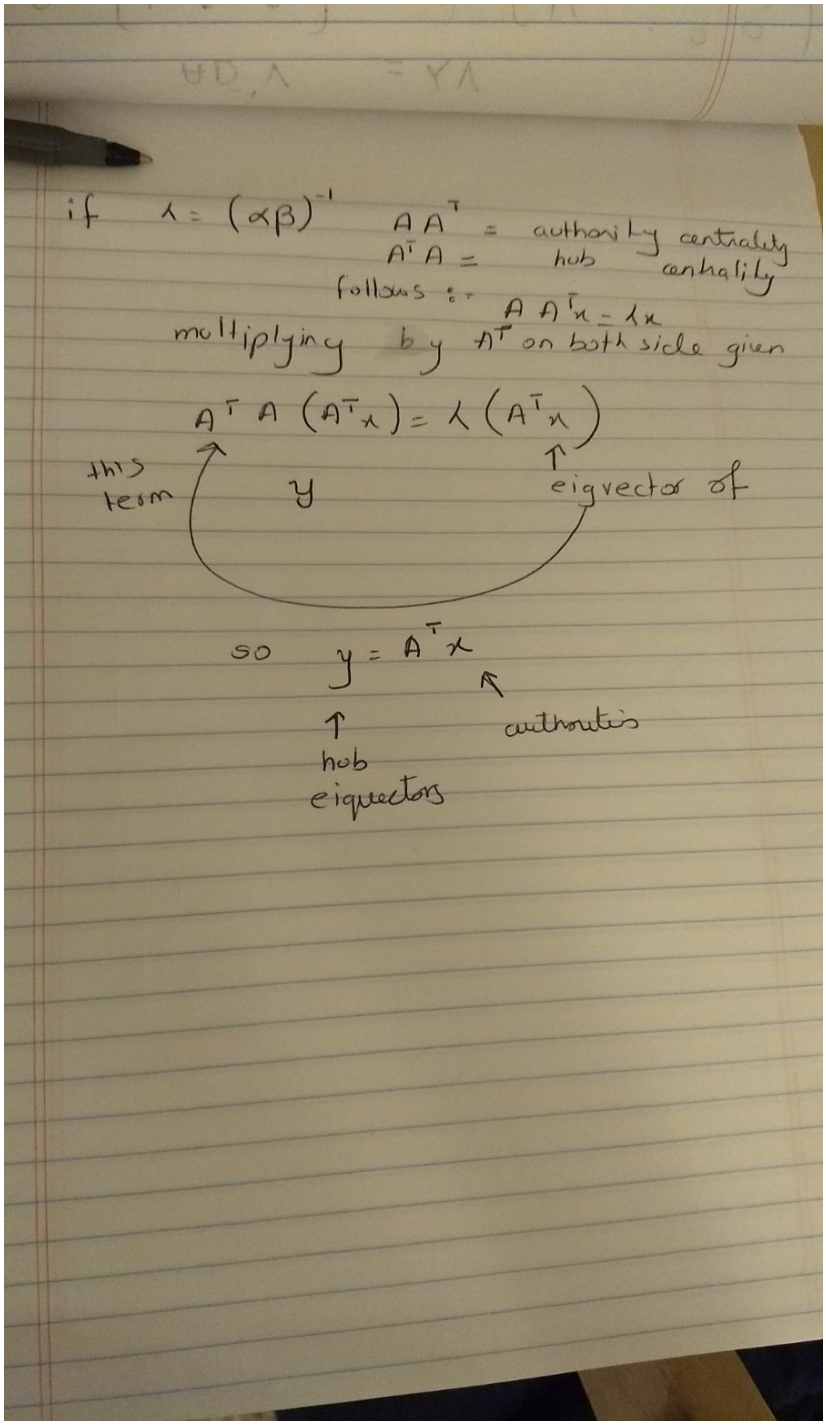
$$= \begin{bmatrix} \overbrace{\alpha_{11} + \alpha_{13} + \alpha_{14} \cdots}^{\text{sum of logics}} \\ \alpha_{21} + \alpha_{23} + \cdots \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \Leftarrow V_1$$

$\lambda = 1$

Q)Page Rank centrality Show that the vector is an eigenvalue of AD-1 ….

