FAST IMITATION SIMULATION OF THE GENERAL MODEL OF THE REDUNDANT SYSTEM WITH RECOVERY

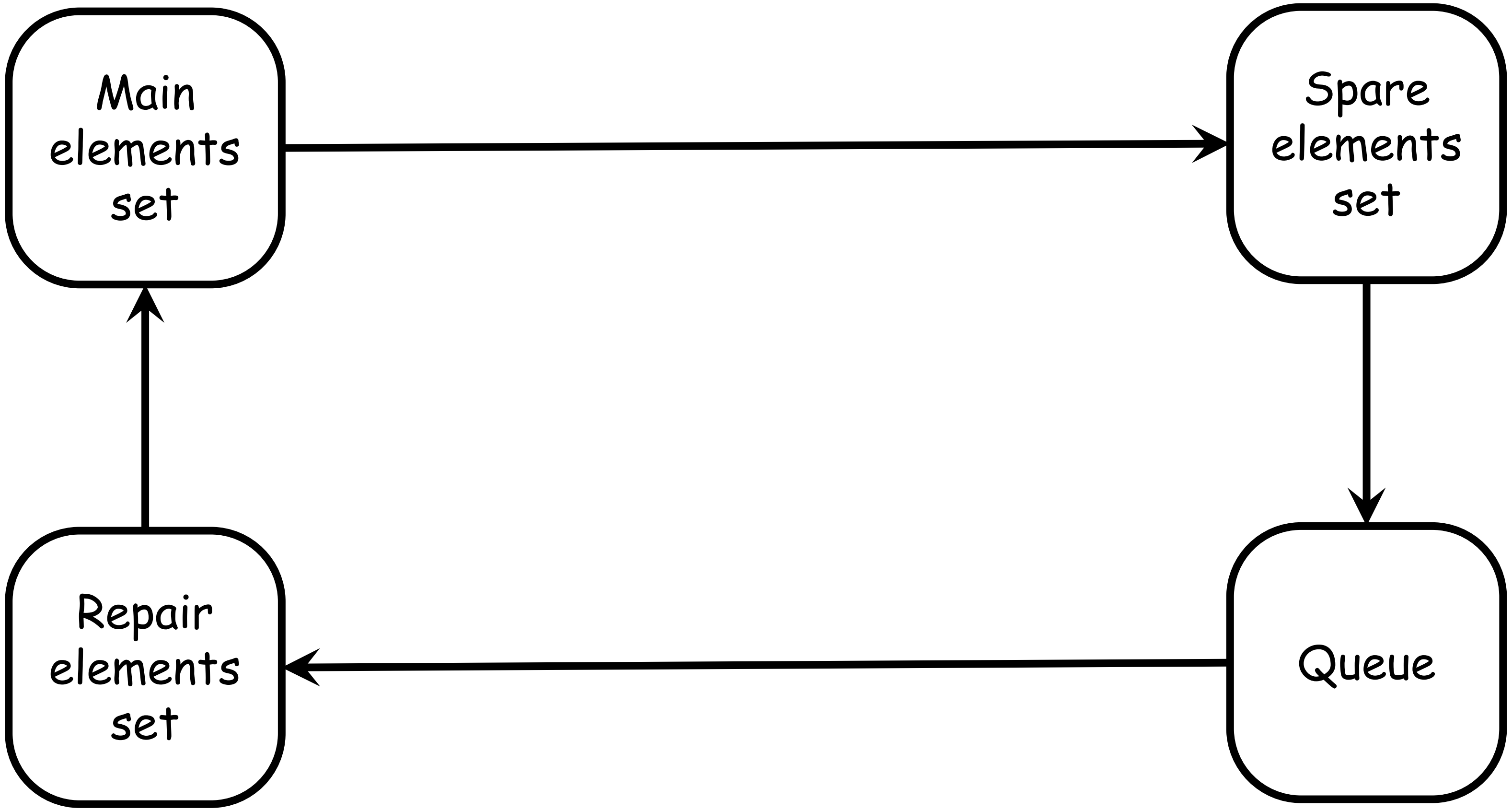
Candidate: Anastasiia Kasian

Supervisor: Ivan Matsak

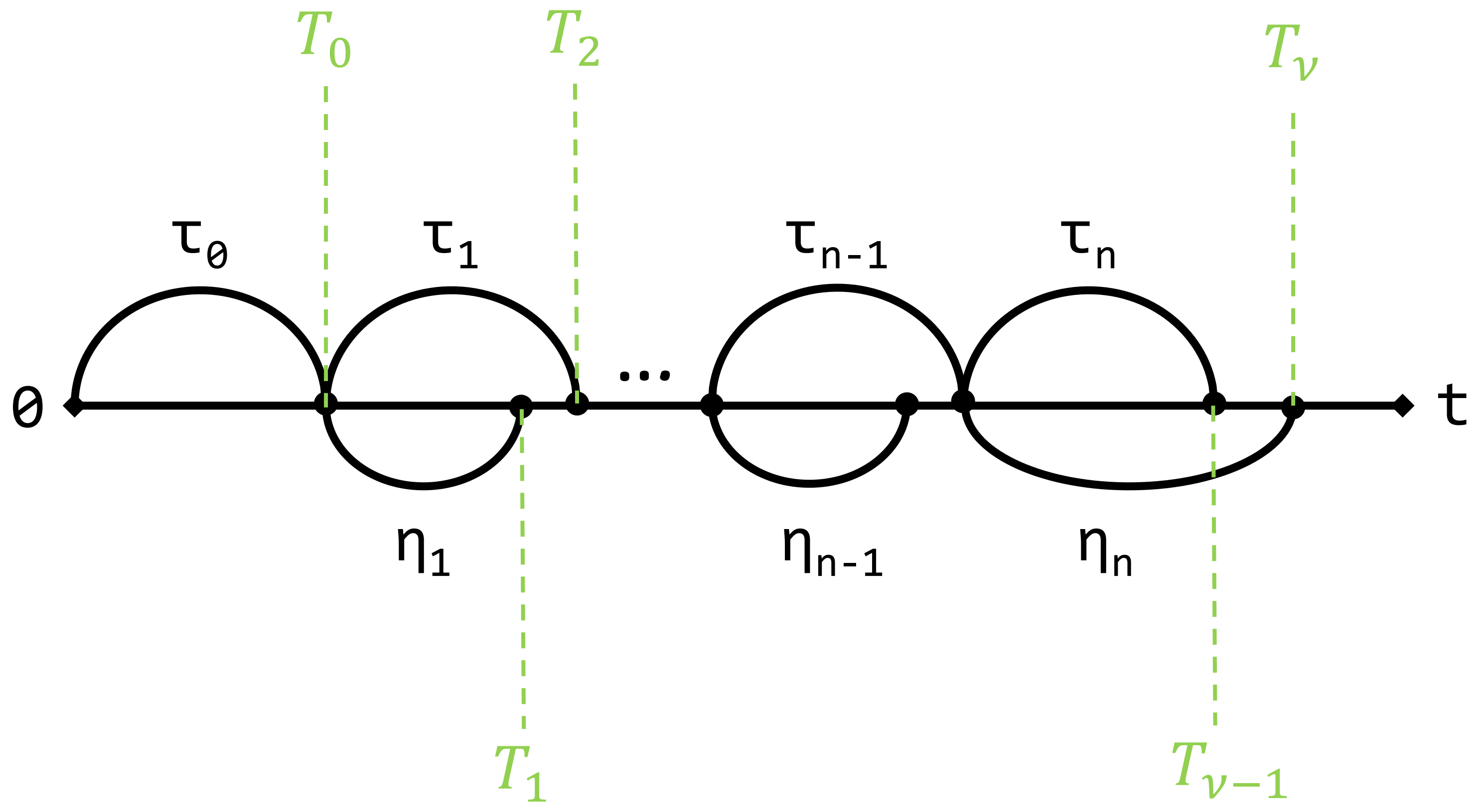
Taras Shevchenko National University of Kyiv

24.05.2022

SYSTEM SCHEME



TIME MOMENTS SCHEME



PROBLEM STATEMENT

Consider redundant system with recovery which contains:

- $n_1, n_2, .., n_s$ working elements of s types;
- $m_1, m_2, .., m_s$- the number of spare elements of each type;
- r – number of repair units;
- τ – work time (exponential distribution with parameters $\lambda_1, \lambda_2, .., \lambda_s$);
- η – repair time ($\eta = const$; η – exponential distribution with parameter μ, $\lambda = 10 \times$ μ).

