* Fluid Dyanamics *

11/8/22

* **Types of flow:** (This type of flow occurs at low velocity where layer of fluid seems to slide with one another without any lateral mixing)

① **Laminar flow:** No lateral mixing [Initial flow rate = final flow rate]

② **Turbulant flow:** Lateral mixing of particles.
⇒ At very high velocity of fluid, where the lateral mixing of fluid are start taking place & eddy's formation occurs will be known as turbulant flow.

③ **Steady & Unsteady flow:**
⇒ A steady state is one in which fluid properties are not changing w.r.to time, but a given section if properties are changing that will be unsteady flow.

$\left(\frac{\partial v}{\partial t}\right)_s = 0$ Steady state. $\left(\frac{\partial v}{\partial t}\right)_s \neq 0$ Unsteady state.

④ **Uniform & Non-uniform flow:**

⇒ Uniform flow is the one in which velocity of flow is not changing w.r. to section, but at given interval of time if velocity is changing then it will be non-uniform flow.

$\left(\frac{\partial v}{\partial s}\right)_t = 0$ Uniform flow $\left(\frac{\partial v}{\partial s}\right)_t \neq 0$ Non-uniform flow

⑤ **Rotational & Irrotational flow:**

⇒ A rotational flow is one in which fluid particles rotate about their own axis. for the fluid flow rotational shear force is required which is created by high viscosity.
⇒ Nonviscous fluid ($\mu = 0$) will be ideal fluid.

Acc. = I − O + G − C

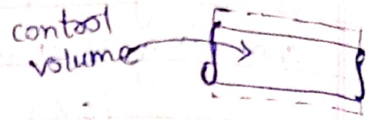
⑥ Potential flow:

⇒ The flow of ideal fluid ($\mu = 0$) which is incompressible in nature is called potential flow.

⑦ Compressible & Non compressible flow:

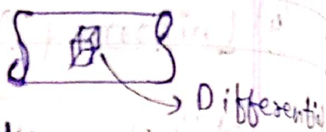
⇒ compressible is one in which density of fluid is changing. & Non-compressible in which density of fluid remains constant.

I Macroscopic balance:



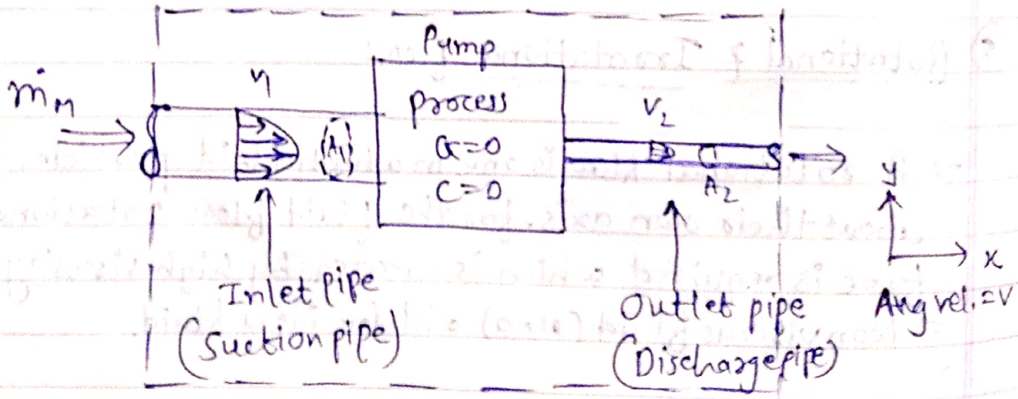
⇒ Control volume will be big
⇒ Algebric equation will be obtained
Disadvantage ⇒ Detail of flow inside the control volume

II Microscopic balance:



⇒ Differential volume element will taken
⇒ Differential eq^n will be obtained
Advantage ⇒ Detail of flow inside the pipe is obtained.

