

Lab -6

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MC312, Modeling and Simulation

In this lab, we studied the radioactive decay problem using a probabilistic model i.e. assuming that the decay of a single nuclei is a random process and there is a certain decay probability associated with the decay of a nuclei. And we analysed the results by simulating the decay for starting with larger no of initial no of nuclei and different values of the probability of decay. We found that the results found from the stochastic model converges with the deterministic differential model when no of initial no of nuclei is large and the probability of decay is a very low number.

I. INTRODUCTION

Here we are trying to plot no of remaining nuclei at any time t by assuming decay process as a stochastic one i.e. initially if there are N_0 no of nuclei at $t=0$ and at each time step the remaining nuclei can decay with a certain probability. And we will analyse the results by trying with different values of probability of decay and initial no of nuclei. Also we will find average no of nuclei at any time step by taking average of a no of samples. We will also analyze the effect of taking different sample sizes.

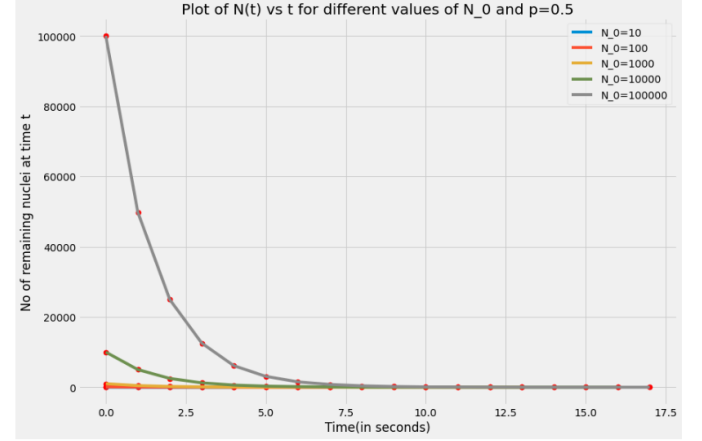


FIG. 1: N_t vs t plot for different values of N_0 ; $p = 0.5$

In the above figure, we have tried to plot N_t vs t plot for different values of N_0 . As this is a random process by simulating the experiment a no of times, we will get different possibilities i.e. there can be different no of nuclei present at each time step in different samples (or experiments). So we will take a lot of samples and will find the sample average of the no of nuclei remaining at each time step.

II. MODEL: STOCHASTIC MODEL FOR RADIOACTIVE DECAY

Suppose initially we have N_0 no of nuclei at $t=0$. And each undecayed nuclei has a probability of decay at each time step. So for simulating this we declared a boolean array of size $=N_0$, initialised with 'false', denoting the situation at $t=0$ i.e. at $t=0$, all of the N_0 nuclei have not decayed. And in another array we store the no of nuclei remaining at each integral time step t , so for $t=0$ we store N_0 in that array. Now at each time step we ask each undecayed nuclei to decay and it decays according to the probability of decay which is decided beforehand.

III. RESULTS

Now, we compare the results obtained by the stochastic model with that obtained from deterministic differential eqn based model. We plot and compare the plots for different values of N_0 and different decay probabilities.

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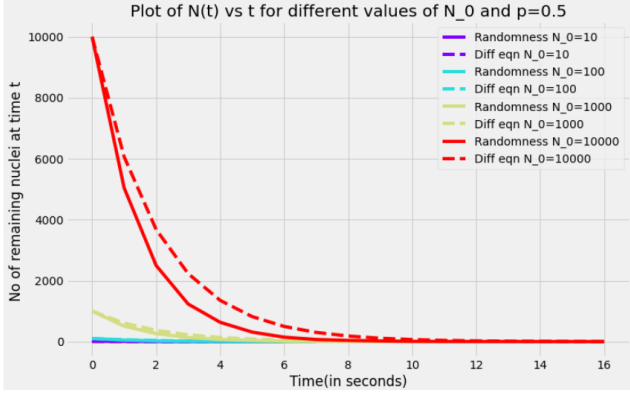