PROBLEM STATEMENT

The system fails when the working element fails, and all spare elements of its type are in the repair.

Let T_ϑ – time to system failure, then

$$P_\vartheta(t_z) = P(T_\vartheta > t_z)$$ - system reliability.

Find $P_\vartheta(t_z)$ with the fast imitational modeling.

IMITATIONAL MODELING

Simulate system work on interval [0, t].

If system breaks, we have $p = 1$, otherwise p $= 0$.

Repeat $n = 10^6$ times.

Then counting the reliability of the system:

$$Q = \frac{1}{n}\sum_{j=1}^{n} p$$

FAST SIMULATION

- τ - random variable with distribution function:

$$F_\tau(x) = \begin{cases} \dfrac{F(x)}{F(\Delta)}, & x < \Delta \\ 1, & x \geq \Delta \end{cases}$$

- Thus main element work time

$$\tau = -\frac{1}{\lambda}\ln\left[1 - u\left(1 - e^{-\lambda\Delta}\right)\right], \qquad (*)$$

FAST SIMULATION

- η – random variable with exponential distribution:

$$F_\eta(x) = \begin{cases} \lambda e^{-\lambda\Delta}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

- Repair time

$$\eta = 1 - e^{-\lambda\Delta}, \qquad (**)$$

FAST SIMULATION

- Probability on each step:

$$p = P(\tau < t) = 1 - e^{(n_1\lambda_1 + .. + n_s\lambda_s)\Delta_\mathrm{k}}$$

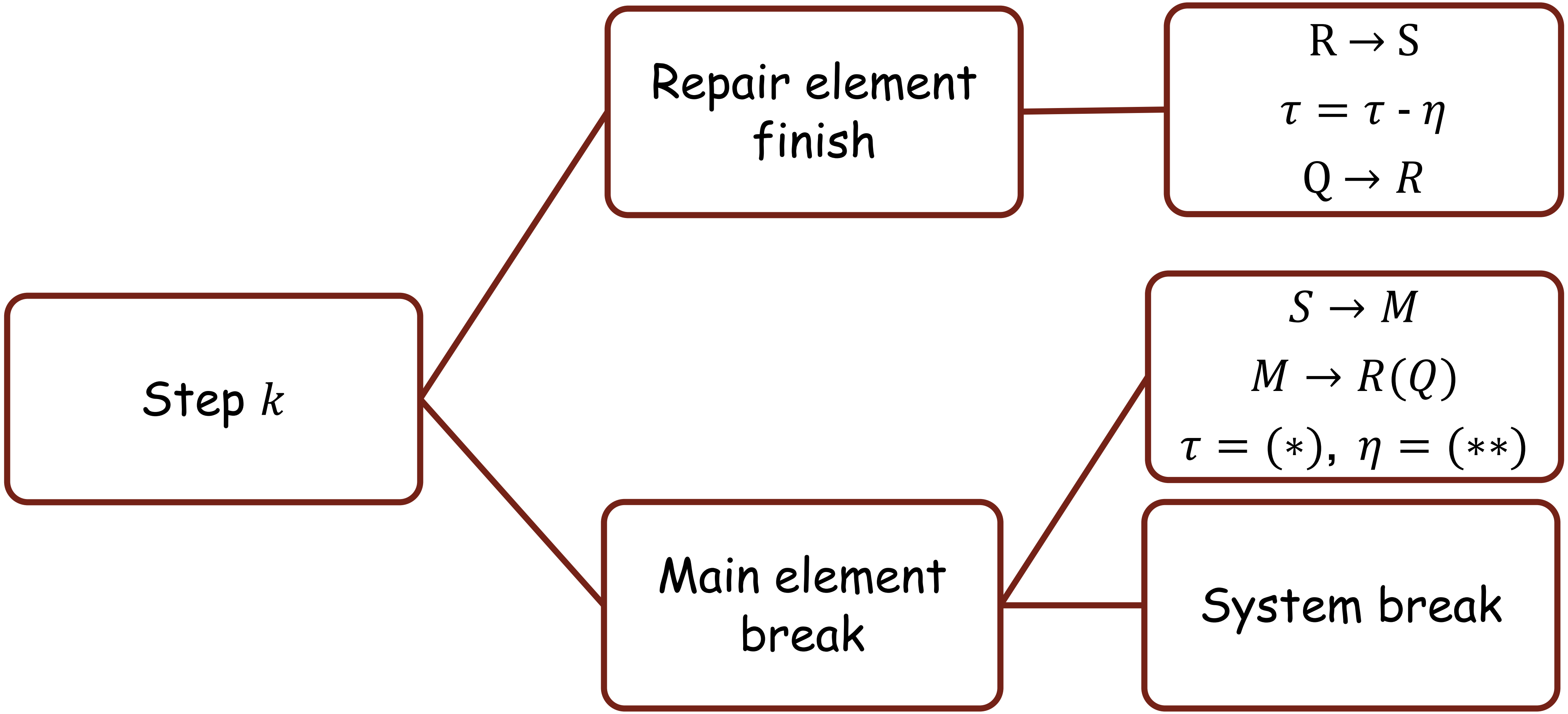
- Probability of system break during iteration of eternal cycle:

$$P = \prod_{i=1}^{n} p_i$$

- System reliability

$$Q = \frac{1}{n}\sum_{j=1}^{n} P_j$$

FAST SIMULATION ALGORITHM



RESULTS

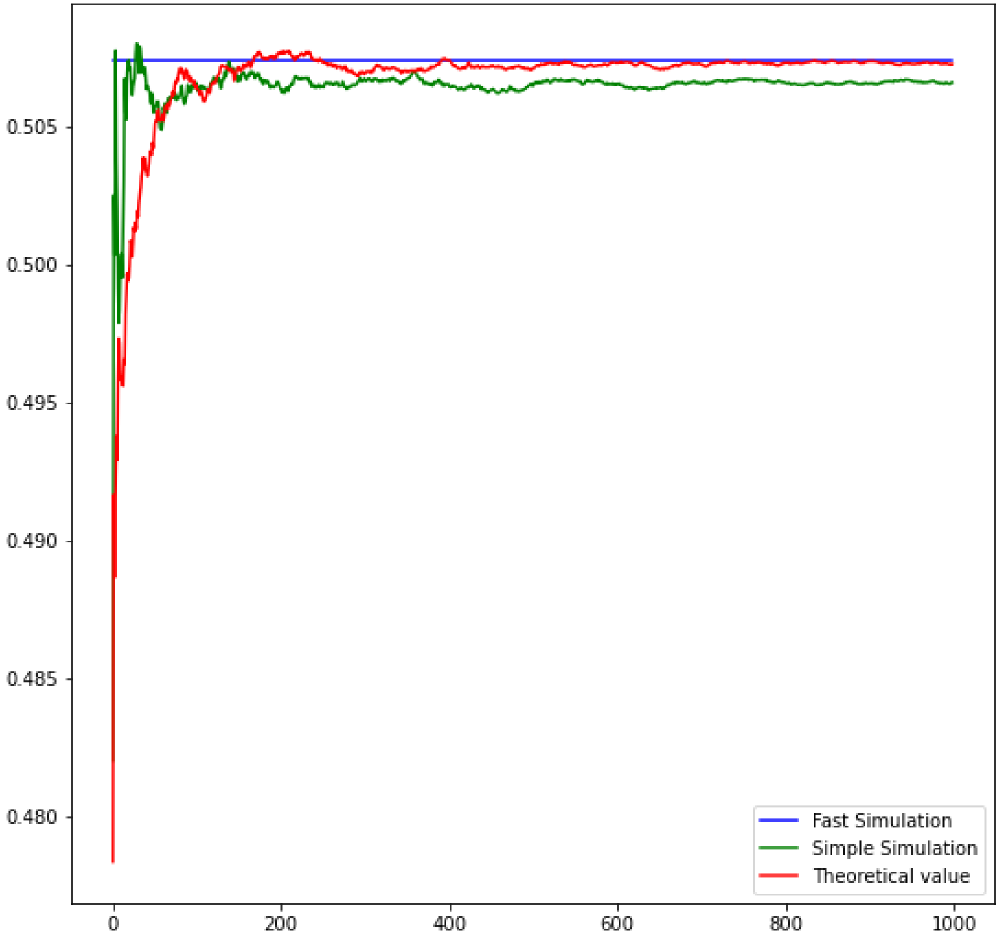
$\lambda = 1.0, \mu = 5.0, t = 5.0$

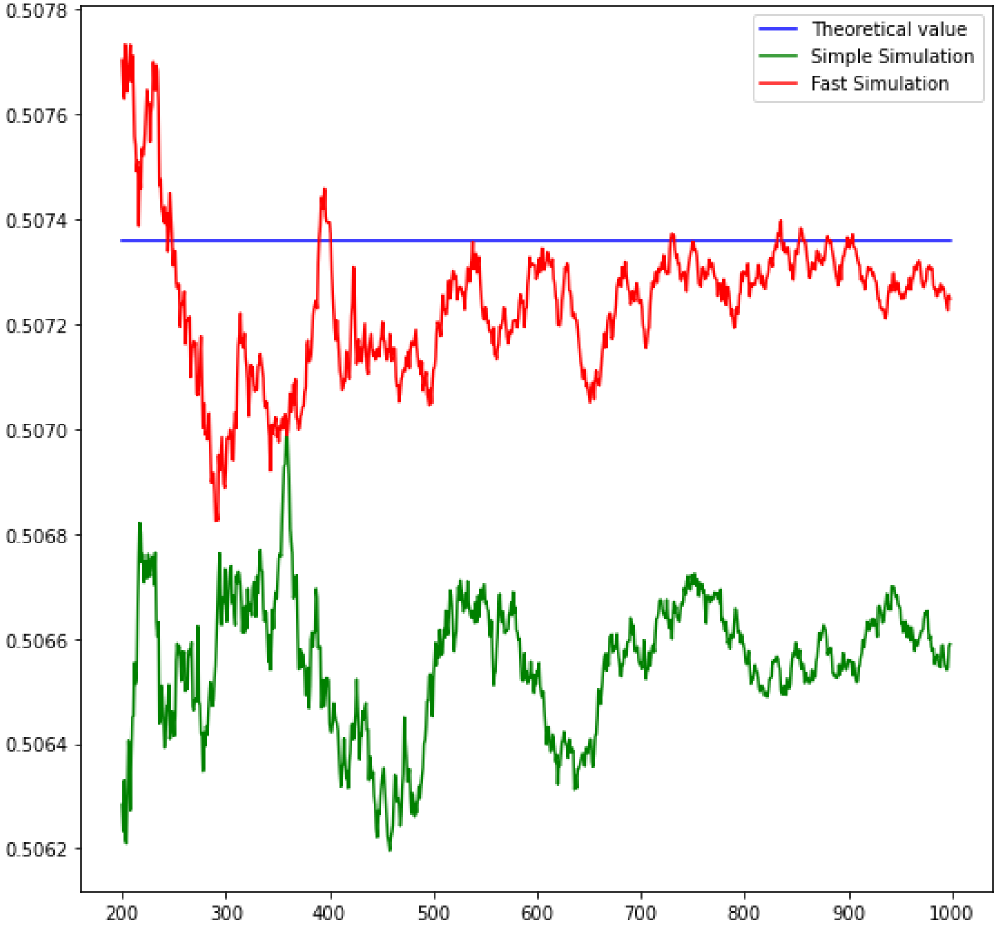
- Theoretical value is 0.507359
- Number of iterations:
  $n_U = 10^5, \qquad n_F = 10^3$

Usual Simulation	Fast Simulation
0.50953	0.502224
0.50977	0.509022
0.50829	0.508088
0.50789	0.505641
0.5071	0.509664
0.50767	0.503931
0.51108	0.506086
0.50602	0.50946
0.50661	0.511239
0.50699	0.508602
avg: 0.508095	avg: 0.507575

