Que Let A set of all line plane define
$A R b \iff a \perp b$.

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1. Symmetric ✓
2. ~~Re~~ Reflexive
3. Transitive ?

$a \perp b$, $a \perp a$, $a \perp c$ ✗

Types of Rela

$a \perp b$, $b \perp c$

BOOK Example

aRb, bRa, cRa

propertie of using matrices.

Reflexive ← how to find relation of Reflexive

1. Reflexive — Diagonal ~~throw~~ should be 1.
dway 1 ~~one होना चाहिए~~.

$a_{ii} = 1 \ \forall \ i$

ii) Symmetric — transpose करना पड़ेगा।

$a_{ij} = a_{ji} \ \forall \ i \neq j$

$M_R = M_R^T$

(iii) How to identify Antisymmetric

$a_{ij} \cdot a_{ji} = 0 \ \forall \ i \neq j$

$\left.\begin{matrix} a_{ij} = 1 \\ a_{ji} = 0 \end{matrix}\right\}$ alway or $\left.\begin{matrix} a_{ij} = 0 \\ a_{ji} = 1 \end{matrix}\right\}$ always.

or both zero $\left.\begin{matrix} a_{ij} = 0 \\ a_{ji} = 0 \end{matrix}\right\}$ always.

(iv) Transitive — There is no direct observation mathematica any condition nahi है।

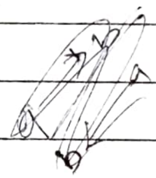
Que $A = \{2, 3, 4, 6, 9\}$

aRb = a divide b.

$$
\begin{array}{c|ccccc}
 & 2 & 3 & 4 & 6 & 9 \\
\hline
2 & 1 & 0 & 1 & 1 & 0 \\
3 & 0 & 1 & 0 & 1 & 1 \\
4 & 0 & 0 & 1 & 0 & 0 \\
6 & 0 & 0 & 0 & 1 & 0 \\
9 & 0 & 0 & 0 & 0 & 1
\end{array}
$$

Transitivity → $(2,4)\ (4,4)$
→ $(2,4)$

$(2,6)\ (6,6)$
→ $(2,6)$



digraph.

→ Reflexivity → ~~also~~ self loop होना चाहिए
हरेक पर होना चाहिए

$A \rightleftarrows b$

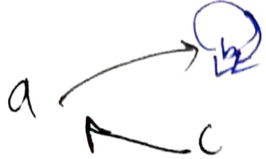
Symm → parallel edge in opposite direction

$A \nearrow$

$a \rightarrow b$ है
$b \rightarrow a$ होना चाहिए।

Anti sy → parallel edge present नहीं होना चाहिए

$a \rightarrow b$
$b \rightarrow a$ नहीं होना चाहिए।

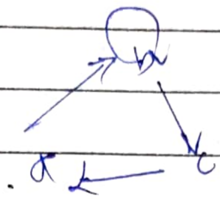


anti symetric

symmetric नहीं है

← Reflex नहीं है

Transitive a → b b → c c → a होगा



← transitive

$A = \{a, b, c, d\}$.

Example $R = \{(a,c), (a,b), (b,b), (c,d)\}$.

minimum No add करना हो तो उसे कहते है closer of the Relation.

$S = \{(a,c), (a,b), (b,b), (c,d)\ (a,a),\ (c,c)\ (d,d)\}$

Reflexive closer. $R \subset S$

and s is the smallest ~~Reflex~~ Relation that contain R.

~~(b,d), (d,b)~~

Symmetric closer → $S_1 = \{(a,c), (c,a), (a,b), (b,a), (c,d), (d,c)\}$.

Smallest Relation that contain R.

