Rational Number

Concepts

Introduction

Natural Numbers: All counting Number are called natural number.

Example: 1, 2, 3, 4, 5, ...etc.

- It is denoted by Capital Letter 'N'.
- Smallest Natural Number is 1
- Largest Natural Number is infinite

Whole Number: All Counting Number together with zero are called whole number.

Example: 0, 1, 2, 3, 4, 5,.....etc.

It is denoted by 'W'.

Largest Whole Number is infinite.

Smallest Whole Number is 0

Note: All-natural number is a whole number but all whole number is not natural number.

Integers: All positive, negative and zero are called integers.

Example: ..... -3, -2, -1, 0, 1, 2, 3, ... etc.

- It is denoted by 'I' OR Z
- Largest integer is infinite.
- Smallest integers are infinite.
- Positive integer {1, 2, 3, 4, 5,.....up to infinite}
- Negative Integer { -1, -2, -3, -4, -5 .....up to infinite}

Note: 0 is neither Positive and nor negative.

Rational Number: Those number is the form $\frac{p}{q}$, where q ≠ 0 are called Rational number.

Example: $\frac{1}{2}$, $\frac{4}{5}$, $\frac{11}{13}$, 0, 3, ...etc.

Natural Number: N = (1, 2, 3, 4, 5, 6,.....up to ∞.)

Whole Number: W = (0, 1, 2, 3, 4, 5, 6,.....up to ∞)

Integer Number: I or Z = (..... -3, -2, -1, 0, 1, 2, 3,.....up to ∞)

Even Number: (2, 4, 6, 8, 10, 12, 14, 16,..... Up to ∞)

Odd Number: (1, 3, 5, 7, 9, 11, 13, 15,.....Up to ∞)

Prime Number: (2, 3, 5, 7, 11, 13, 17, 19, 23,.....Up to ∞)

Compositive Number (4, 6, 8, 10, 12, 14..... Up to ∞)

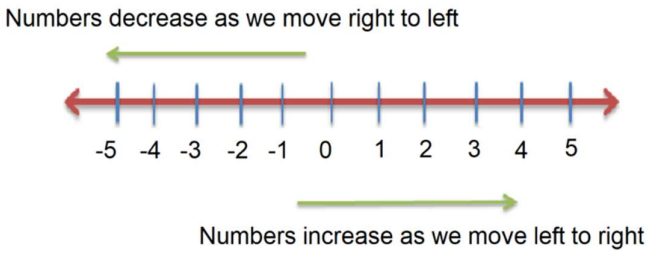
Twin Prime Number: (3, 5), (5, 7), (11, 13), ...Up to ∞)

All are Rational Number

Standard Form of a Rational Number: Those number said to be in its Standard form or Simplest form or lowest terms if its numerator and denominator have no common factors other then one.

Example: $\frac{20}{25}$

Rational Numbers on the Number Line: The set of Positives, 0 and negative integers can be grouped on a number line.



Absolute Value: The Absolute value of a rational number without any regard to its Sign.

- Its Symbol is | |
- The absolute value of 0 is 0.

Examples: The absolute values of $\frac{-10}{15}$ and $\frac{8}{6}$

Comparing and Ordering Rational Numbers

Method 1: By Using a number line

- If $\frac{p}{q}$ lies to the right of $\frac{r}{s}$, Then $\frac{p}{q} > \frac{r}{s}$
- If $\frac{p}{q}$ lies to the left of $\frac{r}{s}$, Then $\frac{p}{q} < \frac{r}{s}$
- All Rational numbers lying to the right of 0 are Positive
- All Rational Numbers lying to the left of 0 negative
- A positive rational number is always greater than a negative rational number.

Method 2. By Making Their Denominators Same and them Compare the Numerators.

Example: Comparing $\frac{2}{5}$ and $\frac{5}{7}$

Method 3. By Changing into Decimals

Example: Compare $\frac{3}{7}$ and $\frac{11}{21}$

Method 4. By Using Cross Product Rule

If ps > qr then $\frac{p}{q} > \frac{r}{s}$

If ps < qr then $\frac{p}{q} < \frac{r}{s}$

Math's Alert: Before Applying Cross Product rule, Make the Denominators Positive if not So.

