Module 02: Vectors and Matrices

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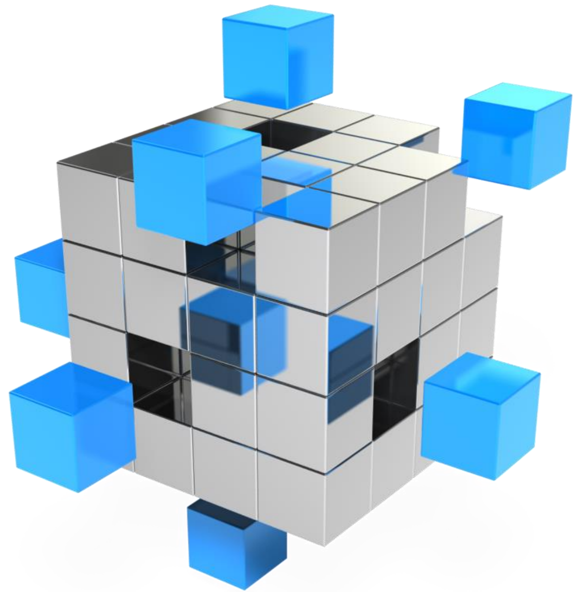

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Module 2: Intended Learning Outcomes

- Create an array in MATLAB, which includes a scalar, vector, and matrix.
- Refer elements in an array using index or subscript.
- Modify and concatenate an array.
- Perform array operations (element-wise operation and matrix operation)
- Define and evaluate a character and character vector
- Create a 3D matrix and refer its element.
- Understand a linear indexing method

Matrix, Row Vector, and Column Vector

Remind

A matrix is an ordered rectangular array of numbers. A general matrix with *m* rows and *n* columns has the following structure:

$$
\begin{array}{cc}
 & \text{column, } j \\
 & \begin{array}{ccccc} 1 & 2 & 3 & \cdots & n \end{array} \\
\text{row, } i \;\; \begin{array}{c} 1 \\ 2 \\ \vdots \\ m \end{array} & \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & \cdots & a_{mn} \end{bmatrix}_{m \times n}
\end{array}
$$

where a_{ij} is each number in the array indexed by its row position i and column position j.

Matrix, Row Vector, and Column Vector in MATLAB

- A ***matrix*** is used to store a set of values of the same type; every value is stored in an ***element.***
- A matrix looks like a table; it has both rows and columns
- A matrix with m rows and n columns is called ***m x n***; these are called its ***dimensions***;
- A ***vector*** is a special case of a matrix in which one of the dimensions is 1
- The term ***array*** is frequently used in MATLAB to refer generically to a matrix or a vector
  - a row vector with n elements is `1 x n`, e.g. `1 x 4`:
  - a column vector with m elements is `m x 1`, e.g. `3 x 1`:
- A ***scalar*** is an even more special case; it is `1 x 1`, or in other words, just a single value

Creating Row Vectors

- Direct method: put the values you want in square brackets, separated by either **commas** or **spaces**
- **Colon operator**: iterates through values in the form `first:step:last`
  - If no step is specified, the default is `1` so for example `2:4` creates the vector `[2 3 4]`
  - Can go in reverse e.g. `4:-1:1` creates `[4 3 2 1]`
  - Will not go beyond last e.g., `1:2:6` creates `[1 3 5]`

```
rvec1 = [1 2 3 4]; % sepr. by space
rvec2 = [1,2,3,4]; % sepr. by comma
rvec3 = [1   2   3 4];
rvec4 = 1:4; % use a colon operator
rvec5 = 1:1:4; % step by 1
rvec6 = [1:4]; % okay with put. brackets
```

Name	Value
rvec1	[1 2 3 4]
rvec2	[1 2 3 4]
rvec3	[1 2 3 4]
rvec4	[1 2 3 4]
rvec5	[1 2 3 4]
rvec6	[1 2 3 4]

Creating Row Vectors (Continue)

```
rvec1 = [1 2 3 4 5 6];
rvec2 = [1:6];

rvec3 = [1:2:6]; % step by 2

rvec4 = [1:3:6]; % step by 3

rvec5 = [6:-1:1]; % step by -1
```

