Lab -5

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MC312, Modeling and Simulation

In this lab,we studied the spread of a disease in a closed and small population (like that in a hostel) using a differential equation based model and analyzed the outcomes by varying the intrinsic factors like R_0(Basic Reproduction number) and S_0(initial no of susceptibles). Also, we analyzed the effect of vaccination rates on the intensity and size of the epidemic.

Larger value of R_0, will increase the intensity and size of the epidemic(total no of people who got infection at any time t), also larger R_0 means the speed at which infection is spreading is high. We say that the disease has gone when the no of infected persons = 0 , there can be cases when disease has gone not because there were no susceptibles, but because there were no infected. Also in vaccination effect cases, we see that no of susceptibles become 0 before the no of infected becomes 0.

I. INTRODUCTION

In first part we had to study how the change in R_0 will result in change in the total no of infections. Also we analyzed how the change in the initial no of susceptibles will result in change in the total infections. We adjusted the SIR Model for considering the vaccination effect.

II. MODEL

A. SIR Model

We used SIR model for the first part of problem, where there are three compartments of susceptibles, infected and recovered. Assumptions made under this model is that the disease is spread in a closed environment i.e. no births, deaths, immigration and migration is possible in such environments. Initially, everybody is assumed to be prone to disease. In addition to that, we further assume that a recovered patient has immunity towards the disease and recovered patient can not be infected again in future. In such situations we can classify the population into three categories:-

- Susceptible (S): Population that has no immunity
- Infected (I): Population that has disease and can spread it to others.
- Recovered (R): Population that has recovered and now immune towards disease.

We can write the following equations :-

$$\frac{ds}{dt} = -\beta si \tag{1}$$

$$\frac{di}{dt} = \beta si - \alpha i \tag{2}$$

$$\frac{dr}{dt} = \alpha i \tag{3}$$

Where s,i,r is the proportion of population which is susceptible,infected and recovered, respectively at any time t . Also β is transmission coefficient ,it is measure of infectiousness of disease and α is the recovery rate of any arbitrary infected person from the disease.

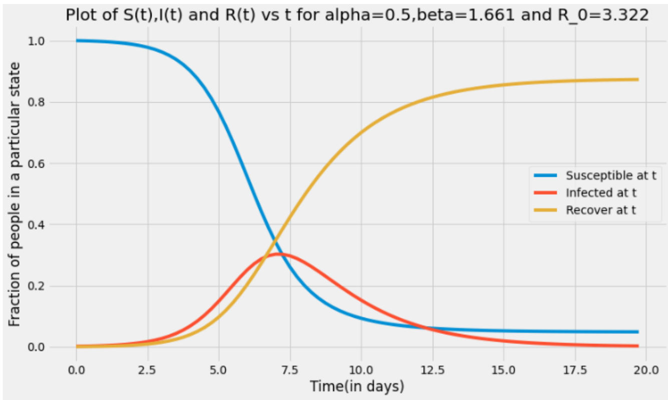

FIG. 1: $S(t), I(t) and R(t)$ vs t plot when $R_0 = 3.322$, N = 762, $I_0 = 1$

B. Vaccination effect

1. With immediate immunization

In this case, we assume that immunization starts immediately after vaccination.

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2. *Possibility 1: In which constant no of people are vaccinated per day*

. For eg if 15% is vaccination rate per day, it means that a constant no of people i.e. 15% of the entire population which are susceptible are vaccinated each day . So we can form another compartment for these people which have become immune due to vaccination.

3. *Possibility 2: In which vaccination depends on no of susceptible*

In this case , the no of people vaccinated at each day depends on the no of susceptible at that day.That is vaccination is happening at a constant rate. We can consider the people who are vaccinated will become immediately immune and hence we can move those people to "recovered" compartment from "susceptible" compartment. We can write the following eqns:-

$$\frac{ds}{dt} = -\beta si - vs \tag{4}$$

$$\frac{di}{dt} = \beta si - \alpha i \tag{5}$$

$$\frac{dr}{dt} = \alpha i + vs \tag{6}$$

where v is the vaccination rate

C. Partially effective vaccination

In this case, we assume that vaccine is partially effective and vaccinated population becomes susceptible at rate of . In order to incorporate partial vaccination, we have to add a compartment that represents vaccinated population. Suppose V denotes the total vaccinated population at any point of time. We write the new equations for each compartments as following:-

$$\frac{ds}{dt} = -\beta si - qs + \mu v \tag{7}$$

$$\frac{di}{dt} = \beta si - \alpha i \tag{8}$$

$$\frac{dr}{dt} = \alpha i \tag{9}$$

$$\frac{dv}{dt} = qs - \mu v \tag{10}$$

where q is the vaccination rate and μ is the rate at which vaccinated people are again becoming susceptible.