Example: Compare $\frac{3}{10}$ and $\frac{1}{-4}$

Self Practice 1A

1. Write the absolute value of each of the following.

   [a] $\frac{-3}{2}$ [b] $\frac{-5}{-9}$ [c] $\frac{7}{-8}$ [d] $\frac{5}{7}$

2. Arrange the following rational numbers in ascending order:

   [a] $\frac{17}{25}, \frac{17}{13}, \frac{17}{19}, \frac{17}{27}$ [b] $\frac{5}{6}, \frac{4}{6}, \frac{1}{6}, \frac{9}{6}$

3. Arrange the following rational numbers in descending Order:

[a] $\frac{4}{5}, \frac{5}{5}, \frac{1}{5}, \frac{4}{5}$ [b] $\frac{4}{8}, \frac{4}{1}, \frac{4}{3}, \frac{4}{5}$

4. Write two rational numbers equivalent to each of the following rational numbers

[a] $\frac{3}{5}$ [b] $\frac{-7}{9}$ [c] $\frac{5}{11}$ [d] $\frac{13}{-15}$

5. Write the Standard form of following rational numbers

[a] $\frac{4}{40}$ [b] $\frac{125}{400}$ [c] $\frac{22}{121}$ [d] $\frac{-54}{17}$

6. Compare.

[a] $\frac{3}{10}$ .....$\frac{5}{12}$ [b] $\frac{3}{15}$ .....$\frac{6}{-15}$ [c] $\frac{24}{48}$ .....$\frac{30}{60}$ [d] $\frac{2}{5}$ .....$\frac{2}{5}$

Operations on Rational Numbers

Case 1: Adding Like Rational Numbers

Rule: To add rational numbers with same denominator, add the numerator keep the denominator same.

$$\left[\ \frac{N}{D} + \frac{N}{D} = \frac{N+N}{D}\ (\mathrm{D} \neq 0)\right] \left[\ \frac{P}{Q} + \frac{R}{Q} = \frac{P+Q}{Q}\ (\mathrm{Q} \neq 0)\right]$$

Example: $\frac{4}{5} + \frac{2}{5}$

Case 2: Adding Unlike Rational Number

Algorithm Step 1 Find least Common Denominator (LCD). Its also Known as LCM.

Step 2 $\frac{P}{Q} + \frac{R}{S} = \frac{P\ X\ \{LCM\ of\ (Q\ and\ S) \div Q\} + R\ X\ \{LCM\ of\ (Q\ and\ S) \div S\}}{LCM\ of\ (Q\ and\ S)}$

Example 1 $\frac{5}{-7} + \frac{2}{3}$

Case 1: Subtract Like Rational Numbers

Rule: To Subtract rational numbers with same denominator, Subtract the numerator keep the denominator same.

$$\left[\ \frac{N}{D}-\frac{N}{D}=\frac{N-N}{D}\ (D\neq 0)\right]\ \left[\ \frac{P}{Q}-\frac{R}{Q}=\frac{P-Q}{Q}\ (Q\neq 0)\right]$$

Example $\frac{6}{7}-\frac{3}{7}$

Case 2: Subtract Unlike Rational Number

Algorithm Step 1 Find least Common Denominator (LCD). It's also Known as LCM.

Step 2 $\frac{P}{Q}-\frac{R}{S}=\frac{P\ X\ \{LCM\ of\ (Q\ and\ S)\div Q\}-R\ X\ \{LCM\ of\ (Q\ and\ S)\div S\}}{LCM\ of\ (Q\ and\ S)}$

