Parametric Compositional Data Types

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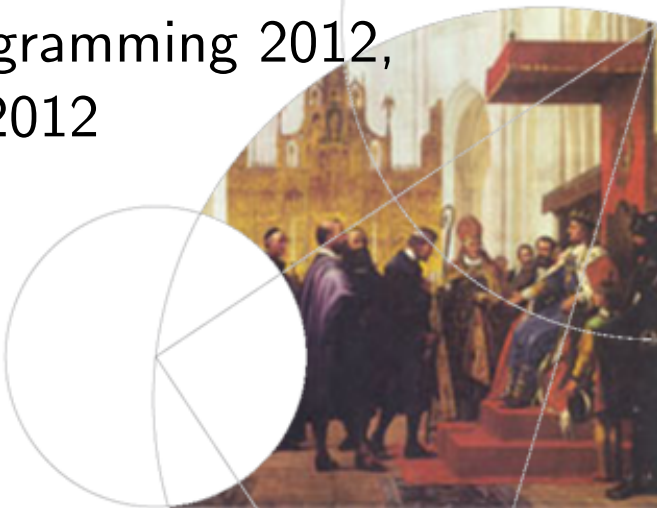

Outline

The Issue

Implementation/Prototyping of DSLs

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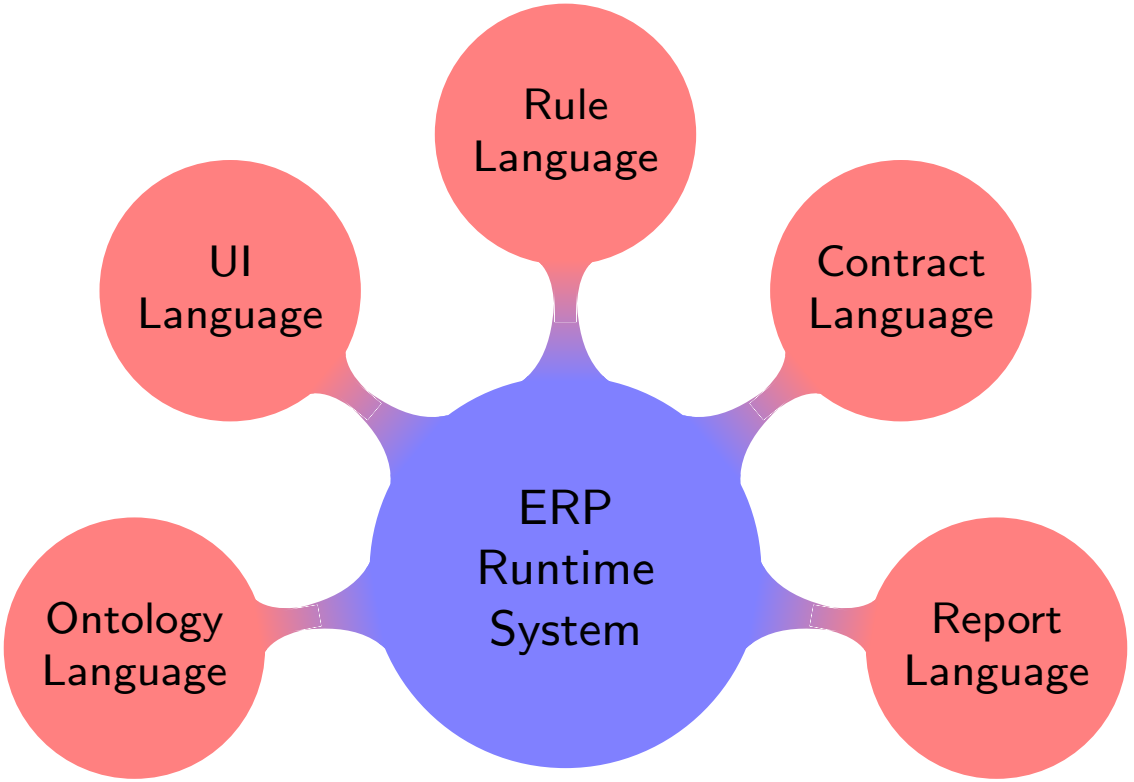

