2. A Trilateral Weighted Sparse Coding Scheme

Topics

01. Real-world (RW) noisy image vs synthetic noisy image with additive white Gaussian noise(AWGN)

02. Sparse coding (SC) and Trilateral weighted sparse coding (TWSC)

03. Proposed denoising algorithm

    3.1. The Trilateral Weighted Sparse Coding Model

    3.2. The Setting of Weight Matrices

    3.3. Model Optimization

    3.4. The Denoising Algorithm

04. Visualization of The Weight Matrices

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

1. RW noisy image vs noisy image with AWGN

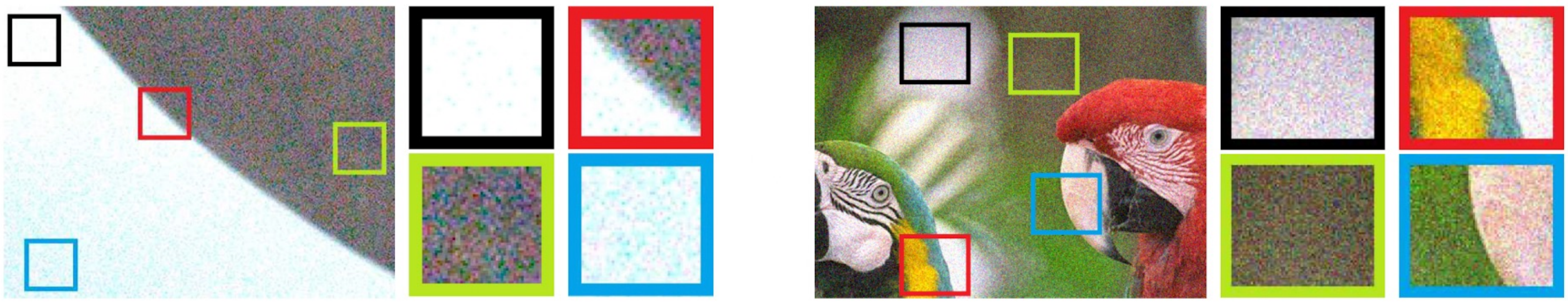
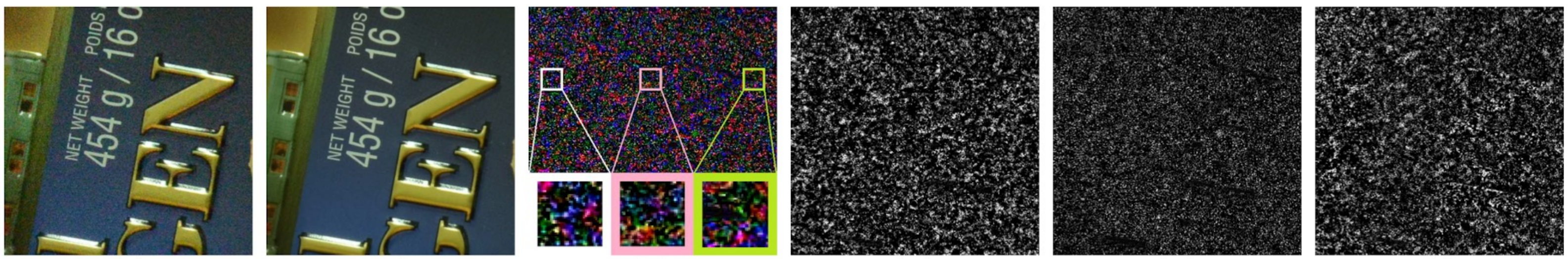


Fig. 1: Comparison of noisy image patches in real-world noisy image (left) and synthetic noisy image with additive white Gaussian noise (right).

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

1. RW noisy image vs noisy image with AWGN

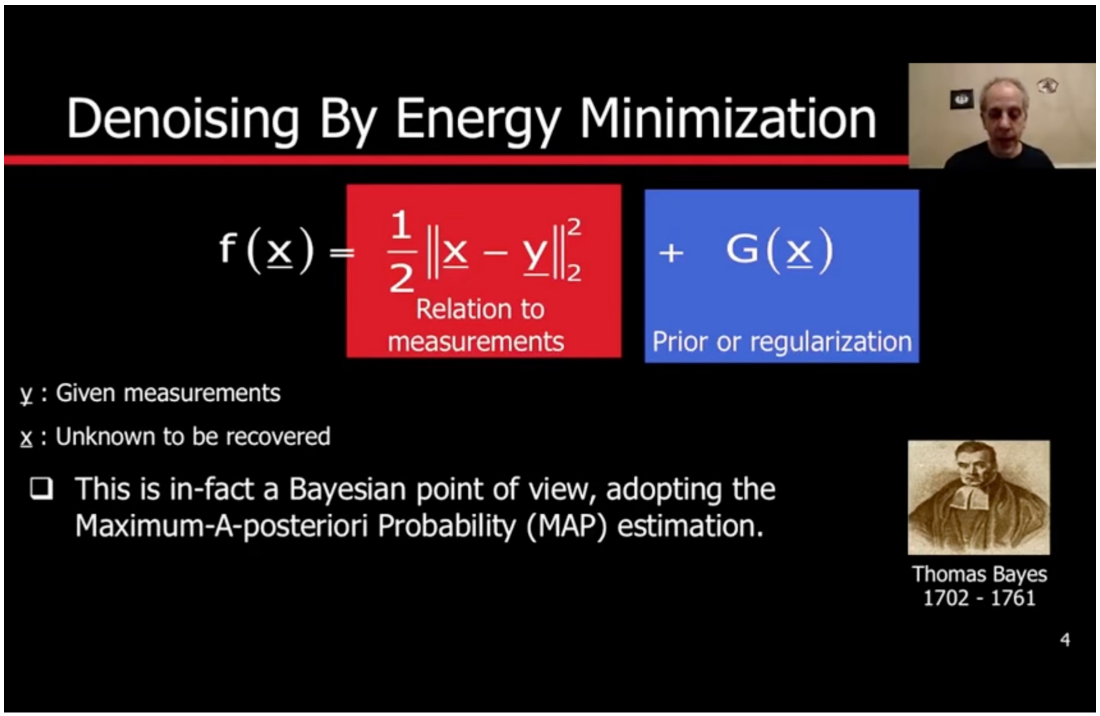


(a) (b) (c) (d) (e) (f)

Fig. 2: An example of realistic noise. (a) A real-world noisy image captured by a Nikon D800 camera with ISO = 6400; (b) the "Ground Truth" image (please refer to Section 4.3) of (a); (c) difference between (a) and (b) (amplified for better illustration); (d)-(f) red, green, and blue channel of (c), respectively. The standard deviations (stds) of noise in the three boxes (white, pink, and green) plotted in (c) are 5.2, 6.5, and 3.3, respectively, while the stds of noise in each channel (d), (e), and (f) are 5.8, 4.4, and 5.5, respectively.

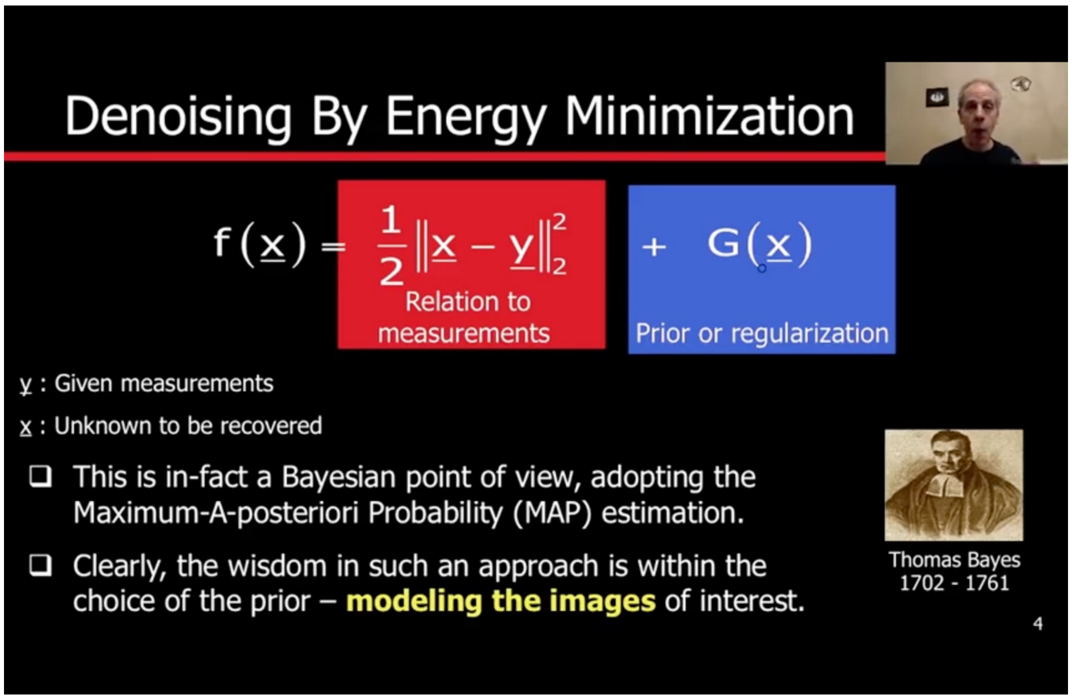
[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

