General Theory of Relativity:

Space-Time-Mind → ~~In~~ complete

Reality

↳ How mind interacts with time?

Just Curious

From differential Geometry - 2 & 3

$$\left(R^{mn} - \frac{1}{2} g^{mn} R\right)_{;n} = 0 = \Phi T^{mn}$$

The Stress Energy Momentum Tensor

↳ Dust & it's per volume components in 4d space.

↳ Cauchy stress Tensor — 4d extension.

↳ Components of Stress Tensor in 4d.

↳ Derivation for $\Phi = \frac{8\pi G}{c^4}$ from newton's gravitational laws.

Stress Energy Tensor:

↳ Stress imposed on a manifold of 4 dimensional spacetime.

covariant derivative of stress tensor = 0

↳ Covariant derivative

$$\boxed{\text{vector Form} = T(\vec{V}, \vec{W})}$$ $\vec{V}$ → direction inside unit 3d space time box ⊥ar to $\vec{W}$

$T^{\alpha\beta} = T(\vec{e}_\alpha, \vec{e}_\beta)$

$(T^{mn}{}_{;n}) = 0$ Torsion free Space time.

↳ 4 Momentum directional Components??

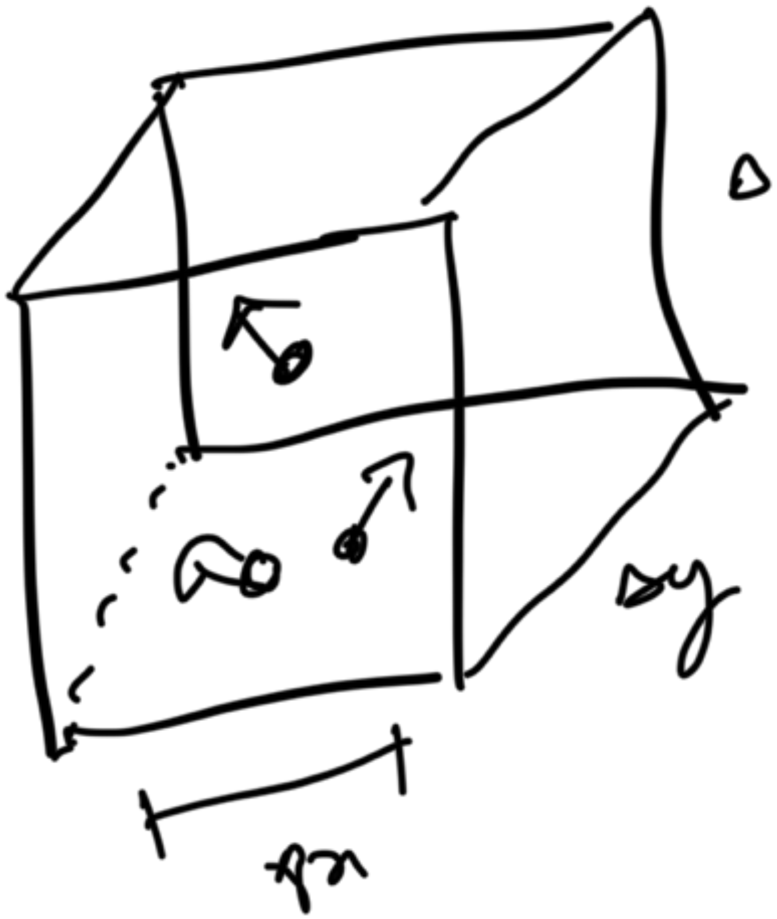
$T^{\alpha\beta}$ → Component of Stress energy tensor.

Meaning → Component of Momentum - Energy α

in the direction of β.

so, $T^{\alpha t} = \dfrac{p^{\alpha}}{dx\,dy\,dz}$ if $\alpha = \alpha$, $\beta = t$.

$\underline{p^{\alpha t}}$.