* Macroscopic mass balance:



$\dot{m}_{in} = \rho_1 A_1 V_1$ & $\dot{m}_{out} = \rho_2 V_2 A_2$

Rate of mass accumulated = Rate of mass in − Rate of mass out

$$\Rightarrow \boxed{\frac{\partial m}{\partial t} = \dot{m}_{in} - \dot{m}_{out}}$$

$$\Rightarrow \underbrace{\frac{\partial(\rho v)}{\partial t}}_{0\ (\because \rho \text{ will constant})} = \rho_1 A_1 V_1 - \rho_2 A_2 V_2$$

for steady state, $\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$

- for incompressible fluid, $(\rho_1 = \rho_2)$

$$\Rightarrow \boxed{A_1 V_1 = A_2 V_2}$$

$$\Rightarrow \underbrace{\boxed{Q = AV}}_{\text{(Volumetric flow rate)}} \quad \& \quad \underbrace{\boxed{\dot{m} = \rho AV}}_{\text{(mass flow rate)}}$$

Mass flow rate:

$$\dot{m} = \rho AV$$

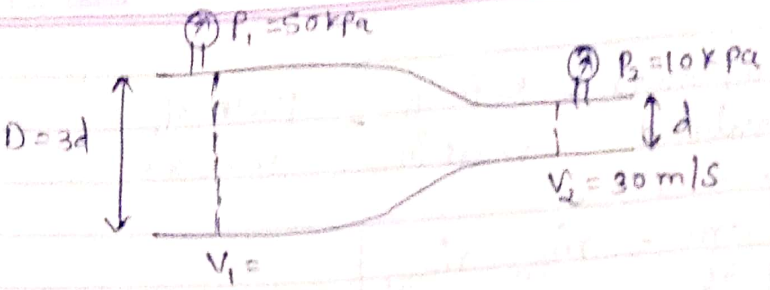
$$\Rightarrow \dot{m} = \rho \int_A u\,dA$$

$$\Rightarrow \rho AV = \rho \int_A u\,dA$$

$$\Rightarrow AV = \int_A u\,dA$$

$$\Rightarrow \boxed{\underbrace{V}_{\text{(Average velocity)}} = \frac{1}{A} \int_A u.dA}$$

Q.1



$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\Rightarrow \frac{P_1 (mw)}{\cancel{RT_1}} A_1 V_1 = \frac{P_2 (mw)}{\cancel{RT_1}} A_2 V_2$$

$$\Rightarrow P_1 A_1 V_1 = P_2 A_2 V_2$$

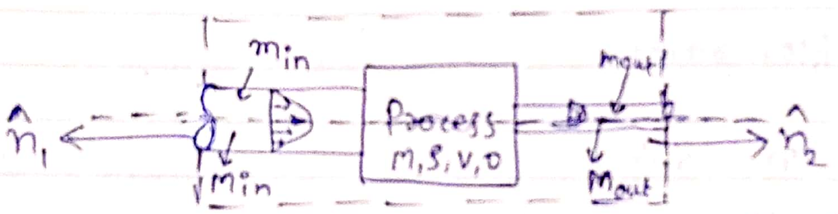
$$\Rightarrow 50 \times \frac{\pi}{\cancel{4}}\, 9 (d)^2 \times V_1 = 10 \times \frac{\pi}{\cancel{4}} d^2 \times 30$$

$$\Rightarrow V_1 = \frac{\cancel{30}^{6} \times 1\cancel{0}}{9 \times \cancel{50}}$$

$$\Rightarrow \boxed{V_1 = 0.67 \ m/s}$$

19/12/22

* Macroscopic momentum balance:



Rate of momentum accumulated = Rate of momentum In − Rate of momentum out + sum of all forces around condition

$$\Rightarrow \boxed{\left(\frac{\partial M}{\partial t}\right) = \dot{M}_{In} - \dot{M}_{out} + \sum F. \quad \left(\frac{kg\,m}{s^2}\right)}$$

for steady state flow, $\frac{\partial M}{\partial t} = 0$

$$\therefore \boxed{\sum F = \dot{M}_{out} - \dot{M}_{in}}$$

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$$\Rightarrow \sum F = \dot{m}_{out} + (-\dot{m}_{in})$$

