BT6270: Computational Neuroscience

Assignment 1

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A BT6270: Computational Neuroscience Assignment

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1 Hodgkin-Huxley Model

Hodgkin-Huxley Model is a mathematical model that explains how the dynamics of the ion channels (Na^{+}, K^{+}, etc.) contribute to the generation of an Action Potential in a Neuron. It is based on the parallel thought of a simple circuit with batteries, conductances, and capacitors.
The typical Hodgkin–Huxley model treats each component of a Neuron as an electrical element. The lipid bilayer is represented as a capacitance (C_m). Voltage-gated ion channels and Leakage channels are represented as electrical conductances (g_{Na}, g_K, g_L) that depend on both voltage and time. The electrochemical gradients (Nernst Potential at resting condition) driving the flow of ions are represented as Batteries (E_{Na}, E_K, E_L). Finally, The membrane potential is denoted by V_m.
Now to see the variance of this Vm with externally applied current:

Problem 1

Threshold values for the external applied currents I1, I2, and I3 in which shift of dynamical behavior from one to another is seen, such as no AP, finite number of AP's, Continuous firing and then followed by distortion resulting in no more APs.

1.1 Threshold Values

The threshold currents (computed at a precision of 0.0001) are as follows:

- $I_1 = 0.0224\ \mu A/mm^2$
- $I_2 = 0.0625\ \mu A/mm^2$
- $I_3 = 0.4578\ \mu A/mm^2$

$$\text{Voltage v/s Time} = \left\{ \begin{array}{ll} \text{No APs are seen,} & \text{if } i \leq \mathrm{I}_1 \\ \text{Limited No. of APs are seen,} & \text{if } \mathrm{I}_1 \geq i \leq I_2 \\ \text{APs are seen periodically,} & \text{if } \mathrm{I}_2 \geq i \leq I_3 \\ \text{No APs are seen,} & \text{if } \mathrm{I}_3 \geq i \end{array} \right\} \tag{1}$$