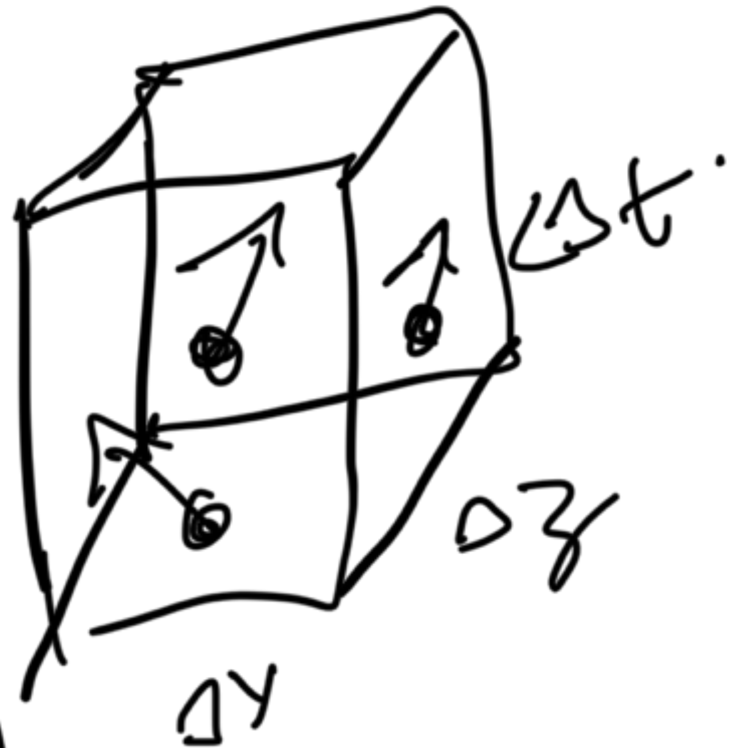


In the direction of time, the value of energy at a given point in time.

↳ A cube slice of 3d space

in 4d space time.

$$\boxed{p^{\alpha x} = \frac{p^{\alpha}}{c\Delta t\,\Delta y\,\Delta z}}$$



at a given 'x'

There will 16 components (10 independent if symmetry is considered)

T^{tx} T^{ty} T^{tz}

$$T = \begin{pmatrix} T^{tt} & T & T & T \\ T^{xt} & T^{xx} & T^{xy} & T^{xz} \\ T^{yt} & T^{yx} & T^{yy} & T^{yz} \\ T^{zt} & T^{zx} & T^{zy} & T^{zz} \end{pmatrix}$$

$T^{t\alpha} \rightarrow$ Energy Flux.

$T^{tt} \rightarrow$ Energy density.

$T^{\alpha t} \rightarrow$ Momentum density.

Revisiting Relativistic Momentum Equations:

distance 4-vector in space time.

$$\vec{X} = \begin{bmatrix} ct \\ dx \\ dy \\ dz \end{bmatrix} \qquad \text{4-velocity} \Rightarrow \vec{U} = \frac{d\vec{X}}{d\tau} = \begin{bmatrix} c\frac{dt}{d\tau} \\ \vec{U}_x \\ \vec{U}_y \\ \vec{U}_z \end{bmatrix} \qquad \frac{dt}{d\tau} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \boxed{\frac{dt}{d\tau} = \gamma}$$

4-Momentum vector $= m\vec{U}$, where m = relativistic mass.

$m = \gamma m_0$

$$P_0 = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}$$, on using binomial expansion.

$$p_0 = mc\left(1 + \frac{v^2}{2c^2} + \dots\right) \quad \text{at rest } v = 0 \quad \left.\begin{array}{l} |p_0| \approx mc \\ |p_{space}| = p \end{array}\right\} \text{say.}$$

Magnitude of Momentum in 4d

$$p_{tot}^2 = m^2c^2 - p^2$$

multiplying both sides with c^2

$$(cp_{tot})^2 = m^2c^4 - p^2c^2.$$

$cp_{tot} \rightarrow$ some energy form.

$$\boxed{E^2 = m^2c^4 - p_{space}^2 c^2}$$

Energy density is now calculated as:

$$T_{dust} = \rho\, u \otimes u \quad ;$$

To find the constant in Einstein field equation

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\Lambda = K\, T_{\mu\nu}.}$$

Scaling constant.