Name	Value
rvec1	[1 2 3 4 5 6]
rvec2	[1 2 3 4 5 6]
rvec3	[1 3 5]
rvec4	[1 4]
rvec5	[6 5 4 3 2 1]

```
my_rvec1 = [1:5:6];

my_rvec2 = [5:-2:1];
```

Name	Value
rvec1	[1 6]
rvec2	[5 3 1]

📖: **Remember !** `variable = first:step:last`

Remind: Transpose

The **transpose** of a matrix is the operation of flipping the rows to columns and vice versa across the diagonal of the matrix. For example,

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times 2} \quad \text{has transpose} \quad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$$

where the power of *T* indicates the transpose of a matrix. Note that in the example, the dimensions of the matrix changes from 3x2 to 2x3 (with the same total number of elements) and diagonal elements remain the same.

Creating Column Vectors

- A **column** vector is an `m x 1` vector
- **Direct method**: can create by separating values in square brackets with ***semicolons*** (e.g., `[4; 7; 2]`)
- You cannot directly create a column vector using methods such as the colon operator, but you can create a row vector and then ***transpose*** it to get a column vector using the transpose operator (e.g., `[4  7  2]’`)

```
rvec1 = [1;2;3;4];
rvec2 = [1 2 3 4]’;
rvec3 = [1:4];
rvec4 = rvec3’;
```

Name	**Value**
rvec1	**[1; 2; 3; 4]**
rvec2	**[1; 2; 3; 4]**
rvec3	**[1 2 3 4]**
rvec4	**[1; 2; 3; 4]**

Referring to Elements

- The elements in a vector are numbered sequentially; each element number is called the ***index***, or ***subscript*** and are shown above the elements here:

```
myvec = [5 33 11 -4 2];
```

Index	1	2	3	4	5
Element	5	33	11	-4	2

- Refer to an element using its ***index*** or ***subscript*** in parentheses, e.g. `vec(4)` is the 4th element of a vector `vec` (assuming it has at least 4 elements)
- Can also refer to a subset of a vector by using an ***index vector*** which is a vector of indices e.g. `vec([2 5])` refers to the 2nd and 5th elements of `vec`.

⚠: **The index in MATLAB starts from 1 !!**

Example: Referring to Elements

```
rvec = [5:10];
cvec = rvec';

val1 = rvec(2);
val2 = rvec(5);

vec1 = rvec([1 4]);
vec2 = rvec([1 2 6]);

vec3 = cvec([1 4]);
vec4 = cvec([1 2 6]);
```

Name	Value
rvec	**[5 6 7 8 9 10]**
cvec	**[5; 6; 7; 8; 9; 10]**
val1	**6**
val2	**9**
vec1	**[5 8]**
vec2	**[5 6 10]**
vec3	**[5; 8]**
vec4	**[5; 6; 10]**

Index	1	2	3	4	5	6	
Element	5	6	7	8	9	10	rvec

Modifying Vectors

- Elements in a vector can be changed.

`variable(index) = expression`

```
rvec = 1:4;
rvec1 = rvec;
rvec1(4) = 10;
```

Name	Value
rvec	[1 2 3 4]
rvec1	[1 2 3 10]

📖: Value(s) computed from *expression* are assigned to *variable* at *index* location(s).

```
rvec = 1:5;
rvec1 = rvec;
rvec2 = rvec;
rvec3 = rvec;

rvec1([1 3]) = [0 0];
rvec2(1:3) = 4:6;
rvec3([1 3]) = rvec4([3 1]);
```

Name	Value
rvec	[1 2 3 4 5]
rvec1	[0 2 0 4 5]
rvec2	[4 5 6 4 5]
rvec3	[3 2 1 4 5]

Modifying Vectors (Continue)

- A vector **can** be extended by referring to elements that do not yet exist
- A vector **cannot** be read by an index that does not yet exist

variable(index) = expression

```
rvec1 = 1:4;
rvec1(5) = 5;

rvec2 = 1:4;
rvec2([5 6]) = [3 2];
```

Name	Value
rvec1	[1 2 3 4 5]
rvec2	[1 2 3 4 3 2]

```
rvec1 = 1:4;
val1 = rvec1(5)
```

Error: Index exceeds the number of array elements (4)

```
rvec1 = 1:4;
rvec1(1:2) = [1 2 3];
```

Error: Unable to perform assignment because the left and right sides have a different number of elements.