D. Lockdown effect

Lockdown affects the rate at which contacts happen. Reduction in the number of contacts helps in reducing the spread rate i.e. it will basically decrease β(no of contacts per unit time which results in transmission). .As $R(reproductionnumber) = \frac{\beta}{\alpha}$, it results in lower values of reproduction number as well.

E. Lockdown effect with people's behaviour

Lockdown does not lead to an immediate reduction in the number of contacts. is at a high value but slowly goes down to a lower value due to implementation of lockdown by the govt. And after some time unlock is initiated and people then start to venture out slowly due to which value of β increases slowly.So we can assume *beta* to a function of time and incorporate this effect into our model.

III. RESULTS

By simulating the equation (1), (2), (3) numerically on computer we got the following plots for the dependence of total infected on R_0(Basic Reproduction Number) and the dependence of the time of maximum number of people infected on R_0.

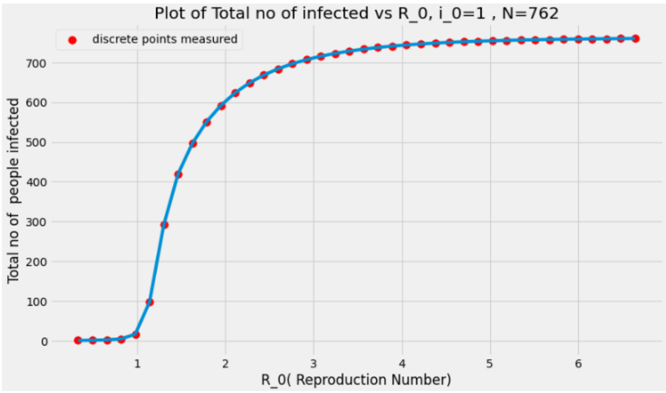

FIG. 2: Plot of size of epidemic Vs R_0, N(total population) = 762, i_0(initial number of infected cases) = 1

We observe that the size of epidemic increases sharply when we increase R_0 slightly above 1 and after certain

R_0 size of epidemic has a decreasing rate of growth and eventually saturates to the total population value that is N when R_0 has a very high value. We see that it gives a S shaped curve. Also we see that for R_0 less than 1 total number of people infected are close to 0(there is no epidemic).

Below is the plot for the time at which the peak of number of people infected at any time t occurs for various different values of R_0.

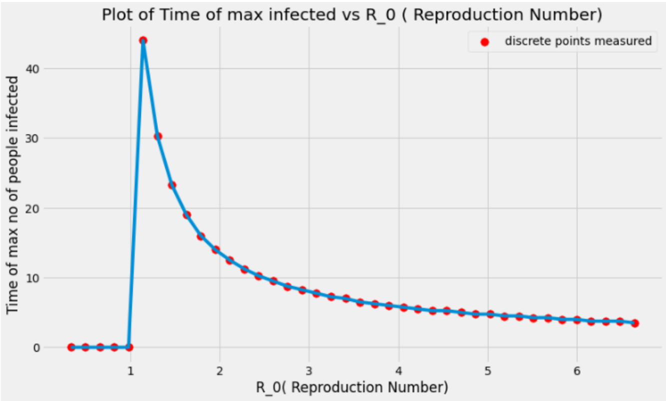

FIG. 3: Plot of time at which max people are infected Vs R_0, N = 762, $I_0 = 1$

For R_0 less than 1 number of people infected remains close 0 for all time so no considerable peak occurs at any time so we assume it to be at time = 0.

As we increase R_0 from initial value of 1 the time at which the max no of people gets infected sharply decreases(that is the peak occurs at an earlier time). As R_0 takes very large value the time at which peak occurs converges to some value.