$$\Rightarrow \boxed{\sum F = \sum \dot{m}_{cv.}}$$

$$\Rightarrow \sum F = \sum m.\vec{V}$$

$$\Rightarrow \sum F = \sum (\rho AV).(V.\hat{n})$$

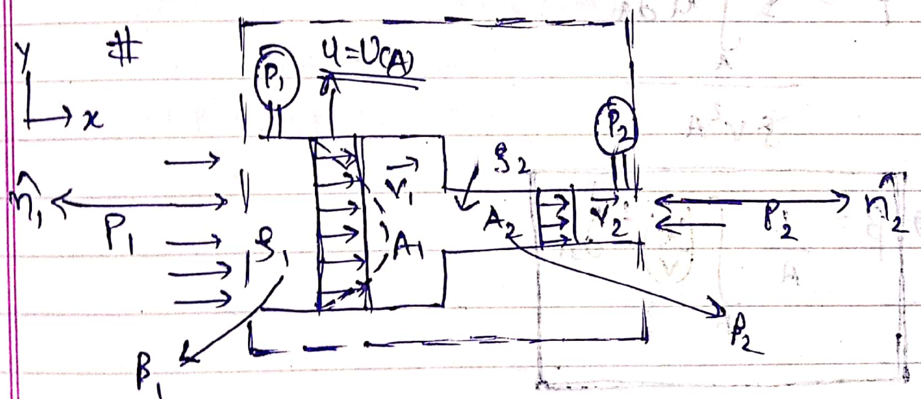
$$\star \Rightarrow \boxed{\sum F = \sum (\rho V)(A\vec{V}.\hat{n})} \star$$

$$\Rightarrow \sum F = -\rho_1 V_1^2 A_1 + \rho_2 V_2^2 A_2$$

$$\Rightarrow \boxed{\sum F = \rho_2 A_2 V_2^2 - \rho A_1 V_1^2}$$

sum of All forces:

- → Body force (gravitational force)
- → Surface force → pressure force
  - → Shearforce



$$\rightarrow \sum F = P_1 A_1 - P_2 A_2$$

$$\rightarrow \sum M_{cv} = \sum (\rho AV)(\vec{V}.\hat{n})$$

$$= \rho_1 V_1 A_1 (\vec{V}_1.\hat{n}) + \rho_2 V_2 A_2 (\vec{V}_2.\hat{n})$$

$$= \rho_1 V_1 A_1 (-V_1) + \rho_2 V_2 A_2 (V_2)$$

$$\sum M_{cv} = \rho_2 V_2^2 A_2 - \rho_1 V_1^2 A_1$$

\# $\sum F = \sum \dot{M}_{cv}$

$$\Rightarrow \boxed{P_1A_1 - P_2A_2 = \rho_2 V_2^2 A_2 - \rho_1 V_1^2 A_1} \longrightarrow (*)$$

Momentum correction factor: (β)

$$\partial \dot{M}_{actual} = \partial(\rho A u) u$$

$$\Rightarrow \partial M_{act.} = \partial(\rho A u^2)$$

$$\Rightarrow \partial M_{act.} = \rho u^2 (\partial A)$$

$$\Rightarrow \boxed{M_{act.} = \rho \int_A u^2 \partial A}$$

$$\Rightarrow \dot{M} = \rho V^2 A$$

\# $\beta = \dfrac{\dot{M}_{actual}}{\dot{M}_{ideal}}$

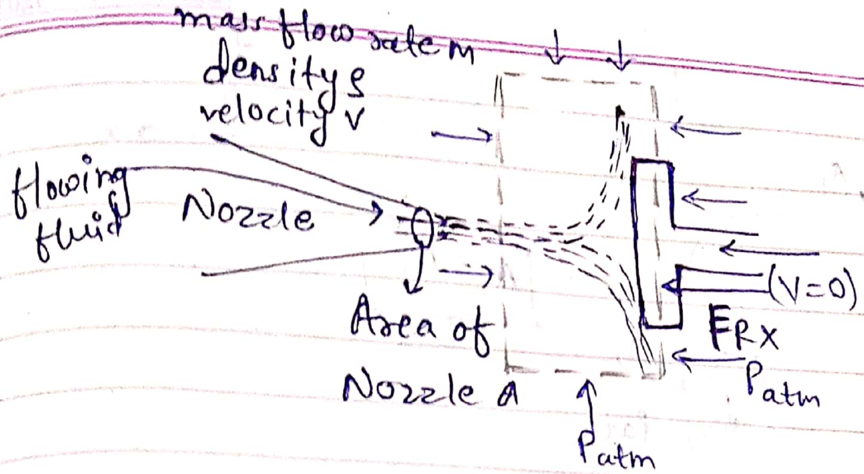
$$\Rightarrow \beta = \frac{\cancel{\rho} \int_A u^2 \partial A}{\rho V^2 A}$$

$$\Rightarrow \boxed{\beta = \frac{1}{A} \int_A \left(\frac{u}{V}\right)^2 \partial A}$$

\# from eq^n (*),

$$\boxed{P_1A_1 - P_2A_2 = \beta_2 \rho_2 V_2^2 A_2 - \beta_1 \rho_1 V_1^2 A_1}$$

Q.