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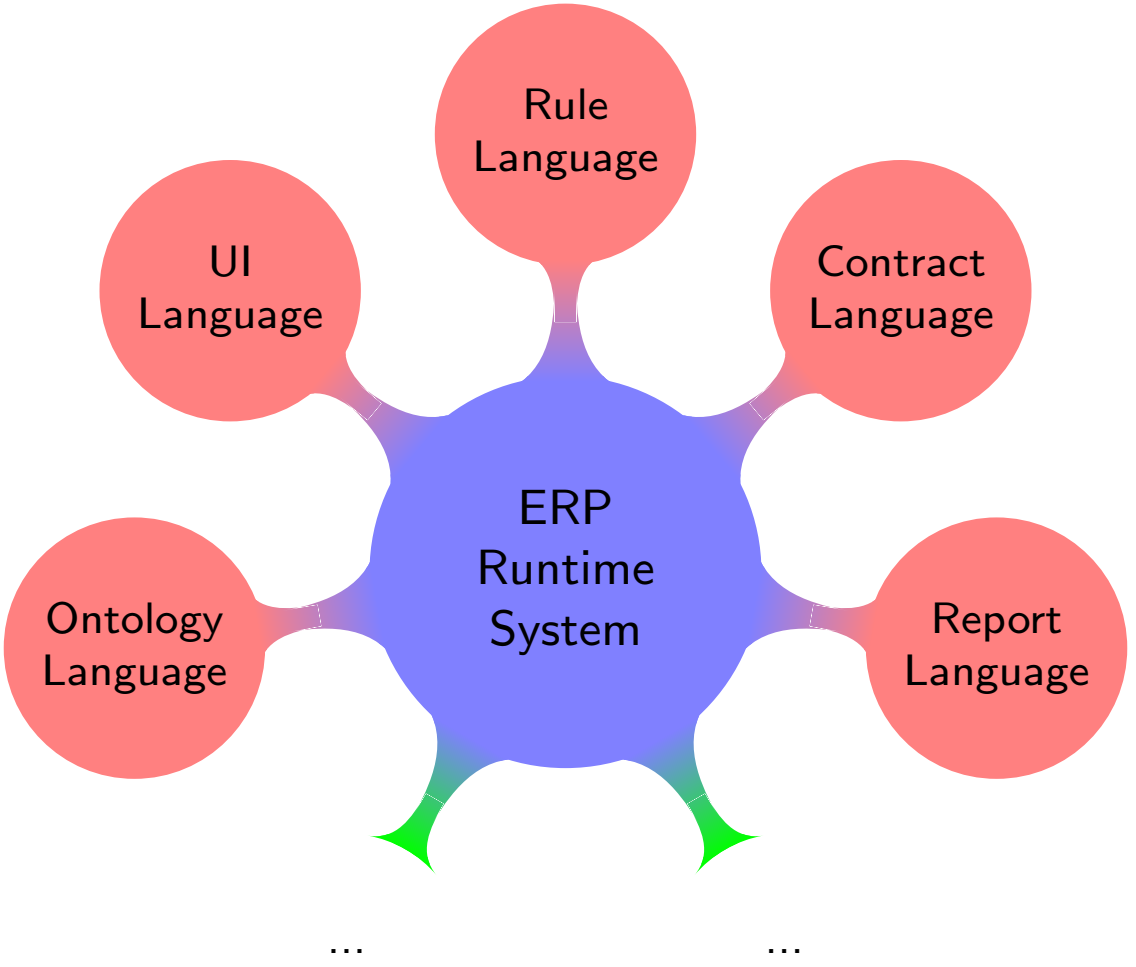

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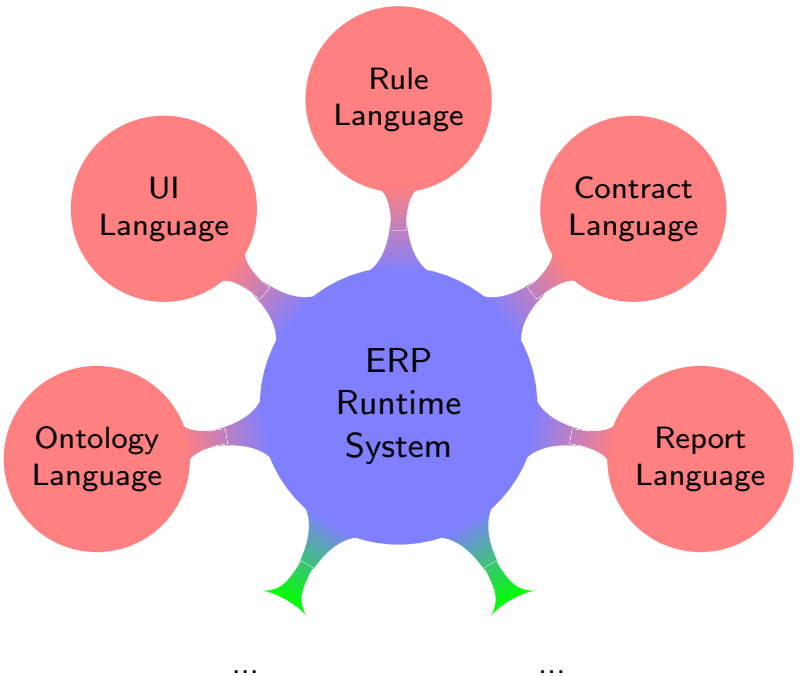

The abstract picture

- We have a number of domain-specific languages.
- Each pair of DSLs shares some common sublanguage.
- All of them share a common language of values.
- We have the same situation on the type level!