2. SC and TWSC



[3] https://www.coursera.org/learn/image-processing

2. SC and TWSC



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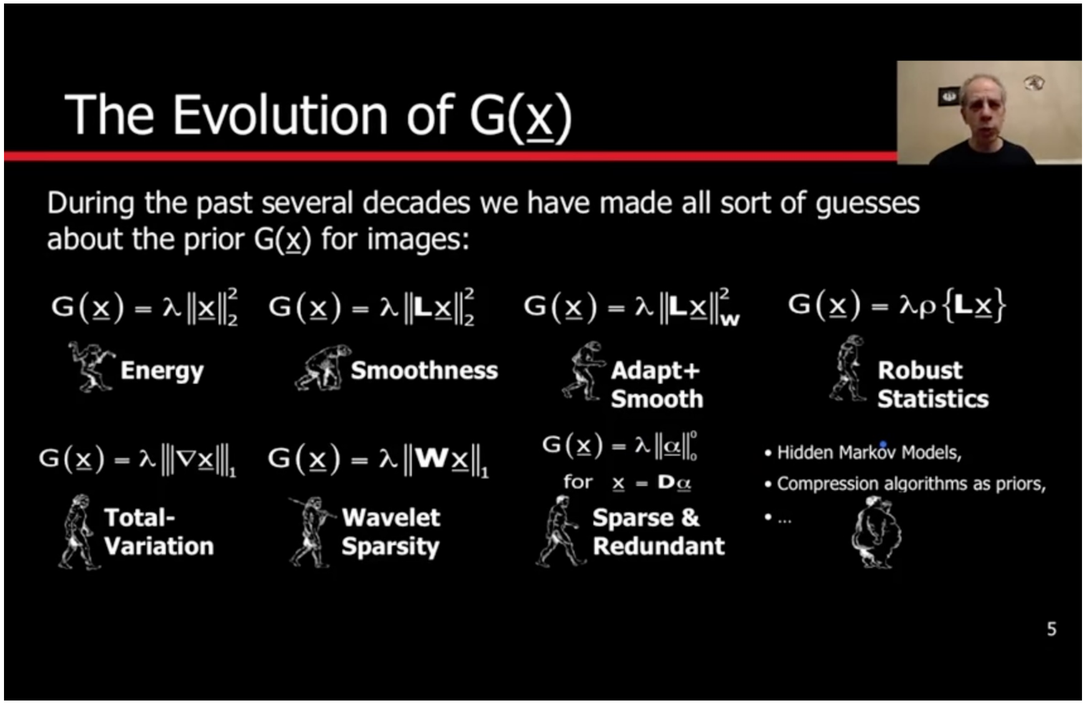

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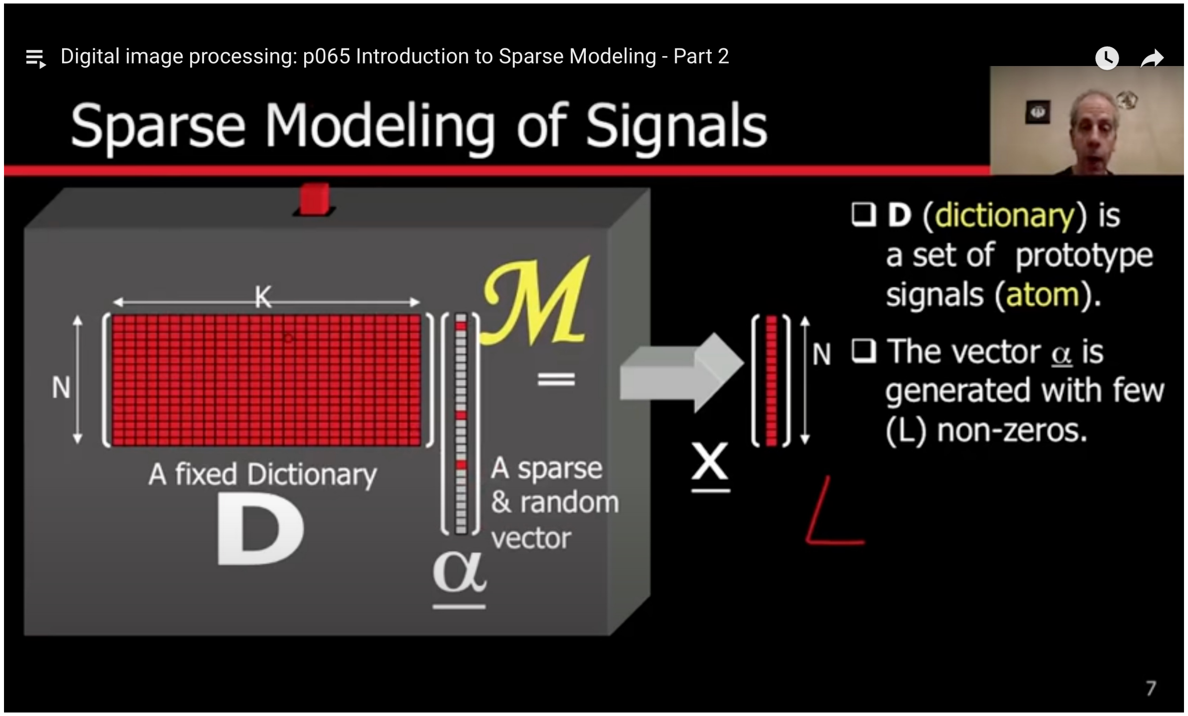

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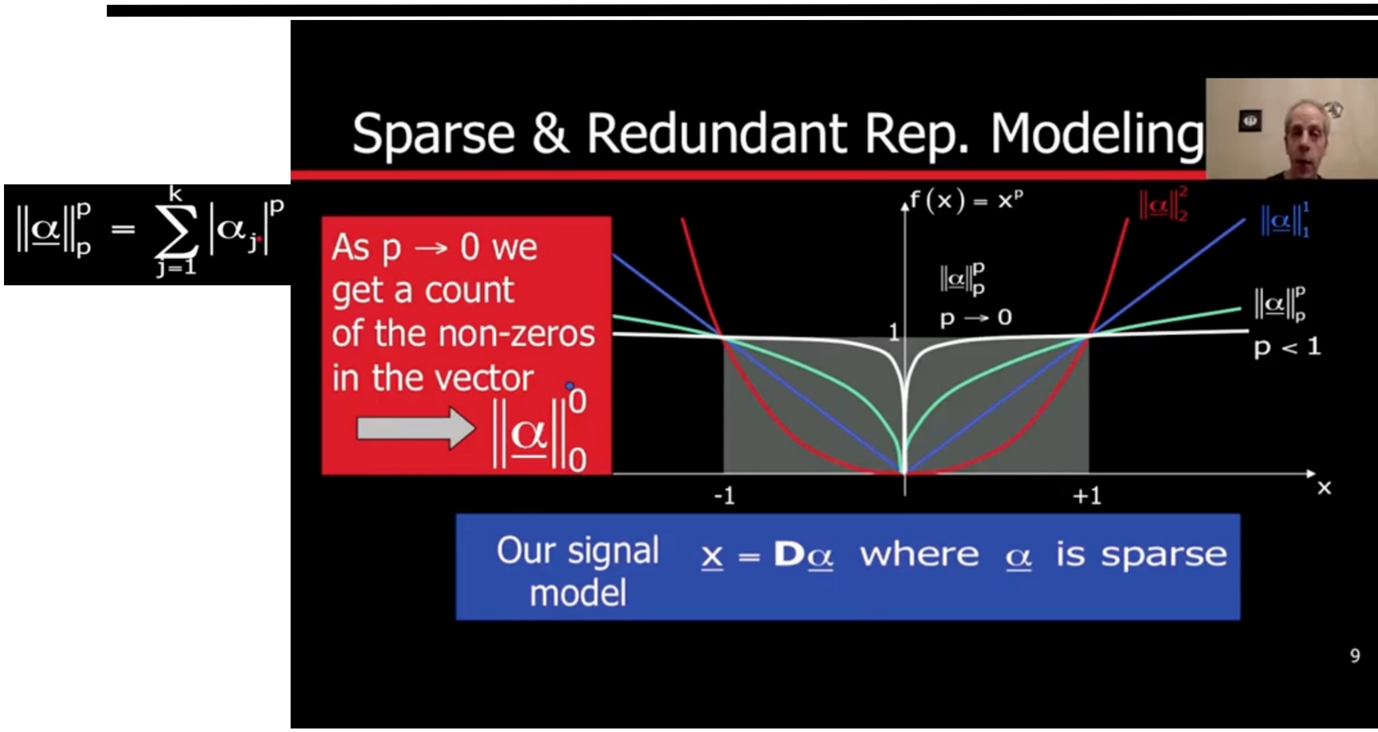

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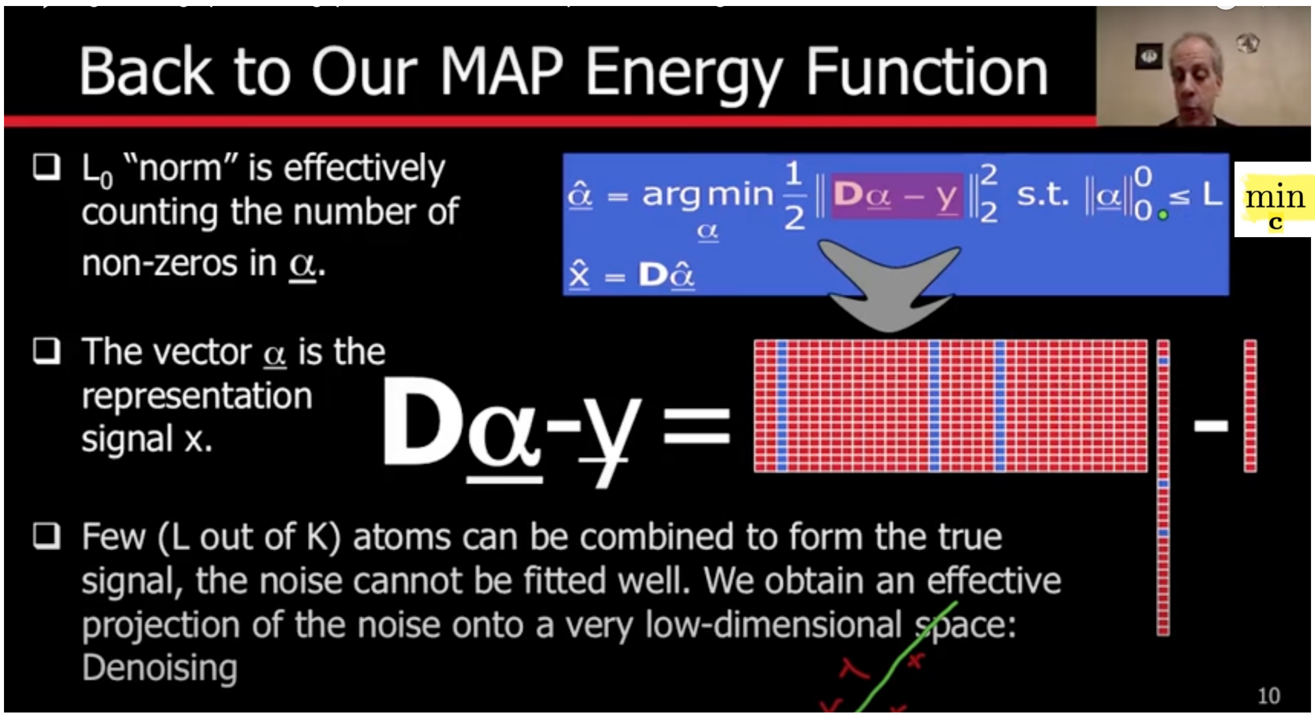