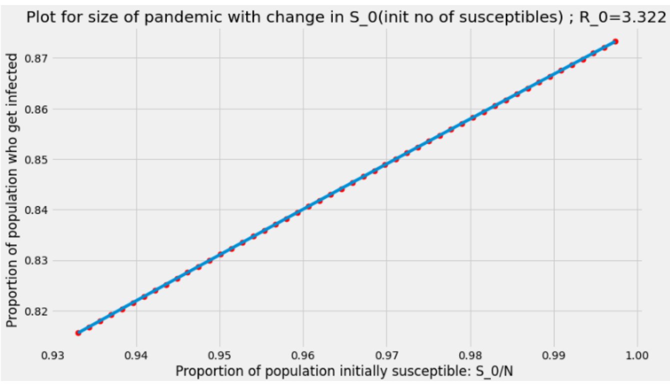

FIG. 4: Plot of size of pandemic Vs s_0,N = 762, $I_0 = 1$

In the above figure we have kept the total population(N) constant and are changing s_0 and i_0 to change the initial number of susceptible. We see that the proportion of the total number of people infected linearly increases with the proportion of initially susceptible population.

Vaccination Effect: Immediate immunization

We explore the vaccination effect in the below plots. First we consider the case of immediate immunization that is after taking the vaccine, the person immediately becomes immunized to the disease.

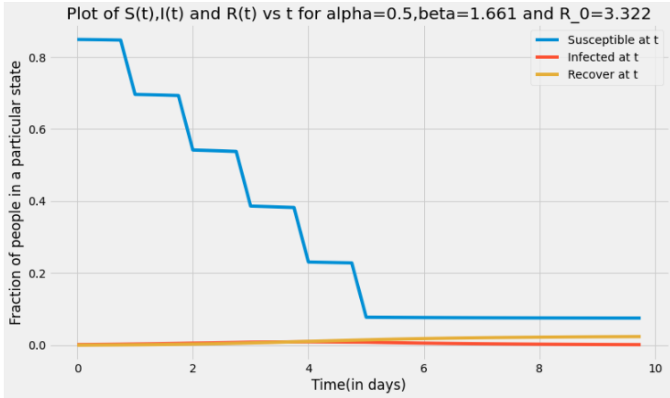

FIG. 5: $i(t), r(t), s(t)$ Vs time plot for 15% vaccination rate per day, N = 762, $I_0 = 1$

Above plot shows vaccination rate of 15% per day which means that 15% of the population(which is susceptible) is getting vaccinated. It means that after each day 15% of further population gets immunity from the disease and will never get the disease. This decreases the number of susceptible sharply after each day, which in turn results in number of susceptible becoming 0%, before the number of infected becomes 0%. Which means that after certain point spread of disease will stop because there will not be any susceptible to be infected, and the currently infected people will eventually recover.