RESULTS

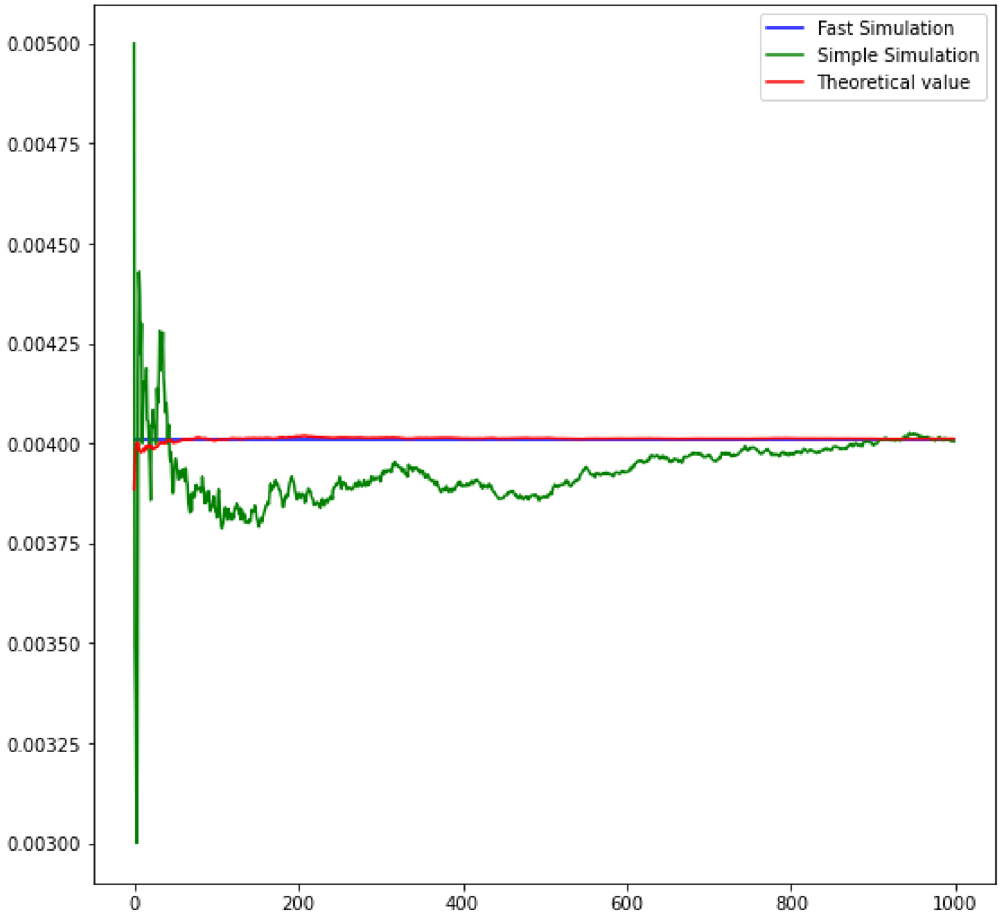
$$\lambda = 1.0, \qquad \mu = 5.0, \qquad t = 5.0$$