✓ step 1: Assume a case where Earth / Mass with space density ρ is moving through spacetime "But"!? stationary in space.

✓ step 2: Derive and obtain scalar gravitational potential from Newton's law. Get Poisson's equation.

✓ Step 3: Assume "step 1" and Find out $R_{\mu\nu}$ using boundary conditions of $T_{\mu\nu}$. Get $R_{00} = f(K)$. & $R = f(K)$

✓ step 4: Use geodesic equation (in local inertial frame) to calculate Christoffel symbols (Γ^k_{00}). Compare it with "Equation from Equivalence Rule".

✓ step 5: Calculate "R_{00}" from "Γ^k_{00}" such that $R_{00} = f(\phi)$.

✓ Final Step: Equate "R_{00}" from Step-5 and Step-3 to obtain the value of "K".

Basic static time condition: A mass moving through space-time but stationary in space. Density = ρ.

Gravitational Scalar potential (ϕ):

$$\vec{F} = \frac{GmM}{r^2}\left(-\vec{e_r}\right) \quad ; \quad g = \frac{GM}{r^2}\left(-\vec{e_r}\right)$$ ⇒ acceleration due to gravity.

Taking divergence on both sides.

$$\nabla\cdot\vec{g} = \frac{-GM}{r^2} \times \frac{\nabla\cdot\left(d\vec{x}+d\vec{y}+d\vec{z}\right)}{r} = \frac{-GM}{r^3} \times \left(\frac{dx}{dx}+\frac{dy}{dy}+\frac{dz}{dz}\right)$$

