Lab -3

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MC312, Modeling and Simulation

In this lab we numerically and analytically analyze the diffusion of innovation models. We will try to analyze the diffusion of innovation through mixed influence model(bass model). We have done qualitative analysis of how the speed of adoption of the new product changes as we vary the parameters p and q.

I. INTRODUCTION

Diffusion of Innovations is a framework that studies the spread of new ideas or behaviour of new products in social system. People usually adopt any new idea or product through mutual exchange and communication in social systems. This process of adoption of new ideas and products is knows as diffusion of innovations.

II. MODELS

The diffusion of innovations is mathematically represented by differential equation,

$$\frac{dN}{dt} = \alpha(t) \cdot (C - N(t)) \tag{1}$$

Here, $N(t)$ represents the number of adopters of the product at time t. C is the total number of potential adopters. And $\alpha(t)$ is coefficient of diffusion.
It is quiet clear that, the solution of Eq. (1) depends on definition of $\alpha(t)$. Each model discussed here differs only by the definition of $\alpha(t)$. We solve the differential equation to find the fractional adopters of innovations of total population as function of time. We would analyze how the fraction changes as the definition of $\alpha(t)$ changes.

A. Innovators Dominate

In this case, we define $\alpha(t) = p$, where p is a constant which depicts the probability of initial purchase. The mathematical modelling for this can be done as:-

$$\frac{dn}{dt} = p \cdot (C - N(t)) \tag{2}$$

Here diffusion coefficient is constant. It implies that the diffusion is independent of other adopters of the innovations. Hence, it captures the spread in innovators.

B. Internal Influence or Imitators Dominate

For the internal influence ,the value of $\alpha(t) = \frac{q.N(t)}{C}$, for which the mathematical modelling can be done as follows,

$$\frac{dN(t)}{dt} = \frac{q.N(t)}{C}(C - N(t)) \tag{3}$$

Diffusion coefficient in this model is function of the current fractional population of adopters. This means that the spread is influenced by the adopters of innovation.

C. Mixed Influence Model: Bass Model

This model was proposed by Bass. Here, $\alpha(t) = p + \frac{q.N(t)}{C}$. So, the differential equation is given by,

$$\frac{dN(t)}{dt} = (p + \frac{q.N(t)}{C})(C - N(t)) \tag{4}$$

As the name suggests, mixed influence model incorporates both external and internal influence in calculations. This is the usual case observed in the social system. We have just added the coefficient of both models to capture mixed behaviour.

III. RESULT

We had plotted the no of adopters i.e. N(t) wrt time for different values of p and q. The below plot shows this:-

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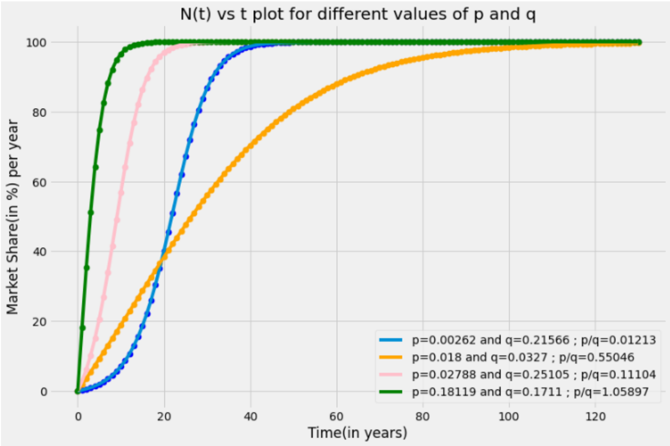

FIG. 1: Plot of N(t) vs t for different values of p and q

We can observe from the above plot that each of them reaches to 100 % market share at different values of time. Also the slope of the graph for each of them is varying in different manner.For eg, for p/q=0.01213 , the slope first increases , then reaches some peak value and then starts decreasing while for p/q= 1.05897, the slope is decreasing throughout the time(as initially only it had its maximum value). For analyzing the position of the peak of the slope(i.e. at what time the speed of adoption is maximum), we have plotted dN(t)/dt for the above 4 cases in the below figure:-
We can observe from the above figure that when p/q

