

ETF Project 1

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1 Abstract

This project investigates the statistical properties of four exchange-traded funds (ETFs): *AGG*, *VAW*, *IWN*, and *SPY*. This will be done using a descriptive analysis and a statistical analysis of the weekly return in the period 2006-2015. The results show that the selected ETFs do not follow a strict normal distribution, that the mean weekly return of *AGG* is not significantly different from zero, and that there is no significant difference between the mean weekly return of *AGG* and *VAW*. Furthermore, a strong correlation between *VAW* and *IWN* is found.

2 Descriptive analysis

a) Description of the dataset

The dataset consists of 95 different ETFs and their weekly return in the period 05/05-2006 - 05/08-2015. In total, that is 454 trading weeks. The weekly returns are a quantitative variable, and the variable 't' is a date object. In this project, the ETFs *AGG*, *VAW*, *IWN*, and *SPY* are selected and will be analyzed throughout this project.

b) Density histogram

Now, let's look at the empirical density histogram of the weekly returns from *AGG*. The empirical density looks symmetrical around the median. The majority of the observations fall between -1% and 1% weekly return, indicating a small variation. It is also apparent that the weekly returns can both be positive and negative. From the histogram, it looks like the weekly returns may be following a normal distribution. This will be examined later on.

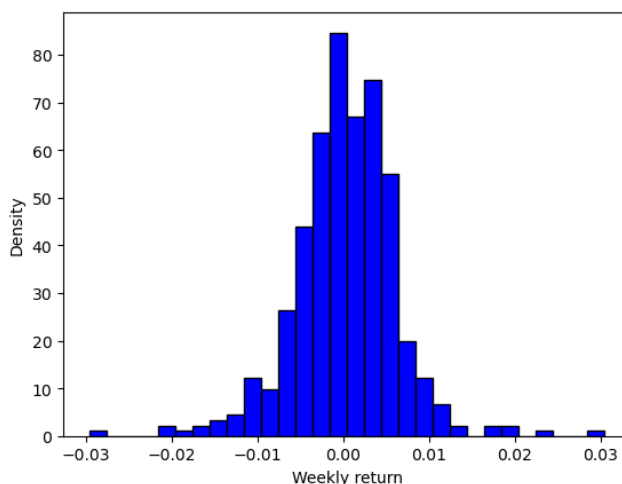


Figure 1: Density histogram of weekly returns for AGG

c) Weekly return over time

Below, the weekly returns over time for the selected ETFs are plotted. The weekly return of *AGG* is way less volatile compared to the three others. Especially during the financial crisis of 2008 and 2011, where *VAW*, *IWN* and *SPY* dropped between 10 and 20% in a single week. Overall *AGG* stands out against the three other ETFs as being less volatile.