$$\vec{\nabla}\cdot g = \frac{-GM}{r^3} \times 3$$ &

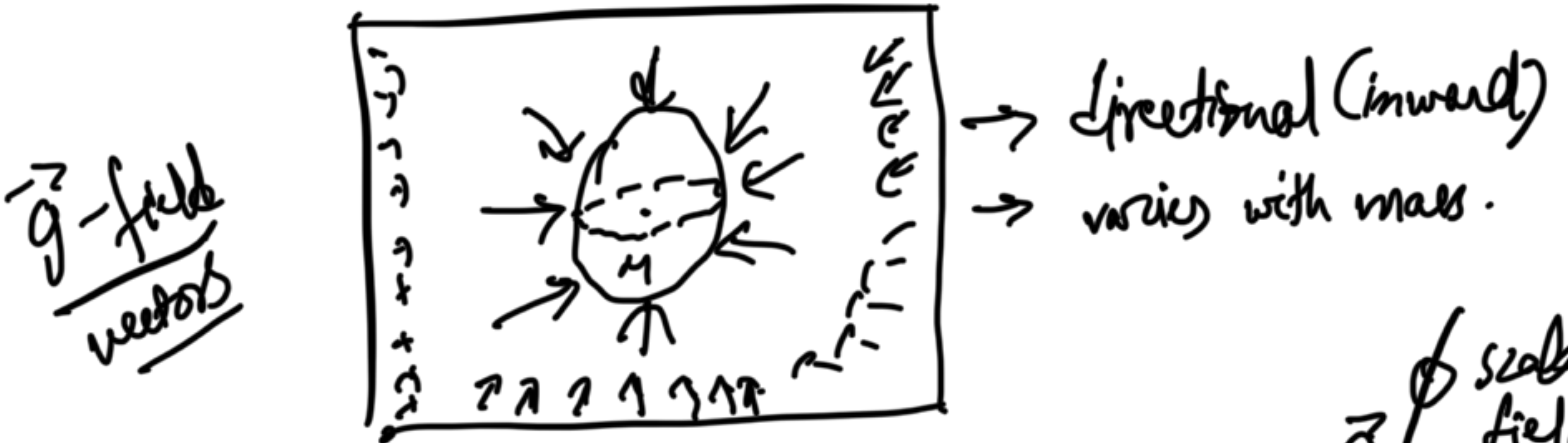
Multiply & divide by 4π

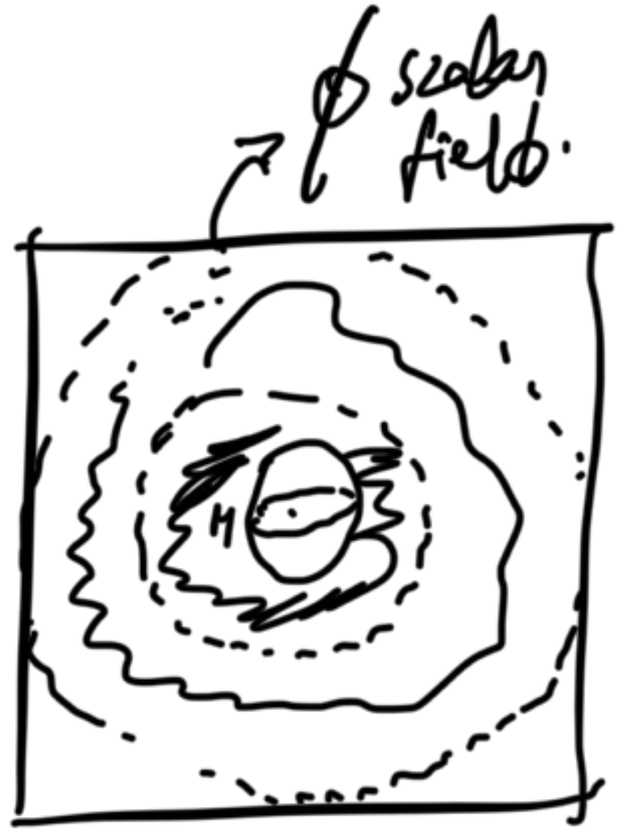
$$= \frac{-4\pi G \times M}{\frac{1}{3} \times 4\pi r^3} = -4\pi G \times \frac{M}{V} = -4\pi G\rho$$

$$\boxed{\vec{\nabla}\cdot g = -4\pi G\rho}$$

Here $\vec{g}$ is a vector field representing the strength of gravitational field.

Here $\vec{g}$ is a vector field ...





Defining a scalar function ϕ as

$$\boxed{\vec{g} = \frac{GM}{r^2}(-\vec{e_r}) = (-\nabla\phi)}$$

Now, substituting this equation in $\nabla\cdot\vec{g}$ equation.

$$\nabla\cdot\vec{g} = -4\pi G\rho \; ; \; \nabla\cdot(-\nabla\phi) = -4\pi G\rho \; ; \; -\nabla^2\phi = -4\pi G\rho$$

$$\boxed{\nabla^2\phi = 4\pi G\rho}$$

$\therefore \boxed{\nabla^2 \phi = 4\pi G \rho} \rightarrow$ Poisson's Equation

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Finding Components of "Ricci Tensor" using step-1 condition.

↳ So, the space here is stationary so, space components of the stress energy tensor are zero.

↳ Only "T_{00}" component is non zero as the earth is moving through time and mass causes distortion/stress in that direction.

$$T_{dust} = \rho \vec{U} \otimes \vec{U} \; ; \; T_{00} = \rho \begin{bmatrix} c \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} c & 0 & 0 & 0 \end{bmatrix} \Rightarrow \boxed{\rho c^2 = T_{00}}$$

$\therefore T_{\mu\nu} = \begin{bmatrix} \rho c^2 & 0 & 0 & 0 \end{bmatrix}$; now calculating spacetime.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

curvature from Ricci curvature tensor i.e "$R_{\mu\nu}$"