Concatenation

- Vectors can be created by joining together existing vectors, or adding elements to existing vectors
- This is called ***concatenation***

```
rvec = [1 2];
rvec1 = [rvec 8 9];
rvec2 = [rvec rvec1];


cvec = [4;5];
cvecp = [6;7]
cvec1 = [cvec; cvecp];
```

Name	Value
rvec	**[1 2]**
rvec1	**[1 2 8 9]**
rvec2	**[1 2 1 2 8 9]**
cvec	**[4 5]’**
cvecp	**[6 7]’**
cvec1	**[4 5 6 7]’**

```
rvec = [1 2];
rvec1 = [rvec; 8];
```

```
Error: using vertcat
Dimensions of arrays being concatenated
are not consistent.
```

Creating a Matrix

- Separate values within rows with ***blanks*** or ***commas***, and separate the rows with ***semicolons***
- Can use any method to get values in each row (any method to create a row vector, including colon operator)
- **There must <u>ALWAYS</u> be the same number of values in every row!!**

```
m1 = [1 2 3;4 5 6];

r1 = [1 2 3];
r2 = [4 5 6];

m2 = [r1;r2];
m3 = [r1;4 5 6];
% it works not recommend
```

Name	Value
m1	**[1 2 3;4 5 6]**
r1	**[1 2 3]**
r2	**[4 5 6]**
m2	**[1 2 3;4 5 6]**
m3	**[1 2 3;4 5 6]**

m1, m2, m3

1	2	3
4	5	6

Functions that Create Matrices

There are many built-in functions to create matrices

- **`zeros(n)`** creates an $n x n$ matrix of all zeros
- **`zeros(n,m)`** creates an $n x m$ matrix of all zeros
- **`ones(n)`** creates an $n x n$ matrix of all ones
- **`ones(n,m)`** creates an $n x m$ matrix of all ones
- **`eye(n)`** creates an $n x n$ identity matrix.

⚠: **`zeros(n)`** or **`ones(n)`** is a matrix, not a column or row vector.

```
m1 = zeros(3);
m2 = zeros(3,2);

m3 = ones(2,1);
m4 = eye(3);
```

Name	Value
m1	[0 0 0;0 0 0;0 0 0]
m2	[0 0; 0 0; 0 0]
m3	[1;1]
m4	[1 0 0; 0 1 0; 0 0 1]

m1

0	0	0
0	0	0
0	0	0

m2

0	0
0	0
0	0

m3

1
1

m4

1	0	0
0	1	0
0	0	1

Matrix Elements

- To refer to an element in a matrix, you use the matrix variable name followed by ***the index of the row, and then the index of the column***, in parentheses
- ALWAYS refer to the <u>row first</u>, <u>column second</u>
- This is called <u>***subscripted indexing***</u>***.***

☺: column second →

row first ↓

Variable (row indexes, column indexes)

```
m1 = [1 2 3;4 5 6];
```

1	2	3
4	5	6

<u>**Element**</u>

(1,1)	(1,2)	(1,3)
(2,1)	(2,2)	(2,3)

<u>**Indexing**</u>

Matrix Elements (Continue)

- You can index and refer an entire row or column using a colon (:).
- Can also refer to any subset of a matrix
  - To refer to the entire `m`th row: `m1(m,:)`
- To refer to the entire `n`th column: `m1(:,n)`
- To refer to the last row or column use **`end`**, (e.g. `m1(end,m)` is the mth value in the last row)
- Can modify an element or subset of a matrix in an assignment statement

Variable (row indexes, column indexes)

Used for **referring** **values** and **reading** **values**

Example: Matrix Elements

```
m1 = [1 2 3;4 5 6;7 8 9];

c1 = m1(:,1);
r1 = m1(2,:);

c2 = m1(:,end);
c3 = m1(1:3, end);
c4 = m1(1:end, end);

r2 = m1(end,:);
r3 = m1(3,:);

m2 = m1(2:3, 2:3);
m3 = m1(2:end, 2:end);
m4 = m1([1 3], [1 3]);
```

m1 $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ c1 $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$ r1 $\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$

c2 $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ c3 $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ c4 $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

r2 $\begin{bmatrix} 7 & 8 & 9 \end{bmatrix}$ r3 $\begin{bmatrix} 7 & 8 & 9 \end{bmatrix}$

m2 $\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}$ m3 $\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}$ m4 $\begin{bmatrix} 1 & 3 \\ 7 & 9 \end{bmatrix}$

Modifying Matrices

- An individual element in a matrix can be modified by assigning a new value to it.
- Entire rows and columns can also be modified.
- Any subset of a matrix can be modified, as long as what is being assigned has the same dimensions as the subset being modified.
- Exception to this: a scalar can be assigned to any size subset; the same scalar is assigned to every element in the subset.