$$\min_{\mathbf{c}} \|\mathbf{y} - \mathbf{D}\mathbf{c}\|_2^2 + \lambda\|\mathbf{c}\|_q,$$

[3] https://www.coursera.org/learn/image-processing

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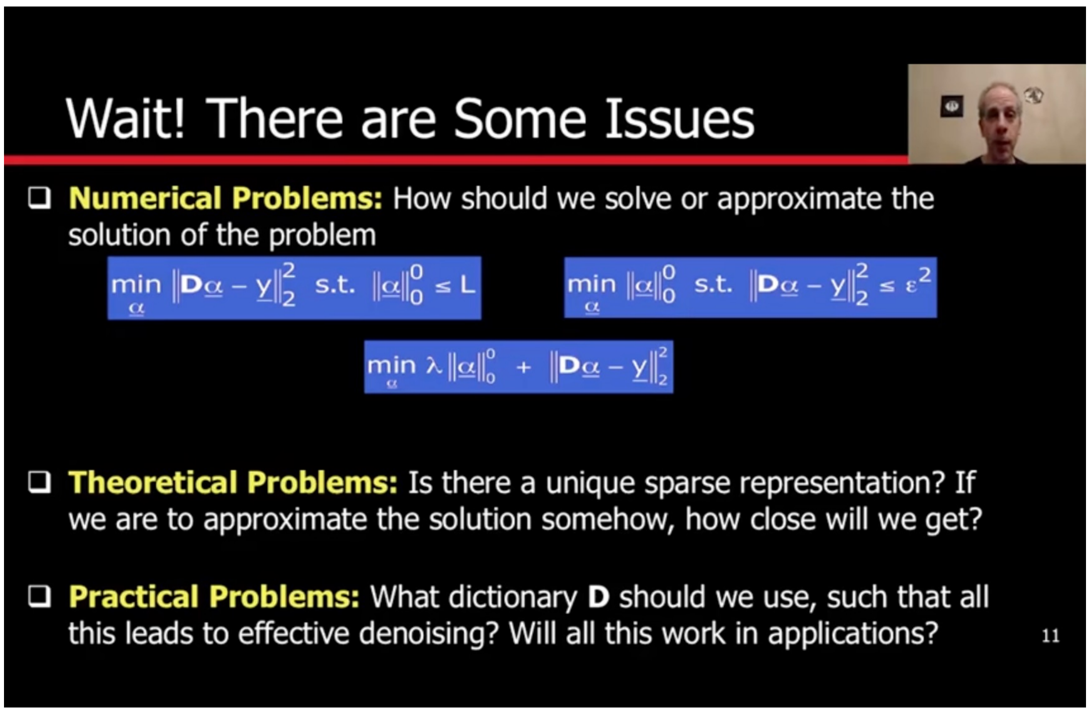

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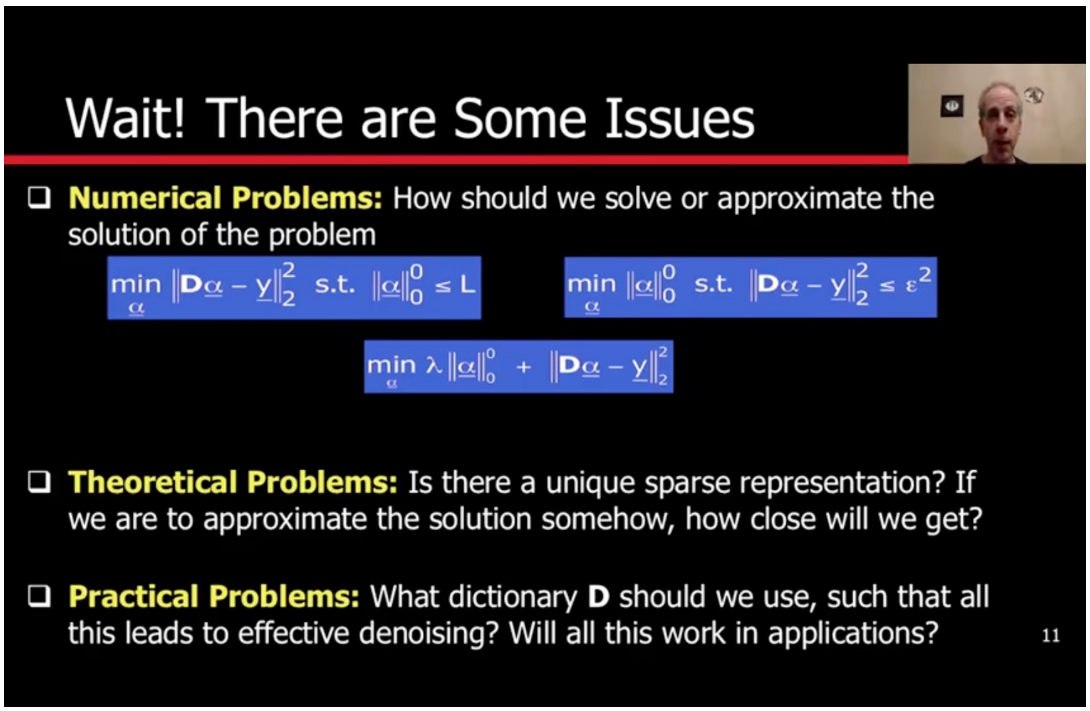

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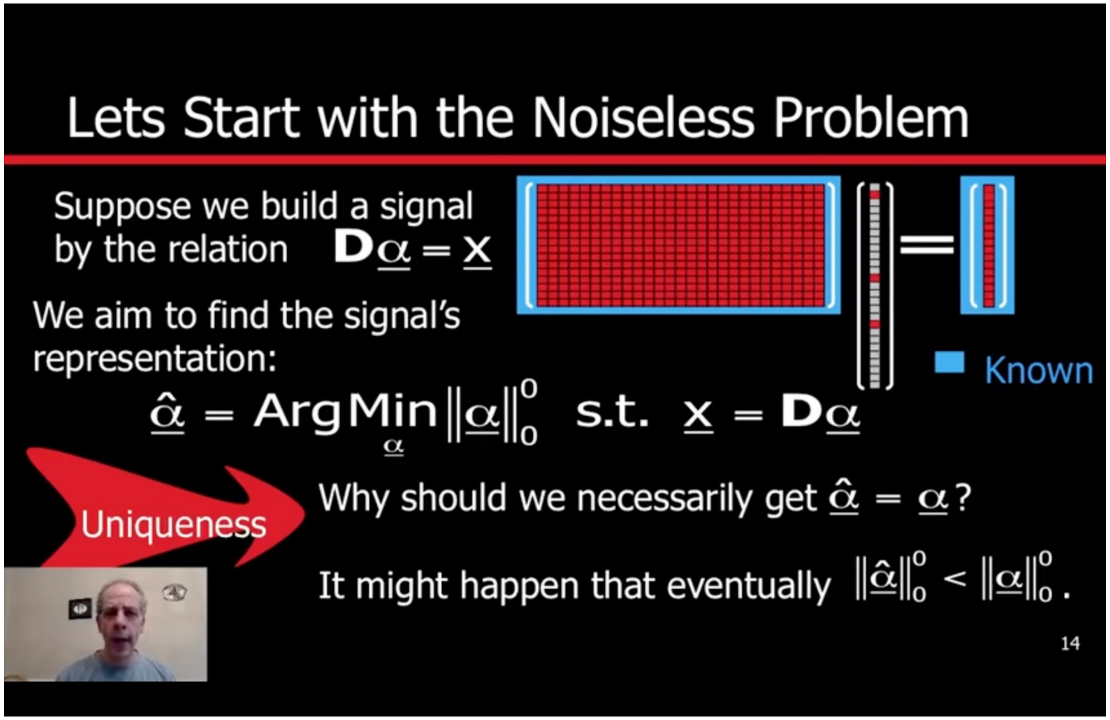

