Robot Perception and Control

Robot Perception in 3D

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Homogeneous Transformations

Rigid motions can be represented in set of matrices of the following form so that composition of rigid motions can be reduced to matrix multiplication.

$$\mathrm{H} = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}, R \in SO(3), d \in \mathbb{R}^3$$

This represents a homogeneous transformation matrix H, where R is a rotation matrix from the special orthogonal group SO(3), and d is a translation vector in 3D. The inverse transformation is given by:

$$\mathrm{H}^{-1} = \begin{bmatrix} R^\intercal & -R^\intercal d \\ 0 & 1 \end{bmatrix}$$

Homogeneous Transformations

The most general homogeneous transformation that we consider may be written as:

$$\mathrm{H}_1^0 = \begin{bmatrix} n_x & s_x & a_x & d_x \\ n_y & s_y & a_y & d_y \\ n_z & s_z & a_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{n} & \mathbf{s} & \mathbf{a} & \mathbf{d} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here, $\mathbf{n}$ is a vector representing the direction of x_1 (x axis of new frame) in the original frame, $\mathbf{s}$ represents the direction of y_1, and $\mathbf{a}$ represents the direction of z_1. The vector $\mathbf{d}$ represents the position of the new origin in the original frame.

Rotation Matrices

Translation along the axis:

$$\mathrm{Trans}_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}_{y,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}_{z,c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation around the axis:

$$\mathrm{Rot}_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Rot}_{y,\beta} = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

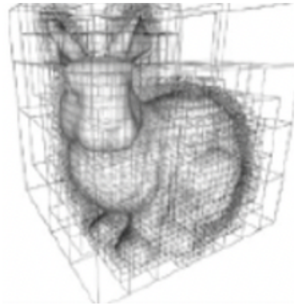
$$\mathrm{Rot}_{z,\gamma} = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Traditional 3D representations

Voxel: simple extension of concept of pixel into 3D

✓ we can reuse the thechniques (CNNs, etc.) used in images

occupies too much memory (thus usually limited to $\sim 256^3$)



Octree: hierarchical voxel

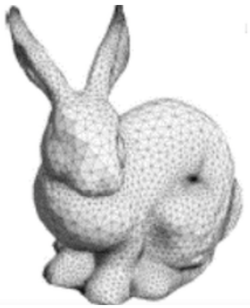
✓ high quality 3D with less memory

hard to generate and store

Point Cloud: group of points represents the 3D scene

✓ much compact compared to voxel

cannot represent the surface



Mesh: group of triangles (polygons) represents the 3D scene

✓ very compact

hard to obtain the mesh

Neural Fields

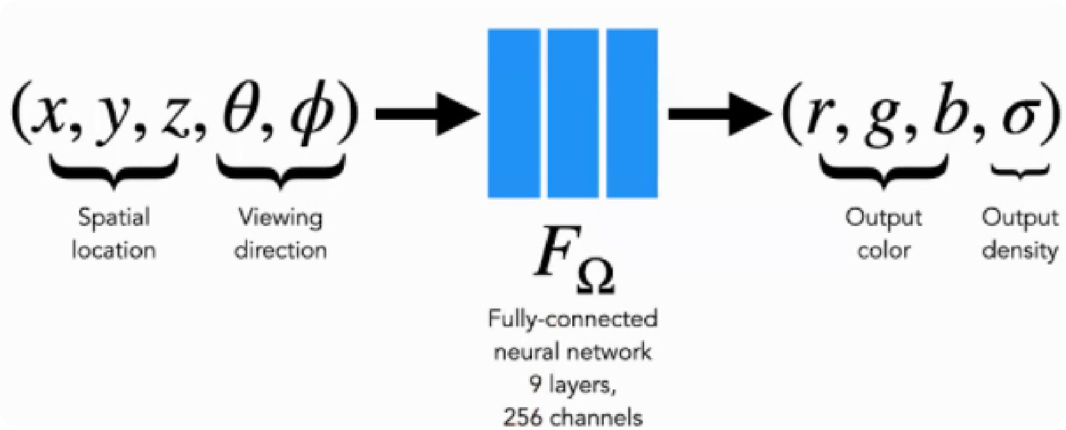
A field is a physical quantity that has a value for each point in space and time. A field can be expressed as a function that takes **spacial coordinates as independent variables**. A neural field is a field that is parameterized fully or partially by neural networks.

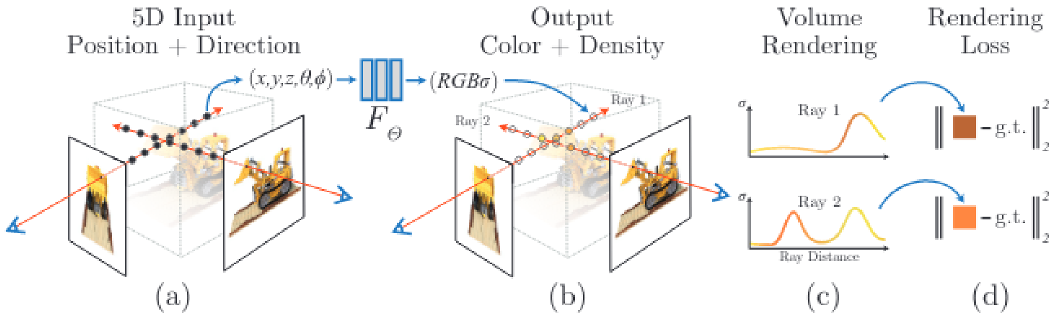
fields	input/output	example
Occupancy Field	position $\longrightarrow$ existance	Occupancy Networks
Distance Field	position $\longrightarrow$ distance	DeepSDF, PIFu
Radiance Field	position + direction $\longrightarrow$ color + density	NeRF
Scene Flow Field	position $\longrightarrow$ scene flow	Neural Scene Flow Fields
Semantic Field	position $\longrightarrow$ semantics	LeRF