Variable (row indexes, column indexes) = Expression

Used for referring **elements** and reading **values**

Example: Modifying Matrices

```
>> m1 = [1 2 3;4 5 6;7 8 9];
>> m2 = m1
m2 =

     1     2     3
     4     5     6
     7     8     9

>> m2(1,1) = 10
m2 =

    10     2     3
     4     5     6
     7     8     9

>> m2(1,end) = 30
m2 =

    10     2    30
     4     5     6
     7     8     9
```

... continue ...

```
>> m2(:,1) = [10; 10; 10];
m2 =

    10     2    30
    10     5     6
    10     8     9

>> m2(end,:) = zeros(1, 3);
m2 =

    10     2    30
    10     5     6
     0     0     0

>> m2(3,:) = m2(1,:)
m2 =

    10     2    30
    10     5     6
    10     2    30
```

Example: Modifying Matrices (Continue)

... continue ...

```
>> m2(:,1) = [10; 10; 10];
m2 =

    10     2    30
    10     5     6
    10     8     9

>> m2(end,:) = zeros(1, 3);
m2 =

    10     2    30
    10     5     6
     0     0     0

>> m2(2,:) = 1;
m2 =

    10     2    30
     1     1     1
     0     0     0
```

... continue ...

```
>> m2(:, end) = 3;
m2 =

    10     2     3
     1     1     3
     0     0     3

>> m2([1 3], [1 3]) = 10;
m2 =

    10     2    10
     1     1     3
    10     0    10
```

📖: Exception to this: a scalar can be assigned to any size subset; the same scalar is assigned to every element in the subset.

Example: Modifying Matrices

Q: Swap the 2nd and 3rd columns in `mat1`

```
mat1 = [1 2 3; 4 5 6; 7 8 9];

% write your code here
vec = mat1(:, 2);

mat1(:,2) = mat1(:,3);
mat1(:,3) = vec;
```

```
mat1 = [1 2 3; 4 5 6; 7 8 9];

mat1(:, [3 2]) = mat1(:, [2 3]);
```

1	2	3
4	5	6
7	8	9

`mat1`

1	3	2
4	6	5
7	9	8

`mat1`

Matrix Dimension

- There are several functions to determine the dimensions of a vector or matrix:
  - `size` returns the # of rows and columns for a vector or matrix
  - Important: capture both of these values in an assignment statement

    `[r, c] = size(mat)`
  - `numel` returns the total # of elements in a vector or matrix

⚠: Very important to generalize your script: do not assume that you know the dimensions of a vector or matrix – use `size` or `numel` to find out!

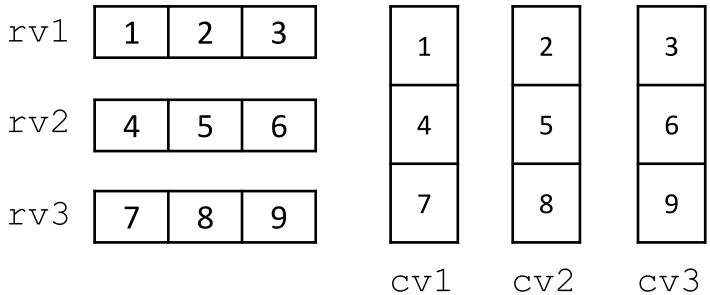
```
m1 = [1 2 3;4 5 6];

[r1, c1] = size(m1);

n_m1 = numel(m1);
```