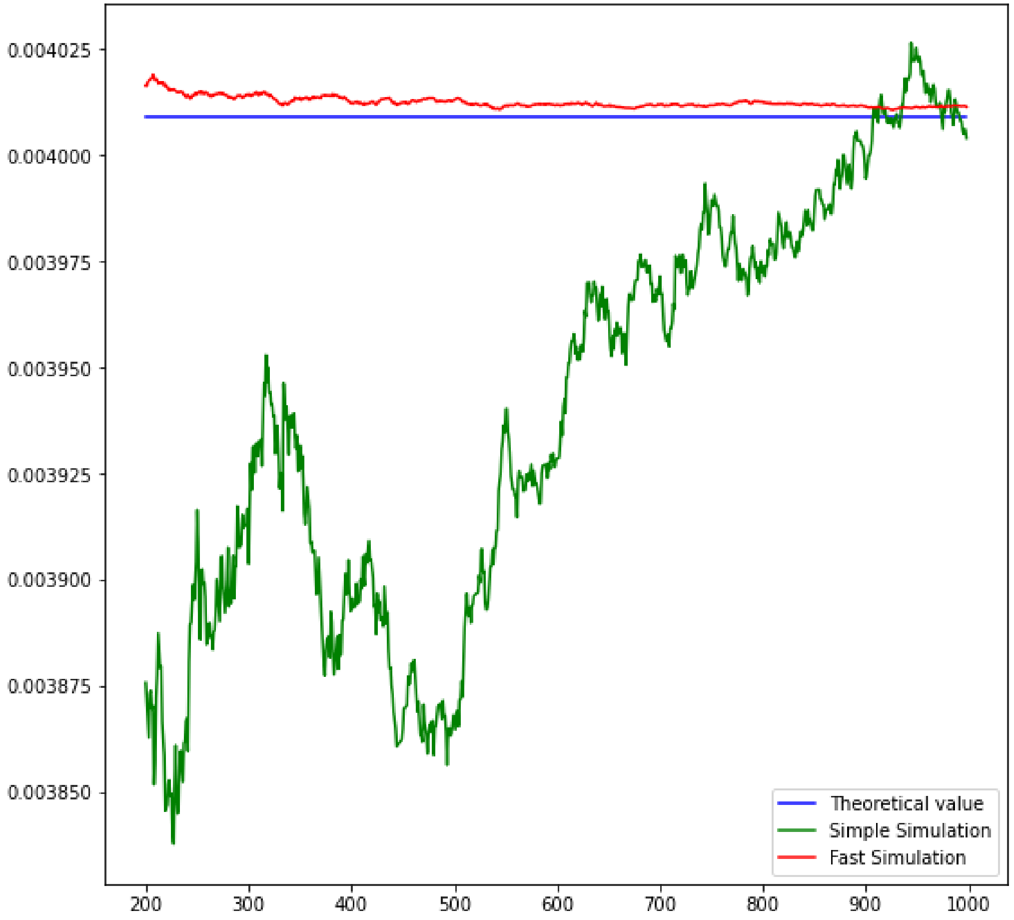




RESULTS

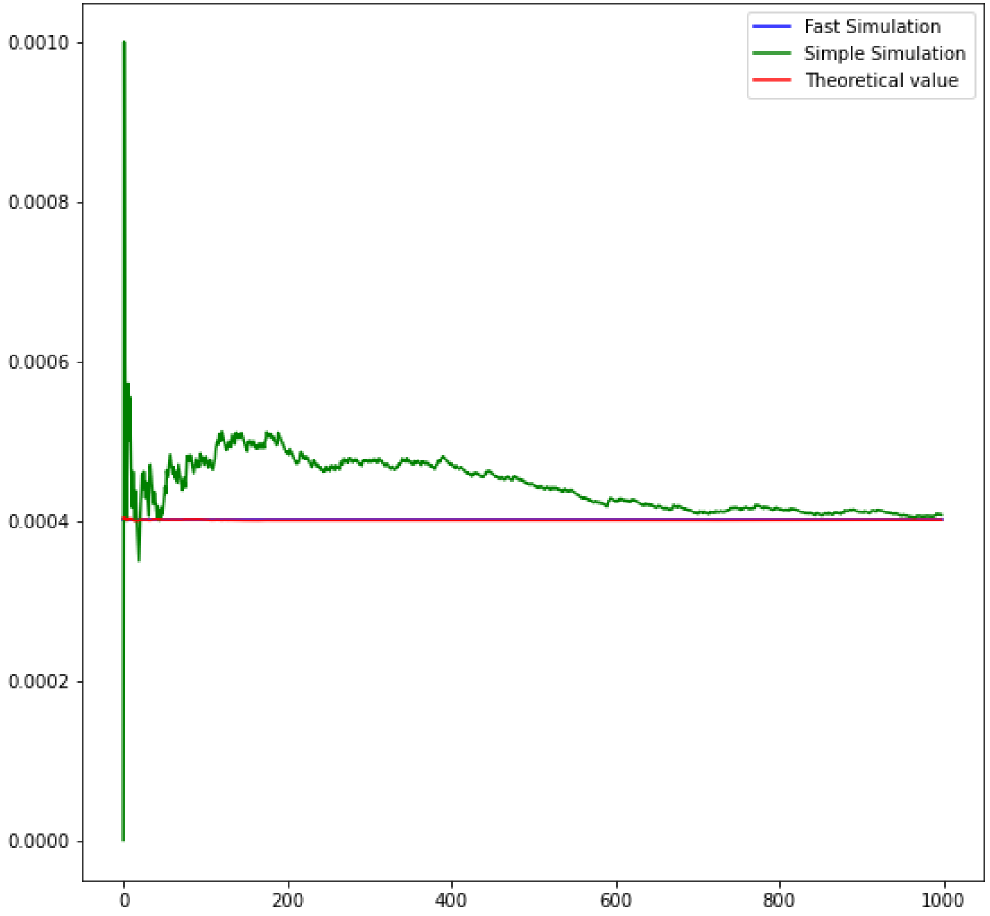
$$\lambda = 1.0, \qquad \mu = 5.0, \qquad t = 0.1$$

