COL215 : Assignment 2 Report

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1 Problem Description

Given a function with 0s, 1s, and x's for each input combination, print the terms of the expanded function, where each cell is expanded to a legal region of maximum size.

- Expanded terms set should cover all the cells with '1'. No such condition for cells with 'x'.
- You can use the K-map to display the given function and expanded terms (if the number of variables is small enough), but your implementation should work for a function with any number of input variables.

2 Approach

Corresponding to any term of length n, we can expand it in n ways, by removing, at a time only one input variable. But expansion is possible only when corresponding the input variable that we are removing, there exist min-terms in the given function which when added give the term in which that particular input variable that is being removed is primed.
So we start with a term, consider all possible expansions of it, and then recursively find which expansion will have the least number of literals, and then return that particular expansion. Formally, Lets consider N input variables, and let term have n input variables.
So, term = $x_1x_2 \dots x_n$
Let A$(x_1x_2 \dots x_a)$ =$\{n : x_1x_2 \dots x_a$ can be expanded to $x_1 \dots x_{n-1}x_{n+1} \dots x_a\}$
Let $MinTerms(x_1x_2 \dots x_a)$ =$\{x_1 \dots x_a b_{a+1} \dots b_N : b_i$ is either an input variable not in term or its prime$\}$
A term $x_1x_2 \dots x_a$ can be expanded only if there exists an i such that $MinTerms(x_1x_2 \dots x_i' \dots x_a)$ is a subset of the function. Where function is a set of minterms.
In such a case, term can be expanded to term'=$x_1x_2 \dots x_{i-1}x_{i+1} \dots x_n$ So Let F be the function that gives the maximally expanded term for a given function.
We can define F recursively as:-
F$(x_1x_2 \dots x_n)$=$x_1x_2 \dots x_n$ if term can not be expanded, else

F$(x_1x_2\dots x_n)$ = F$(x_1x_2\dots x_{a-1}x_{a+1}\dots x_n)$
where $a \in A(x_1x_2\dots x_n)$ and

$$\mathrm{len}(F(x_1x_2\dots x_{a-1}x_{a+1}\dots x_n)) \leq \mathrm{len}(F(x_1x_2\dots x_{i-1}x_{i+1}\dots x_n)) \quad (1)$$

for all $i \in A(x_1x_2\dots x_n)$

We then evaluate F for every term in the function and output the result.

3 Test Cases

```
Input:
func_TRUE: ["a'bc'd'", "abc'd'", "a'b'c'd", "a'bc'd", "a'b'cd"]
func_DC: ["abc'd"]
Output:
["bc'", "bc'", "a'c'd", "bc'", "a'b'd"]
```

```
Input:
func_TRUE: ["a'b'c'", "a'bc'", "ab'c'", "a'b'c", "a'bc"]
func_DC: ["ab'c"]
Output:
["b'", "a'", "b'", "b'", "a'"]
```

```
Input:
func_TRUE: ["a'bc'd'", "abc'd'", "ab'c'd", "a'b'cd", "ab'cd", "a'b'cd'"]
func_DC: ["a'b'c'd'", "a'b'c'd", "a'bcd'"]
Output:
["a'd'", "bc'd'", "b'd", "b'd", "b'd", "a'd'"]
```

4 Comments

4.1 Do All Expansions result in an identical set of terms?

All expansions don't result in an identical result of terms. We can see this in the following figure. Here we can see that the term $a'b'c'd$ can be expanded to either $a'c'd$ or $a'b'd$, Both of which are maximal expansions for $a'b'c'd$.

cd \ ab	00	01	11	10
00	0	1	1	0
01	1	1	x	0
11	1	0	0	0
10	0	0	0	0

4.2 Are all expansions equally good, assuming that our objective is to maximally expand each term?

No, all expansions are not equally good, because some expansions may further expand. According to our output specification, we ought to choose the expansion which can further expand the maximum. Our algorithm ensures this by recursively choosing the expansion which will give the least literals in the final answer. In the above figure, we can see $a'bc'd$ can be expanded to $a'c'd$, $a'bc'$ or $bc'd$. Among these, $a'c'd$ can not be expanded further. But $a'bc'$ and $bc'd$ can be further expanded to bc' which is the maximally expanded region. Hence, we can see that all expansions are not equally good.