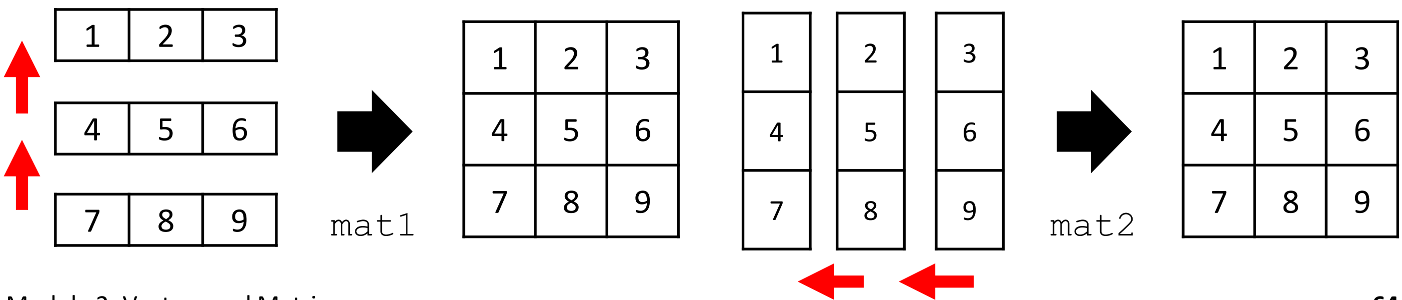
Name	Value
m1	[1 2 3;4 5 6]
r1	2
c1	3
n_m1	6

Matrix Concatenation

```
rv1 = [1 2 3];
rv2 = [4 5 6];
rv3 = [7 8 9];

mat1 = [rv1; rv2; rv3];

cv1 = [1; 4; 7];
cv2 = [2; 5; 8];
cv3 = [3; 6; 9];

mat2 = [cv1 cv2 cv3]
```

Matrices can be created by joining, vectors or adding elements.





Empty Vectors and Matrices

- An ***empty vector*** is a vector with no elements; an empty vector can be created using square brackets with nothing inside [ ]
- To **delete** an element from a vector, assign an empty vector to that element
- Delete an entire row or column from a matrix by assigning [ ]
  - Note: **<u>cannot delete an individual element from a matrix</u>**

```
>> v1 = 2:6
v1 =

     2     3     4     5     6

>> v1(end) = []
v1 =

     2     3     4     5

>> v1(3) = []
v1 =

     2     3     5
```

```
>> m1 = [1 2 3;4 5 6;7 8 9]
m1 =

     1     2     3
     4     5     6
     7     8     9

>> m1(2,:) = []
m1 =

     1     2     3
     7     8     9
```

Scalar Operations

- Numerical operations can be performed on every element in a vector or matrix
- For example, ***Scalar multiplication:*** multiply every element by a scalar

```
>> v1 = [4  0  11] *  3
v1 =

    12     0    33
```

- Another example: ***Scalar addition***; add a scalar to every element

```
>> v2 = zeros(1,3) + 5

v2 =

     5     5     5
```

Array Operations

Array operations on two matrices `A` and `B`.

- These are applied term-by-term, or element-by-element
- In MATLAB:
  - matrix addition: `A + B`
  - matrix subtraction: `A - B` or `B - A`
- For operations that are based on multiplication (multiplication, division, and exponentiation), a dot must be placed in front of the operator
  - array (element-wise) multiplication: `A .* B`
  - array (element-wise) division: `A ./ B`, `A .\ B`
  - array (element-wise) exponentiation `A .^ 2`
- **Matrix multiplication: NOT an array operation:** `A .* B` is not equal to `A * B`.

Example: Array Operations

```
m1 = [1 2 3; 4 5 6; 7 8 9];
m2 = [1 1 1; 2 2 2; 1 1 1];
sv = 3;

mat1 = m1 + m2;
mat2 = m1 .* m2;
mat3 = m1 - m2;
mat4 = m1 ./ m2;

mat5 = m1 + sv;
mat6 = m1 * sv;
```

$$\texttt{m1}\ \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \texttt{m2}\ \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \qquad \texttt{sv}\ \begin{bmatrix} 3 \end{bmatrix}$$

$$\texttt{mat1} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 7 & 8 \\ 8 & 9 & 10 \end{bmatrix}$$

$$\texttt{mat2} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \ .* \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 8 & 10 & 12 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\texttt{mat3} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 6 & 7 & 8 \end{bmatrix}$$

$$\texttt{mat4} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \ ./ \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2.5 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\texttt{mat5} = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

$$\texttt{mat6} = \begin{bmatrix} 3 & 6 & 9 \\ 12 & 15 & 18 \\ 21 & 24 & 27 \end{bmatrix}$$