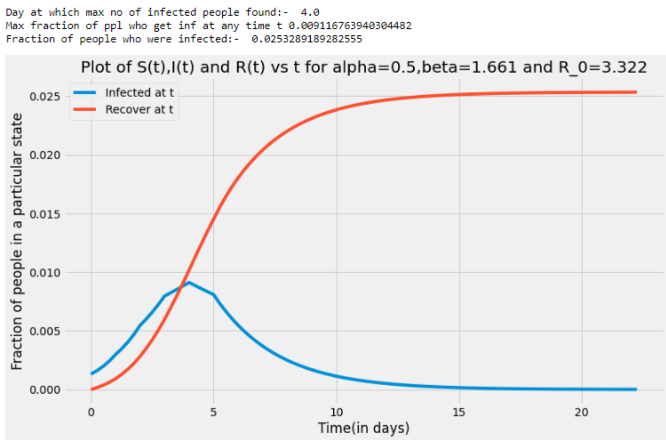

FIG. 6: $i(t)$,$r(t)$ Vs time plot for 15% vaccination rate per day

Above figure is zoomed in version of 5. From 5 and 6 we see that number of infected goes to 0 at about 20th day, whereas the number of susceptible becomes 0 at the 5th day only.As we see that the proportion of recovered people converge to 0.025 as $t \to \infty$, i.e. the size of the epidemic in this case is 2.5% (means that only 2.5% of the population gets infected at any point in time).And proportion of max infected people at any point in time is 0.0091 and the duration the epidemic(when no of infected people becomes 0) is about 5 days(the plot for no of infected people is close to 0 only, for the entire duration, but become exactly 0 after 5 days) .But in this vaccination case we have to note that no of susceptible becomes 0 before no of infected. When no vaccination was done on the same population size(N=762) and same no of initially infected people(i.e. $i_0 = 1$) and same $R_0 = 3.222$, as in 1, the size of epidemic was 0.9, which means 90% of the total population was infected at any point in time. Also we see that the max no of infected at any point in time for no vaccination case was 0.3 and duration of epidemic(time it takes to make no of infected to become 0) was more than 20 days.The intensity , size and duration of epidemic in no vaccination case is large compared to the vaccination case of immediate immunization.

Vaccination Effect: Delayed immunization

In the below plot we are considering that immunization happens after 3rd day of vaccination. 15% of the population takes vaccine each day(which are susceptible)i.e constant no of people will take the vaccine each day , and all of that people will be susceptible i.e. we are giving vaccine to only susceptible people .

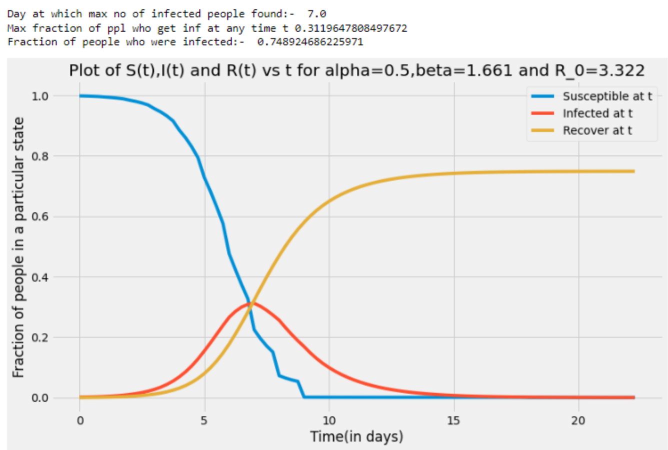

FIG. 7: $i(t), r(t), s(t)$ Vs time plot for 15% vaccination rate per day(immunization happens after 3rd day of vaccination), N = 762, $I_0 = 1$

So here the critical part is that the 15% of the population which had taken the vaccine on 1st day will be immunized at the 4th day, during this time interval some of these vaccinated people can become infected. So at the 4th day,only the uninfected people who had taken vaccine on 1st day will be immunized. So those who had taken vaccine on dth day will get immunized on (d + 4)th day if they remain uninfected. In this case the size of the epidemic is 0.75(i.e. 75% of the population), which is much greater than immediate immunization case and lower than no-vaccination case . We see that the proportion of max no of infected is about 0.3, same as in the no-vaccination case and much higher than immediate immunization case. The duration of the epidemic in this delayed immunization case is about 15 days(as no of infected people becomes 0 after 15th day., which is much greater than that in immediate immunization case(5 days) and lower than the no-vaccination case(20 days).

A. Special case of Delayed Vaccination

In the below plot we consider the case when the entire population is vaccinated at the 0th day and all of them will get immunized at the 4th day. Additionally, one person with the infection comes in the closed environment of the population at the 2nd day. So by rescaling we consider that the infectious person will be arriving at the 0th day and population becoming immune at the 2nd day.

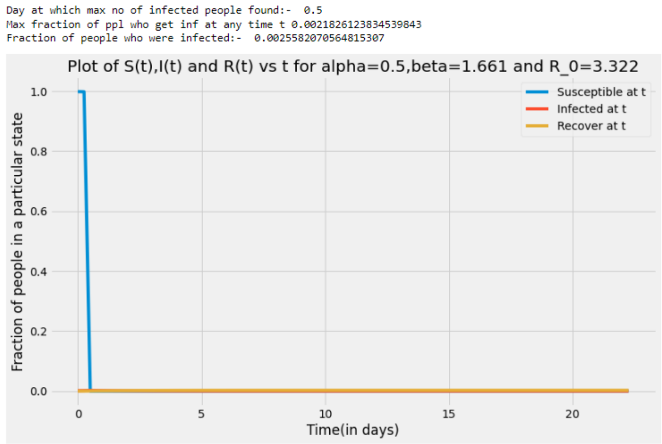

FIG. 8: $i(t), r(t), s(t)$ Vs time plot for $i_0 = 1$ and entire population becomes immune at the 2nd day

Below is the zoomed in view of 8, so that we can see $r(t)$ and $s(t)$ plot clearly.