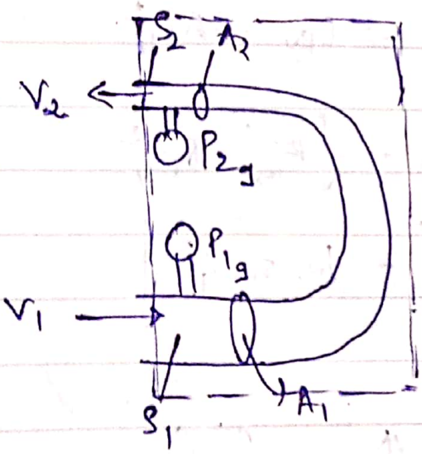


$$\Rightarrow \left(F_{atm_0} - F_{Rx}\right) = \left[(\rho A V)(\vec{V}\cdot\hat{n})\right]_{in}$$

$$\Rightarrow -F_{Rx} = (\rho A V)(V \times 1 \times \cos 180)$$

$$\Rightarrow \boxed{F_{Rx} = \rho A V^2}$$

Q2



$$P_1 A_1 + P_2 A_2 = \sum (\rho A V)(\vec{V}\cdot\hat{n})$$

$$\Rightarrow \boxed{P_1 A_1 + P_2 A_2 = -\rho_2 A_2 V_2^2 - \rho_1 A_1 V_1^2}$$

$\Sigma F = \Sigma \dot{M}_{cv}$

$$\Rightarrow \boxed{P_1A_1 - P_2A_2 = \rho_2 V_2^2 A_2 - \rho_1 V_1^2 A_1} \longrightarrow (*)$$

Momentum correction factor: (β)

$$\partial \dot{M}_{actual} = \partial(\rho A u)u$$

$$\Rightarrow \partial M_{act.} = \partial(\rho A u^2)$$

$$\Rightarrow \partial M_{act.} = \rho u^2 (\partial A)$$

$$\Rightarrow \left[M_{act.} = \rho \int_A u^2 \partial A \right]$$

$$\Rightarrow \dot{M} = \rho V^2 A$$

$\beta = \dfrac{\dot{M}_{actual}}{\dot{M}_{ideal}}$

$$\Rightarrow \beta = \frac{\rho \int_A u^2 \partial A}{\rho V^2 A}$$

$$\Rightarrow \boxed{\beta = \frac{1}{A} \int_A \left(\frac{u}{V}\right)^2 \partial A}$$

from eq^n (*),

$$\boxed{P_1A_1 - P_2A_2 = \beta_2 \rho_2 V_2^2 A_2 - \beta_1 \rho_1 V_1^2 A_1}$$

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Q.3

$\Rightarrow A_1V_1 + A_2V_2 = A_3V_3$

$\Rightarrow A_1V_1 + Q = A_3V_3$

$\Rightarrow Q = A_3V_3 - A_1V_1$

$= 0.075(6) - 0.01(30)$

$= 0.45 - 0.3$

$\boxed{Q = 0.15 \ m^3/s}$ or $\boxed{150 \ \ell/s}$

Q.4

$\Rightarrow \rho_1A_1V_1 + \rho_2A_2V_2 = \rho_3A_3V_3$

$\Rightarrow \rho_1Q_1 + \rho_2Q_2 = \rho_3Q_3$

$\# \Rightarrow Q_1 + Q_2 = Q_3$

$\therefore \underset{\text{water}}{\rho_1}Q_1 + \underset{\text{oil}}{\rho_2}Q_2 = \underset{\text{mixture}}{\rho_3}(Q_1+Q_2)$