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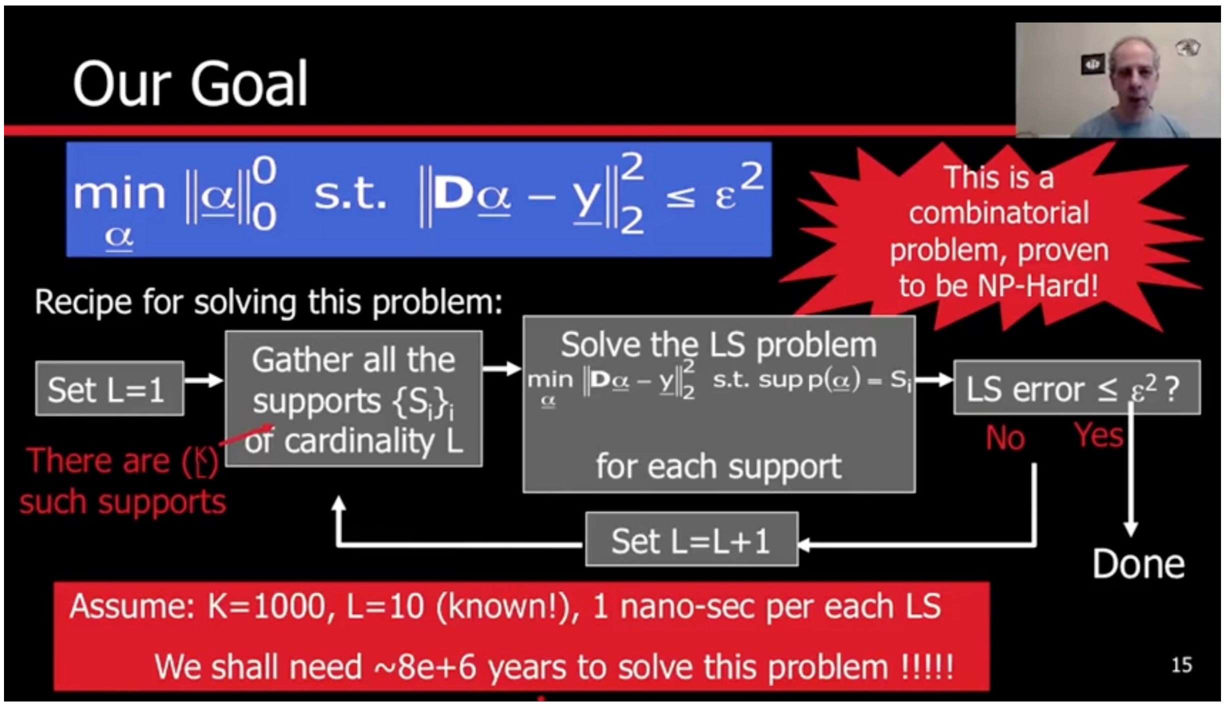

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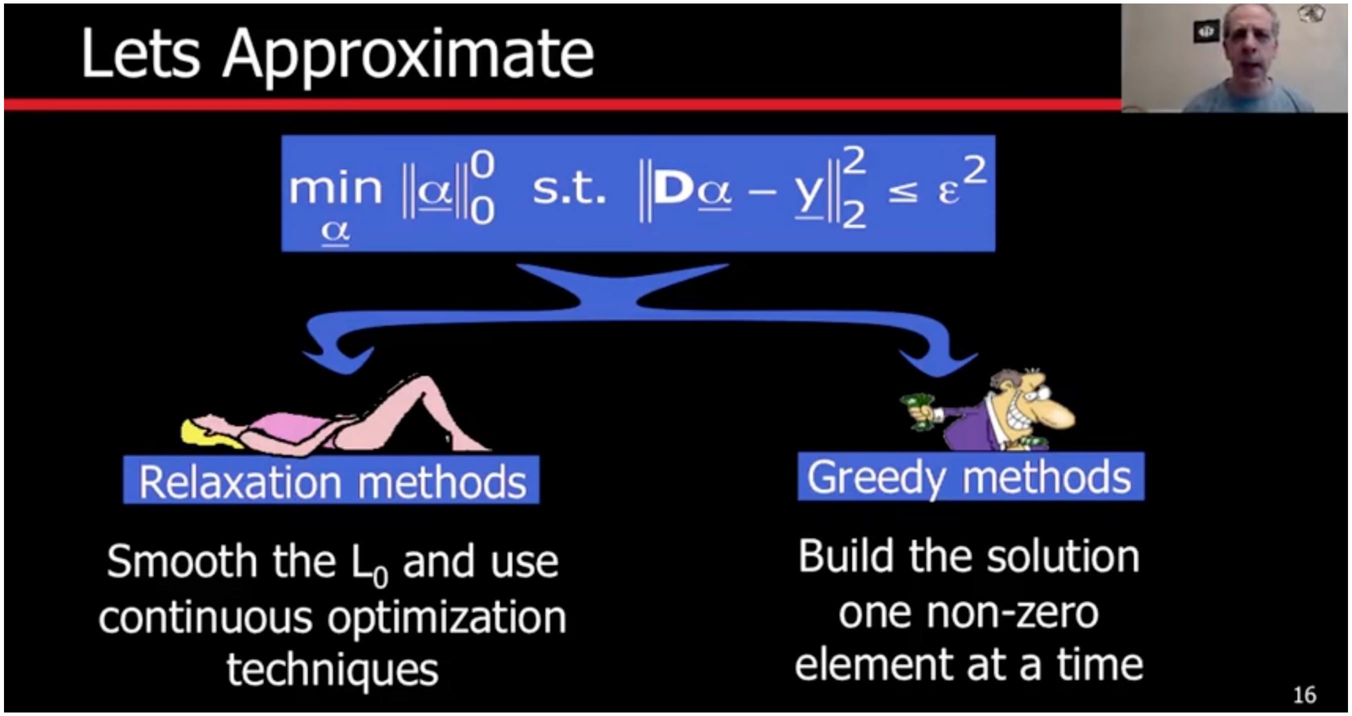

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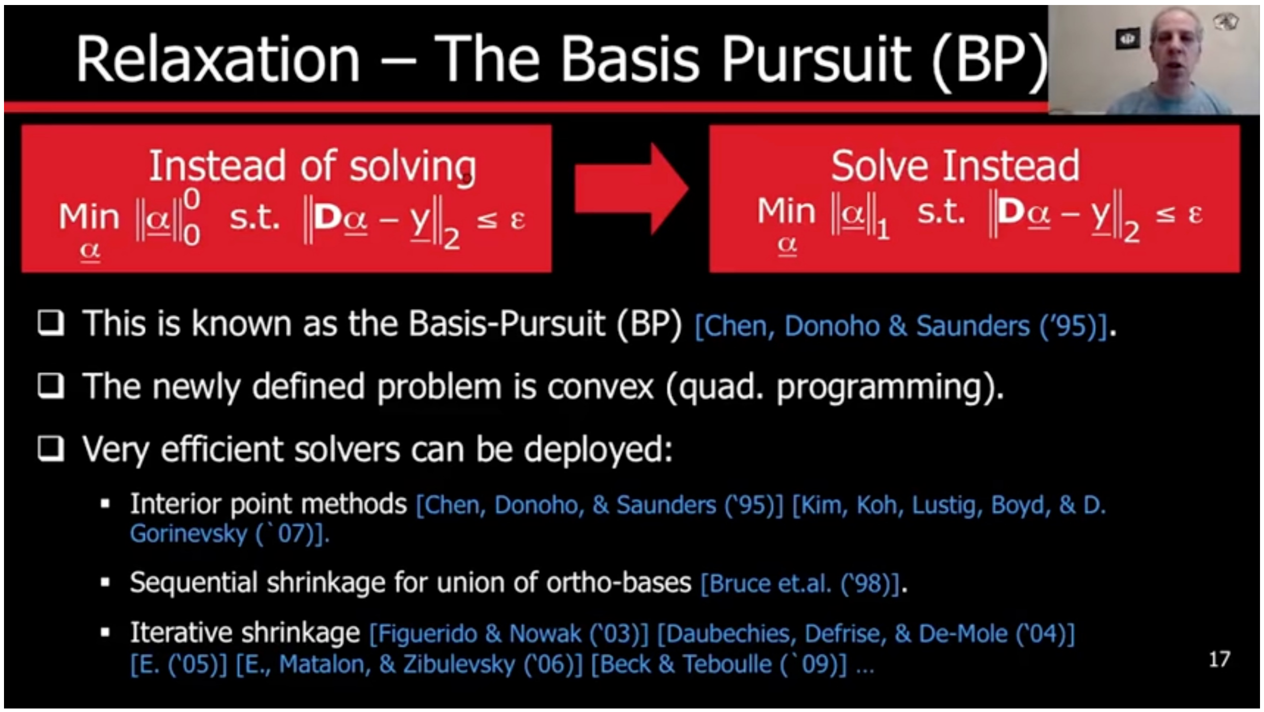