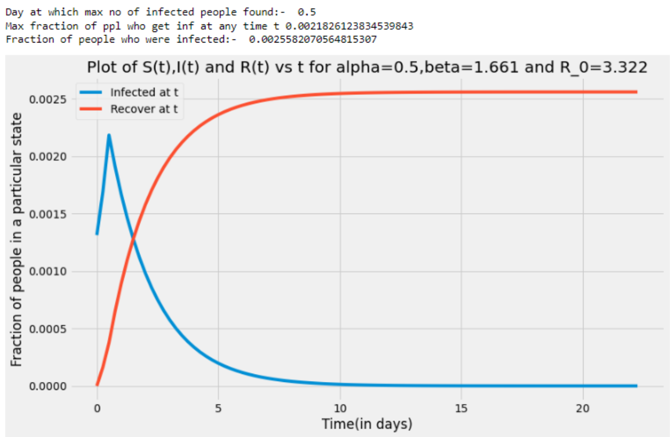

FIG. 9: $i(t), r(t)$Vs time plot for above situation

Infectious person will spread the infection till the 2nd day, after which the entire uninfected population will become immunized. Here also the number of susceptible person become 0(after 2 days) . No of infected is almost close to 0 only all the time.Proportion of max no of infected people at any time t converges to 0.0021(higher than immediate immunization) and fraction of people infected at any time is 0.0025(same as immediate immunization)

B. Partially effective vaccination

By simulating (4),(5) and (6), we get the following plot for the same initial values of i_0, s_0 and r_0 as it was in (1) :-

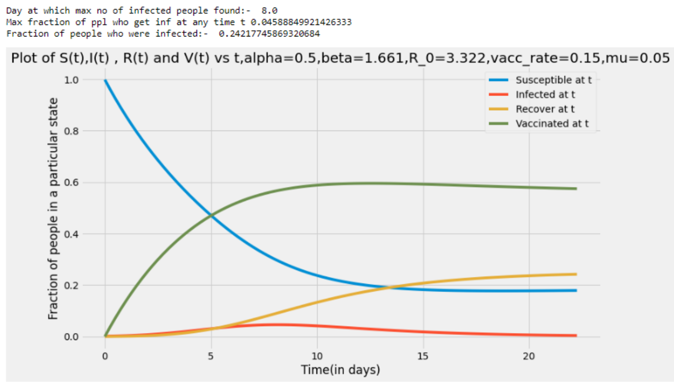

FIG. 10: $s(t), i(t), r(t)v(t)$ plot when vaccination rate=0.15 and mu=0.05

In the above case, vaccinated people are again becoming susceptible with a rate $\mu = 0.05$, we can see that partially effective vaccine is definitely better than no vaccination at all as in this plot we can see that total no of infected during the epidemic has decreased and the time at which peak no of cases occur has increased . Also we can conclude that lesser the μ, better the vaccine. Below plot is for vaccine whose $\mu = 0.1$(twice that of the previous one):-

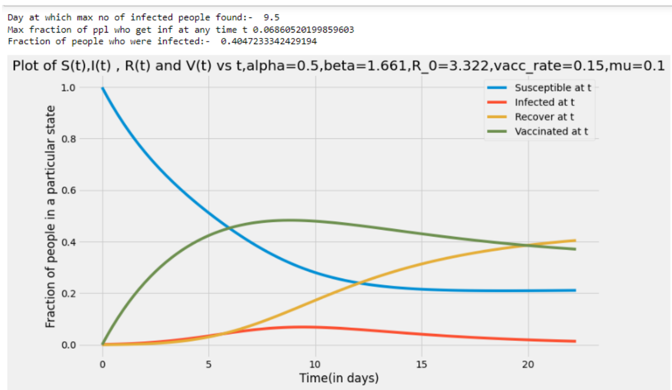

FIG. 11: $s(t), i(t), r(t)v(t)$ plot when vaccination rate=0.15 and mu=0.1

C. Role of Lockdown

As we had mentioned earlier that lockdown decreases value of *beta* as it decreases no of contacts people have among themselves. So more stricter the lockdown is , lower will be the value of *beta* and hence lower will be R(Reproduction no). We had already seen effect of varying R_0 in (2) and (3). This is same because in cases of (2) and (3) also, we were varying R_0 by varying α or β.