$\therefore Q_1 + (S.G.)_2Q_2 = (S.G.)_3(Q_1+Q_2)$

$\cancel{\therefore \dfrac{Q_1(1-(S.G.)_3)}{(S.G.)_2} = Q_2}$

$\cancel{\therefore Q_2 = \left[\dfrac{1-0.95}{0.9}\right]2}$

$\therefore Q_1 + (S.G.)_2Q_2 = (S.G.)_3Q_1 + Q_2(S.G.)_3$

$\therefore \dfrac{Q_1(1-(S.G.)_3)}{(S.G.)_3 - (S.G.)_2} = Q_2$

$\therefore Q_2 = \dfrac{2\times(1-0.95)}{0.95-0.9} = \dfrac{2\times\cancel{0.05}}{\cancel{0.05}}$

$Q_2 = 2 \ m^3/s$

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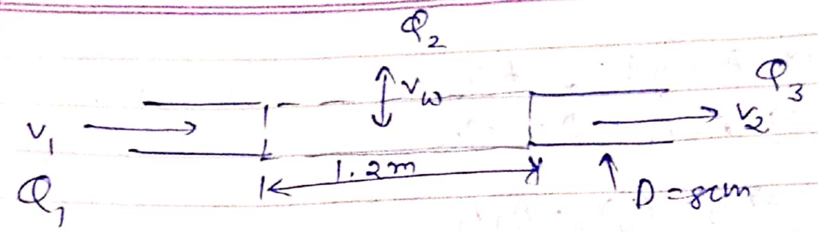
Q.6



$Q_1 = Q_2 + Q_3$

$A_1 V_1 = A_2 V_2 + A_1 V_2$

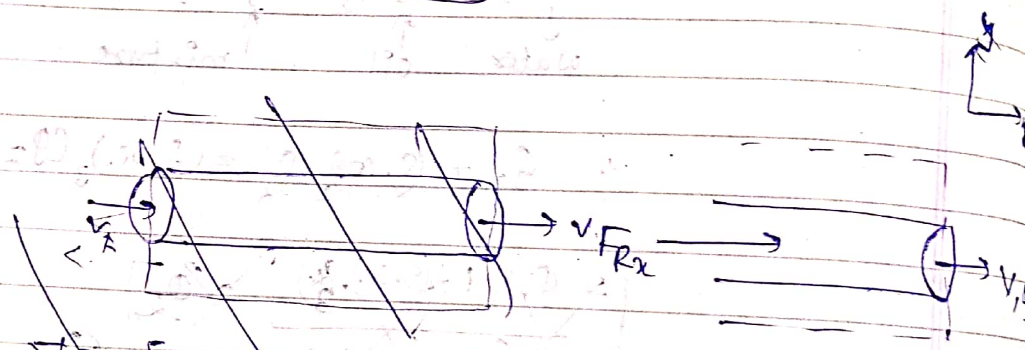
$\Rightarrow \frac{\pi}{4}\left(\frac{8}{100}\right)^2 \times 12 = \left(\pi\left(\frac{8}{100}\right)(1.2) \times \frac{15}{100}\right) + \frac{\pi}{4}\left(\frac{8}{100}\right)^2 \times$

$\Rightarrow 0.060 = 0.0452 + 0.005\, V_2$

$\Rightarrow \frac{0.0148}{0.005} = V_2$

$\Rightarrow \boxed{V_2 = 2.96\ m/s}$

Q.7



$\sum F = \sum \rho AV(V.\hat{n})$

$= \rho AV[-V$

$F_{Rx} = \rho AV(V.\hat{n})$

$= \rho AV(V \times 1 \times \cos 0)$

$= \rho AV^2$

$=$

$\Rightarrow Q = AV$

$V = \frac{Q}{A} = \frac{12\ m^3}{60\ sec} \times \frac{100}{8\ m^2}$

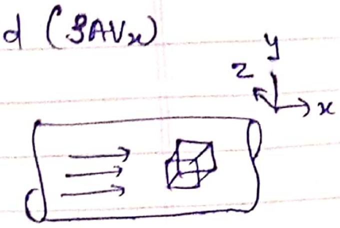
$= \frac{12 \times 100 \times 4}{60 \times 8 \times \pi}$

$\boxed{v = 2.5\ m/s}$

1/12/22 * Microscopic momentum balance in x direction:

Rate of momentum Accumulated = Rate of momentum in − Rate of momentum out + sum of all forces

* (Rate of momentum in) ⇒ Due to bulk flow of fluid ($\rho A V_x$)