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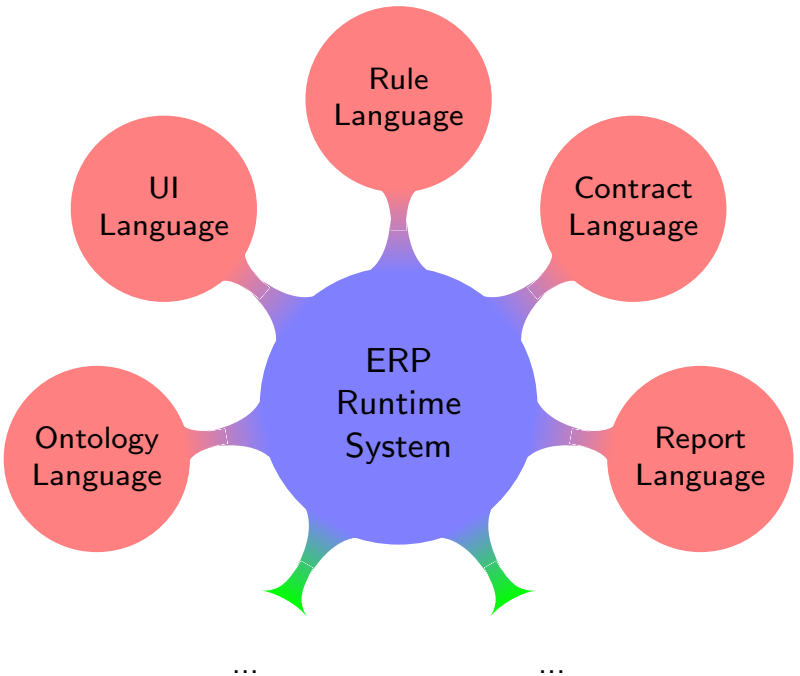

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- We have a number of domain-specific languages.
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How do we implement this system without duplicating code?!

More General Application

Even with only one language to implement this issue appears!

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Different stages of a compiler work on different languages.

- Desugaring: $\mathit{FullExp} \rightarrow \mathit{CoreExp}$
- Evaluation: $\mathit{Exp} \rightarrow \mathit{Value}$

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Manipulating/extending syntax trees

- annotating syntax trees
- adding/removing type annotations

Compositional Data Types

$$
\begin{aligned}
\textbf{data}\ \mathit{Exp} = {} & \mathit{Lit}\ \ \mathit{Int} \\
& \mid \mathit{Add}\ \ \mathit{Exp}\ \mathit{Exp} \\
& \mid \mathit{Mult}\ \mathit{Exp}\ \mathit{Exp}
\end{aligned}
$$

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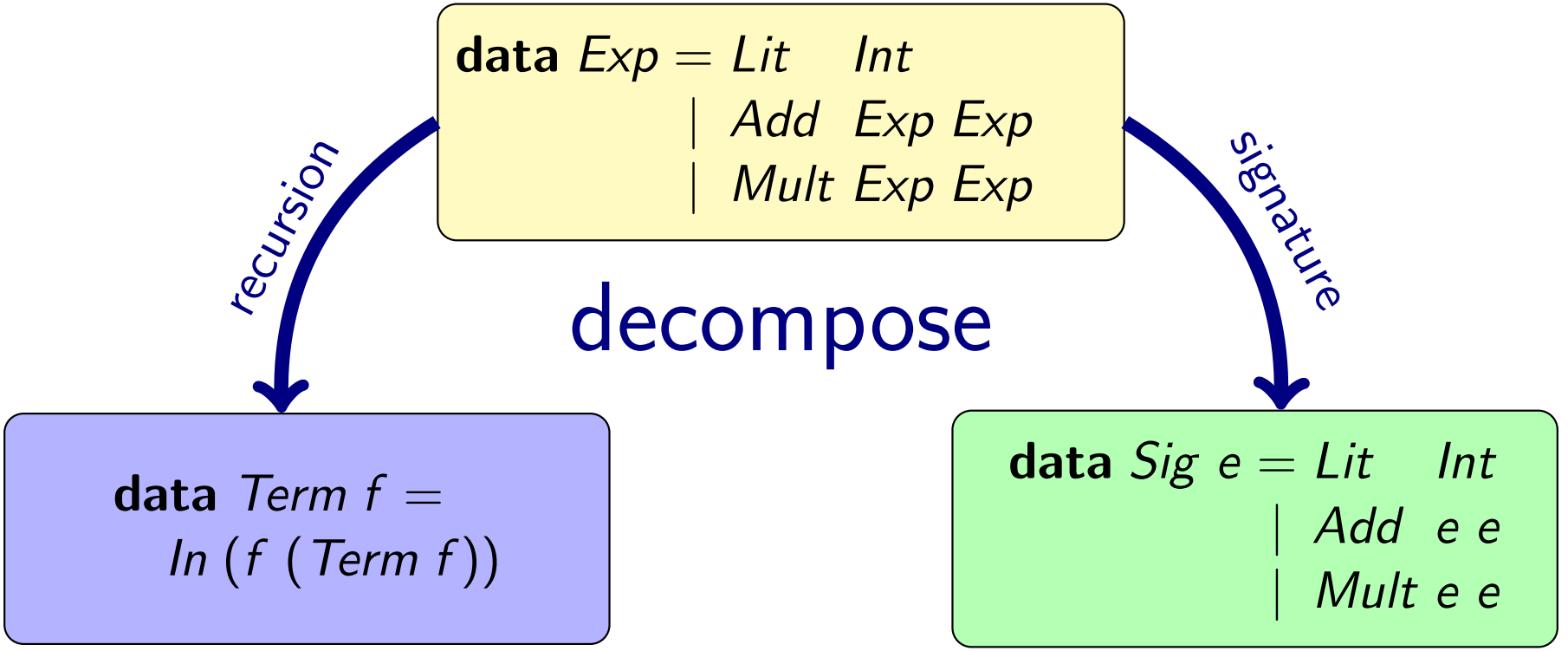