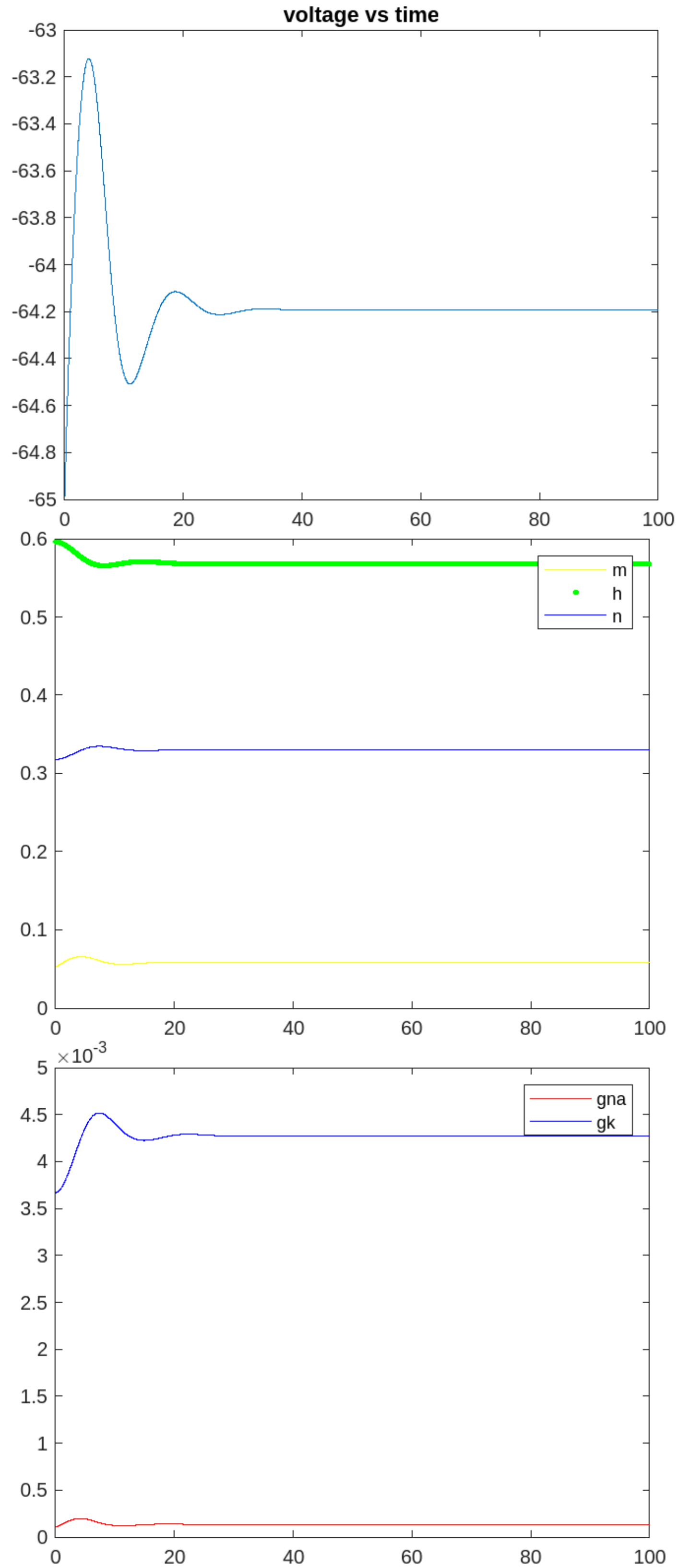

Figure 1: Variation of Voltage, Gating variables and Conductance v/s Time for a current range of i $< I_1$. No AP observed.

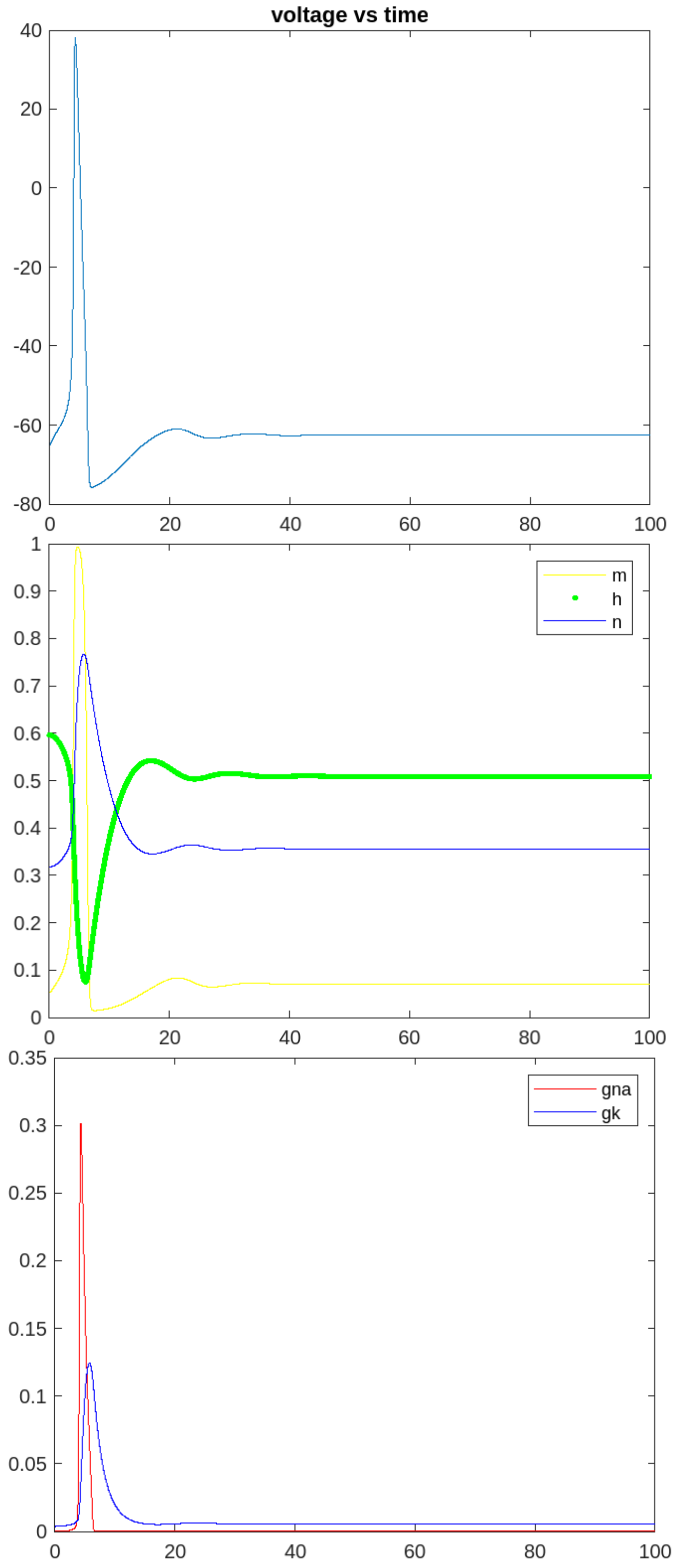

Figure 2: Variation of Voltage, Gating variables and Conductance v/s Time for a current range of $I_1 < \mathrm{i} < I_2$. Limited number of APs observed.