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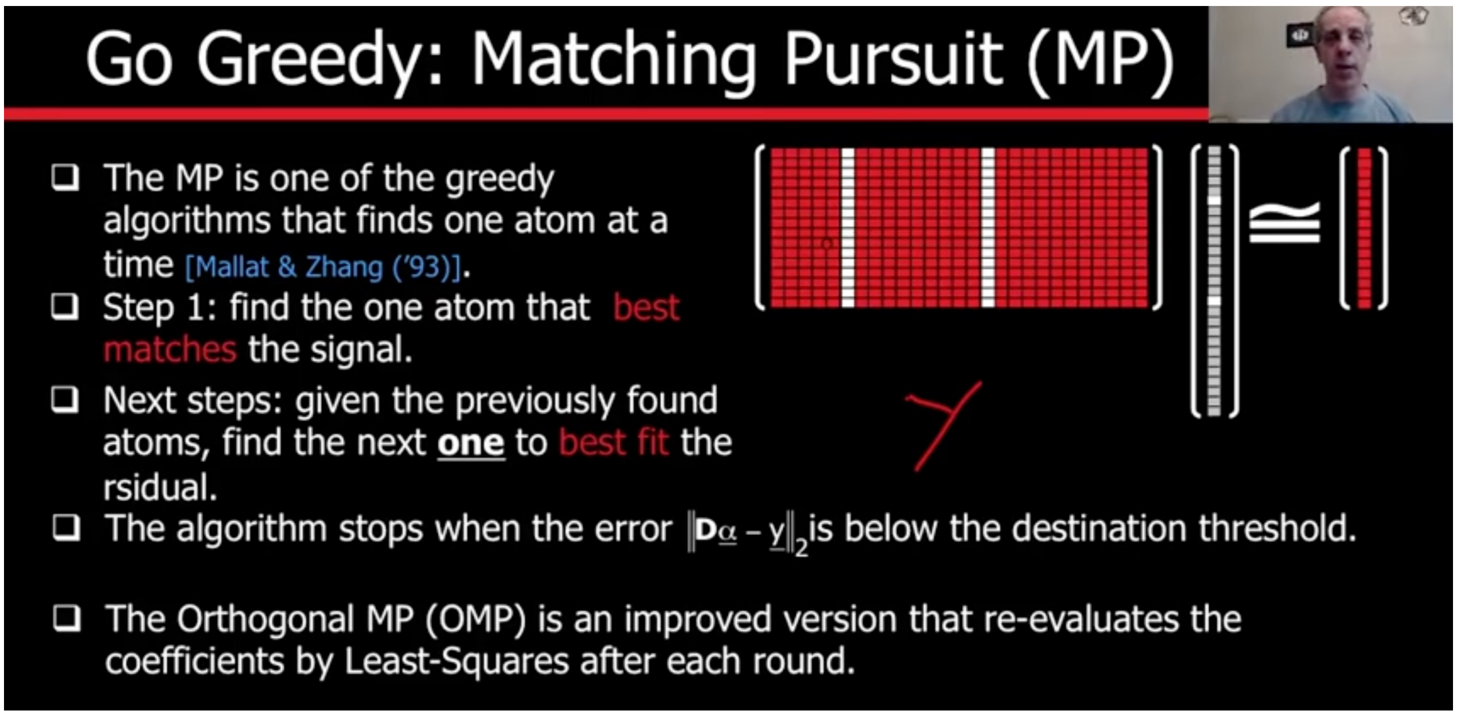

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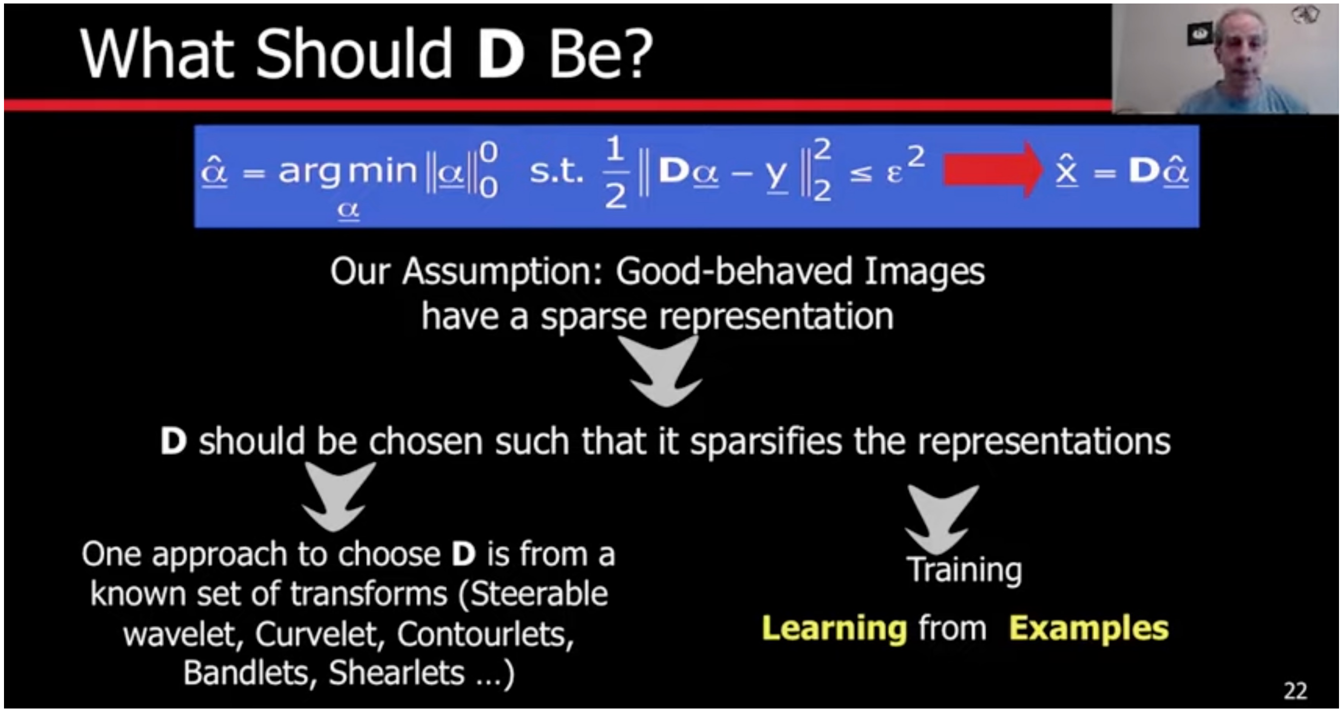

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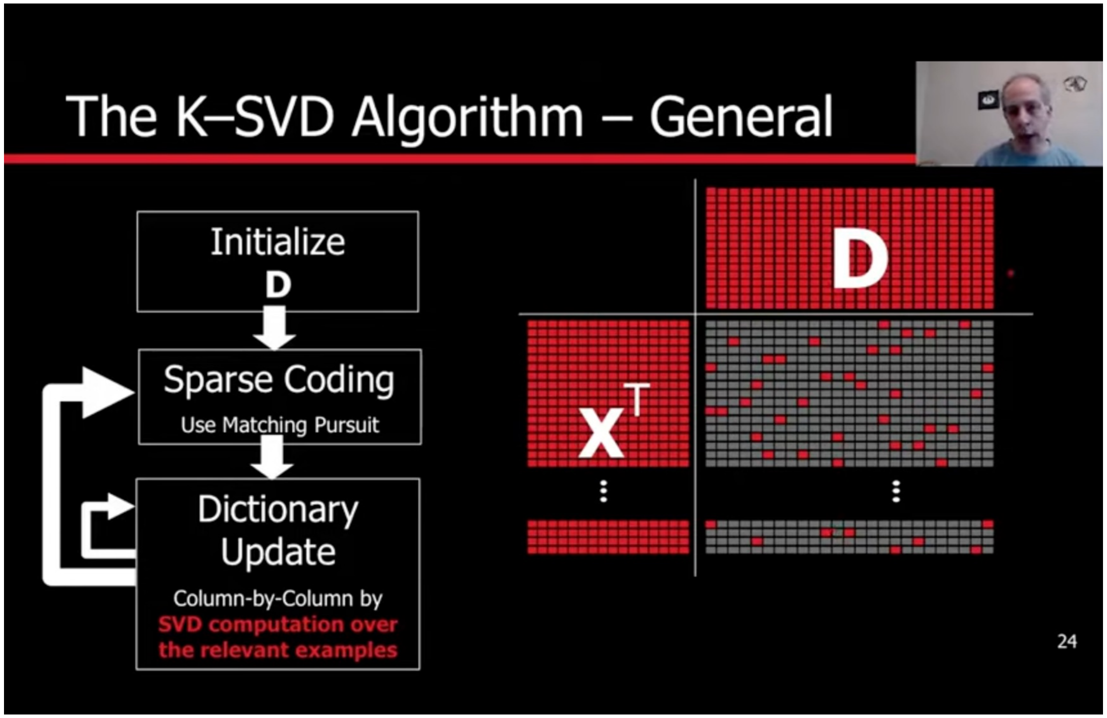

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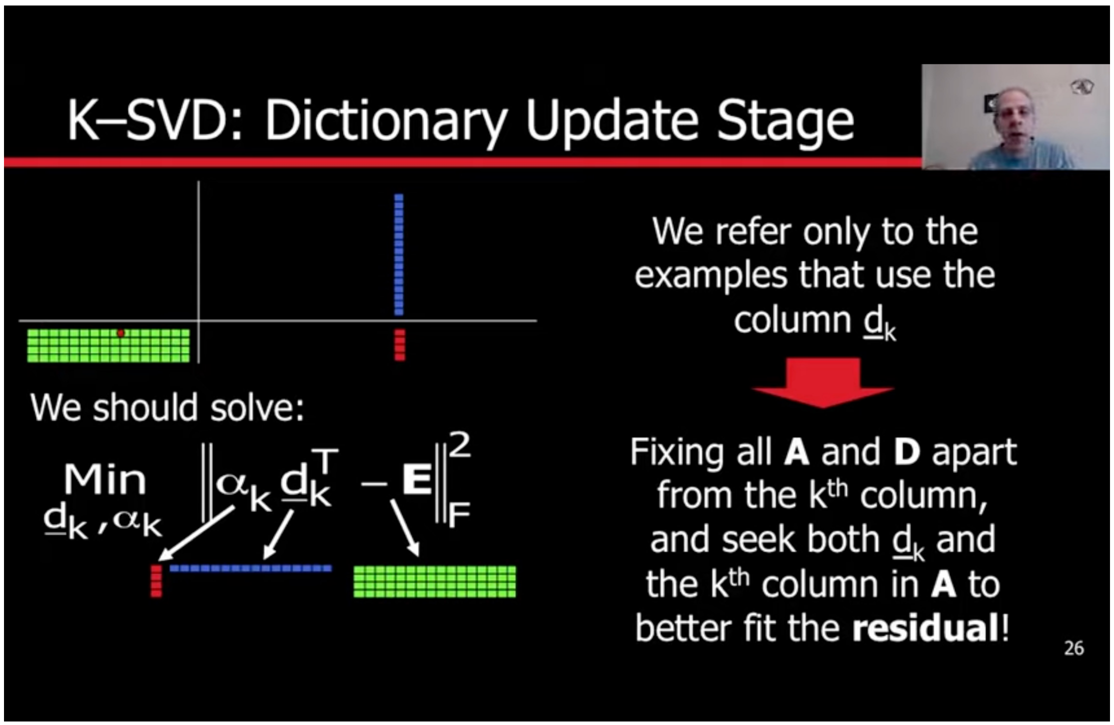