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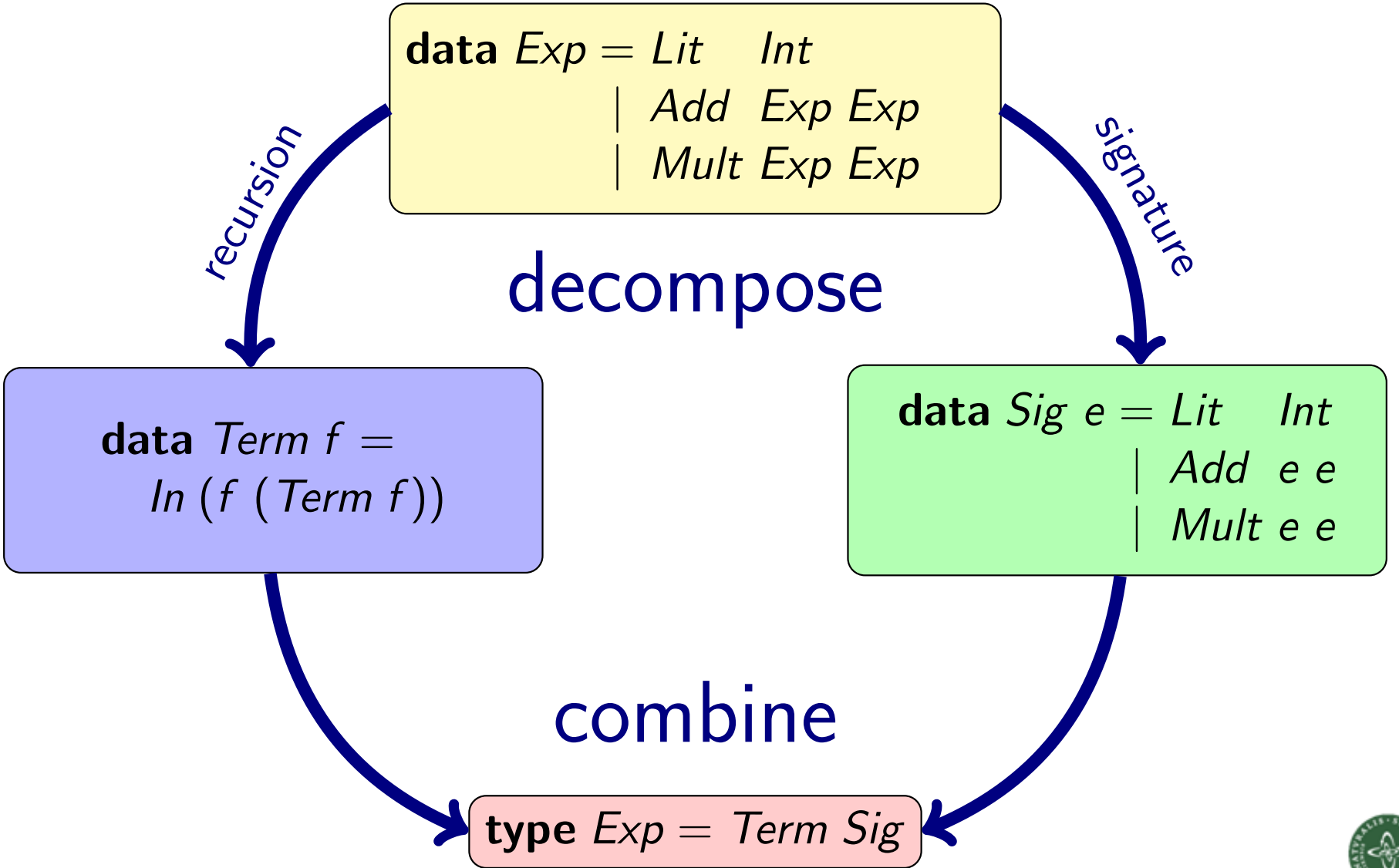

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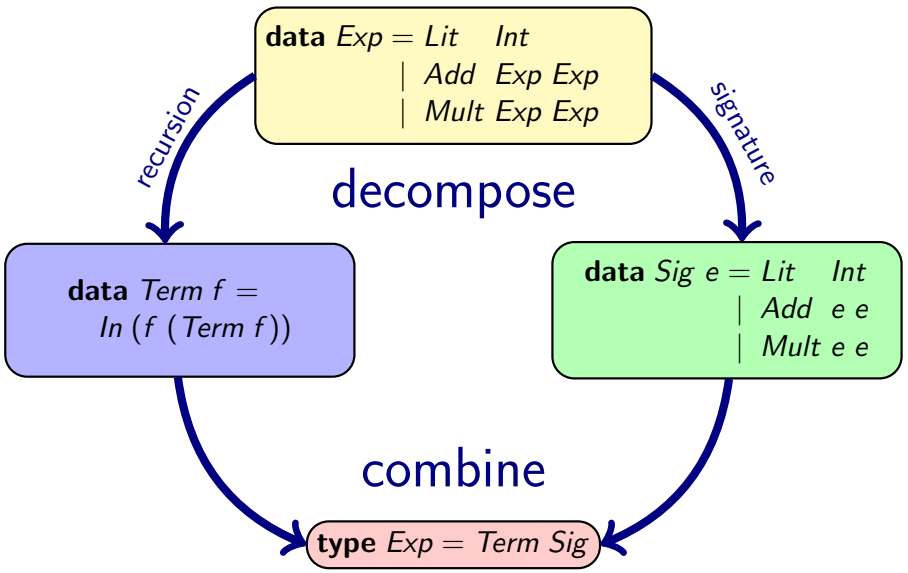

