Øving 6

(1) $b_1 = 4$ $b_2 = 12$ $b_n = b_{n-2} + b_{n-1}$

$4 \mid b_n$ for alle heltall $n \geq 1$

Setter $n = 1$: $P(n)$

$b_1 = 4$ $\underline{4 \mid 4}$

Setter $n = k$

Antar $P(1), P(2), P(n)$ sanne.

$b_k = b_{k-2} + b_{k-1}$

Vil vite om $P(k+1)$ er delelig med 4.

Setter $n = k+1$

$b_{k+1} = b_{k-1} + b_k$

$P(k-1)$: $\exists\, q_1 \in \mathbb{Z}$ slik at $b_{k-1} = 4 q_1$.

$P(k)$: $\exists\, q_2 \in \mathbb{Z}$ slik at $b_k = 4 q_2$

Bevis. $b_{k+1} = b_{(k+1)-2} + b_{(k+1)-1}$

$= b_{k-1} + b_k$

$= 4 q_1 + 4 q_2$

$= \underline{4(q_1 + q_2)}$

$\underline{\underline{4 \mid 4(q_1 + q_2)}}$

2

$g_k = g_{k-1} + 5k \quad , \quad g_1 = 5$

$g_2 = g_{2-1} + 5 \cdot 2 = 5 + 10 = 15$

$g_3 = g_{3-1} + 5 \cdot 3 = 15 + 15 = 30$

$g_4 = g_{4-1} + 5 \cdot 4 = 30 + 20 = 50$

$g_5 = g_{5-1} + 5 \cdot 5 = 50 + 25 = 75$

$\sum_{i=1}^{n} 5i$

3 1 $c_k = \dfrac{c_{k-1}}{2+c_{k-1}}$ $\quad k \geq 2$

2 $c_1 = 1$

$$c_2 = \frac{c_{2-1}}{2+c_{2-1}} = \frac{c_1}{2+c_1} = \frac{1}{2+1} = \frac{1}{3}$$

$2^n - 1$

$$c_3 = \frac{c_{3-1}}{2+c_{3-1}} = \frac{c_2}{2+c_2} = \frac{\frac{1}{3}}{2+\frac{1}{3}} = \frac{1}{7}$$

$2^3 - 1$

$$c_4 = \frac{c_{4-1}}{2+c_{4-1}} = \frac{c_3}{2+c_3} = \frac{\frac{1}{7}}{2+\frac{1}{7}} = \frac{1}{15}$$

$2^4 - 1$

$$c_5 = \frac{c_{5-1}}{2+c_{5-1}} = \frac{c_4}{2+c_4} = \frac{\frac{1}{15}}{2+\frac{1}{15}} = \frac{1}{31}$$

$$c_n = \frac{1}{2^n - 1}$$

$n^2 + 6$

$n = 1$:

$$c_1 = \frac{1}{2^1 - 1} = \frac{1}{1} = 1$$

$n = k$:

$$c_k = \frac{1}{2^k - 1}$$

$n = k+1$

$$c_{k+1} = \frac{1}{2^{k+1} - 1}$$

Sett inn :

$$c_{k+1} = \frac{c_k}{2 + c_k} = \frac{\frac{1}{2^k - 1}}{2 + \frac{1}{2^k - 1}}$$

$$= \frac{\frac{1}{2^k - 1} \cdot (2^k - 1)}{2 + \frac{1}{2^k - 1} \cdot (2^k - 1)} = \frac{1}{2^{k+1} - 2 + 1}$$

$$= \underline{\underline{\frac{1}{2^{k+1} - 1}}}$$

④ $t_k = t_{k-1} + 3k + 1 \qquad k \geq 1$

$t_0 = 0$

4

$t_1 = t_{1-1} + 3 \cdot 1 + 1 = 0 + 3 + 1 = 4$

7

$t_2 = t_{2-1} + 3 \cdot 2 + 1 = 4 + 6 + 1 = 11$

10

$t_3 = t_{3-1} + 3 \cdot 3 + 1 = 11 + 9 + 1 = 21$

13

$t_4 = t_{4-1} + 3 \cdot 4 + 1 = 21 + 12 + 1 = 34$

16

$t_5 = t_{5-1} + 3 \cdot 5 + 1 = 34 + 15 + 1 = 50$

$$t_k = 3(1 + 2 + \ldots k) + k = 3\frac{k(k+1)}{2} + k$$

$$= \frac{3k^2 + 5k}{2}$$

$k = 0$:

$$t_0 = 3\frac{0(0+1)}{2} + 0 = \underline{0}$$

Antar $k \geq 0$

Må vise at $k+1$ holder for

$$t_{k+1} = \frac{3(k+1)^2 + 5(k+1)}{2} =$$

$$t_{k+1} = t_k + 3k + 1 = \frac{3k^2 + 5k}{2}$$

$$= 3(k+1) + 1 = \frac{3k^2 + 5k}{2} + \frac{2(3k+4)}{2}$$

$$= \frac{3k^2 + 5k + 6k + 8}{2} = \frac{3k^2 + 6k + 3 + 5k + 5}{2}$$

$$= \frac{3(k+1)^2 + 5(k+1)}{2} \quad \text{qed}$$

5) a) $a_k = 7a_{k-1} - 10a_{k-2} \quad k \geq 2$

$t^2 - 7t + 10 = 0$

$t^2 - 7t + 10 = (t-2)(t-5)$

$t_1 = 2 \wedge t_2 = 5$

$a_n = C \cdot 2^n + D \cdot 5^n$

b) $b_k = b_{k-1} + 6b_{k-2} \quad k \geq 2$

$t^2 - t - 6 = 0$

$t^2 - t - 6 = (t+2)(t-3)$

$t_1 = -2 \wedge t_2 = 3$

$b_n = C(-2)^n + D \cdot 3^n$

c) $c_k = 2c_{k-1} - c_{k-2} \qquad k \geq 2$

$t^2 - 2t + 1 = 0$

$t^2 - 2t + 1 = (t-1)(t-1)$

Dobbelrot: $t = 1$, 1

$c_n = C \cdot 1^n + Dn \cdot 1^n = C + Dn$

(6) a) $e_k = 9e_{k-2} \qquad k \geq 2 \qquad e_0 = 0, \; e_1 = 3$

Karakteristisk likn: $t^2 - 9t = 0$

$t^2 - 9t = (t+3)(t-3) \qquad t_1 = -3, \; t_2 = 3$

$e_n = C \cdot 3^n + D(-3)^n$

$e_0 = 0 = C \cdot 3^0 + D(-3)^0 = C + D$

$e_1 = 3 = C \cdot 3^1 + D(-3)^1 = 3C - 3D$

$C = \frac{1}{2}, \quad D = -\frac{1}{2}$

$e_n = \frac{1}{2} \cdot 3^n - \frac{1}{2}(-3)^n = \frac{3^n}{2} - \frac{(-3)^n}{2}$

b) $s_k = -4s_{k-1} - 4s_{k-2} \quad k \geq 2 \quad s_0 = 0, s_1 = -1$

Karakteristik Denk: $t^2 + 4k^2 + 4 = 0$

$\Rightarrow (t+2)^2$

$s_n = C \cdot (-2)^n + Dn \cdot (-2)^n$

$s_0 = 0 = C \cdot (-2)^0 + D \cdot 0 \cdot (-2)^0 = C$

$s_1 = -1 = C \cdot (-2)^1 + D \cdot 1 \cdot (-2)^1 = -2C - 2C$

$C = 0 \quad , \quad D = \frac{1}{2}$

$$s_n = \frac{n}{2}(-2)^n = \underline{\underline{\frac{n(-2)^n}{2}}}$$

c) $t_k = 6t_{k-1} - 9t_{k-2} \quad k \geq 2 \quad t_0 = 1, t_1 = 3$

Karakteristik Denk: $t^2 - 6t + 9 = 0$

$\Rightarrow (t-3)^2$

$t_n = C 3^n + Dn \cdot 3^n$

$t_0 = 1 = C \cdot 3^0 + D \cdot 0 \cdot 3^0 = C + D$

$t_1 = 3 = C \cdot 3^1 + D \cdot 1 \cdot 3^1 = 3C + 3D$

$C = 1 \quad D = 0$

$$\underline{\underline{t_n = 3^n}}$$