[3] https://www.coursera.org/learn/image-processing

2. SC and TWSC

$$\hat{\mathbf{C}} = \arg\min_{\mathbf{C}} \underbrace{\|\mathbf{Y} - \mathbf{DC}\|_F^2}_{\text{Data Fidelity term}} + \underbrace{\lambda\|\mathbf{C}\|_1}_{\text{Regularization term}} \quad \text{Eq. (2)}$$

- SC based denoising methods are essentially limited by the data fidelity term described by L_2 norm which actually assumes White Gaussian Noise (WGN) and is not able to characterize the signal dependent & realistic noise.

$$\mathbf{Y} = \mathbf{X} + \mathbf{N} \in \mathbb{R}^{3p^2 \times M}$$

$$\mathbf{Y} = [\mathbf{Y}_r^\top \ \mathbf{Y}_g^\top \ \mathbf{Y}_b^\top]^\top$$

$$\mathbf{D} = [\mathbf{D}_r^\top \ \mathbf{D}_g^\top \ \mathbf{D}_b^\top]^\top$$

$$\boxed{\hat{\mathbf{X}} = \mathbf{D}\hat{\mathbf{C}}}$$

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.1 The Trilateral Weighted Sparse Coding Model

- We introduce two weight matrices $W_1 \in R^{3p2\times 3p2}$ and $W_2 \in R^{M\times M}$ to characterize the SC residual ($Y - DC$) in the data-fidelity term of Eq. (2). Besides, to better characterize the sparsity priors of the natural images, we introduce a third weight matrix W3, which is related to the distribution of the sparse coefficients matrix C, into the regularization term of Eq. (2)
- For the dictionary D, we learn it adaptively by applying the SVD to the given data matrix Y as:

$$\mathbf{Y} = \mathbf{D}\mathbf{S}\mathbf{V}^\top$$

- Note that the aim of this paper wasn't proposing a new dictionary. Once obtained from SVD, the dictionary D is fixed and not updated iteratively.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.1 The Trilateral Weighted Sparse Coding Model

- Finally, the proposed trilateral weighted sparse coding (TWSC) model is formulated as:

$$\min_{\mathbf{C}} \|\mathbf{W}_1(\mathbf{Y} - \mathbf{DC})\mathbf{W}_2\|_F^2 + \|\mathbf{W}_3^{-1}\mathbf{C}\|_1$$

- λ has been implicitly incorporated into the weight matrix W_3.

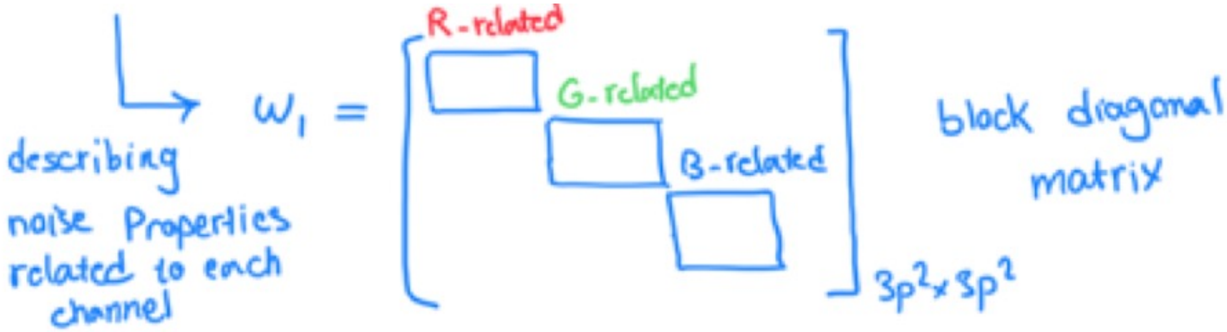
W_1, W_2 → Realistic Noise

W_3 → Sparsity priors of natural images

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.2. The Setting of Weight Matrices

- In this paper, we set the three weight matrices W_1, W_2, and W_3 as diagonal matrices and grant clear physical meanings to them.



[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.2. The Setting of Weight Matrices

$$W_2 = \begin{bmatrix} \ddots \end{bmatrix}_{M \times M}$$

Describing noise variance in the corresponding patch y.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.2. The Setting of Weight Matrices

$W_1 \longrightarrow$ regulizes row discrepancy of $(Y-DC)$

$W_2 \longrightarrow$ regulizes column discrepancy of $(Y-DC)$

$W_3 \longrightarrow$ each diagonal element is set based on sparsity priors on C

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.2. The Setting of Weight Matrices

We determine the three weight matrices W_1, W_2, and W_3 by employing the Maximum A-Posterior (MAP) estimation technique:

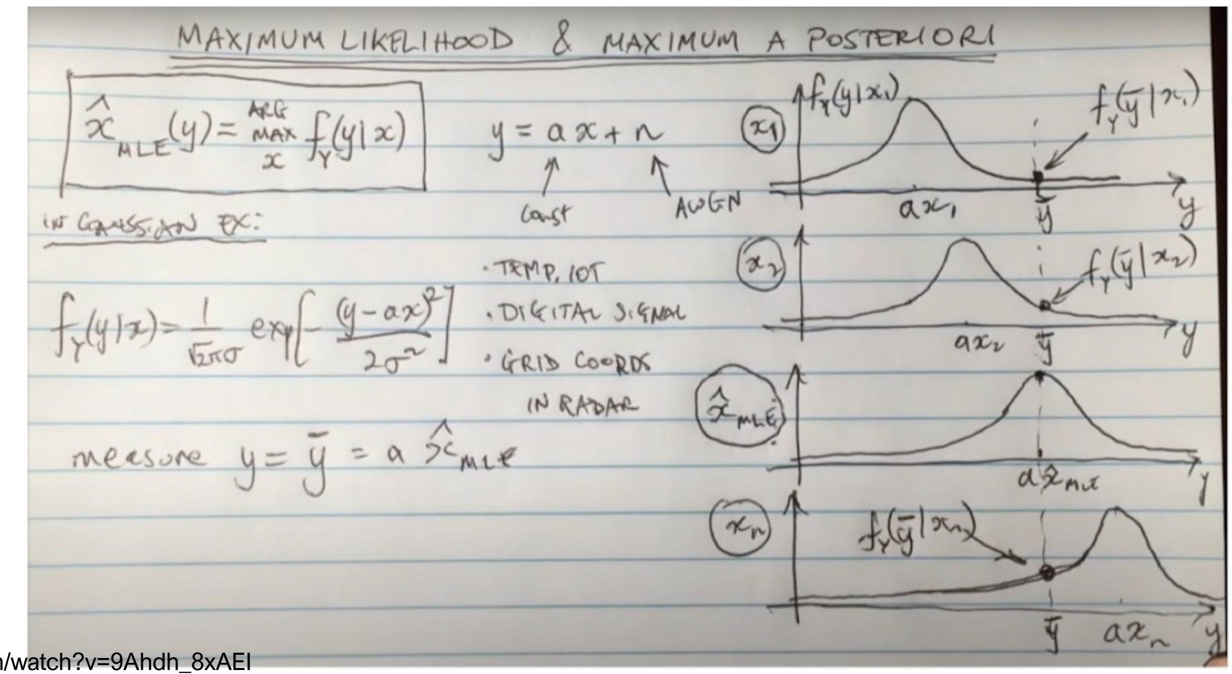
$$\hat{\mathbf{C}} = \arg\max_{\mathbf{C}} \ln P(\mathbf{C}|\mathbf{Y}) = \arg\max_{\mathbf{C}} \{\ln P(\mathbf{Y}|\mathbf{C}) + \ln P(\mathbf{C})\}. \quad \text{Eq. (5)}$$

The log-likelihood term ln(P(Y|C)) is characterized by the statistics of noise. According to, it can be assumed that the noise is independently and identically distributed (i.i.d.) in each channel and each patch with Gaussian distribution.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.2. The Setting of Weight Matrices