Ans) As we can see that the largest eigen value is equal to 1
Alpha = 1/largest eigenvalue. In this case the 1/1 = 1 hence it wont exceed 1
and I have also shown in the image that i is an eigenvector of AD-1 by proving lambda =1

Q) relationship between hub and authority eigen vectors



The Code and the printouts for unweighted network:

kashif@kashif-Inspiron-5423:~/Desktop/Systems_Networks/Lab3$ python lab3_stub.py

==========

Degree Centrality:
The degree centrality vector is :
[78. 140. 88. 80. 47. 124. 144. 65. 102. 63. 125. 90. 71. 71.
58. 38. 81. 27. 99. 153. 69. 67. 77. 111. 27. 108. 94. 58.
40. 44. 86. 48. 40. 40. 40. 40. 40. 40. 74. 113. 40. 40.
91. 137. 110. 104. 40. 53. 50. 51. 105. 66. 40. 31. 64. 45.
11. 26. 24. 24. 26. 17. 39. 14. 32. 55. 16. 16. 84. 7.
7. 7. 17. 17. 57. 17. 76. 19. 64. 9. 7. 23. 21. 64.
79. 52. 68. 61. 21. 24. 62. 24. 24. 24. 24. 24. 62. 54.
58. 58. 96. 49. 92. 53. 43. 43. 47. 43. 49. 49. 49. 45.
20. 31. 11. 25. 13. 16. 16. 21. 18. 11. 23. 18. 28. 24.
22. 22. 13. 38. 20. 18. 24. 12. 24. 20. 20. 6. 23. 11.
18. 18. 28. 22. 22. 28. 37. 22. 22. 20. 30. 31. 22. 8.
26. 8. 21. 21. 21. 17. 12. 15. 15.]
1. gandalf (153.0000)
2. frodo (144.0000)
3. aragorn (140.0000)
4. pippin (137.0000)
5. elrond (125.0000)

==========

Eigenvector Centrality (by Zen):
1. gandalf (0.0153)
2. aragorn (0.0149)
3. frodo (0.0147)
4. elrond (0.0140)
5. pippin (0.0139)

Eigenvector Centrality (by linear algebra):
1. gandalf (0.1682)
2. aragorn (0.1641)
3. frodo (0.1618)
4. elrond (0.1541)
5. pippin (0.1533)

Confirming that eigenvector centrality is a steady-state of sorts for node 60 (lobelia):
lobelias centrality is
0.00300267745743
sum of lobeliases neighbors centrality is
0.189604834908
normalized sum of lobelias neigbors is
[0.00300256]

Showing the convergece of eigenvector centrality...
lab3_stub.py:91: ComplexWarning: Casting complex values to real discards the imaginary part
  cs[j,i] = numpy.dot(x , v[:,j]) # project x onto each of the eigenvectors