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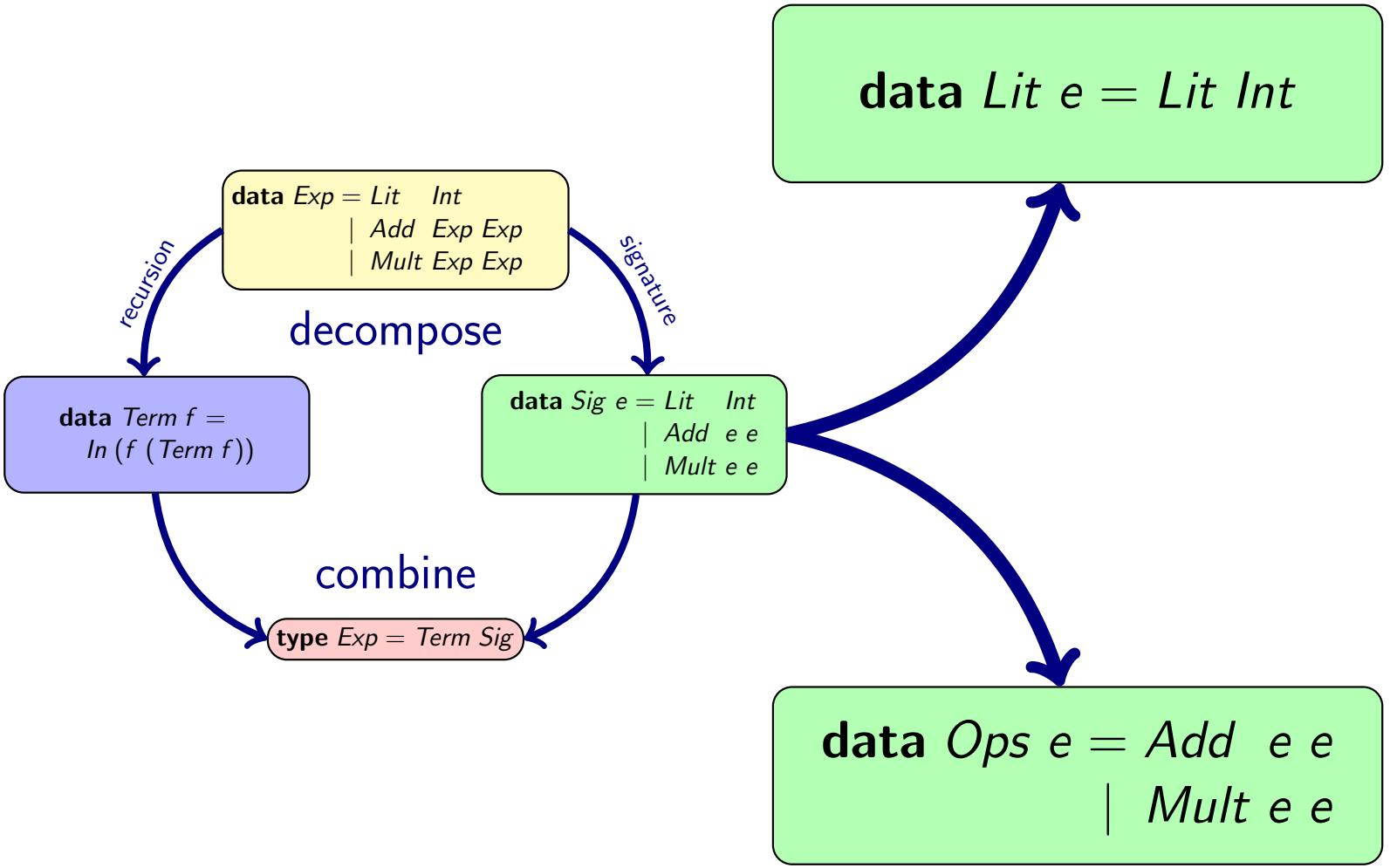