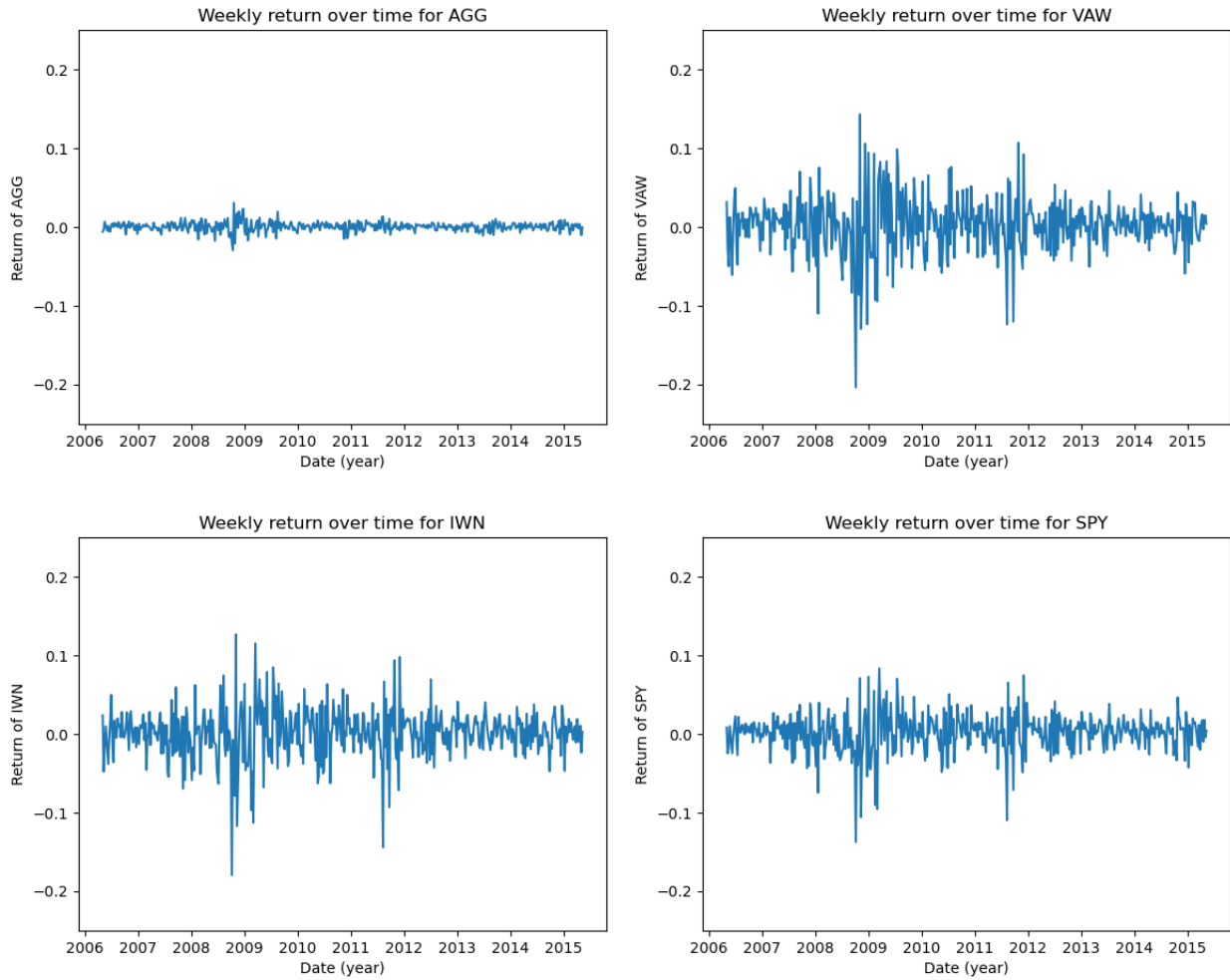


Figure 2: Weekly returns for selected ETFs

d) Boxplot

Below is a boxplot of the weekly returns for the selected ETFs. The empirical distribution seems to be symmetrical, and the boxplot for AGG is way narrower than the rest (lower IQR). This indicates a smaller spread and therefore less volatility compared to the rest. Each of the ETFs has multiple extreme observations in both the positive and negative directions. This indicates trading weeks with big movements, such as a market crash or a big surge in the stock market.

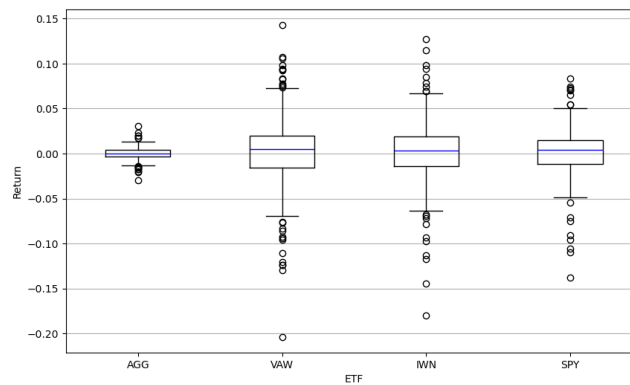


Figure 3: Boxplots of the selected ETFs

e) Summary statistics

Table 1 shows important statistics regarding the selected ETFs. It is now possible to compare the ETFs based on statistical parameters such as mean and standard deviation. It is noticeable that the mean weekly return for *AGG* is significantly lower than that of the others, with a mean of 0.00027 compared to 0.00119 – 0.00179. This means that *AGG* on average will yield a lower return compared to the other ETFs. *AGG* also has a lower standard deviation with just 0.00598. This means that the ETF has low volatility compared to the other ETFs.

ETF	Number of obs. (n)	Sample mean (\bar{x})	Sample variance (s^2)	Std. dev. (s)	Lower quartile (Q_1)	Median (Q_2)	Upper quartile (Q_3)
AGG	454	0.00027	0.00006	0.00598	-0.00297	0.00024	0.00389
VAW	454	0.00179	0.00130	0.03608	-0.01610	0.00480	0.01969
IWN	454	0.00119	0.00102	0.03202	-0.01431	0.00312	0.01906
SPY	454	0.00136	0.00061	0.02479	-0.01133	0.00422	0.01450

Table 1: Summary statistics table

3 Statistical analysis

f) Model validation

It is now assumed that the four ETFs follow a normal distribution. This can be expressed by the model: $X_i \sim N(\mu, \sigma^2)$ and i.i.d, where μ is the mean, σ^2 is the variance and $i = 1, \dots, n$.