==========

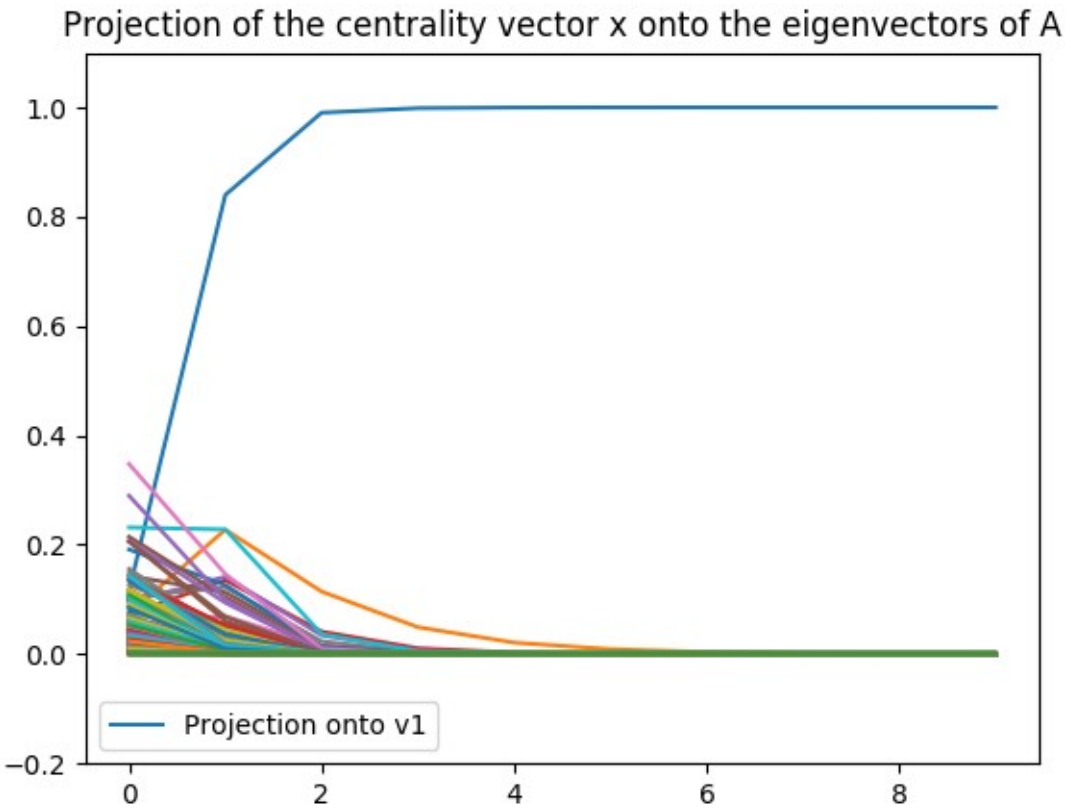
Katz Centrality:
katz for alpha = 0
  1. amroth (1.0000)
  2. aragorn (1.0000)
  3. arathorn (1.0000)
  4. arwen (1.0000)
  5. beregond (1.0000)
katz for alpha = 0.2
  1. sauron (2.6987)
  2. peregrin (2.2340)
  3. tobold (1.9659)
  4. gamling (1.9140)
  5. erkenbrand (1.9140)
katz for alpha = 0.4
  1. goatleaf (107.0039)
  2. smeagol (85.9850)
  3. hamfast (77.1504)
  4. anborn (72.5080)
  5. shadowfax (68.2373)
katz for alpha = 0.6
  1. angbor (4.6228)
  2. hamfast (4.4787)
  3. goatleaf (3.7800)
  4. theoden (3.1754)
  5. orome (2.9831)
katz for alpha = 0.8
  1. rumil (1.9585)
  2. baran (1.4471)
  3. muzgash (1.0382)
  4. arathorn (0.9588)
  5. ungoliant (0.9035)

==========

PageRank
pagerank for alpha = 0.8
  1. gandalf (16.3241)
  2. frodo (15.4448)
  3. pippin (14.5563)
  4. aragorn (14.2308)
  5. bilbo (13.0523)
pagerank for alpha = 0.6
  1. gandalf (7.3384)

  2. frodo (6.9785)
  3. pippin (6.5548)
  4. aragorn (6.2468)
  5. bilbo (5.8556)
pagerank for alpha = 1
  1. gandalf (3743842370233084928.0000)
  2. frodo (3523616348454680064.0000)
  3. aragorn (3425738116553143296.0000)
  4. pippin (3352329442627014656.0000)
  5. elrond (3058694746922460160.0000)
==========

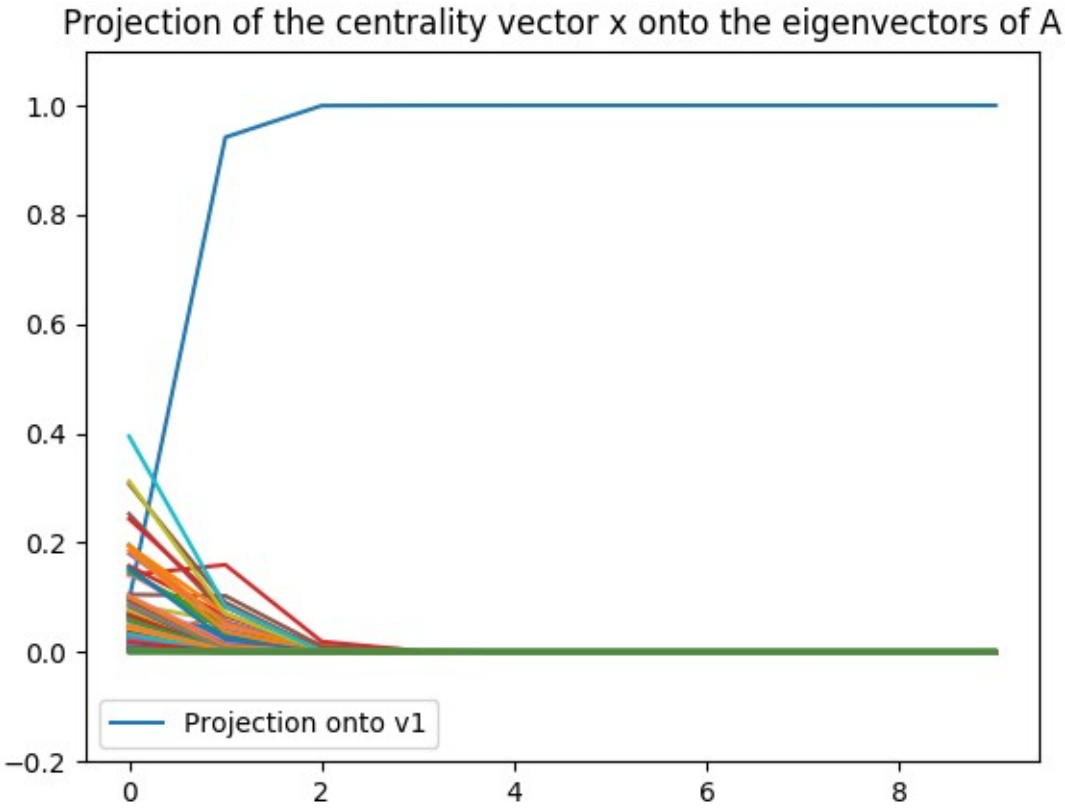
Betweenness Centrality
  1. gandalf (0.0797)
  2. frodo (0.0675)
  3. pippin (0.0613)
  4. aragorn (0.0486)
  5. bilbo (0.0393)