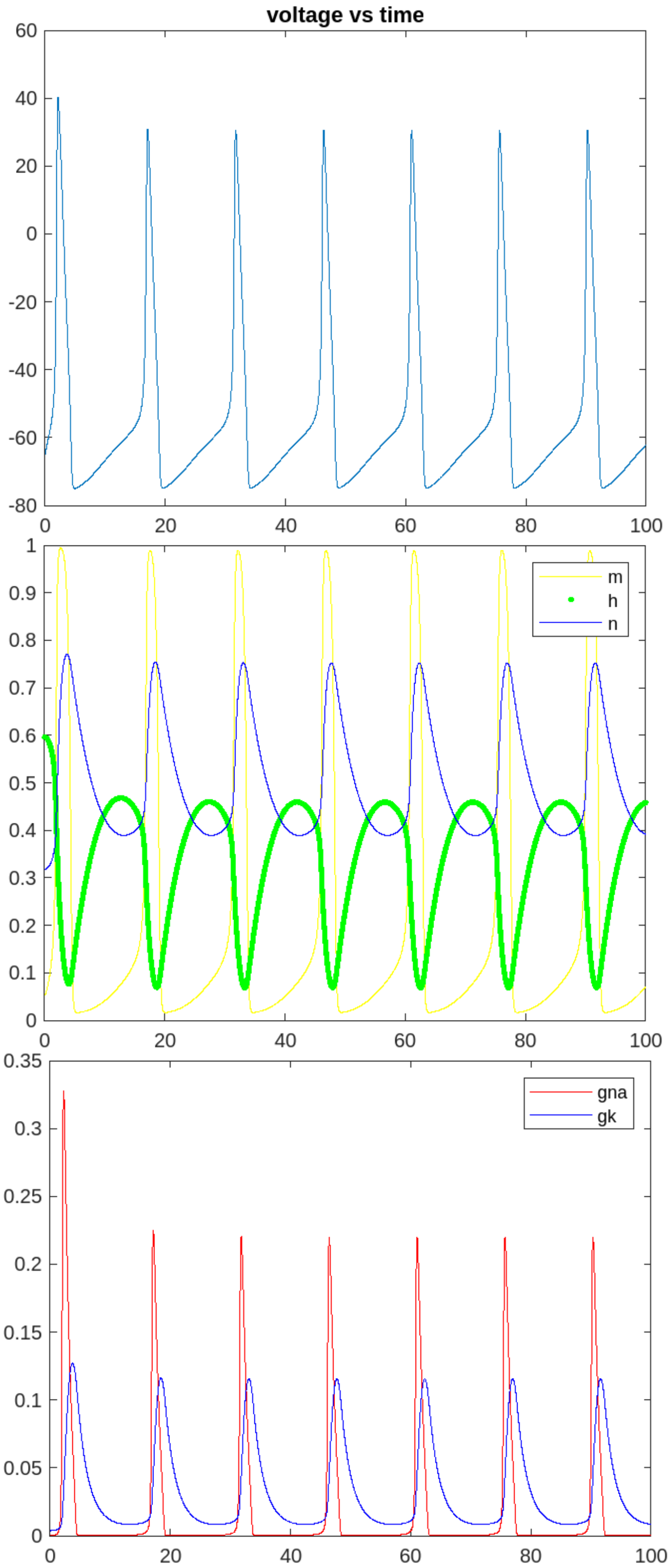

Figure 3: Variation of Voltage, Gating variables and Conductance v/s Time for a current range of $I_2 < \mathrm{i} < I_3$. Periodic APs observed.