- Maximum Likelihood & Maximum A Posteriori



[4] https://www.youtube.com/watch?v=9Ahdh_8xAEI

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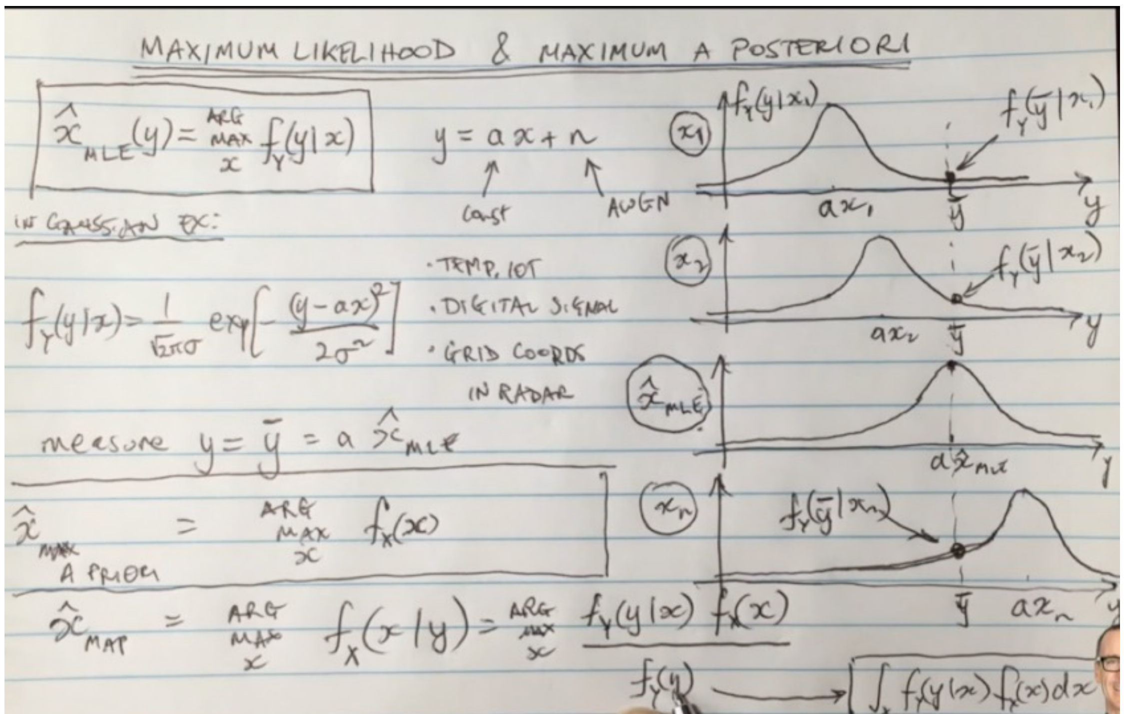

[4] https://www.youtube.com/watch?v=9Ahdh_8xAEI

3.2. The Setting of Weight Matrices

- Denote by y_{cm} and c_m the mth column of the matrices Y_c and C, respectively, and denote by σ_{cm} the noise std of y_{cm}. We have:

$$P(\mathbf{Y}|\mathbf{C}) = \prod_{c\in\{r,g,b\}} \prod_{m=1}^{M} (\pi\sigma_{cm})^{-p^2} e^{-\sigma_{cm}^{-2}\|\mathbf{y}_{cm}-\mathbf{D}_c\mathbf{c}_m\|_2^2}. \quad \text{Eq. (6)}$$

y_{cm} : m_{th} column of Y_c

c_m : m_{th} column of C

σ_{cm} : std of y_{cm}

$\{\sigma_{cm}\}_{3\times M} \xrightarrow[\text{estimated}]{\text{Can be}} \sigma_c,\ \sigma_m$, $\begin{cases} \sigma_{cm} = \sigma_c^{l_1}\sigma_m^{l_2} \\ l_1 + l_2 = 1 \end{cases}$ (we consider: $l_1 = l_2 = 0.5$)

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.2. The Setting of Weight Matrices

- The sparsity prior is imposed on the coefficients matrix C, we assume that each column c_m of C follows i.i.d. Laplacian distribution. Specifically, for each entry c mi, which is the coding coefficient of the mth patch y_m over the ith atom of dictionary D, we assume that it follows distribution of:

$$\left[\frac{1}{2S_i} e^{\left(-\frac{|c_m^i|}{S_i}\right)} \right]$$

- Where S_i is the ith diagonal element of the singular value matrix S in :

$$\mathbf{Y} = \mathbf{D}\mathbf{S}\mathbf{V}^\top$$

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.2. The Setting of Weight Matrices

- Note that we set the scale factor of the distribution as the inverse of the ith singular value S_i. This is because the larger the singular value S_i is, the more important the ith atom (i.e., singular vector) in D should be, and hence the distribution of the coding coefficients over this singular vector should have stronger regularization with weaker sparsity. The prior term in Eq. (5) becomes:

$$P(\mathbf{C}) = \prod_{m=1}^{M} \prod_{i=1}^{3p^2} (2\mathbf{S}_i)^{-1} e^{-\mathbf{S}_i^{-1} |\mathbf{c}_m^i|} \qquad \text{Eq. (7)}$$

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.2. The Setting of Weight Matrices

- Put (7) and (6) into (5) and consider the log-linear model σ cm= $\sigma_c^{1/2}\sigma_m^{1/2}$, we have:

$$\begin{aligned}\hat{\mathbf{C}} &= \arg\min_{\mathbf{C}} \sum_{c\in\{r,g,b\}} \sum_{m=1}^{M} \sigma_{cm}^{-2} \|\mathbf{y}_{cm} - \mathbf{D}_c \mathbf{c}_m\|_2^2 + \sum_{m=1}^{M} \|\mathbf{S}^{-1}\mathbf{c}_m\|_1 \\ &= \arg\min_{\mathbf{C}} \sum_{c\in\{r,g,b\}} \sigma_c^{-1} \|(\mathbf{Y}_c - \mathbf{D}_c\mathbf{C})\mathbf{W}_2\|_F^2 + \|\mathbf{S}^{-1}\mathbf{C}\|_1 \\ &= \arg\min_{\mathbf{C}} \|\mathbf{W}_1(\mathbf{Y} - \mathbf{DC})\mathbf{W}_2\|_F^2 + \|\mathbf{W}_3^{-1}\mathbf{C}\|_1,\end{aligned}$$

- Where:

$$\begin{aligned}\mathbf{W}_1 &= \mathrm{diag}(\sigma_r^{-1/2}\mathbf{I}_{p^2}, \sigma_g^{-1/2}\mathbf{I}_{p^2}, \sigma_b^{-1/2}\mathbf{I}_{p^2}), \\ \mathbf{W}_2 &= \mathrm{diag}(\sigma_1^{-1/2}, ..., \sigma_M^{-1/2}), \mathbf{W}_3 = \mathbf{S},\end{aligned}$$

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.3. Model Optimization

- Letting C∗ = W 3−1C, we can transfer the weight matrix W 3into the data fidelity term of (4). Thus, the TWSC scheme (4) is reformulated as:

$$\min_{\mathbf{C}^*} \|\mathbf{W}_1(\mathbf{Y} - \mathbf{D}\mathbf{W}_3\mathbf{C}^*)\mathbf{W}_2\|_F^2 + \|\mathbf{C}^*\|_1$$

- By introducing an augmented variable Z, the problem (10) is reformulated as a linear equality-constrained problem with two variables C and Z:

$$\min_{\mathbf{C},\mathbf{Z}} \|\mathbf{W}_1(\mathbf{Y} - \mathbf{D}\mathbf{W}_3\mathbf{C})\mathbf{W}_2\|_F^2 + \|\mathbf{Z}\|_1 \text{ s.t. } \mathbf{C} = \mathbf{Z}.$$

- Since the objective function is separable w.r.t. the two variables, the problem can be solved under the alternating direction method of multipliers (ADMM) framework.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.3. Model Optimization

- Since the objective function is separable w.r.t. the two variables, the problem can be solved under the alternating direction method of multipliers (ADMM) framework.
  - The *alternating direction method of multipliers* (ADMM) is an algorithm that solves convex optimization problems by breaking them into smaller pieces, each of which are then easier to handle. It has recently found wide application in a number of areas.
- The augmented Lagrangian function is:

$$\mathcal{L}(\mathbf{C}, \mathbf{Z}, \boldsymbol{\Delta}, \rho) = \|\mathbf{W}_1(\mathbf{Y} - \mathbf{D}\mathbf{W}_3\mathbf{C})\mathbf{W}_2\|_F^2 + \|\mathbf{Z}\|_1 + \langle \boldsymbol{\Delta}, \mathbf{C} - \mathbf{Z} \rangle + \frac{\rho}{2}\|\mathbf{C} - \mathbf{Z}\|_F^2,$$

- Where Δ is the augmented Lagrangian multiplier and $\rho > 0$ is the penalty parameter.

- We initialize the matrix variables C_0, Z_0, and $\Delta 0$ to be comfortable zero matrices and $\rho_0 > 0$. Denote by (C_k, Z_k) and Δ_k the optimization variables and Lagrange multiplier at iteration k (k = 0, 1, 2, ...), respectively.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.3. Model Optimization

- By taking derivatives of the Lagrangian function L w.r.t. C and Z, and setting the derivatives to be zeros, we can alternatively update the variables as follows:

1. **Update C by fixing Z and Δ:**

$$\mathbf{C}_{k+1} = \arg\min_{\mathbf{C}} \|\mathbf{W}_1(\mathbf{Y} - \mathbf{D}\mathbf{W}_3\mathbf{C})\mathbf{W}_2\|_F^2 + \frac{\rho_k}{2}\|\mathbf{C} - \mathbf{Z}_k + \rho_k^{-1}\mathbf{\Delta}_k\|_F^2$$

- This is a two-sided weighted least squares regression problem with the solution satisfying that:

$$\mathbf{A}\mathbf{C}_{k+1} + \mathbf{C}_{k+1}\mathbf{B}_k = \mathbf{E}_k, \qquad \text{Eq. (14)}$$

Where:

$$\mathbf{A} = \mathbf{W}_3^\top\mathbf{D}^\top\mathbf{W}_1^\top\mathbf{W}_1\mathbf{D}\mathbf{W}_3, \mathbf{B}_k = \frac{\rho_k}{2}(\mathbf{W}_2\mathbf{W}_2^\top)^{-1},$$

$$\mathbf{E}_k = \mathbf{W}_3^\top\mathbf{D}^\top\mathbf{W}_1^\top\mathbf{W}_1\mathbf{Y} + (\frac{\rho_k}{2}\mathbf{Z}_k - \frac{1}{2}\mathbf{\Delta}_k)(\mathbf{W}_2\mathbf{W}_2^\top)^{-1}$$