$S_2 = \{(a,c)\ (c,a), (a,b)\ (b,a)\ (c,d)\ , (c,d)\ (d,c)\ (b,d)\ (d,d)\}$

→ thing biger than S_1

$S_2 > S_1$

Antisymmetric → nahi होगा ~~का~~ because diff...
हर किसी remove करें

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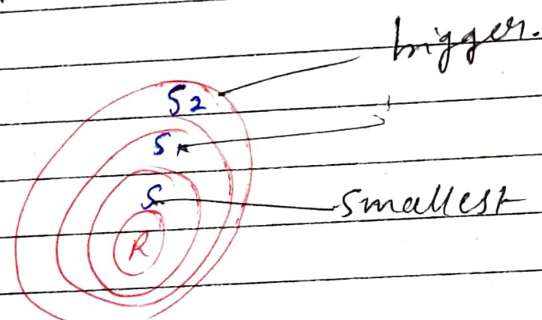
closer of Relation → ~~set~~ Let R be a relation on a set A that means $R \subseteq A \times A$ without the property P than the smallest relation S on A $S \subseteq A \times A$ that contain R and having the property P and contain in any other relation. containing R is called the closer of the Relation R.



a b c

→ Reflexive Closer find
$S = R \cup I_R$ Identity के साथ union.
- (a,a) (b,b) (c,c).

symmetric closer find
$\boxed{S = R \cup R^{-1}}$

(a,b) | (b,a) आ जाएगा
है | नहीं ↗

R में (c,d) है तो
(d,c) आ जाएगा

Transitive find. (d,a) (a,c)

dc

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Ex — $R = \{(a,c), (c,d), (b,c), (c,a), (d,a)\}$

(a,d) (a,a) new element generate होगा

(d,a) (a,c)

(d,c) (d,d) new Element

Washer Algorithem

warshall's

$W_0 = M_R$

Initially. matries of Relation R.

sequene of ~~matrix~~ vertic genrate करेंगे

All Elements आते हैं

~~warshal Algori~~

→ The warshall's Algorithem consist of generating of the seque of matrics $W_0, W_1, W_2, W_3 \ldots\ldots W_n$ succesivelly using the Diagraph and matrics M_R of a non-transitive Relation R. on A of ~~an~~ elements.

Let, v_1 v_2 v_3 .... v_n be an arbitray listing of the elements of A.

$A = \{a, b, c, d\}$
$\downarrow \; \downarrow \; \downarrow \; \downarrow$
$v_1 \; v_2 \; v_3 \; v_3$

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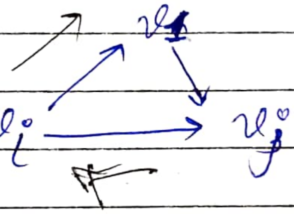
then $\boxed{w_0 = M_R}$ matric of the Relation and the matric

w_K, $K = 1, 2, 3 \ldots n$ are obtaine succesing as follow.

$$w_{ij} = \begin{cases} 1, \text{ if there is an path from } v_i \text{ to } v_j \text{ directly or through some or all } v_1, v_2, v_3 \ldots v_K \\ 0, \text{ otherwise.} \end{cases}$$

w_1 अगर बनाना है तो directly हो या किसी के through हो

w_1



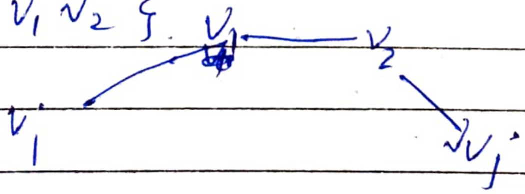
then

w_2 बनाएंगे $\{v_1, v_2\}$ fix करके देखा

direct अगर path है तो वह w_2 को already होगा। then नहीं है direct तो v_1 v_2 के through देखेंगे

w_2

$- \{v_1, v_2\}$



In this way last vals elements [illegible] is [illegible] transitive

Ques
Ex Let $A = \{a, b, c, d\}$

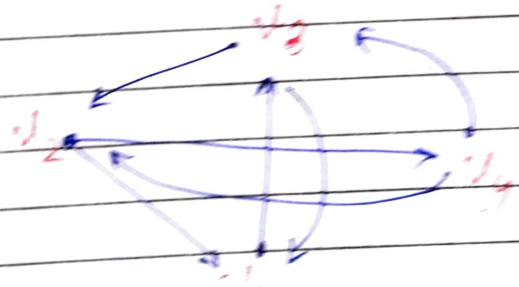
$R \subseteq A \times A$ is $R = \{(a,c), (b,d), (c,b), (d,b), (b,a), (c,a), (d,c)\}$.

~~then find the~~ transitive closure of R using Warshall's Algorithm.