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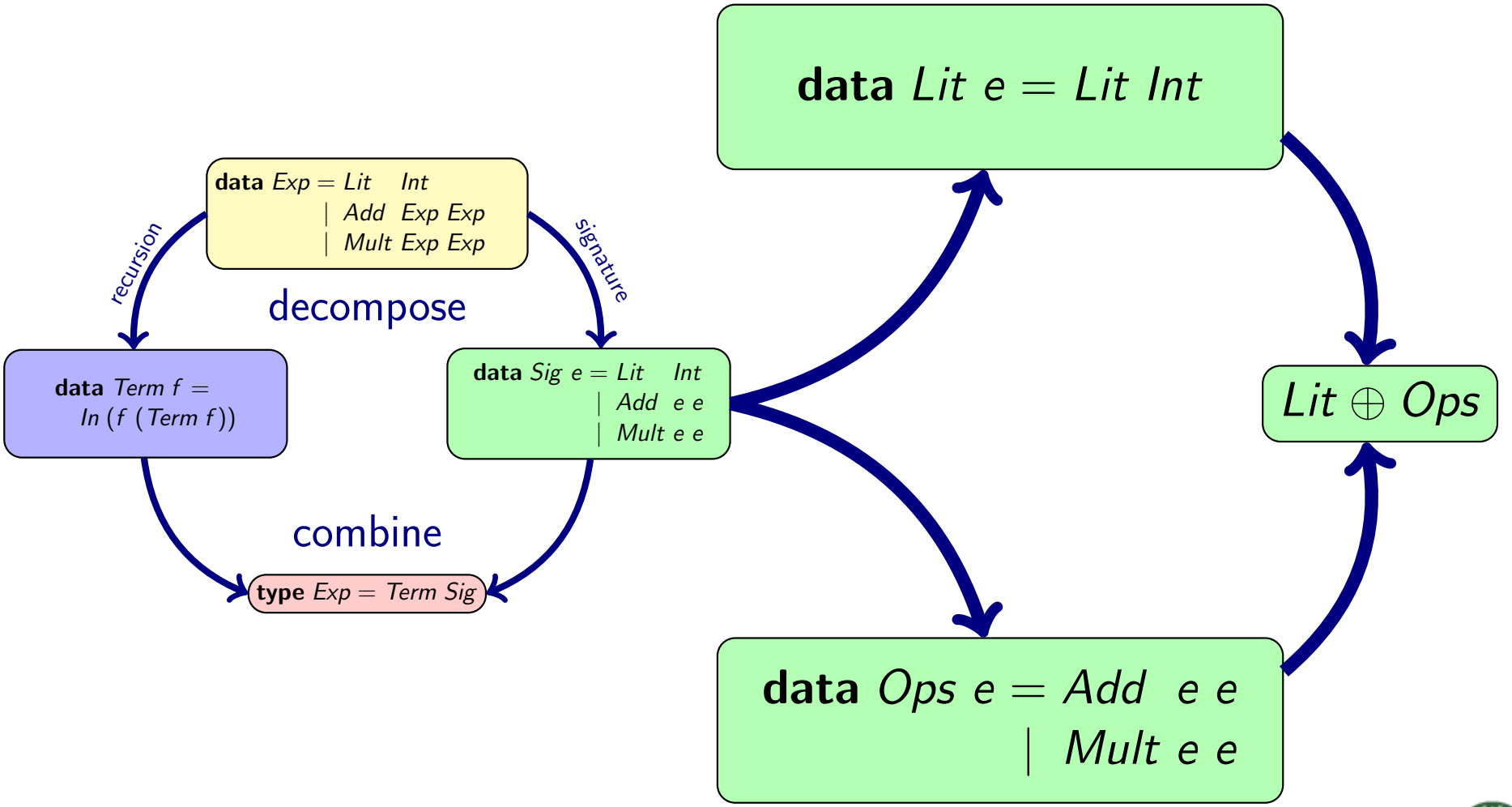

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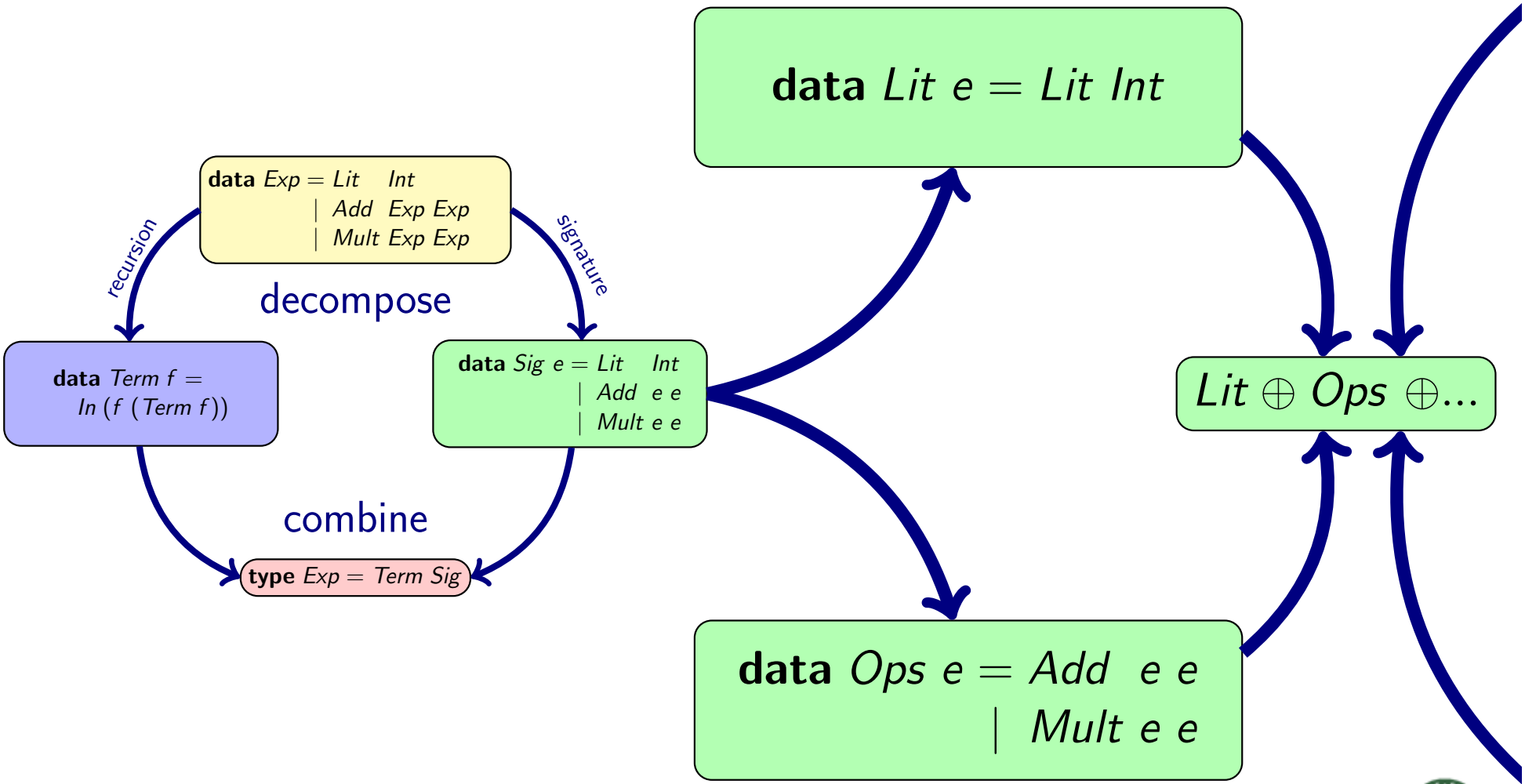