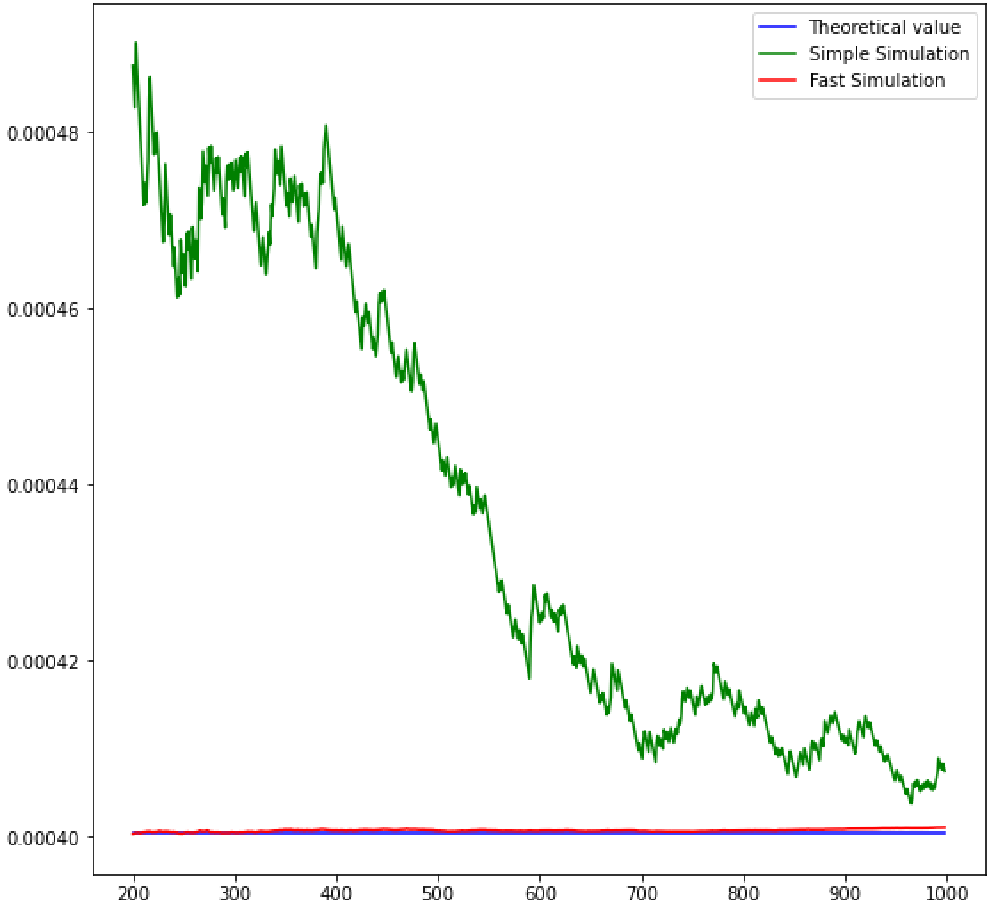




RESULTS

$$\lambda = 3.0, \qquad \mu = 30.0, \qquad t = 0.01$$





CONCLUSIONS

- The method of fast simulation allows to estimate the reliability of the system in fewer computational cycles.

- Usual method does not give a stable exact approximation of the theoretical value, while the method of fast modeling reaches a value very close to the theoretical for a small number of iterations.