⇒ In in x @ $x \rightarrow (\rho V_x) V_x (\Delta y \Delta z)$

y @ $y \rightarrow (\rho V_x) V_y (\Delta x \Delta z)$

z @ $z \rightarrow (\rho V_x) V_z (\Delta x \Delta y)$

⇒ Out in x @ $x \rightarrow (\rho V_x) V_{x+\Delta x} (\Delta y \Delta z)$

y @ $y \rightarrow (\rho V_x) V_{y+\Delta y} (\Delta x \Delta z)$

z @ $z \rightarrow (\rho V_x) V_{z+\Delta z} (\Delta x \Delta y)$

* (Rate of momentum In) ⇒ Due to velocity gradient.

⇒ In @ $x \Rightarrow \tau_{xx}|_x \ \Delta y \Delta z$

@ $y \rightarrow \tau_{yx}|_y \ \Delta x \Delta z$

@ $z \rightarrow \tau_{zx}|_z \ \Delta x \Delta y$

$\tau_{xx} = \mu \left(\frac{\partial V_x}{\partial x} \right)$

⇒ Out @ $x \rightarrow \tau_{xx}|_{x+\Delta x} \ \Delta y \Delta z$

@ $y \rightarrow \tau_{yx}|_{y+\Delta y} \ \Delta x \Delta z$

@ $z \rightarrow \tau_{zx}|_{z+\Delta z} \ \Delta x \Delta y$

* Pressure force @ $x \rightarrow P_x (\Delta y \Delta z)$

pressure force @ $x+\Delta x \rightarrow (-P_{x+\Delta x} (\Delta y \Delta z))$

* Body force @ $x \rightarrow mg_x = V\rho g_x = \rho g_x (\Delta x \Delta y \Delta z)$

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$$\frac{\partial}{\partial t}(\rho V_x) = -\left[\frac{\rho V_x \cdot V_{x+\Delta x} - \rho V_x \cdot V_x}{\Delta x}\right] - \left[\frac{\rho V_x V_{y+\Delta y} - \rho V_x V_y}{\Delta y}\right]$$

$$-\left[\frac{\rho V_x V_{z+\Delta z} - \rho V_x V_z}{\Delta z}\right] - \left[\frac{\tau_{xx}|_{x+\Delta x} - \tau_{xx}|_x}{\Delta x}\right]$$

$$-\left[\frac{\tau_{yx}|_{y+\Delta y} - \tau_{yx}|_y}{\Delta y}\right] - \left[\frac{\tau_{zx}|_{z+\Delta z} - \tau_{zx}|_z}{\Delta z}\right]$$

$$-\left[\frac{P_{x+\Delta x} - P_x}{\Delta x}\right] + \rho g x.$$

$$\Rightarrow \frac{\partial}{\partial t}(\rho V_x) = -\frac{\partial}{\partial x}(\rho V_x V_x) - \frac{\partial}{\partial y}(\rho V_x V_y) - \frac{\partial}{\partial z}(\rho V_x V_z)$$

$$- \frac{\partial}{\partial x}\tau_{xx} - \frac{\partial}{\partial y}\tau_{yx} - \frac{\partial}{\partial z}\tau_{zx} - \frac{\partial P}{\partial x} + \rho g_x$$

$$\Rightarrow \rho\frac{\partial V_x}{\partial t} = -\rho\left\{\left[V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z}\right]\right\}$$

$$-\left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial \tau_{zx}}{\partial z}\right] - \frac{\partial P}{\partial x} + \rho g_x$$

$$\Rightarrow \boxed{\rho\left(\frac{D\vec{V}}{Dt}\right) = -\nabla\tau - \Delta P + \rho\vec{g}}$$ Equation of motion in 3D for cartesian co-ordinates

In x direction

$$\rho\left(\frac{DV_x}{Dt}\right) = \rho\left[\frac{\partial V_x}{\partial t} + V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z}\right]$$

$$\nabla\tau = \frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}$$

$$\nabla P = \frac{\partial P}{\partial x}$$

In y direction

$$\rho\left(\frac{DV_y}{Dt}\right) = \rho\left[\frac{\partial V_y}{\partial t} + V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_z\frac{\partial V_y}{\partial z}\right]$$

$$\nabla\tau = \frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}$$

$$\Delta P = \frac{\partial P}{\partial y}$$

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