Example $\frac{3}{7}-\frac{1}{3}$

Self Practice 1B

1. Find the Sum of the following Rational Numbers
   [a] $\frac{7}{25}+\frac{11}{25}$ [b] $5\frac{2}{7}+3\frac{3}{7}$ [c] $\frac{7}{18}+\frac{5}{6}$ [d] $\frac{7}{9}+\frac{1}{6}$
2. Find the difference of the following Numbers
   [a] $9-11\frac{4}{7}$ [b] $8\frac{4}{11}-9$ [c] $\frac{5}{-7}+\frac{3}{4}$ [d] $5\frac{3}{15}-3\frac{13}{30}$
3. Simplify
   [a] $2\frac{3}{4}+\frac{11}{7}+\frac{9}{14}$ [b] $5\frac{7}{9}-2\frac{3}{27}-\frac{7}{18}$ [c] $5\frac{5}{13}\ 2\frac{7}{13}-3\frac{11}{13}$

Multiplying Rational Number

If $\frac{P}{Q}$ and $\frac{R}{S}$ $(Q, S\neq 0)$ are two rational numbers,

then $\frac{P}{Q}\times\frac{R}{S}=\frac{PR}{QS}=\frac{Product\ of\ Numerators}{Product\ of\ Denominators}$

(+) x (+) = (+)
(-) x (-) = (+)
(+) x (-) = (-)

Keep in Mind | **(-) x (+) = (-)**

Note: Product of Same Signs is Positive and Product of diff Signs is Negative.

Examples 1. $\frac{3}{4} \times \frac{1}{5}$ 2. $\frac{3}{-7} \times \frac{-3}{11}$ 3. $\frac{5}{9} \times \frac{-6}{4}$ 4. $-2\frac{1}{3} \times \frac{3}{5}$

Dividing Rational Number

If $\frac{P}{Q}$ and $\frac{R}{S}$ (Q, S ≠ 0) are two rational numbers,

$$\text{then } \frac{P}{Q} \div \frac{R}{S} = \frac{PS}{QR} = \frac{Product\ of\ Numerators}{Product\ of\ Denominators}$$

Keep in Mind

(+) ÷ (+) = (+)
(-) ÷ (-) = (+)
(+) ÷ (-) = (-)
(-) ÷ (+) = (-)

Examples 1. $10 \div 2$ 2. $\frac{-1}{2} \div \frac{4}{-6}$ 3. $\frac{3}{2} \div \frac{6}{-8}$ 4. $\frac{-7}{9} \div \frac{14}{3}$

Self Practice 1C

1. Multiply

[a] $\frac{3}{27} \times \frac{81}{27}$ [b] $\frac{-24}{49} \times \frac{42}{36}$ [c] $2\frac{1}{9}$ by $\frac{-27}{57}$ [d] $\frac{8}{-9} \times \frac{-7}{-16}$

2. Divide

[a] 1 by $\frac{1}{3}$ [b] 0 by $\frac{-3}{5}$ [c] $\frac{-7}{4}$ by -6 [d] $\frac{2}{3} \div \frac{12}{18}$

3. Simplify

[a] $\frac{3}{11} \times \frac{-5}{6} \times \left(-\frac{22}{9}\right) \times \left(-\frac{9}{5}\right)$ [b] $\left(\frac{3}{2} \times \frac{1}{6}\right) + \left(\frac{5}{3} \times \frac{7}{2}\right) - \left(\frac{13}{8} \times \frac{4}{6}\right)$

4. The product of two number is 15. If one of the numbers is -10, find the other.

Properties of Operations on Rational Numbers

Closure Property

Operation	Formula		Remarks
Addition Sum of any two Rational numbers is a Rational Number	**A + B = C** Example 2 + 4 = 6	**R. No+R. No = R. No** Example $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$	Rational Number are **Closed** Under Addition.
Subtraction Difference of any two Rational numbers is a Rational Number	**A - B = C** 2 - 4 = - 2	**R. No - R. No = R. No** $\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$	Rational Number are **Closed** Under Subtraction
Multiplication Product of any two Rational numbers is a Rational Number	**A X B = C** 2 x 4 = 8	**R. No x R. No = R. No** $\frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$	Rational Number are **Closed** Under Multiplication
Division Division of two Rational numbers is not always a Rational Number	**A ÷ B ≠ C** **4** ÷ 2 = 2	**R. No ÷ R. No ≠ R. No** $\frac{3}{5} \div \frac{1}{5} = \frac{3}{1}$	Rational Number are **Not Closed** Under Division