∴ Assume $\Lambda = 0$ (i.e non expanding universe).

from $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \Lambda = K T_{\mu\nu}$.

$$\begin{bmatrix} R_{00} & R_{01} & R_{02} & R_{03} \\ R_{10} & R_{11} & R_{12} & R_{13} \\ R_{20} & R_{12} & R_{22} & R_{23} \\ R_{30} & R_{13} & R_{32} & R_{33} \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} (R_{00} - R_{11} - R_{22} - R_{33})$$

$$= K \begin{bmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so, Non zero components $= R_{00}, R_{11}, R_{22}, R_{33}$

... of the non diagonal terms go to zero.

i.e $\boxed{R_{a0}, R_{0b}, R_{ab} (a \neq b) = 0}$

Calculating diagonal elements of the Ricci Tensor. (spatial components)

$$R_{ij} - \frac{1}{2} g_{ij} R = 0$$

here for spatial components.

$$g_{ij} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = -\delta_{ij}$$

so, $R_{ij} - \left(\frac{R}{2} (-\delta_{ij}) \right) = 0 \Rightarrow \boxed{R_{ij} = -\frac{1}{2} R \delta_{ij}} \Rightarrow$ non diagonal $= 0$.
diagonal elements $= -\frac{1}{2} R$

Also, Curvature Scalar R = Trace of the Ricci tensor matrix.

$$R = R_{00} - R_{11} - R_{22} - R_{22}.$$

$$R = R_{00} - 3(R_{ii}) \Rightarrow R = R_{00} - 3\left(-\frac{1}{2}R\right)$$

$$\Rightarrow R_{00} = R - \frac{3R}{2} \Rightarrow \boxed{R_{00} = -\frac{1}{2}R}$$

$$\boxed{R_{00} = -\frac{1}{2}R \;;\; R_{a0} = 0 \;;\; R_{0b} = 0 \;;\; R_{ij} = -\frac{1}{2}R\delta_{ij}}$$

Now the field equation can be written as.

$$\underbrace{-\frac{1}{2}R\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_{\mu\nu}} - \frac{1}{2}R\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}}_{g_{\mu\nu}} = \underbrace{\begin{bmatrix} K\rho c^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{T_{\mu\nu}}$$

Adding and comparing Top left terms

on both sides.

$$-\frac{1}{2}R - \frac{1}{2}R = k\rho c^2$$

$$\Rightarrow \boxed{R = -k\rho c^2} \quad \& \quad \boxed{R_{00} = \frac{k\rho c^2}{2}}$$

Calculate R_{00} from Geodesic equation

Assuming the path parameter is one of the co-ordinates, the geodesic equation will be

$$\boxed{\frac{d^2 x^\sigma}{d{x^k}^2} + \Gamma^\sigma_{\mu\theta} \frac{dx^\mu}{dx^k} \frac{dx^\theta}{dx^k} = 0}$$

For our assumption in Case-1: velocity through space

..., velocity through space = 0

$$u_x = u_y = u_z = 0$$

$$x^k = \tau$$

the equation becomes

$$\boxed{\frac{d^2 x^\sigma}{d\tau} + \Gamma^\sigma_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0}$$

→ This equation is under summation convention.

Also, under low velocity limit.

$r \approx 1;\ t \approx \tau;\ U^0 \simeq c$

$U^i = 0$

so, Calculate all Christoffel Symbols under "step-1 condition"

Case 1: $\mu = 0;\ \nu = 0$ then $x^\mu, x^\nu = t$, equation becomes $\Gamma^\sigma_{00}(c)(c) = \Gamma^\sigma_{00} c^2$; since $\frac{dx^\mu}{dt}, \frac{dx^\nu}{dt} = c$	Case 2: $\mu = i;\ \nu = 0$ then $\frac{dx^\mu}{d\tau} = \frac{dx^i}{d\tau}$ & $\frac{dx^\nu}{d\tau} = \frac{dt}{d\tau} = C$ $\frac{dx^i}{dt} = 0$; Since space velocity = 0 so, $\Gamma_{i0}(0)(c) = 0$; $\Gamma_{i0} = 0$