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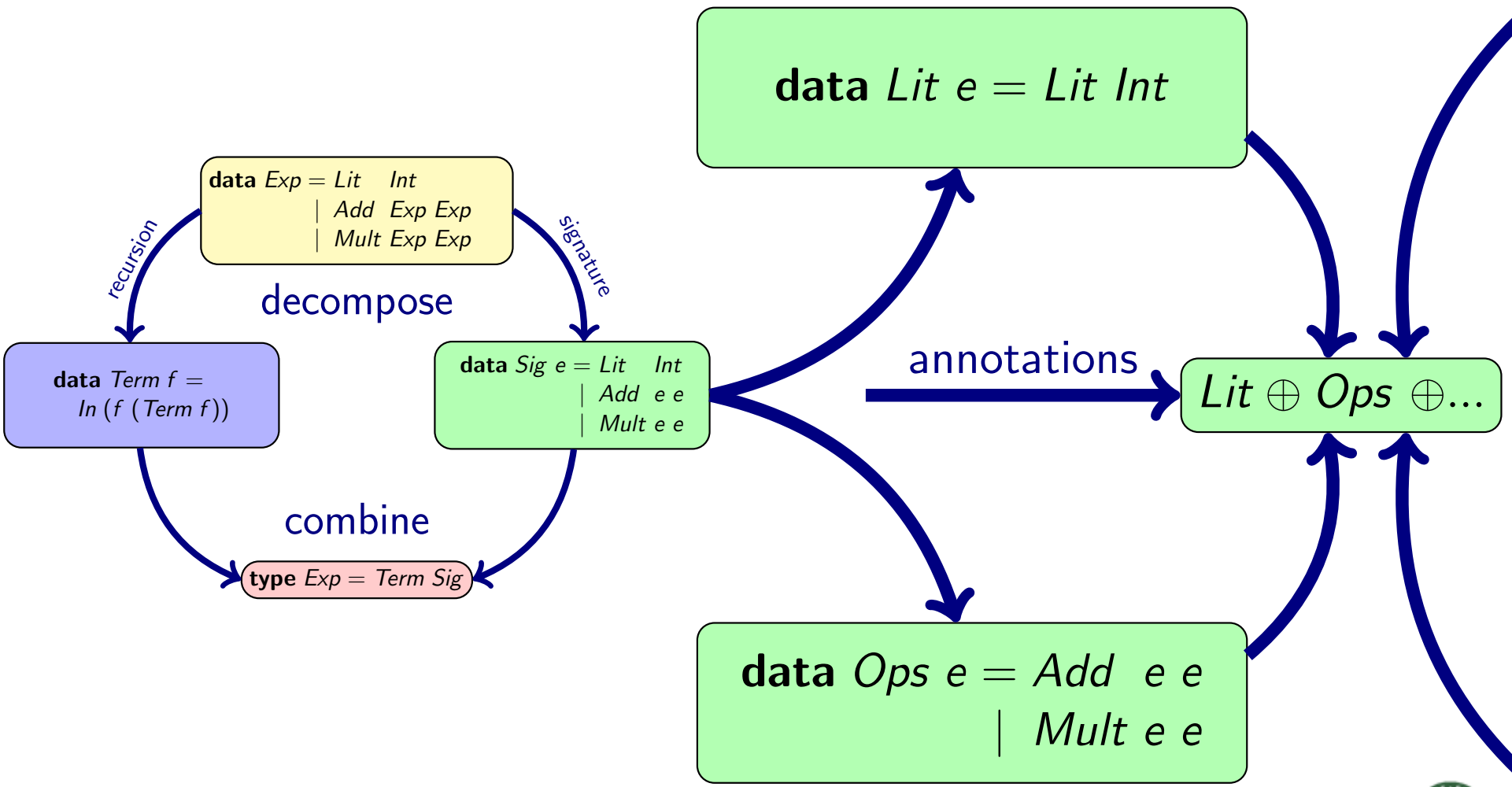

Variable Binding

A straightforward solution

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type Name = String
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data Var e = Var Name
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- Capture-avoiding substitutions
- Implementing embedded languages

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Goal

Use higher-order abstract syntax to leverage the variable binding mechanism of the host language.