NeRF arxiv ⮥

Neural Rediance Field (NeRF) is a field represented by 5D vector (3D location (x, y, z) and 2D viewing direction (θ, ϕ)) and has color $c = (r, g, b)$ and volume density σ for each point in space. NeRF approximate this continuous 5D scene representation with an MLP.

- the weights of the MLP are the *model of the world* (overfits the model to one scene).
- the most famous instance of neural fields.





Dex-NeRF arxiv

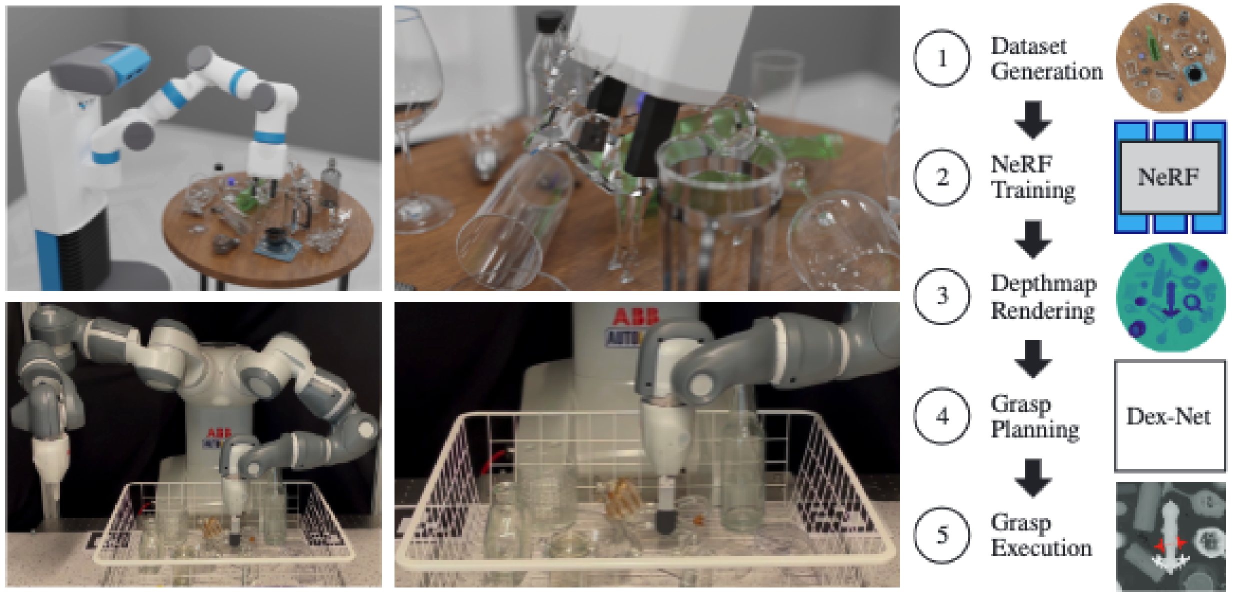
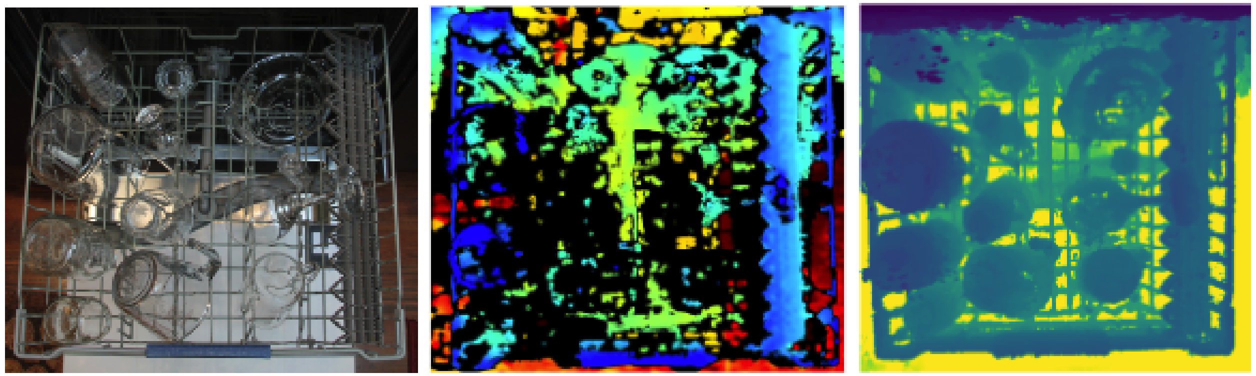


Figure 1: **Using NeRF to grasp transparent objects** Given a scene with transparent objects (left column), we the pipeline on the right to compute grasps (middle column). The top row shows Dex-NeRF working in a simulated scene while the bottom row shows it working in a physical scene.



Real Image | RealSense Depth | Depth (Dex-NeRF)

Gaussian Splatting