The printout with the weighted graphs is

```
kashif@kashif-Inspiron-5423:~/Desktop/Systems_Networks/Lab3$
python lab3_stub.py

============================================

Degree Centrality:
The degree centrality matrix is  :
[227. 632. 251. 141.  98. 423. 661. 131. 304. 115. 484. 329. 225.
178.
  70.  47. 228.  27. 343. 762. 129.  91. 191. 443.  27. 480. 342.
70.
  40.  58. 235.  76.  40.  40.  40.  40.  40.  40. 164. 454.  40.
40.
 174. 606. 390. 297.  40.  84.  92.  60. 316. 152.  40.  46. 146.
102.
  15.  43.  38.  38.  46.  23.  55.  14.  52.  59.  16.  16. 265.
7.
   7.   7.  17.  17.  81.  17. 199.  34. 129.   9.  15.  27.  21.
138.
 167.  64. 126.  88.  21.  24. 113.  24.  24.  24.  24.  24. 119.
67.
 110.  90. 354.  63. 278.  71.  43.  43.  66.  43.  63.  63.  56.
54.
  20.  38.  11.  50.  13.  16.  16.  21.  25.  11.  37.  18.  64.
44.
  33.  33.  13.  49.  20.  18.  32.  12.  32.  20.  20.   6.  41.
11.
  30.  30.  39.  22.  22.  37.  43.  22.  22.  20.  41.  39.  26.
8.
  29.   8.  21.  21.  21.  17.  12.  15.  15.]
  1. gandalf (762.0000)
  2. frodo (661.0000)
  3. aragorn (632.0000)
  4. pippin (606.0000)
  5. elrond (484.0000)

============================================

Eigenvector Centrality (by Zen):
  1. gandalf (0.0469)
  2. frodo (0.0401)
  3. aragorn (0.0400)
  4. pippin (0.0390)
  5. sauron (0.0316)

Eigenvector Centrality (by linear algebra):
  1. gandalf (0.3412)
  2. frodo (0.2920)
  3. aragorn (0.2908)
  4. pippin (0.2839)
```

```
  5. sauron (0.2300)

Confirming that eigenvector centrality is a steady-state of sorts
for node 60 (lobelia):
lobelias centrality is
0.00284915533195
sum of lobeliases neighbors centrality is
0.330196423292
normalized sum of lobelias neigbors is
[0.00110261]

Showing the convergence of eigenvector centrality...
```




```
==========================================

Katz Centrality:
katz for alpha = 0
  1. amroth (1.0000)
  2. aragorn (1.0000)
  3. arathorn (1.0000)
  4. arwen (1.0000)
  5. beregond (1.0000)
katz for alpha = 0.2
```

```
  1. gorbag (18.5698)
  2. shagrat (18.5698)
  3. dain (14.9485)
  4. beren (13.4605)
  5. meneldor (12.4077)
katz for alpha = 0.4
  1. thorondor (3.4615)
  2. meneldor (3.4615)
  3. ungoliant (3.0283)
  4. angbor (2.9851)
  5. dain (2.1912)
katz for alpha = 0.6
  1. varda (4.7428)
  2. celebrian (4.7428)
  3. goldberry (4.4167)
  4. baldor (3.0469)
  5. dunhere (2.8928)
katz for alpha = 0.8
  1. thorondor (9.8387)
  2. meneldor (9.8387)
  3. cirion (5.4409)
  4. hador (5.4409)
  5. arvedui (4.5345)

==========================================

PageRank
pagerank for alpha = 0.8
  1. gandalf (31.6524)
  2. frodo (28.3370)
  3. aragorn (25.6157)
  4. pippin (25.5455)
  5. elrond (20.1194)
pagerank for alpha = 0.6
  1. gandalf (12.5674)
  2. frodo (11.5585)
  3. pippin (10.3196)
  4. aragorn (10.0375)
  5. elrond (8.0976)
pagerank for alpha = 1
  1. anborn (-9472086958211502.0000)
  2. proudfoot (-11050768117913408.0000)
  3. adelard (-11050768117913408.0000)
  4. twofoot (-11050768117913412.0000)
  5. orome (-12629449277615308.0000)

==========================================

Betweenness Centrality
  1. gandalf (0.0437)
```

```
  2. pippin (0.0366)
  3. aragorn (0.0345)
  4. frodo (0.0289)
5. bilbo (0.0270)
```