FIG. 2: Dotted line depicts the deterministic model and filled line depicts stochastic one

In the above figure , we plotted the no of nuclei at any time t using both stochastic and deterministic model. To clearly highlight the difference between the two plots , we take logarithm of the no of nuclei and which results in the below figure:-

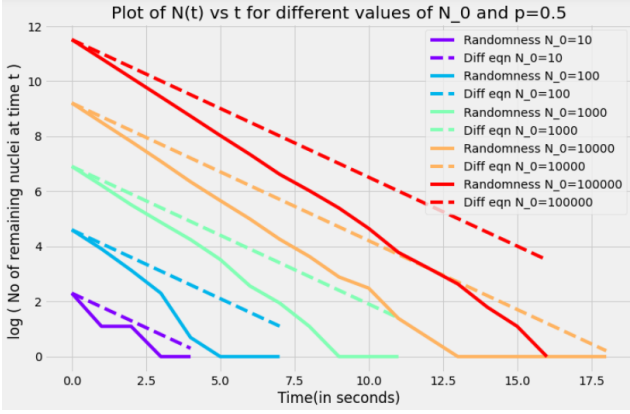


FIG. 3: Compared using semi-log plot for $p=0.5$

Now in the below plots we will compare assuming decay probability of $p=0.1$:-

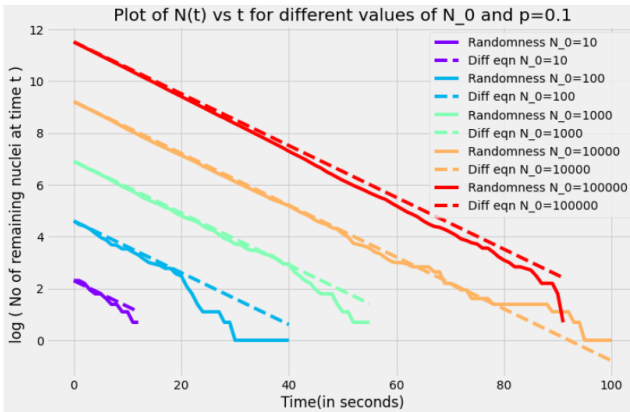


FIG. 4: Compared using semi-log plot for $p=0.1$

We find that as we decrease the decay probability the results obtained from deterministic and stochastic model converges. Below plot shows for decay probability of 0.01:-

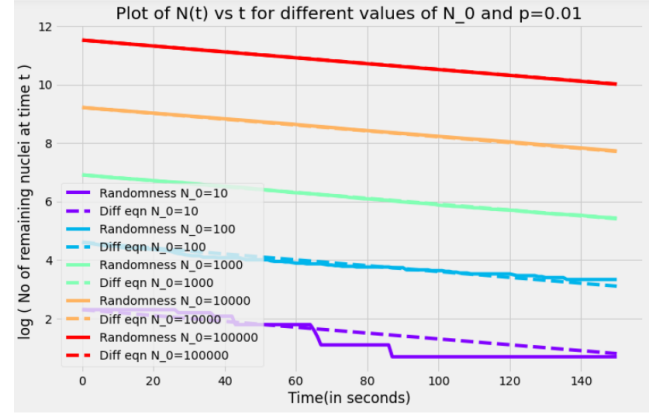


FIG. 5: Compared using semi-log plot for $p=0.01$

From above plots we can conclude that as we decrease the probability of decay the results from the two models converges. Also we conclude that it converges better if we start with large no of initial nuclei.