Matrix Multiplication: Dimensions

- In MATLAB, the multiplication operator * performs matrix multiplication
- In order to be able to multiply a matrix A by a matrix B, the number of columns of A must be the same as the number of rows of B
- If the matrix `A` has dimensions $m \times n$, that means that matrix `B` must have dimensions $n \times something$;
  - In mathematical notation, $[A]_{m \times n}\ [B]_{n \times p}$
  - We say that the ***inner dimensions*** must be the same
- The resulting matrix `C` has the same number of rows as `A` and the same number of columns as `B`
  - in other words, the ***outer dimensions*** $m \times p$
  - In mathematical notation, $A]_{m \times n}\ [B]_{n \times p} = [C]_{m \times p}$.

⚠: Matrix multiplication is NOT an array operation. It does NOT mean multiplying term by term (element by element)

Matrix Times a Vector

Remind

A linear system of equations with coefficient matrix A, variable vector $\vec{x}$ and constant term vector $\vec{b}$, can be expressed as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$\underset{(m \times n)}{\text{matrix}} \underset{(n \times 1)}{\text{vector}} = \underset{(m \times 1)}{\text{vector}}$$

$$\boxed{b_i = \sum_j^n x_j a_{ij}}$$

Compatibility – the number of columns in the matrix **must** equal the number of rows in the vector.

Example: Matrix Times a Vector

```
m1 = [1 1 1; 2 2 2; 3 3 3];
v1 = [2 2 2]';

c1 = m1*v1;

c21 = m1(1,:)*v1;
c22 = m1(2,:)*v1;
c23 = m1(3,:)*v1;

c2 = [c21;c22;c23];
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} * \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix}$$

`m1 * v1 = c1`

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$$

`m1(1,:) * v1 = c21`

$$\begin{bmatrix} 2 & 2 & 2 \end{bmatrix} * \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \end{bmatrix}$$

`m1(2,:) * v1 = c22`

$$\begin{bmatrix} 3 & 3 & 3 \end{bmatrix} * \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 18 \end{bmatrix}$$

`m1(3,:) * v1 = c23`

Matrix Times a Matrix

Remind

Matrix multiplication is an extension of matrix and vector multiplication. Consider the product of an $m \times n$ matrix A, and an $n \times p$ matrix B:

$$AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ b_{31} & b_{32} & \cdots & b_{3p} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{bmatrix}$$

$$\overset{\text{matrix}}{(m \times n)}\overset{\text{matrix}}{(n \times p)} = \overset{\text{matrix}}{(m \times p)}$$

Compatibility – the number of columns in the first matrix **must** equal the number of rows in the second matrix.

Example: Matrix Times a Matrix

```
m1 = [1 2;3 4];
m2 = [1 2;2 1];

m3 = m1*m2;

m411 = m1(1,:)*m2(:,1);
m421 = m1(2,:)*m2(:,1);
m412 = m1(1,:)*m2(:,2);
m422 = m1(2,:)*m2(:,2);

m4 = [m411 m412;m421 m422];
```

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 11 & 10 \end{bmatrix}$$

`m1 * m2 = m3`

$$\begin{bmatrix} 1 & 2 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \end{bmatrix}$$

`m1(1,:) * m2(:,1) = m411`

$$\begin{bmatrix} 1 & 2 \end{bmatrix} * \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \end{bmatrix}$$

`m1(1,:) * m2(:,2) = m412`

$$\begin{bmatrix} 3 & 4 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \end{bmatrix}$$

`m1(2,:) * m2(:,1) = m421`

$$\begin{bmatrix} 3 & 4 \end{bmatrix} * \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \end{bmatrix}$$

`m1(2,:) * m2(:,2) = m422`

⚠: * and .* are different!

Example: Matrix Times a Matrix or Vector

Q. Write a script to compute `A, B, C, D` using `m1` and `v1`.

$$\text{m1} = \begin{bmatrix} 2 & 4 \\ 3 & 2 \\ 5 & 3 \end{bmatrix} \qquad \text{v1} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \\ 5 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 5 \\ 4 & 2 & 3 \end{bmatrix}$$

```
m1 = [2 4; 3 2; 5 3];
v1 = [1; 3; 1];