Summary: Closure Property

Rational Number are Closed Under		Yes/No
Addition	a + b = c	Yes
Subtraction	a - b = c	Yes
Multiplication	a x b = c	Yes
Division	a ÷ b = c	NO

Try These

Fill in the blanks in the following table

Numbers	Closed under			
	Addition	Subtraction	Multiplication	Division
Rational Numbers	YES	YES	...	No
Integers	.....	YES	....	No
Whole Numbers	.....	.....	YES	...
Natural Numbers	...	NO	.....	....

Commutativity Property

Operation	Formula	Examples	Remarks
Addition	A + B = B + A	3 + 4 = 4 + 3	Addition is Commutative
Subtraction	A – B ≠ B – A	3 – 4 ≠ 4 – 3	Subtraction is not Commutative
Multiplication	A X B = B X A	3 X 4 = 4 X 3	Multiplication is Commutative
Division	A ÷ B ≠ B ÷ A	3 ÷ 4 ≠ 4 ÷ 3	Division is not Commutative

Try These

Complete the following table

Numbers	Commutative for			
	Addition	Subtraction	Multiplication	Division
Rational Numbers	YES	....	...	...
Integers	...	No		...
Whole Numbers	...		YES	...
Natural Numbers	...	...	...	No

Associativity Property

Operation	Formula	Examples	Remarks
Addition	(A + B) +C= A+ (B +C)	(2+3) + 5 = 2 +(3 + 5)	Addition is **associative**
Subtraction	(A - B)-C≠ A+ (B +C)	(2-3) - 5 ≠ 2 -(3 - 5)	Subtraction is not **associative**
Multiplication	(A x B) x C= Ax (B x C)	(2x3) x 5 = 2 x (3 x 5)	Multiplication is **associative**
Division	(A ÷ B) ÷C= A÷ (B ÷C)	(2÷3) ÷ 5 ≠ 2 ÷ (3 ÷ 5)	Division is not **associative**

Complete the following table

Numbers	Associative for

Try These

	Addition	Subtraction	Multiplication	Division
Rational Numbers	…	….	…	No
Integers	…	…	Yes	…
Whole Numbers	YES	…	….	…
Natural Numbers	…	No	…	…

Distributivity Property

Operation	Formula	Examples
Addition	A(B+C) = AB + BC	3(4+2) = 12 + 6
Subtraction	A(B-C) = AB -BC	3(4-2) = 12 -6

Property of Zero (Additive Identity): The Sum of any rational number a and o is the rational number itself

A + 0 = A

Example: 3 + 0 = 3 $\frac{2}{3} + 0 = \frac{2}{3}$

Property of One (Multiplicative identity): the Product of any rational number and 1 is the rational number itself. i.e. a x 1 = 1 x a = a

Example: 1. 4 x 1 = 1 x 4 = 4 Example 2. $\frac{1}{2}$ x 1 = 1 x $\frac{1}{2} = \frac{1}{2}$

Additive Inverse (Negative of a Number): For any rational number a, a + (-a) = (-a) + a = 0. The rational number (-a) is the additive inverse of (a)

Example 3 + (-3) = -3 + 3 = 0

Multiplicative inverse or Reciprocal of a Number

If a is any non – zero rational number and a x $\frac{1}{a} = \frac{1}{a}$ x a = 1, then $\frac{1}{a}$ is called the reciprocal or Multiplicative inverse of a.

Example: The reciprocal of 7 is $\frac{1}{7}$

Self Practice 1D

1. Find the additive inverse of each of the following.

[a] $\frac{-5}{9}$ [b] $\frac{-3}{-2}$ [c] $\frac{5}{9}$ [d] $\frac{7}{-13}$

2. Write the Multiplicative inverse of each of the following.

[a] -5 [b] $\frac{3}{7}$ [d] $\frac{-11}{-13}$ [d] -1 x $\frac{-2}{7}$

3. Name the Property under Multiplication used in each of the following.

[f] $\left(\frac{8}{15} \text{x} \left(\frac{-7}{18}\right)\right)+\left(\frac{8}{15} \text{x} \frac{-11}{18}\right)$

Rational Number Between Two Rational Numbers

Note: There are infinitely many rational numbers between any two different rational numbers.