A. Plotting after taking a large no of samples

As it is a random process, there can be various possible outcomes of the same experiment i.e. no of nuclei at a time step can vary if we simulate the same experiment no of times , i.e no of nuclei at any particular time step will vary as we take different samples of the same experiment. So, in the below figure we have repeated the experiment for different no of times i.e. we have taken different no of samples and had found the average no of nuclei at each time step. So below plots are kind of demonstration of Law of Large Nos because as we take large no of samples , probability that the sample mean will take a value away from population mean tends to 0. We then compare that with the deterministic model semilog plot:-

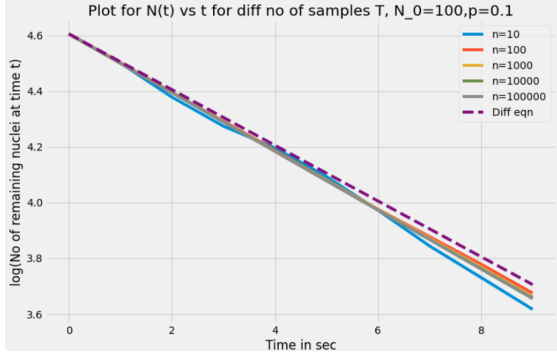


FIG. 6: Compared using semi-log plot for $p=0.1$ and plotted average no of nuclei at each time step by considering different no of samples at each time step

We observe from the above figure that the error between the deterministic and averaged mean plot decreases as we increase the no of samples. Also, in the below figure we have plotted the deviation of the average no of nuclei at each time step from the differential equation based model.

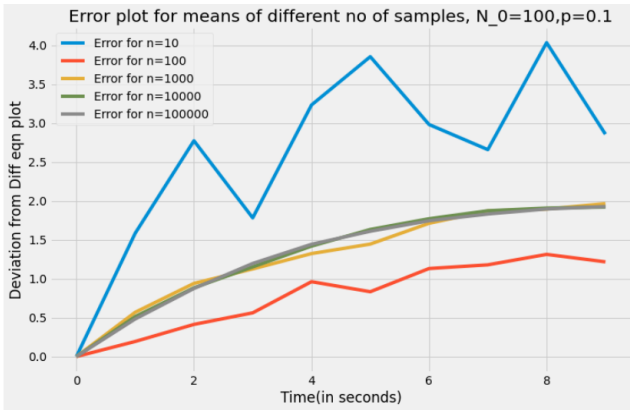


FIG. 7: Plot for the absolute difference between the average no of nuclei at time step t and no obtained from the deterministic model, for different no of samples considered in the average

We observe from the above figure that the absolute error varies a lot from one time step to the other when we take small no of samples. In case when we take large no of samples, the absolute error does not fluctuate a lot from one time step to the other. Also the absolute error is less in cases when we take large no of samples. Below plot is for the standard deviation of the no of nuclei at each time step for different no of samples taken into consideration for each of the plot in the figure:-

We observe that when we take more no of samples into consideration the standard deviation between the values of no of nuclei at each time step decreases.

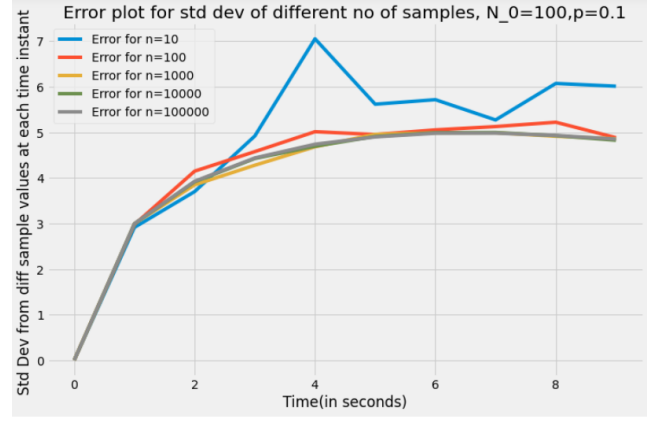


FIG. 8: Standard deviation of the no of nuclei at each time step when we take different no of samples

B. Plotting histogram for No of decays in time interval T

We simulated the number of decays in a time interval T and have repeated the simulation many times and have plotted the histogram of the distribution. As we expect that the distribution should be binomial because chance of happening extreme events i.e. all nuclei decay or none of the nuclei decay in time T is very less. And chance of happening a mediocre event is more. And as we take more and more samples it converges to a normal distribution. This is demonstration of Central Limit theorem because as we increase no of samples, the distribution is converging to a normal distribution. The below plots confirm these:-