– Let $v_1 = a$, $v_2 = b$, $v_3 = c$, $v_4 = d$

The matrix of R and the digraph are as

$$W_0 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



digraph

Find W_1 then using Warshall Algorithm.

$W_0 = M_R$

and

$$w_{ij} = \begin{cases} 1, & \text{if there is a path for } v_i \text{ to } v_j \text{ through } v_1 \text{ or direct} \\ 0, & \text{otherwise.} \end{cases}$$

over

V_1

$$W_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & \boxed{1} & 1 \\ 1 & 1 & \boxed{1} & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$v_2 \to v_3$

0 &

not possible

& 1 through

now

W_2 is given by

$$w_{ij} = \begin{cases} 1 & \text{if there is a path from } v_i \text{ to } v_j \text{ thereafter } v_1 \text{ or } v_2 \text{ or both directly} \\ 0, & \text{otherwise} \end{cases}$$

$$W_2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$v_3 \to v_4$

v_2 through

&

possible &

$v_4 \to v_1$

V_1, W_2

Now W_3 is given by

$w_{ij} = \begin{cases} 1 \text{ if there is a path from } v_i \text{ to } v_j \text{ directly therefore } v_1 \text{ or } v_2 \text{ or } v_3 \text{ or } \cancel{\text{both}} \\ 0, \text{ otherwise.} \end{cases}$

$$W_3 = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 1 & 1 & 1 & 1 \\ v_2 & 1 & 0 & 1 & 1 \\ v_3 & 1 & 1 & 1 & 1 \\ v_4 & 1 & 1 & 1 & 1 \end{array}$$

$v_1 \to v_1 \to$ through v_3.

$v_1 \to v_2 \to v_3$ through

$v_1 \to v_4 \to v_3$ v_2 through

Now W_4 is given by

$w_{ij} = \begin{cases} 1 \text{ if there is a path from } v_i \text{ to } v_j \text{ directly therefore } v_1 \text{ or } v_2 \text{ or } v_3 \text{ or } v_4 \text{ } \cancel{\text{both}} \\ 0, \text{ otherwise.} \end{cases}$

$$W_4 = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 1 & 1 & 1 & 1 \\ v_2 & 1 & 1 & 1 & 1 \\ v_3 & 1 & 1 & 1 & 1 \\ v_4 & 1 & 1 & 1 & 1 \end{array}$$

Equivalence Relation & Partial-order } Type of relation

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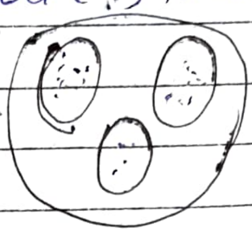
Equivalance – A relation $R \subseteq A \times A$ is called a equivalance relation if R is reflexive, symmetric, transitive



Example –

A = set of all interface

$aRb \Leftrightarrow a-b$ is divisible by 3

i. R is reflexive

$aRa \Leftrightarrow a-a=0$ is divisible by 3

ii. R is symmetric

$aRb \Leftrightarrow a-b$ is divisible by 3

$\Leftrightarrow b-a$ is divisible by 3

$\Leftrightarrow bRa$

iii. R is transitive

aRb, $bRc \Leftrightarrow a-b$ and $b-c$ are divisible by 3

$\Leftrightarrow a-b=3p$, $b-c=3q$

$\Leftrightarrow a-b+b-c=3(p+q)$

$\Leftrightarrow a-c=3(p+q)$

$\Leftrightarrow aRc$

All three property are in the Equivai

partion of set using Equivalance classes

Let $R \subseteq A \times A$ be an equivalance relation on A

then equivalance class ~~on~~ of an element $a \in A$ is the set $[a] = \{x \in A \mid x R a\}$