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.3. Model Optimization

1. **Update C by fixing Z and Δ:**

- Eq. (14) is a standard Sylvester equation (SE) which has a unique solution if and only if $\sigma(A) \cap \sigma(-B_k) = \emptyset$, where $\sigma(F)$ denotes the spectrum, i.e., the set of eigenvalues, of the matrix F. We can rewrite the SE (14) as:

$$(\mathbf{I}_M \otimes \boldsymbol{A} + \mathbf{B}_k^\top \otimes \boldsymbol{I}_{3p^2})\mathrm{vec}(\mathbf{C}_{k+1}) = \mathrm{vec}(\mathbf{E}_k)$$

- and the solution C_{k+1}(if existed) can be obtained via $C_{k+1} = \mathrm{vec}^{-1}(\mathrm{vec}(C_{k+1}))$, where $\mathrm{vec}^{-1}(\bullet)$ is the inverse of the vec-operator $\mathrm{vec}(\bullet)$.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.3. Model Optimization

2. **Update Z by fixing C and Δ:**

$$\mathbf{Z}_{k+1} = \arg\min_{\mathbf{Z}} \frac{\rho_k}{2} \|\mathbf{Z} - (\mathbf{C}_{k+1} + \rho_k^{-1}\mathbf{\Delta}_k)\|_F^2 + \|\mathbf{Z}\|_1.$$

- This problem has a closed-form solution as:

$$\mathbf{Z}_{k+1} = \mathcal{S}_{\rho_k^{-1}}(\mathbf{C}_{k+1} + \rho_k^{-1}\mathbf{\Delta}_k)$$

- Where $S_\lambda(x) = \text{sign}(x) \cdot \max(x - \lambda, 0)$ is the soft-thresholding operator.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.3. Model Optimization

3. **Update Δ by fixing X and Z:**

$$\mathbf{\Delta}_{k+1} = \mathbf{\Delta}_k + \rho_k(\mathbf{C}_{k+1} - \mathbf{Z}_{k+1}).$$

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.3. Model Optimization

(4) Update ρ:

$$\rho_{k+1} = \mu\rho_k, \text{ where } \mu \geq 1.$$

- The above alternative updating steps are repeated until the convergence condition is satisfied or the number of iterations exceeds a preset threshold K 1. The ADMM algorithm converges when $\|C_{k+1} - Z_{k+1}\|_F \leq Tol$, $\|C_{k+1} - C_k\|_F \leq Tol$, and $\|Z_{k+1} - Z_k\|_F \leq Tol$ are simultaneously satisfied, where Tol > 0 is a small tolerance number.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.3. Model Optimization

- **Convergence Analysis.**
  - The convergence of Algorithm 1 can be guaranteed since the overall objective function (11) is convex with a global optimal solution. In Fig. 3, we can see that the maximal values in $|C_{k+1} - Z_{k+1}|$, $|C_{k+1} - C_k|$, $|Z_{k+1} - Z_k|$ approach to 0 simultaneously in 50 iterations.

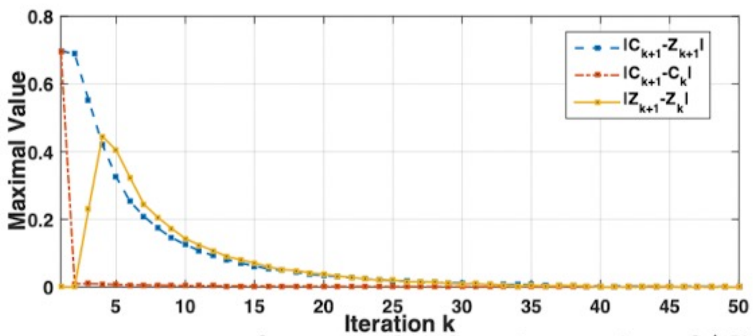

Fig. 3: The convergence curves of maximal values in entries of $|\mathbf{C}_{k+1} - \mathbf{Z}_{k+1}|$ (blue line), $|\mathbf{C}_{k+1} - \mathbf{C}_k|$ (red line), and $|\mathbf{Z}_{k+1} - \mathbf{Z}_k|$ (yellow line). The test image is the image in Fig. 2 (a).

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.3. Model Optimization

- **Summary**

Algorithm 1: Solve the TWSC Model (4) via ADMM

Input: $\mathbf{Y}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3$, μ, Tol, K_1;
Initialization: $\mathbf{C}_0 = \mathbf{Z}_0 = \boldsymbol{\Delta}_0 = \mathbf{0}$, $\rho_0 > 0$, $k = 0$, T = False;
While (T == false) **do**
1. Update $\mathbf{C}_{k+1}$ by solving Eq. (13);
2. Update $\mathbf{Z}_{k+1}$ by soft thresholding (18);
3. Update $\boldsymbol{\Delta}_{k+1}$ by Eq. (19);
4. Update ρ_{k+1} by $\rho_{k+1} = \mu\rho_k$, where $\mu \geq 1$;
5. $k \leftarrow k + 1$;
   if (Converged) or ($k \geq K_1$)
6. T $\leftarrow$ True;
   end if
end while
Output: Matrices $\mathbf{C}$ and $\mathbf{Z}$.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.4. The Denoising Algorithm

- Given a noisy color image, suppose that we have extracted N local patches {y j} j=1Nand their similar patches. Then N noisy patch matrices $\{Y_j\}_{j=1}^{N}$ can be formed to estimate the clean patch matrices $\{X_j\}_{j=1}^{N}$. The patches in matrices $\{X_j\}_{j=1}^{N}$ are aggregated to form the denoised image x_cˆ . To obtain better denoising results, we perform the above denoising procedures for several (e.g., K 2) iterations.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

3.4. The Denoising Algorithm

- **Summary**

Algorithm 2: Image Denoising by TWSC

Input: Noisy image $\mathbf{y}_c$, $\{\sigma_r, \sigma_g, \sigma_b\}$, K_2;
Initialization: $\hat{\mathbf{x}}_c^{(0)} = \mathbf{y}_c$, $\mathbf{y}_c^{(0)} = \mathbf{y}_c$;
for $k = 1 : K_2$ **do**
1. Set $\mathbf{y}_c^{(k)} = \hat{\mathbf{x}}_c^{(k-1)}$;
2. Extract local patches $\{\mathbf{y}_j\}_{j=1}^{N}$ from $\mathbf{y}_c^{(k)}$;
   for each patch $\mathbf{y}_j$ **do**
3. Search nonlocal similar patches $\mathbf{Y}_j$;
4. Apply the TWSC scheme (4) to $\mathbf{Y}_j$ and obtain the estimated $\mathbf{X}_j = \mathbf{DC}$;
   end for
5. Aggregate $\{\mathbf{X}_j\}_{j=1}^{N}$ to form the image $\hat{\mathbf{x}}_c^{(k)}$;

end for
Output: Denoised image $\hat{\mathbf{x}}_c^{(K_2)}$.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

4.1 Visualization of The Weight Matrices

- The three diagonal weight matrices in the proposed TWSC model (4) have clear physical meanings, and it is interesting to analyze how the matrices actually relate to the input image by visualizing the resulting matrices.

- One can see that the matrix W_1 reflects well the noise levels in the images. Though matrix W_2 is initialized as an identity matrix, it is changed in iterations since noise in different patches are removed differently. For real-world noisy images, the noise levels of different patches in Y are different, hence the elements of W_2 vary a lot. In contrast, the noise levels of patches in the synthetic noisy image are similar, thus the elements of W_2 are similar. The weight matrix W_3 is basically determined by the patch structure but not noise, and we do not plot it here.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

4.1 Visualization of The Weight Matrices

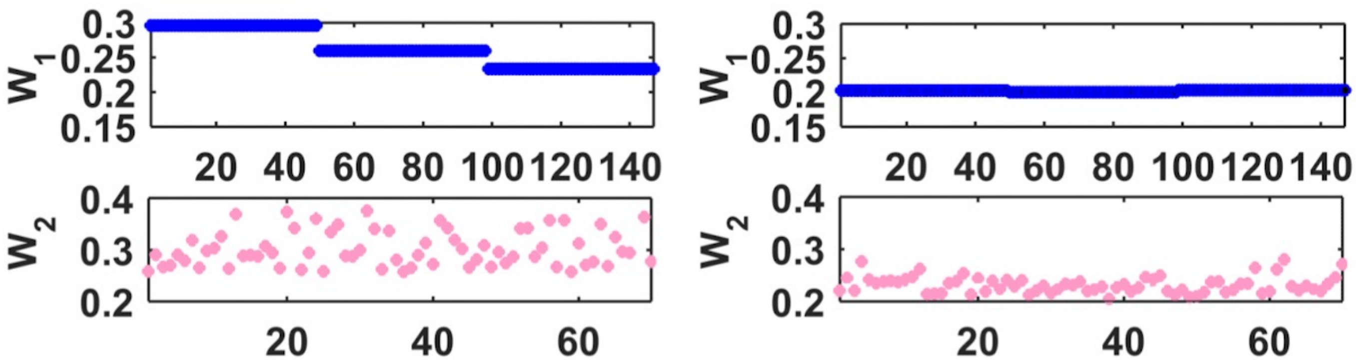

Fig. 7: Visualization of weight matrices $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ on the real-world noisy image (left) and the synthetic noisy image (right) shown in Fig. 1.

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

References:

[1]: Zhu, L., Fu, C.W., Brown, M.S. and Heng, P.A., 2017. A non-local low-rank framework for ultrasound speckle reduction. In Proceedings of the IEEE conference on computer vision and pattern recognition (pp. 5650-5658).

[2]: Xu, J., Zhang, L. and Zhang, D., 2018. A trilateral weighted sparse coding scheme for real-world image denoising. In Proceedings of the European conference on computer vision (ECCV) (pp. 20-36).

[3] https://www.coursera.org/learn/image-processing

[4] https://www.youtube.com/watch?v=9Ahdh_8xAEI