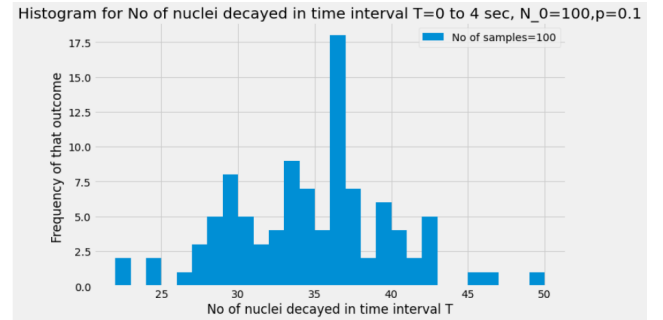


FIG. 9: Histogram for no of decays in time T when we take 100 samples

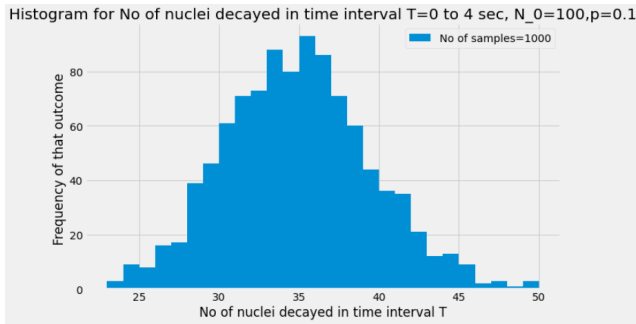


FIG. 10: Histogram for no of decays in time T when we take 1000 samples

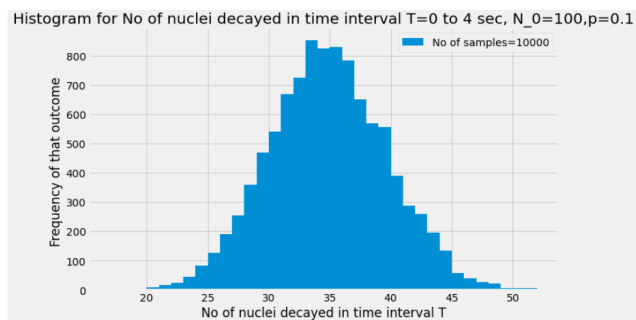


FIG. 11: Histogram for no of decays in time T when we take 10000 samples

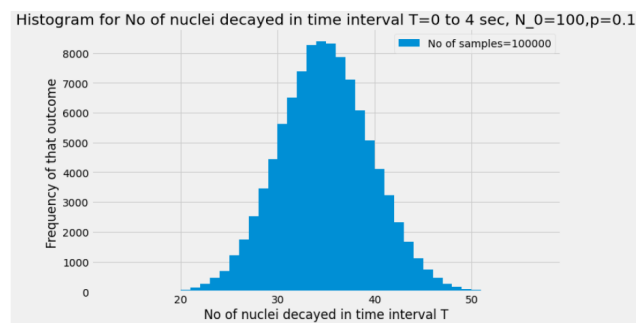


FIG. 12: Histogram for no of decays in time T when we take 100000 samples

IV. CONCLUSIONS

In this lab we studied the radioactive decay problem using a probabilistic model i.e. assuming that the decay of a single nuclei is a random process and there is a certain decay probability associated with the decay of a nuclei. We conclude that the deterministic model and stochastic model converges when decay probability is a very small number and the no of initial no of nuclei taken is large enough. Also we conclude that as we increase the no of samples that we take into consideration to calculate the average no of nuclei at any time t , it gives more accurate result(i.e. converges with the deterministic value at time step t). This is confirmed by lower values of the absolute error between average no of nuclei at time step t and the deterministic value at time step t . Also lower standard deviation between values of no of nuclei decayed at any time step t gives a confirmation.