Case 1. To find 1 Rational Number between Two Given Rational Numbers.

First Method: Mid- Value Method

Let p and q be two rational numbers such that q>p. then $\frac{a+b}{2}$ is a rational number lying between p and q.

Example: Find the rational number between [a] 2 and 3 [b] $\frac{1}{4}+\frac{1}{2}$

Case 2. To find more than one rational number between two given rational numbers

Second Method: Gap Method

The n rational numbers between rational numbers a and b are

(a + 1)$\frac{b-a}{n+1}$, (a + 2)$\frac{b-a}{n+1}$, (a + 3)$\frac{b-a}{n+1}$, ...(a + n)$\frac{b-a}{n+1}$

Algorithm Step 1. Find the gap between the given rational number a and b (a < b)

Gap = b – a.

Step 2. Divide this gap by (n + 1), if you have to find n rational numbers.

Step 3. Multiply $\frac{b-a}{n+1}$ by 1, 2, 3,.....until n, and add each product to a.

Example: Find three rational number between $\frac{1}{4}$ and $\frac{1}{2}$

Third Method: Common Denominator Method

Method: To find n rational numbers between two rational numbers a and b with like Denominators.

Then, we convert the given rational numbers into equivalent rational numbers by multiplying the Numerator and denominator by a suitable number (n + 1)

Example: Find five rational number between $\frac{3}{5}$ and $\frac{4}{5}$.

Self Practice 1E

1. Find a rational number between -2 and 6.
2. Find any four rational numbers less than 2.
3. Find two rational numbers between $\frac{1}{5}$ and $\frac{1}{2}$
4. Find ten rational numbers between $\frac{-1}{5}$ and $\frac{2}{4}$.

NCERT CORNER

Multiples Choice Questions (MCQs)

1. If the rational number $\frac{a}{b}$ is positive, then which of the following must be true? (Cbse)

   [a] a > 0 [b] b > 0 [c] ab > 0 [d] a + b > 0

2. The sum of multiplicative identity of a number and its additive identity is always ____. (IMO)

   [a] 0 [b] 1 [c] -1 [d] itself

3. Which of the following is a rational number? (IMO)

   [a] π [b] 0 [c] $2\sqrt{5}$ [d] $\sqrt{7}$

4. A rational Number lying between $\sqrt{2}$ and $\sqrt{3}$ is (IMO)

   [a] 1.5 [b] 1.7 [c] 1.414 [d] 1. 763

5. Which of the following is true Statement? (IMO)

   [a] Every real number is always rational.

[b] The Sum of two irrational numbers is an irrational number.

[c] The Product of two irrational numbers is an irrational number.

[d] Every real number is either rational or irrational.

6. The Sum of two numbers is $\frac{-1}{3}$. if one of the numbers is $\frac{-12}{3}$, find the other. (IMO)

[a] $\frac{-13}{3}$ [b] $\frac{-13}{6}$ [c] $\frac{11}{3}$ [d] $\frac{11}{-3}$

7. What Should be Added to $\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5}\right)$ to get 3? (IMO)

[a] $\frac{59}{30}$ [b] $\frac{59}{-30}$ [c] $\frac{3}{9}$ [d] $\frac{3}{5}$

8. Divide the sum of $\frac{65}{12}$ and $\frac{12}{7}$ by their difference. (IMO)

[a] $\frac{77}{84}$ [b] $\frac{499}{84}$ [c] $\frac{599}{311}$ [d] $\frac{499}{311}$

9. The Product of two rational numbers is $\frac{-8}{9}$. if one of the numbers is $\frac{-4}{15}$, find the other. (IMO)

[a] $\frac{32}{135}$ [b] $\frac{-3}{135}$ [c] $\frac{10}{3}$ [d] $\frac{3}{10}$

10. The Multiplicative inverse of -2 $\frac{3}{7}$ is (IMO)

[a] $\frac{17}{7}$ [b] $\frac{7}{17}$ [c] $\frac{-7}{17}$ [d] $-\frac{17}{7}$

Chapter Test

Concepts Review

1. Fill in blanks.

[a] There are ..... rational numbers between any two rational numbers.