The best estimate for the parameters of the model is the sample mean and sample variance found in section c). After inserting these values, we get the following models:

$$\begin{aligned}
X_{AGG} &\sim N(0.00027, 0.00006) \\
X_{VAW} &\sim N(0.00179, 0.00130) \\
X_{IWN} &\sim N(0.00119, 0.00102) \\
X_{SPY} &\sim N(0.00136, 0.00061)
\end{aligned}$$

To assess the normality assumption, four QQ-plots are constructed for the mean weekly return. The QQ-plots below show a relatively good fit in the central samples. The tails, however, in both the positive and negative sides, are clearly deviating from a normal distribution. The extremes in both directions represent extreme market movements. Therefore, it can be concluded that the ETFs do not follow a strict normal distribution.

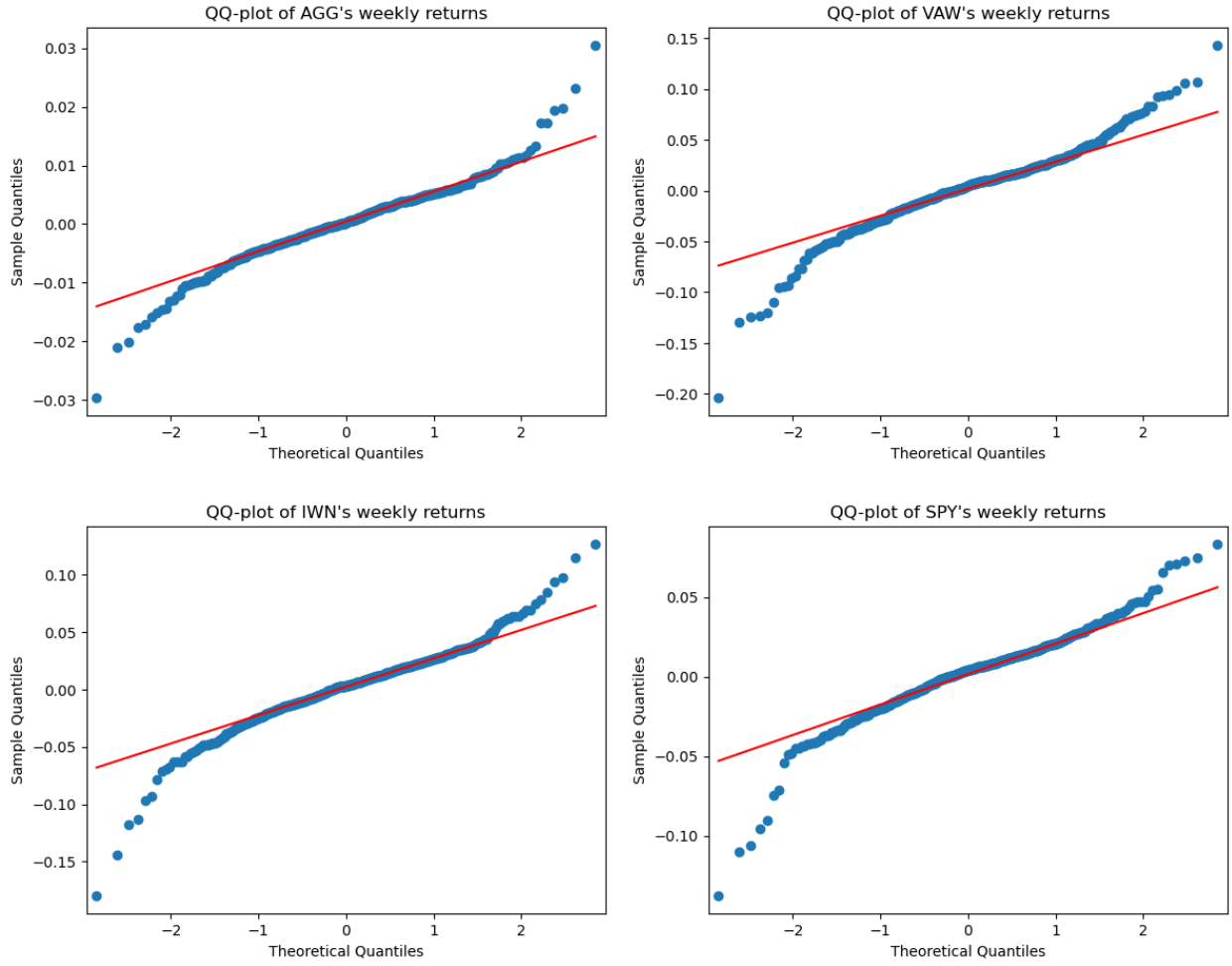


Figure 4: QQ plot for selected ETFs

g) Confidence intervals

Now the 95% confidence intervals (CI) for the weekly returns are computed with the formula: $\bar{x} \pm t_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}}$, where \bar{x} is the sample mean, $t_{1-\alpha/2}$ is the $(1 - \alpha/2)$ quantile from the t-distribution with $n - 1$ degrees of freedom, s is the standard deviation and n is the sample size. In the table below are the confidence intervals for the selected ETFs. Here *AGG* stands out again with a narrower confidence interval compared to the rest.

