A short note on 2D integrator implementation

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Abstract

In this note I shortly describe the ideas behind my two-dimensional integrator implementation.

1 Introduction

The task is to numerically calculate two-dimensional integrals of the form

$$I = \int_a^b \mathrm{d}x \int_{d(x)}^{u(x)} \mathrm{d}y \, f(x, y), \tag{1}$$

by consecutively applying an adaptive one-dimensional integrator along each of the two dimensions. Let

$$F(x) := \int_{d(x)}^{u(x)} \mathrm{d}y \, f(x, y), \tag{2}$$

so that

$$I = \int_a^b \mathrm{d}x \, F(x). \tag{3}$$

The integral I is estimated by a weighted sum of the form

$$I \approx \sum_i w_i F(x_i), \tag{4}$$

where the function values $F(x_i)$ are given by the integral

$$F(x_i) = \int_{d(x_i)}^{u(x_i)} \mathrm{d}y \, f(x_i, y), \tag{5}$$

and estimated as a weighted sum in a similar fashion

$$F(x_i) \approx \sum_j \omega_j f(x_i, y_j). \tag{6}$$

Both Eq. 4 and 6 are estimated using a one-dimensional recursive adaptive integrator.