A = v1'*m1

B = m1'*v1

C = m1(:,1)'*v1

D = v1(1:2)'*m1'
```

📖: `A'` is a transpose of `A`.

Character and Character Vector

- A ***character*** is a single character in single quotes
- A character vector is sequences of characters in single quotes, e.g. `'hello and how are you?'`
- **A character vector is a vector in which every element is a single character.**

```
ch_vec = 'MATLAB';

n_ch = numel(ch_vec);

ch3 = ch_vec(3);
ch4 = ch_vec(1:3);

ch_vec1 = [ch_vec ' is fun.']
```

Name	Value
ch_vec	'MATLAB'
n_ch	6
ch3	'T'
ch4	'MAT'
ch_vec1	'MATLAB is fun.'

Character Type Casting

Challenging

- The numeric type cast function can also convert a character to its equivalent numeric value.
- All characters are mapped to equivalent numeric values.

```
ch1 = 'a';
ch2 = 'b';
ch3 = 'd';

ch_vec1 = 'abc';
```

`double(ch1)`	`97`
`char(97)`	`‘a’`
`char(98)`	`‘b’`
`double(‘d’)`	`100`
`char([97 99])`	`‘ac’`
`‘ab’ + 1`	`[98 99]`
`‘cd’ - 1`	`[98 99]`
`char('cd'-1)`	`'bc‘`
`ch_vec1 + 1`	`[98 99 100]`
`char(ch_vec1 + 1)`	`‘bcd’`

3D Matrices

Challenging

- A three dimensional matrix has dimensions `m x n x p`
- For example, we can create a `3 x 4 x 2` matrix of random integers; there are 2 layers, each of which is a `3 x 4` matrix

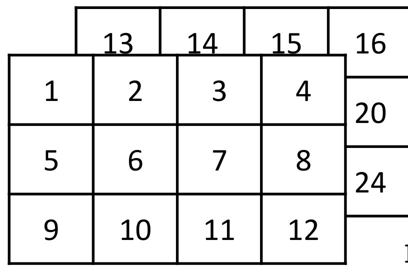
```
mat3D = zeros(3, 4, 2);

mat1 = [1 2 3 4; 5 6 7 8; 9 10 11 12];
mat2 = [13 14 15 16; 17 18 19 20; 21 22 23 24];

mat3D(:,:,1) = mat1;
mat3D(:,:,2) = mat2;
```

1	2	3	4
5	6	7	8
9	10	11	12

`mat1`

13	14	15	16
17	18	19	20
21	22	23	24

`mat2`



`mat3D`

3D Matrices (Continue)

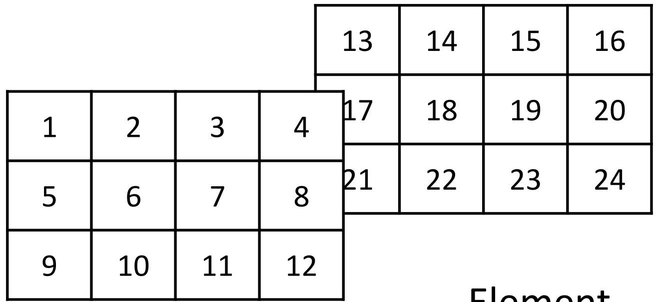
Challenging

```
val1 = mat3D(1, 3, 1);

val2 = mat3D(1, 1, 2);

mats = mat3D(1:2, 1:2, 1);
```

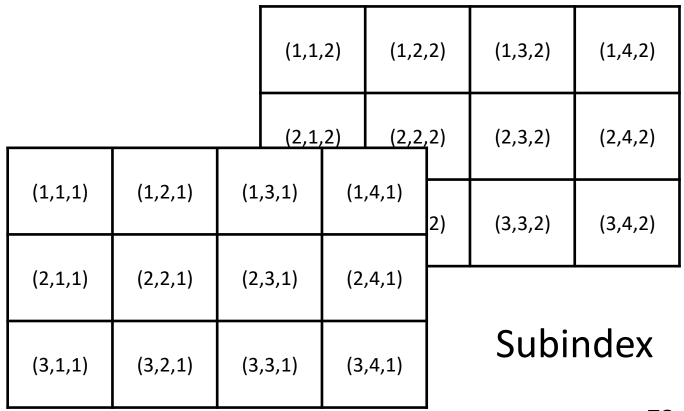
Name	Value
`mat3D`	`3x4x2` *`double`*
`val1`	`3`
`val2`	`13`
`mats`	`[1 2; 5 6]`

📖: Large matrices are not printed out in *Workspace*.

`mat3D`



Element



Subindex

Linear Indexing

Challenging

- Linear indexing: only using one index into a matrix
- MATLAB will unwind it column-by column going from top to bottom.