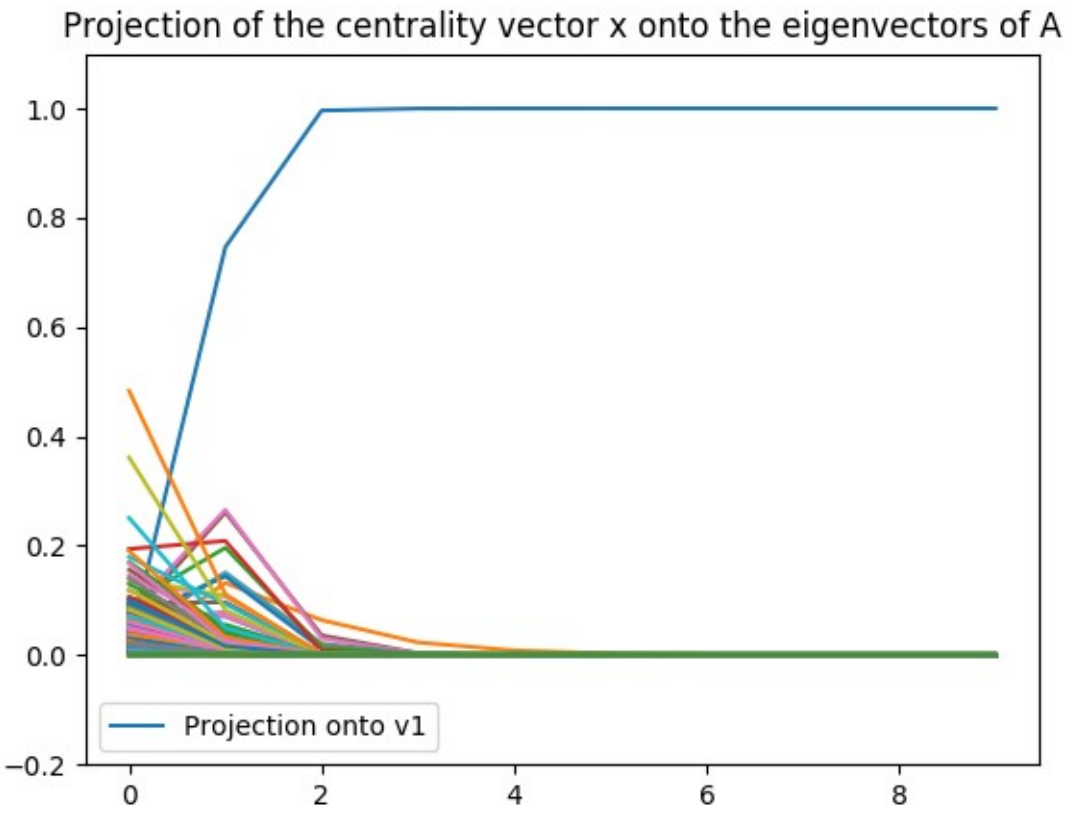
The change that I noticed is that the centralities and moreover the degree centrality has increased overall as expected as the graph weights are now incorporated in size. Yes it is backwards compatible as the Adjacency matrix just has weights now instead of the ones and my code adds up the matrix rows in the same way.