D. People's behaviour with Role of Lockdown

As mentioned in above section, value of *beta* will decrease slowly from the time when lockdown is announced

and after some time when unlock phase starts , *beta* will increase slowly.
In the below case we are linearly decreasing value of *beta* from 1.661 to 0.1 and then linearly increasing it from that minimum value again to the highest value.

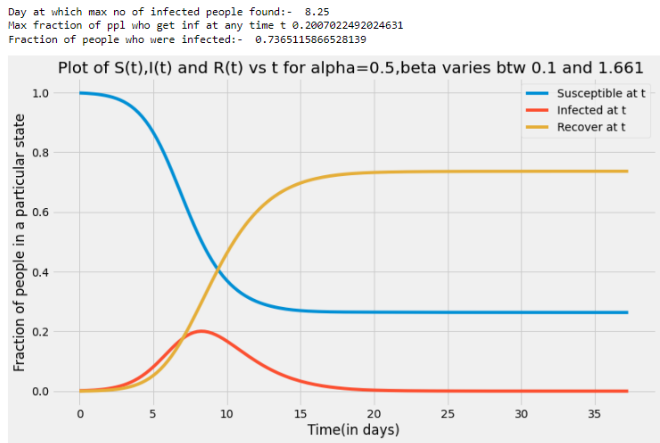

FIG. 12: $s(t), i(t), r(t)$ plot when *beta* varies from 0.1 to 1.661

We can see that lockdown has pushed the time at which peak of infected people comes and also that peak is lower than the case in which there was no lockdown, and also total no of infected people has also decreased than in the no lockdown case. We see that nature of graph is of the same nature as when there was no lockdown.

Below plot is for the case when *beta* values range from 0.5 to 1.661, i.e. the lockdown is less strict as the lower value of *beta* is 0.5(¿0.1):-

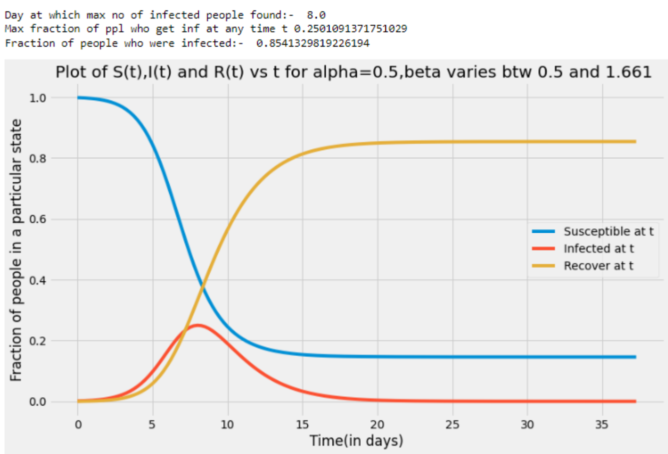

FIG. 13: $s(t), i(t), r(t)$ plot when *beta* varies from 0.5 to 1.661

IV. CONCLUSIONS

In this lab we modelled the SIR model, both computationally and analytically along with graphical visualizations for for better understanding of the model of epidemic spread. **Based on our models we observed the following points and would suggest policy makers to keep in mind the following things:-**

- $R0 > 1$ implies epidemic spread, hence all the policies and intervention methods of the government should be focused on keeping the R_0 less than 1.

- Vaccination is the key to keep the peak infected population low and hence the rate of vaccination should be scaled rapidly.

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- Lockdown is also a quite effective measure,as it reduces the no of contacts per person drastically and hence reduces *beta* and therefore R(Reproduction No).Hence it can be effective measure to reduce the spread of infection.