Case 3: $\mu = 0;\ \nu = j$ then $\frac{dx^0}{d\tau} = c;\ \frac{dx^\nu}{d\tau} = \frac{dx^j}{d\tau} = 0$ so, $\Gamma_{0j}(c)(0) = 0;\ \Gamma_{0j} = 0$	Case 4: $\mu = i;\ \nu = j$ then $\frac{dx^i}{d\tau} = 0$ & $\frac{dx^j}{d\tau} = 0$ so, $\Gamma_{ij}(0)(0) = 0;\ \Gamma_{ij} = 0$

Summing all the values from the table into the summation convention, the geodesic equation becomes

$$\frac{d^2x^\sigma}{dt^2} + \left(c^2\Gamma_{00}^{\sigma} + 0 + 0 + 0\right) = 0$$

$$\Rightarrow \boxed{\frac{d^2x^\sigma}{dt^2} + \left(\Gamma_{00}^{\sigma}\right)c^2 = 0} \rightarrow$$ Geodesic equation for step 1 condition.

From the "Principle of Equivalence", gravity is just fake force caused due to space time curvature.

So,

$$\vec{F} = m\vec{a}$$

$$m\vec{g} = m\vec{a}$$

Here "inertial mass" is assumed to be equal to the "gravitational mass"

so $\vec{a} - \vec{g} = 0$

but $\vec{g} = -(\nabla\phi)$ ← ϕ = scalar gravitation potential

$$a = \frac{d^2 x^\sigma}{d(x^0)^2}$$ From step-2

Also, $\dfrac{d^2 x^\sigma}{dt^2} + \Gamma^\sigma_{00} c^2 = 0$

if $\sigma = 0$; then

$\dfrac{d^2(ct)}{dt^2} = 0$; $\Gamma^0_{00} c^2 = 0$

So, only the non-temporal components are non-zero.

the equation becomes.

$$\boxed{\frac{d^2 x^k}{dt^2} + \Gamma^k_{00} c^2 = 0}$$

On Low velocity limit, the spatial part is following newton's gravity law.

$a - g = 0 \Rightarrow a + \nabla\phi = 0$

$\Rightarrow \boxed{\dfrac{d^2x^k}{dt^2} + \nabla\phi = 0}$

Comparing this equation with the geodesic equation obtained from condition in step. 1.

$$\boxed{\frac{d^2x^\sigma}{dt^2} + \Gamma_{00}^{\sigma} c^2 = 0}$$

$$\therefore \boxed{\Gamma_{00}^{\sigma} = \frac{1}{c^2}\nabla\phi} \text{ or } \boxed{\Gamma_{00}^{\sigma} = \frac{1}{c^2}\frac{\partial\phi}{\partial x^\sigma}}$$

↳ Now calculating R_{00} component of the Ricci Curvature Tensor

↳ To get R_{00}, we need to solve Riemann Curvature tensor and contract it on second index.

$\boxed{R^{d}_{\ 0d0} = R_{00}}$ → Trace summation on d index.

Calculating $R^{d}_{\ 0d0}$ from Component form of Riemann Tensor.

$$\boxed{R^{d}_{\ cab} = \partial_a\left(\Gamma^{d}_{bc}\right) - \partial_b\left(\Gamma^{d}_{ac}\right) + \Gamma^{i}_{bc}\Gamma^{d}_{ai} - \Gamma^{j}_{ac}\Gamma^{d}_{bj}}$$

here $c, b = 0$

$$\text{so,}\quad R^{d}_{\ 0d0} = \partial_d\left(\Gamma^{d}_{00}\right) - \underbrace{\partial_0\left(\Gamma^{d}_{d0}\right)}_{0} - \underbrace{\Gamma^{i}_{00}\Gamma^{d}_{di}}_{0} - \underbrace{\Gamma^{\beta}_{i0}\Gamma^{i}_{0\beta}}_{0}$$

<u>Assumptions and</u> <u>Conditions</u>:

↳ Connection co-efficients are small, their products are smaller.