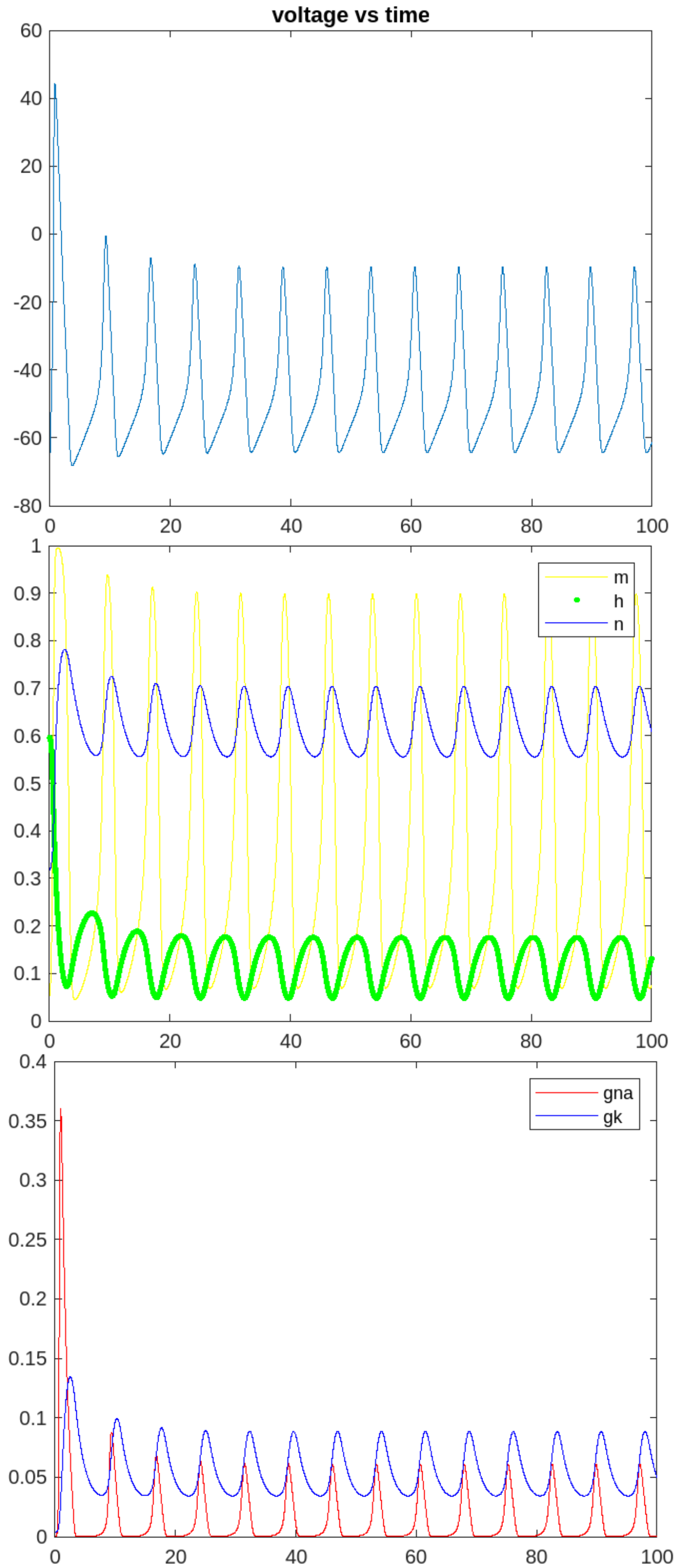

Figure 4: Variation of Voltage, Gating variables and Conductance v/s Time for a current range of I_3 $<$ i . No APs observed.

Problem 2

A graph that depicts the firing rate (frequency) as you change the applied external current (i.e. Iext vs. Firing rate (f)). You can make this plot either in Matlab or Python.