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Explicit Variables

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Lam ($\lambda x \rightarrow$ ...x...)

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  (catamorphism needs an algebra and the inverse coalgebra)
- Full function space allows for exotic terms

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  $\rightsquigarrow$ Washburn & Weirich (2008): polymorphism & abstract type of terms

Parametric Higher-Order Abstract Syntax

[Chlipala 2008]

Idea

- Signature for lambda bindings:

  data $Lam\ a\ e = Lam\ (a \to e)$

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data *Sig a e* = *Lam* (*a* → *e*) | *App e e*

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Recursion Schemes

Generalising functors

class *Functor f* **where**

$fmap :: (a \to b) \to f\ a \to f\ b$

Recursion Schemes

Generalising functors to difunctors

class *Difunctor* f **where**

$$dimap :: (a \to b) \to (c \to d) \to f\ b\ c \to f\ a\ d$$

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Catamorphisms

```
cata :: Difunctor f ⇒ Alg f c → Term f → c
cata φ (Term t) = cat t
where cat :: Trm f c → c
      cat (In t)  = φ (dimap id cat t)
      cat (Var x) = x
```

Example

Declaring a catamorphism

class *Count f* **where**

ϕ_{Count} :: *Alg f Int*

count :: (*Difunctor f*, *Count f*) ⇒ *Term f* → *Int*

count = *cata* ϕ_{Count}

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Instantiation

instance *Count Lam* **where**

ϕ_{Count} (*Lam f*) = *f* 1

instance *Count App* **where**

ϕ_{Count} (*App* e_1 e_2) = $e_1 + e_2$

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Let expressions

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$$e, e' :: \mathit{Term}\ \mathit{Sig}'$$
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Example

let $y = \lambda x.x\,x$ in $y\,y$

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$e = Term\ \$\ iLam\ (\lambda x \to x\ `iApp`\ x)$

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Extending the variable counter

instance *Count Let* **where**

$$\phi_{\mathrm{Count}}\ (\mathit{Let}\ e\ f) = e + f\ 1$$

But Wait, There is More!

Term transformations

- functions of type $\mathit{Term}\ f \to \mathit{Term}\ g$
- e.g. desugaring, constant folding, type inference, annotations
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Generalised Algebraic Data Types

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- algebras $\rightsquigarrow$ many-sorted algebras

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Generalised Algebraic Data Types

- difunctors $\rightsquigarrow$ indexed difunctors
- algebras $\rightsquigarrow$ many-sorted algebras
- mutually recursive data types (with binders)
- for simple type systems (e.g. simply typed lambda calculus)

Current Work

We use our library constantly. $\rightsquigarrow$ We extend it constantly.

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Other extensions

- algebras with nested monadic effect
- tree homomorphisms
- tree transducers
- attribute grammars

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Try it yourself

- `http://hackage.haskell.org/package/compdata`
- `cabal install compdata`

An example – Language Definition & Desugaring

```
data Lam a b  = Lam (a → b)
data App a b  = App b b
data Lit a b  = Lit Int
data Plus a b = Plus b b
data Let a b  = Let b (a → b)
data Err a b  = Err

$(derive [smartConstructors, makeDifunctor, makeShowD, makeEqD, makeOrdD]
         [''Lam, ''App, ''Lit, ''Plus, ''Let, ''Err])

e :: Term (Lam :+: App :+: Lit :+: Plus :+: Let :+: Err)
e = Term (iLet (iLit 2) (λx → (iLam (λy → y `iPlus` x) `iApp` iLit 3)))

-- * Desugaring
class Desug f g where
  desugHom :: Hom f g

$(derive [liftSum] [''Desug]) -- lift Desug to coproducts

desug :: (Difunctor f, Difunctor g, Desug f g) ⇒ Term f → Term g
desug (Term t) = Term (appHom desugHom t)

instance (Difunctor f, Difunctor g, f :<: g) ⇒ Desug f g where
  desugHom = In . fmap Hole . inj -- default instance for core signatures

instance (App :<: f, Lam :<: f) ⇒ Desug Let f where
  desugHom (Let e1 e2) = inject (Lam (Hole . e2)) `iApp` Hole e1
```

An example – Call-By-Value Evaluation

```
data Sem m = Fun (Sem m → m (Sem m)) | Int Int

class Monad m ⇒ Eval m f where
  evalAlg :: Alg f (m (Sem m))

$(derive [liftSum] [''Eval]) -- lift Eval to coproducts

eval :: (Difunctor f, Eval m f) ⇒ Term f → m (Sem m)
eval = cata evalAlg

instance Monad m ⇒ Eval m Lam where
  evalAlg (Lam f) = return (Fun (f . return))

instance MonadError String m ⇒ Eval m App where
  evalAlg (App mx my) = do x ← mx
                           case x of Fun f → my >>= f
                                     _     → throwError "stuck"

instance Monad m ⇒ Eval m Lit where
  evalAlg (Lit n) = return (Int n)

instance MonadError String m ⇒ Eval m Plus where
  evalAlg (Plus mx my) = do x ← mx
                            y ← my
                            case (x,y) of (Int n,Int m) → return (Int (n + m))
                                          _             → throwError "stuck"

instance MonadError String m ⇒ Eval m Err where
  evalAlg Err = throwError "error"
```