```
mat1 = [1 5 3 7;3 2 2 4;4 2 8 9];
val1 = mat1(2, 1)
val2 = mat1(2)
val3 = mat1(3, 4)
val4 = mat1(12)

vec1 = mat1(3:5)
```

Name	Value
`val1`	`3`
`val2`	`3`
`val3`	`9`
`val4`	`9`
`vec1`	`[4 5 2]`

1	5	3	7
3	2	2	4
4	2	8	9

Element

(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)

Subscripted indexing

1	4	7	10
2	5	8	11
3	6	9	12

Linear indexing

Function: `reshape(x, sz)`

Challenging

`B = reshape(A,sz)` reshapes `A` using the size vector, `sz`, to define `size(B)`. `sz` must contain at least 2 elements, product of elements in `sz` must be the same as `numel(A)`.

```
vec = 3:14; % 12 elements

mat0 = reshape(vec, [2 6]);
mat1 = reshape(vec, [3 4]);
mat2 = reshape(mat1, [6 2]);
```

mat1

3	6	9	12
4	7	10	13
5	8	11	14

vec

3	4	5	6	7	8	9	10	11	12	13	14

mat0

3	5	7	9	11	13
4	6	8	10	12	14

mat2

3	9
4	10
5	11
6	12
7	13
8	14

Types of Errors

Syntax error

```
a1 = [1 2 3];
3 = a1 + 2;
```

Error: Incorrect use of '=' operator. To assign a value to a variable, use '='. To compare values for equality, use '=='.

```
a@2 = [1 2 3];
```

Error: Invalid expression. Check for missing multiplication operator, missing or unbalanced delimiters, or other syntax error.

Runtime error

```
a2 =[11 2 3];
b2 = a2(4) + 1;
```

Index exceeds the number of array elements (3).

```
A = [1 2;3 4];
val1 = A(3,4);
```

Index in position 1 exceeds array bounds (must not exceed 2).

Logical error

```
A = [1 2;3 4];
B = [3 4;5 6];

m1 = A.*B;
m2 = A*B;
```

Example: Types of Errors

Q. Which of the following scripts occur run-time or syntax error?

(1)	`mat1 = ones(10,10);` `mat1(1:2, 5:10) = [];`
(2)	`mat1 = ones(10,10);` `mat1(1, 5) = [];`
(3)	`mat1 = ones(10,10);` `mat1(1, 1:2) = 3;`
(4)	`mat1 = ones(10,10);` `mat1 = [1 2; 3 4];`

1. (1), (2)
2. (1), (2), (3)
3. (1), (2), (4)
4. (3), (4)
5. (1), (3), (4)

Example (Midterm in S19) ★★

Q. Write a code to compute `val1`, `val2`, and `mat1` using `A`,`B`,`r`, and `c`

$$\boldsymbol{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix} \quad \boldsymbol{r} = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 \end{bmatrix}$$

$$\boldsymbol{val1} = \boldsymbol{c_1 r_1 + c_2 r_2 + c_3 r_3}$$

$$\boldsymbol{val2} = [\boldsymbol{c_1 r_1} \quad \boldsymbol{c_2 r_2} \quad \boldsymbol{c_3 r_3} \quad \boldsymbol{c_4 r_4}]$$

$$\boldsymbol{mat1} = \begin{bmatrix} b_{11}r_1 & b_{12}r_2 & b_{13}r_3 & b_{14}r_4 \\ b_{21}r_1 & b_{22}r_2 & b_{23}r_3 & b_{24}r_4 \\ b_{31}r_1 & b_{32}r_2 & b_{33}r_3 & b_{34}r_4 \end{bmatrix}$$

```
val1 = r(1:3)*c(1:3);
val2 = r.*c';

mat1 = B.*[r; r; r];
```