1.2 Plot for Frequency v/s I_{ext}

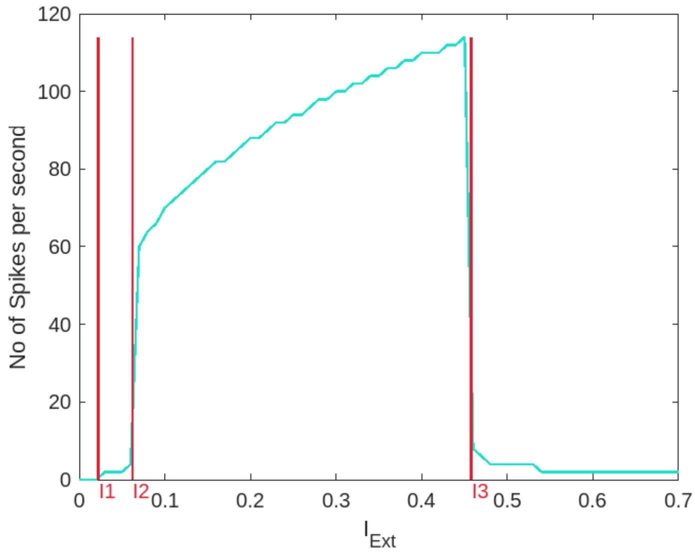

Figure 5: The change in frequency of firing as a function of Input current. The number of iterations performed for each current instance: 5 x 10^4. The red vertical bars indicate the current thresholds - I_1, I_2, and I_3 respectively.

Given below is the code used for getting the Plot of Frequency v/s External Current (in Matlab):

```
close all;
stepSize=0.01; % Kept low for faster computation
freq=zeros(1,60);
ct=1;

for ImpCur=0:stepSize:1
    gkmax=.36;
    vk=-77;
    gnamax=1.20;
    vna=50;
    gl=0.003;
    vl=-54.387;
    cm=.01;

    dt=0.01;
```

```
niter=50000;
t=(1:niter)*dt;
iapp=ImpCur*ones(1,niter);
v=-64.9964;
m=0.0530;
h=0.5960;
n=0.3177;

gnahist=zeros(1,niter);
gkhist=zeros(1,niter);
vhist=zeros(1,niter);
mhist=zeros(1,niter);
hhist=zeros(1,niter);
nhist=zeros(1,niter);


for iter=1:niter
  gna=gnamax*m^3*h;
  gk=gkmax*n^4;
  gtot=gna+gk+gl;
  vinf = ((gna*vna+gk*vk+gl*vl)+ iapp(iter))/gtot;
  tauv = cm/gtot;
  v=vinf+(v-vinf)*exp(-dt/tauv);
  alpham = 0.1*(v+40)/(1-exp(-(v+40)/10));
  betam = 4*exp(-0.0556*(v+65));
  alphan = 0.01*(v+55)/(1-exp(-(v+55)/10));
  betan = 0.125*exp(-(v+65)/80);
  alphah = 0.07*exp(-0.05*(v+65));
  betah = 1/(1+exp(-0.1*(v+35)));
  taum = 1/(alpham+betam);
  tauh = 1/(alphah+betah);
  taun = 1/(alphan+betan);
  minf = alpham*taum;
  hinf = alphah*tauh;
  ninf = alphan*taun;
  m=minf+(m-minf)*exp(-dt/taum);
  h=hinf+(h-hinf)*exp(-dt/tauh);
  n=ninf+(n-ninf)*exp(-dt/taun);
  vhist(iter)=v; mhist(iter)=m; hhist(iter)=h; nhist(iter)=n;
end
j=1;
peaks=zeros;
[mag,loc]=findpeaks(vhist);
for temp=1:length(mag)
    if mag(temp) >=10 % Min value for a spike to be considered a AP
        peaks(j)=mag(temp);
        j=j+1;
    end
end
if peaks ~= 0
    no_of_peaks(ct)=length(peaks);
else
    no_of_peaks(ct)=0;
end
```

```
    ct=ct+1;
end
I1=0.0224;   % Calculated at a stepsize of 0.0001
I2=0.0625;
I3=0.4578;
freq=no_of_peaks*1000/(niter/100);
plot(0:stepSize:1,freq,'Color','#21DECC','LineWidth',1.2);
xlim([0 0.7]);
xlabel('I_{Ext}');
ylabel('No of Spikes per second');
hold on;
I1=0*freq+I1;
plot(I1,freq,'Color','#DE2133','LineWidth',1.2);
text(I1(1),-3,'I1','Color','#DE2133');
I2=0*freq+I2;
plot(I2,freq,'Color','#DE2133','LineWidth',1.2);
text(I2(1),-3,'I2','Color','#DE2133');
I3=0*freq+I3;
plot(I3,freq,'Color','#DE2133','LineWidth',1.2);
text(I3(1),-3,'I3','Color','#DE2133');
```

1.3 Assumptions Made:

- Step size of the external current was kept low (0.01 μA) for the calculation of Frequency vs Time Plot.
- Step size of the external current was kept (0.0001 μA) while finding its threshold values.
- A threshold of 10 mV was taken, for a voltage spike to be considered as an Action Potential.
- Change in time dt was taken as 0.01 ms.