Ex: $\boxed{\Gamma^{i}_{00} = \partial_i^2\phi \approx 0 \; ; \text{ their products} \approx 0.}$

c^2

↳ Connection co-efficients are time independent.

thier time derivatives are zero.

$$\boxed{\partial_0(\Gamma^d_{d0}) = 0}$$

↳ Only remaining non-zero term in the summation

is $R^d_{0d0} = \partial_d(\Gamma^d_{00})$

$$R^d_{0d0} = \underbrace{\frac{\partial(\Gamma^0_{00})}{\partial x^0}}_{0} + \frac{\partial(\Gamma^1_{00})}{\partial x^1} + \frac{\partial(\Gamma^2_{00})}{\partial x^2} + \frac{\partial(\Gamma^3_{00})}{\partial x^3}$$

(Time Independent)

$\Rightarrow R^{\alpha}_{0\alpha0} = \frac{\partial \Gamma^{\alpha}_{00}}{\partial x^{\alpha}} = \nabla \Gamma^{\alpha}_{00}$

$$\boxed{\therefore R_{00} = R^{\alpha}_{0\alpha0} = \nabla \Gamma^{\alpha}_{00} = \frac{\partial \Gamma^{\alpha}_{00}}{\partial x^{\alpha}}}$$

But from results obtained by
comparing Geodesic equation to Equivalence principle equation

$$\boxed{\Gamma^{\sigma}_{00} = \frac{1}{c^2}\frac{d\phi}{dx^{\sigma}}} \quad \text{or} \quad \boxed{\Gamma^{\sigma}_{00} = \frac{1}{c^2}\nabla\phi}$$

Substituting in R_{00} equation, it becomes:

$$R_{00} = \frac{\partial \Gamma^{\alpha}_{00}}{\partial x^{\alpha}} = \frac{\partial}{\partial x^{\alpha}}\left(\frac{d\phi}{dx^{\alpha}}\right) \times \frac{1}{c^2}$$

$$= 1 \times \partial^2\phi = 1 \times \nabla^2\phi$$

$$\frac{\quad}{c^2 (\partial x^d)^2} \quad c^2$$

$$\therefore R_{00} = \frac{1}{c^2} \nabla^2 \phi$$

$$\boxed{\nabla^2 \phi = c^2 R_{00}}$$

Summarizing All equations from All the 5 steps

Step-2 $\rightarrow$ $\boxed{\nabla^2 \phi = 4\pi G \rho}$

Poisson's Equation.

$\boxed{R_{00} = \frac{K \rho c^2}{2}}$ $\rightarrow$ Step 3

From Stress Energy Tensor

$\boxed{\nabla^2 \phi = c^2 R_{00}}$ Step-5

Now substituting "$\nabla^2\phi$" value and "R_{00}" value in step-5 equation.

$$\Rightarrow \nabla^2\phi = c^2 R_{00}$$

$$\Rightarrow 4\pi G\rho = c^2 \times \frac{k\rho c^2}{2}$$

$$\Rightarrow 8\pi G\cancel{\rho} = kc^4 \times \cancel{\rho}$$

$$\boxed{k = \frac{8\pi G}{c^4}}$$

The coupling constant for Stress Energy Tensor

$\therefore$ The Scaling constant for Stress Energy tensor is

$$K = \frac{8\pi G}{c^4} = \frac{8\pi \times 6.67 \times 10^{-11}}{(3\times 10^8)^2}$$

$$\Rightarrow \boxed{K \approx 18.626 \times 10^{-37} \text{ units}}$$

So, it takes a Lot of Mass to cause a significant curvature in space time.