property

1. $[a] \neq \phi$ because ~~[[a]]~~ is $aRa \Rightarrow a \in [a]$

2. $b \in [a] = [b] = [a]$

~~prove~~ ~~$x \in [b]$~~

prove ~~$[b] \in a$~~,

$b \in [a], x \in [b] \Leftrightarrow bRa, xRb$

$\Leftrightarrow aRb, bRx$ symmetric

$\Leftrightarrow aRx$ transitive

$\Leftrightarrow xRa$ symmetric

$\Leftrightarrow x \in [a]$

$[a] = [b]$ both are same

3. $[a] = [b] \Leftrightarrow aRb$

4. $[a] \cap [b] \neq \phi$ then $[a] = [b]$

दोनों element में कोई common है तो equal होगा।

prove $x \in [a] \cap [b] \Leftrightarrow$

$x \in [a]$ and $x \in [b]$

$\Leftrightarrow xRa, xRb$

$\Leftrightarrow aRx, xRb$

$\Leftrightarrow aRb$

$\Leftrightarrow [a] = [b]$

prove iii. $[a] = [b] \Leftrightarrow aRb$

- $x \in [a] = [b] \Leftrightarrow x \in [a]$ and $x \in [b]$

$\Leftrightarrow xRa, xRb$

$\Leftrightarrow aRx, xRb$

$\Leftrightarrow aRb$

Example

Let $A = \{2, 3, 4, 5, 6, 7, \ldots 12\}$

Define $R \subseteq A \times A$ as $aRb \Leftrightarrow a-b$ is divisible by 3

then R is an equivalance relation on A

solu $[2] = \{2, 5, 8, 11\}$

2 के साथ इनका diff निकाला 3 से div

2 का diff divisible है 3 से

$[3] = \{3, 6, 9, 12\}$

$[4] = \{4, 7, 10\}$

$[5] = \{5, 8, 11, 2\}$

$5-2 =$ $\quad$ $5-11 =$

$5-5 =$

$5-8 =$

$[6] = \{3, 6, 9, 12\}$

$[7] = \{4, 7, 10\}$

$[8] = \{2, 5, 8, 11\}$

$[9] = \{3, 6, 9, 12\}$

$[10] = \{4, 7, 10\}$

$[11] = \{5, 8, 2, 11\}$

$[12] = \{3, 6, 9, 12\}$

$A = [2] \cup [3] \cup [4]$, $[2] \cap [3] \cap [4] = 0$

then it is called the

~~disjoint~~ partion of A is to disjoint classes

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<u>Example</u> Let n be an positive ..

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bit for length is n

more than n.

$a R_n b \Leftrightarrow a = b$ or both a and b have atleast n bcb and first n bits of a and b are the same.

<u>prove</u> – R_n is reflexive

– $a R_n a \Leftrightarrow a = a$

II. $a R b \Leftrightarrow a = b$ or both a and b are of length altest n and first n bits of a and b are the same

$\Leftrightarrow b = a$ or both b and have altered n bit and first n bits of a and b ax the same

$\Leftrightarrow b R_n a$

III. R_n is transitive

$a R_n b$, $b R_n c \Leftrightarrow a = b$ or both a and b ..... are the same

$b = c$ or both b and c ... are the same

then $a = c$.

If a and b have atleast n bit and first n bit of a and b are the same

b and c have atleast n bit and first n bit of b and c are the same.

Atleast R bit same होना चाहिए

(R_2)

$\underline{1}101$ — a

$\underline{1}1001$ — b

$\underline{1}101010$ — c — suppose length 10bit कि भी हो सकती है

a, b, c - diff. length के हो सकती है

(2.) what are the equiv ....

$a R_3 b \iff a = b$ or both a and b have atleast 3 bit and first 3 bit the same

ये क्यों होगा है

$[01011] = \{\underline{010}11, \underline{010}00, \underline{010}01, \underline{010}10, \underline{010}000, \underline{010}001, \ldots\}$

same bit string

$010, 0100, 0101, \ldots\}$

$001, 0010, 0011, \ldots$

$[00111] = \{\underline{001}11, 00100, 00110, \ldots\}$

minimum length atleast (3)

③ find the partition of the . . . .

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[0] = {0} {01} = {01}

[1] = {1}. {11} = {11}

[00] = {00}

{10} = {10}

length 3

Total 14 distinct class distinct

[000] = {000, 0000, 0001, 00000, 00001, 00010, 00011 . . . . }.

[001] = {001, 0010, 0011, 00100, ... {00110 . . }.

[011] = {011, 0110, 0111, 00100, 01110 . . . }

1 and 3rd bit if same than same { } class

यह same

से आपको । → ①0① ①1① — same clas