Including the hobbit network:

the printouts are:

```
kashif@kashif-Inspiron-5423:~/Desktop/Systems_Networks/Lab3$
python lab3_stub.py

==============================================

Degree Centrality:
The degree centrality matrix is  :
[  6. 661. 228. 901. 164. 289. 129. 520. 632. 138. 602. 126. 343.
480.
 116. 174.  41.  11.  30.  30.  56. 227.  98.  39. 354. 235. 278.
22.
  22.  37. 304.  43. 191.  22. 606. 297. 342. 316. 152.  22. 251.
20.
 115. 329. 225. 178. 454.  41. 199. 443.  58.  76.  39.  26. 390.
167.
   8.  49.  29.   8. 141.  21.  21.  21.  91.  90.  84.  17. 117.
12.
  38.  38.  15.  15. 131.  70.  47.  27. 129.  27.  70.  40.  40.
40.
  40.  40.  40.  40.  40.  40. 138.  92.  78.  40.  46. 165.  15.
43.
  46.  23.  55.  14.  52.  59.  16.  16.  18. 275.  32. 140. 138.
102.
 164. 234.  18. 139. 155. 288. 139. 145. 169.  85.  81.   4.  42.
80.
  12.  32.  26.  26.   7.   7.   7.  17.  17.  81.  17.  34.   9.
15.
  27.  21.  64.  88.  21.  24.  63.  43.  43.  43.  38.  11.  50.
13.
  16.  16.  21.  25.  11.  37.  18.  64.  44.  33.  33.  13.  20.
18.
  32.  12.  32.  20.  20.]
  1. gandalf (901.0000)
  2. frodo (661.0000)
```

```
  3. aragorn (632.0000)
  4. pippin (606.0000)
  5. bilbo (602.0000)

==========================================

Eigenvector Centrality (by Zen):
  1. gandalf (0.0469)
  2. frodo (0.0372)
  3. aragorn (0.0366)
  4. pippin (0.0358)
  5. bilbo (0.0301)

Eigenvector Centrality (by linear algebra):
  1. gandalf (0.3570)
  2. frodo (0.2827)
  3. aragorn (0.2783)
  4. pippin (0.2723)
  5. bilbo (0.2287)

Confirming that eigenvector centrality is a steady-state of sorts
for node 60 (arwen):
lobelias centrality is
0.00709220260428
sum of lobeliases neighbors centrality is
0.767092649174
normalized sum of lobelias neigbors is
[0.00247167]

Showing the convergece of eigenvector centrality...
```



==========

```
Katz Centrality:
katz for alpha = 0
  1. anborn (1.0000)
  2. frodo (1.0000)
  3. faramir (1.0000)
  4. gandalf (1.0000)
  5. samwise (1.0000)
katz for alpha = 0.2
  1. thorondor (12.4946)
  2. meneldor (12.4946)
  3. barahir (12.3880)
  4. thingol (12.3880)
  5. beren (12.2722)
katz for alpha = 0.4
  1. feanor (8.9862)
  2. baran (8.4856)
  3. bolg (5.9853)
  4. nain (5.9853)
  5. thorondor (5.3404)
katz for alpha = 0.6
  1. meneldor (4.9592)
```

```
  2. thorondor (4.9592)
  3. baran (4.1887)
  4. goldberry (4.1614)
  5. celebrian (3.3496)
katz for alpha = 0.8
  1. thorondor (11.3516)
  2. meneldor (11.3516)
  3. varda (8.3295)
  4. celebrian (8.3295)
  5. cirion (6.7214)

==========================================

PageRank
pagerank for alpha = 0.8
  1. gandalf (34.7997)
  2. frodo (26.5416)
  3. pippin (23.9721)
  4. aragorn (23.9582)
  5. bilbo (23.8723)
pagerank for alpha = 0.6
  1. gandalf (13.8056)
  2. frodo (10.9015)
  3. bilbo (9.7796)
  4. pippin (9.7573)
  5. aragorn (9.4348)
pagerank for alpha = 1
  1. galion (-10839968494401312.0000)
  2. anborn (-16259952741602008.0000)
  3. proudfoot (-18969944865202324.0000)
  4. adelard (-18969944865202328.0000)
  5. twofoot (-18969944865202328.0000)

==========================================

Betweenness Centrality
  1. gandalf (0.0498)
  2. pippin (0.0370)
  3. aragorn (0.0339)
  4. frodo (0.0296)
5. bilbo (0.0292)

I notice that Bilbo has popped up in most of the centralities.

Eigenvector Centrality does not hold in the case of weighted graph
because steady state convergence hasnt been achieved in the
weighted graphs, it will probably take more trials and high
computational power.
```