	Lower bound of CI	Upper bound of CI
AGG	-0.00029	0.00082
VAW	-0.00153	0.00512
IWN	-0.00177	0.00414
SPY	-0.00093	0.00365

Table 2: 95% confidence intervals for selected ETFs

h) Hypothesis test 1

Now we want to investigate whether the weekly return from *AGG* differs from not investing the money at all. Therefore, the following hypothesis is tested:

$$H_0 : \mu_{AGG} = 0.$$

$$H_1 : \mu_{AGG} \neq 0.$$

The significance level $\alpha = 0.05$ is chosen and the p-value is calculated with the formula: $2 \cdot (T > |t_{obs}|)$, where T follows a t-distribution with $(n - 1)$ degrees of freedom and $|t_{obs}| = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$.

The p-value is computed to be 0.34385, which, according to Table 3.1 from the book *Introduction to statistics at DTU*, means that there is little to no evidence against H_0 . Since $0.34385 > \alpha$, the null hypothesis is accepted. This does not mean that we can conclude that the null hypothesis is true, but rather that it may be true. This conclusion could also be obtained from the confidence interval in the previous section $[-0.00029, 0.00082]$, because 0 is within this interval. Therefore, the mean weekly return of *AGG* is not significantly different from zero.

i) Hypothesis test 2

Now we want to investigate whether the weekly return from *AGG* differs from the weekly return of *VAW*. Therefore, the following hypothesis is tested:

$$H_0 : \mu_{VAW} - \mu_{AGG} = 0.$$

$$H_1 : \mu_{VAW} - \mu_{AGG} \neq 0.$$

The significance level $\alpha = 0.05$ is chosen again. To determine the p-value, the following formula is used:

$$\text{p-value} = 2 \cdot (T > |t_{obs}|), \text{ where } T \text{ follows a t-distribution with } v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \text{ degrees of freedom and}$$

$$|t_{obs}| = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}.$$

The p-value is calculated to be 0.37381, which, according to Table 3.1 from the book *Introduction to statistics at DTU*, means that there is little to no evidence against H_0 . Comparing the p-value to the significance level tells us that the null hypothesis is accepted: $0.37381 > \alpha$. This means there is no significant difference between the mean weekly return of *AGG* and *VAW*.

j) Necessity of statistical test

Since the confidence intervals of *VAW* $[-0.00153, 0.00512]$ and of *AGG* $[-0.00029, 0.00082]$ overlap, it would be tempting to accept the null hypothesis on this ground alone. This, however, is not a valid conclusion according to Remark 3.59 from *Introduction to statistics at DTU*. Therefore, it was necessary to perform the Welch t-test to accept the null hypothesis.

4 Correlation

k) Correlation between *VAW* and *IWN*.

In order to calculate the correlation between *VAW* and *IWN* the formula for correlation is used: $\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \cdot \sigma_y}$. The correlation is calculated to be 0.85. This indicates a relatively strong correlation between the two ETFs. Thus, a portfolio consisting of these two ETFs alone would not be considered a diversified portfolio. Below is a scatter plot of the two ETFs. They appear to have a strong correlation, as expected from the calculated correlation value.

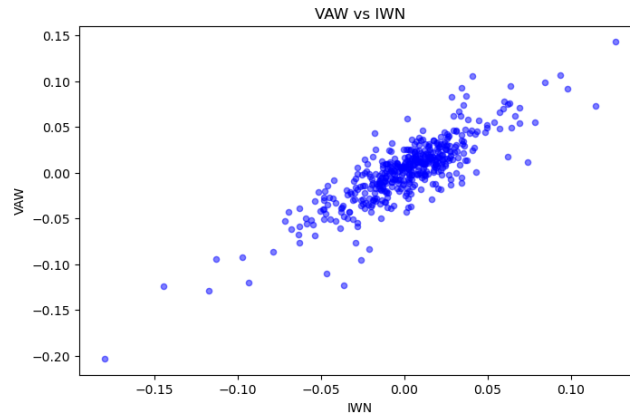


Figure 5: Correlation between VAW and IWN

5 Appendix

This appendix contains the code used for the figures and calculations in this report. The Jupyter Notebook is available in the following file:

`etf_analysis.ipynb`