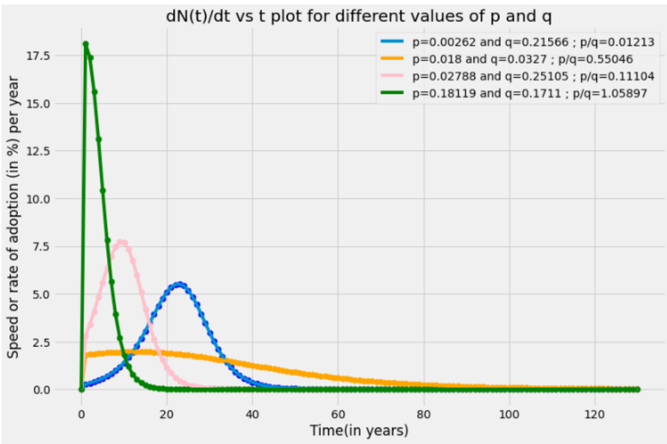

FIG. 2: Plot of dN(t)/dt vs t for different values of p and q

ratio increases , the peak of dN(t)/dt vs t plot(i.e. time at which max value of dN(t)/dt occurs) shifts leftwards i.e. the peak in value of dN(t)/dt occurs at an earlier time.

This behaviour can be analysed and understood by solving for the roots of the equation (4), solving for N(t) where dN(t)/dt=0.So:-

$$\frac{dN(t)}{dt} = (p + \frac{q.N(t)}{C})(C - N(t)) = 0$$

So $dN(t)/dt = 0$ when $N(t) = C$ and $N(t) = \frac{-p.C}{q}$ Hence , we can see from (4) that $dN(t)/dt$ vs $N(t)$ plot will be a downward parabola with roots at C and $\frac{-p.C}{q}$. Also we know that N(t) i.e(no of adopter at time t) will be non-negative.But for analysing the position of peak, I have plotted the parabola for -ve value of N(t) as well.I have plotted for all the above 4 different values of p and q.