Finally the Einstien Field Equation can be written as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = K T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \Lambda$$

Since

$$k = \frac{8\pi G}{c^4}$$

Final Form.

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

Here $+g_{\mu\nu}\Lambda$ is also valid.

Trace Reversed Form:

We have

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \Lambda = k T_{\mu\nu}$$ → cosmological value subtracted

Take inverse metric on

both sides.

$$R_{\mu\sigma} g^{\mu\sigma} - \frac{1}{2} g_{\mu\sigma} g^{\mu\sigma} R - g^{\mu\sigma} g_{\mu\sigma} \Lambda = K T_{\mu\sigma} g^{\mu\sigma}$$

on contracting a tensor with metric, the components left are diagonal elements

$$R^{\mu}_{\mu} - \frac{1}{2} R \delta^{\mu}_{\mu} - \Lambda \delta^{\mu}_{\mu} = K T^{\mu}_{\mu}$$

here R^{μ}_{μ} = Trace of Ricci tensor matrix $= R$ = Curvature scalar.

T = Trace of stress tensor.

Also, $\delta^{\mu}_{\mu} = 1+1+1+1 = 4$.

$$R - \frac{1}{2} R \times 4 - 4\Lambda = KT$$

$$R - 2R - 4\Lambda = KT$$

$$\Rightarrow \boxed{R = -4\Lambda - KT}$$

Substitute R in the first equation (Main EFE).

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} \times -1(4\Lambda + KT) - g_{\mu\upsilon}\Lambda = KT_{\mu\upsilon}$$

$$\Rightarrow R_{\mu\upsilon} + 2\Lambda g_{\mu\upsilon} + \frac{1}{2} g_{\mu\upsilon} KT - g_{\mu\upsilon}\Lambda = KT_{\mu\upsilon}.$$

$$\Rightarrow R_{\mu\upsilon} + \frac{1}{2} g_{\mu\upsilon} KT - g_{\mu\upsilon}\Lambda = KT_{\mu\upsilon}.$$

so, on rearranging.

$$\boxed{R_{\mu\nu} = K\left(T_{\mu\nu} - \frac{T}{2} \times g_{\mu\nu}\right) - \Lambda g_{\mu\nu}}$$

↳ Trace reversed form of Einstein Field equation.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

$$R_{\mu\nu} g^{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\mu\nu} R - g_{\mu\nu} g^{\mu\nu} \Lambda$$

$$= \frac{8\pi G}{\ } T_{\mu\nu} g^{\mu\nu}$$

$$\underline{R^{\mu}_{\mu}} - \frac{1}{2}(4R) - 4\Omega = \underbrace{\frac{8\pi G}{c^4}}_{c^4} \underline{T^{\mu}_{\mu}}$$

$\frac{8\pi G}{c^4} \to K$

$R \to$ Curvature scalar.

$T^{\mu}_{\mu} \to$ Trace of the stress-tensor matrix

$$R - 2R - 4\Omega = KT$$

$$-R = K[T] + 4\Omega$$

$$R = -[K[T] + 4\Omega]$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \Lambda = k T_{\mu\nu}.$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \cdot [kT + 4\Lambda] - g_{\mu\nu} \Lambda = k T_{\mu\nu}.$$

$$R_{\mu\nu} = k T_{\mu\nu} - \frac{g_{\mu\nu} kT}{2} - 2\Lambda + g_{\mu\nu} \Lambda$$

$$R_{\mu\nu} = k \left[T_{\mu\nu} - \frac{kT g_{\mu\nu}}{2} \right] + g_{\mu\nu} \Lambda$$

Formulating a problem in terms of stress tensor and trace reversed form $\rightarrow$?

$$g_{\mu\nu}(\text{Schwarzschild}) = \left[\left(1-\frac{r_s}{r}\right)dt^2, -\left(1-\frac{r_s}{r}\right)d\right.$$

$$\left., r^2 d\theta^2, r^2 \sin^2\theta\, d\phi\right].$$