[b] Zero has .....reciprocal.

[c] The product of two rational number is always a .....

[d] the reciprocal of a negative rational number is.....

[e] $\frac{-9}{14}$ + ... = -1

2. State the Property Shown by the following Numerical Statements.

[a] $\frac{5}{7} \times 1 = 1 \times \frac{5}{7} = \frac{5}{7}$ [b] $\frac{11}{20} \times \frac{3}{7} = \frac{3}{7} \times \frac{11}{20}$ [c] $\frac{-12}{17} \times \frac{17}{-12} = 1$

[d] $\left(\frac{1}{4} \times \frac{1}{6}\right) \times \frac{1}{5} = \frac{1}{4} \times \left(\frac{1}{6} \times \frac{1}{5}\right)$ [e] $\frac{3}{12} \times \frac{5}{9} = \frac{5}{36}$

3. Multiple Choice Questions (MCQs)

[a] Which Number is greater than $\frac{-3}{4}$?

[a] $\frac{-5}{4}$ [b] $\frac{-1}{2}$ [c] -1 [d] -2

[b] Evaluate: $\left|\frac{-5}{7}\right| \div \left|\frac{5}{7}\right|$.

[a] -1 [b] 1 [c] $\frac{-2}{49}$ [d] $\frac{49}{25}$

[c] What is the product of $-5\frac{1}{3}$ and $3\frac{3}{4}$?

[a] $-15\frac{1}{4}$ [b] $15\frac{1}{4}$ [c] -20 [d] 20

[d] How Many pieces of $13\frac{1}{5}$ cm length can be cut from a 330 cm long rod ?

[a] 25 [b] 28 [c] 21 [d] 35

[e] The Additive inverse of $\frac{3}{7}$ is

[a] 3 [b] 7 [c] $\frac{-4}{7}$ [d] $\frac{-7}{4}$

4. Simplify

[a] $\frac{-4}{5} \times \frac{3}{7} \times \frac{15}{16} \times \left(\frac{-14}{9}\right)$ **(NCERT)** [b] $\left\{\frac{9}{16} \times \frac{4}{12}\right\} + \left\{\frac{9}{16} \times \frac{-3}{9}\right\}$ **(NCERT)**

[c] $\frac{5}{6} + \left(\frac{-2}{5}\right) + \left(\frac{-2}{15}\right)$ [d] $\left(\frac{25}{8} \times \frac{2}{3}\right) - \left(\frac{3}{5} \times \frac{-10}{9}\right)$

[e] $\left(5 \times \frac{2}{15}\right) + \left(6 \times \frac{2}{9}\right)$

5. What Should be added to $\frac{5}{7}$ so get $\frac{2}{-3}$.

6. By What number should we multiply $\frac{-8}{13}$ so that the product may be 24?

7. Find the five rational number between -2 and 0. **(NCERT)**

8. Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$. **(NCERT)**

NCERT CORNER

Exercise 1.1

1. Name the property under multiplication used in each of the following

(i) $\frac{-4}{5} \times 1 = 1 \times \frac{-4}{5} = -\frac{4}{5}$ (ii) $-\frac{13}{17} \times \frac{-2}{7} = \frac{-2}{7} \times -\frac{13}{17}$ (iii) $\frac{-19}{29} \times \frac{29}{-19} = 1$

2. Tell what property allows you to compute $\frac{1}{3} \times (6 \times \frac{4}{3})$ as $(\frac{1}{3} \times 6) \times \frac{4}{3}$.

3. he product of two rational numbers is always a .....