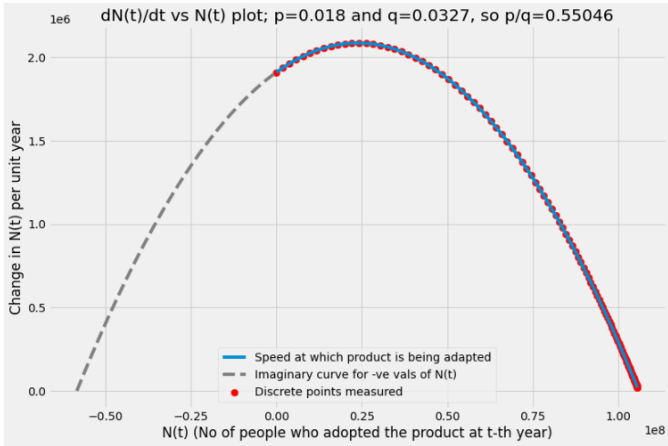

FIG. 3: Rate of adoption vs N(t) for p=0.018 and q=0.0327 and p/q=0.55046

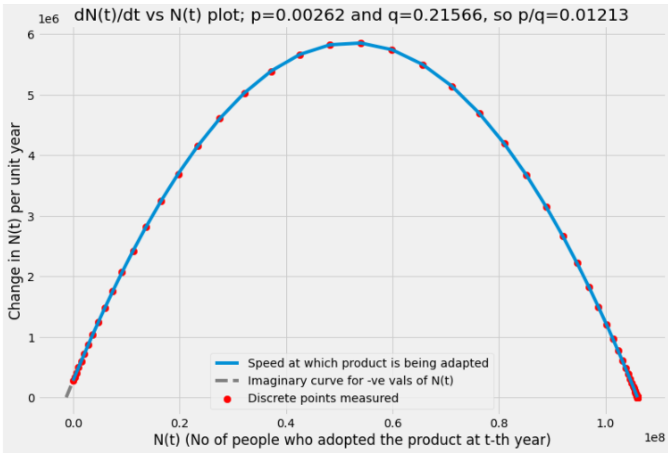

FIG. 4: Rate of adoption vs N(t) for p=0.00262 and q=0.21566 and p/q=0.01213

We observe that the root at the right i.e. N=C is fixed in all the cases, by changing the parameters p and q, we are only changing the location of the left root i.e. $N = \frac{-pC}{q}$. So by this we can say that as p/q increases, the left root shift more towards left and the peak also shifts left, and similarly when p/q decreases, the left root shift towards right and the peak also shifts right.

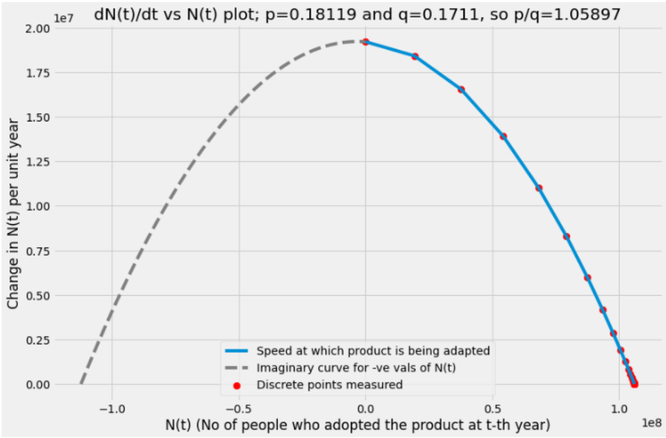

FIG. 5: Rate of adoption vs N(t) for p=0.18119 and q=0.1711 and p/q=1.05897

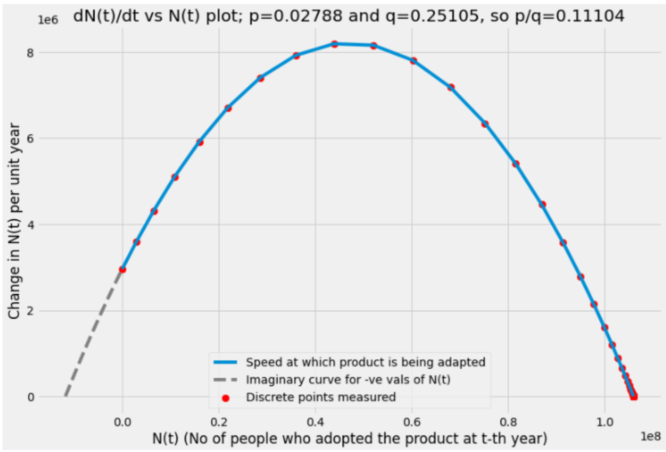

FIG. 6: Rate of adoption vs N(t) for p=0.027877 and q=0.25105 and p/q=0.11104

As dN(t)/dt vs N(t) is a quadratic equation with two roots. We can also quantitatively give the value of N when the peak value of dN(t)/dt will occur i.e. it will be at $N = (1 - \frac{p}{q})C$ and the maximum value of dN(t)/dt itself will be $\frac{-(p+q)^2 C}{4q}$.
Because for a downward parabola($ax^2 + bx + c = 0$) the peak occurs at $x = -b/2a$ and max value $= -D/4a$.

IV. CONCLUSION

The time and the peak value depends upon the interaction between the parameters p and q. Higher p value means more number of innovators in the population due to that the peak value of the rate of adoption occurs at an early stage of time. The higher value of $|p/q|$ leads to the shift of the peak dN/dt(in dN/dt vs $N(t)$) towards the left, which in turn leads to higher initial value of dN/dt.