Operation Research
Final Project Report

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February 2, 2021

In this assignment, we are going to take the power distribution system in Iceland into consideration and calculate the mean power that should be distributed to each city (using only the 10 largest cities) by each nearby powerplant. For this matter, we are going to formulate the problem as a Linear Programming model in the same way as a Transportation Problem. To solve the problem, we will use the PuLP solver to get the optimized answers.

Keywords: Linear Programming, Power Distribution, PuLP, Transportation Problem, LP Problem Formulation

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1 Introduction

In this project, we are going to go through different phases including:

1. Data Collection
2. Problem Formulation
3. Implementation

In the first phase, we will have to gather the data from Iceland's powerplants and their distance from the top 10 large cities in the country. We will then have to formulate a problem, considering a loss based on the data we have gathered from the previous part. Finally, we will implement our model in order to test whether it works.

2 Data Collection

2.1 Finding the Cities

In this section, we first have to identify the 10 largest cities in Iceland which we have done through a simple google search. These cities are listed below in descending order:

1. Höfn
2. Vopnafjörður
3. Fjarðabyggð
4. Djúpivogur
5. Grýtubakkahreppur
6. Grindavik
7. Reykjavík
8. Seydisfjordur
9. Mosfellsbær
10. Blönduós

2.2 Finding the Power Plants

Locating the cities in Google Maps, we can find the latitude and longitude. Then, we search for the given coordinates in openinframap and with that, we can find the nearest powerplants to the given city. After that, we can calculate the distance from point-to-point distance of the towers in a line in the path from each power plant to the corresponding city. A large scale overview of the power lines in Iceland is given below:

Figure 1: Iceland's powerlines map.

As you can see, the powerlines are mostly stretched throughout the sides of the land and very little span the middle of the country. The distances calculated are given in the table below:

Table 1: The cities and their distances in correspondence to their closest power plants.

City	Fljótsdalsstöð	Svartsengi	Hellisheiðar	Blöndu	Sigöldu	Reykjanes	Írafoss	Nesjavellir
Höfn	141.69	∞	∞	∞	236.24	∞	∞	∞
Vopnafjörður	125.17	∞	∞	∞	∞	∞	∞	∞
Fjarðabyggð	32.31	∞	∞	∞	∞	∞	∞	∞
Djúpivogur	70.67	∞	∞	∞	∞	∞	∞	∞
Grýtubakkahreppur	207.61	∞	∞	156.55	∞	∞	∞	∞
Grindavik	∞	3.97	∞	∞	∞	13.31	∞	∞
Reykjavík	∞	317.19	19.69	∞	∞	∞	44.23	∞
Seydisfjordur	66.51	∞	∞	∞	∞	∞	∞	∞
Mosfellsbær	∞	∞	25.83	∞	∞	∞	∞	22.95
Blönduós	∞	∞	∞	55.16	∞	∞	∞	∞

After we have found the distances from each city to its nearest powerplants, we can move on to our next step which is formulating the problem.

3 Problem Formulation

As we mentioned previously, we are going to use the Transportation Problem model to formulate a linear programming example to solve this problem. In this case, we have 8 sources and 10 demands that should be met.
The value of each source is equivalent to the maximum output (per year) for each powerplant that we have taken into consideration. Value of the demands are also the same as average annual usage for each city. The former is given in the openinframap. The latter, however, is estimated using the net usage per year for the whole country, devided by its population to give us net usage per person, and then multiplied by the population of each city to give us the average net usage for each city per year:

$$D_j = \text{Value of demand} j = \frac{\text{Net Usage per Year}}{\text{Country's Population}} \times j\text{'th City's Population} \tag{1}$$

Also, the cost for each powerline that connects a powerplant to a city is estimated by its distance. Therefore, we can say $c_{i,j}$ can be equivalent to the distances gathered in the previous part. All we have to do right now is to formulate the whole problem using the defined variables:

$$minJ = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j} x_{i,j} s.t. \begin{cases} \forall 1 \leq i \leq n : & \sum_{j=1}^{m} x_{i,j} = S_i \\ \forall 1 \leq j \leq m-1 : & \sum_{j=1}^{n} x_{i,j} = D_j \\ \sum_{i=1}^{n} S_i = \sum_{j=1}^{m} D_j & \\ \forall 1 \leq i \leq n, 1 \leq j \leq m : & x_{i,j} \geq 0 \end{cases} \tag{2}$$

4 Implementation

To apply this algorithm, we have used the PuLP solver in python. We defined three set of variables:

1. S_i which gives the output that source i should give.
2. D_j which gives the input to city j.
3. x_{ij} which gives the amount of power transmitted from plant i to city j.

We also read the data and store the maximum outputs to each power plant as $sthreshold_i$, and average power usage for each city as $dthreshold_j$, to make sure that no excess power is drawn from each plant or no little power is transferred to each city, hence:

$$\begin{cases} \forall 1 \leq i \leq n : & S_i \leq sthreshold_i \\ \forall 1 \leq j \leq m : & D_j \leq dthreshold_j \end{cases} \tag{3}$$

Inside our data, we have two matrices named $edge$ and $dist$. The first one indicates if a city is connected to a powerplant or not. The second one gives information on the distance between the two. Eventually, we use each index of their product to indicate the distance between two cities.

5 Results

Using the data we gathered which is stored in file "data3.csv", we ran the code, and the result was pleasingly fine which is shown as follows:

City	Fljótsdalsstöð	Svartsengi	Hellisheiðar	Blöndu	Sigöldu	Reykjanes	Írafoss	Nesjavellir
Höfn	130.73	-	-	-	0	-	-	-
Vopnafjörður	36.90	-	-	-	-	-	-	-
Fjarðabyggð	269.22	-	-	-	-	-	-	-
Djúpivogur	24.21	-	-	-	-	-	-	-
Grýtubakkahreppur	0	-	-	18.74	-	-	-	-
Grindavik	-	175.23	-	-	-	0	-	-
Reykjavík	-	0	6963.51	-	-	-	0	-
Seydisfjordur	35.89	-	-	-	-	-	-	-
Mosfellsbær	-	-	0	-	-	-	-	641.09
Blönduós	-	-	-	47.53	-	-	-	-

Table 2: The amount of power each city recieves from each power plant.