# BT6270: Computational Neuroscience

Assignment 2

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A BT6270: Computational Neuroscience Assignment

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# 1 FitzHugh-Nagumo Model

The FitzHugh-Nagumo model is a simplified mathematical model of the electrical activity in a neuron or excitable medium. It was developed independently by Richard FitzHugh and J. Nagumo in the 1960s and is often used to describe the dynamics of action potentials in nerve cells. The model consists of a system of two coupled differential equations and is a simplification of the more complex Hodgkin-Huxley model.

**Problem 1**

Simulate the two-variable FitzHugh-Nagumo neuron model using the following equations:

1. $\frac{dv}{dt} = f(v) - w + I_{ext}$

2. $\frac{dw}{dt} = bv - rw$

where a=0.5, choose b and r=0.1

**Problem 2**

Use single forward Euler Integration

$\frac{dv}{dt} = \frac{\Delta v}{\Delta t}$

$\Delta$v(t) = v(t+1) - v(t) = [fv(t) - w(t) + Iext(t)]$\Delta$t

given:

v(0) − > v(t ) − > v(2t ) − >....

## 1.1 Case 1: $I_{ext} = 0$

### 1.1.1 Phaseplot and Stability Analysis

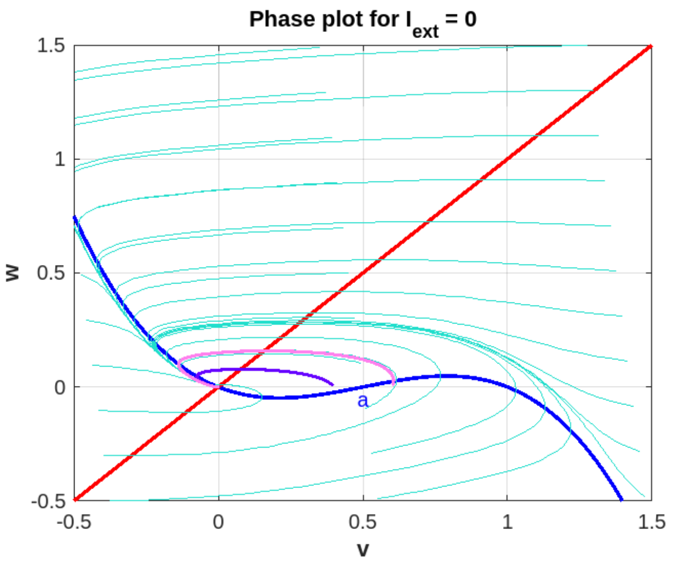


Figure 1: Phase Plot of the system when $I_{ext} = 0$

Analyzing the trajectories by using initial points - v=0.4 and 0.6 ; w = 0, we can see that even for perturbations in the initial start point, we approach the equilibrium point at (0,0). Hence, the point (0,0) is a stable fixed point.

### 1.1.2 V (t); W(t) across t

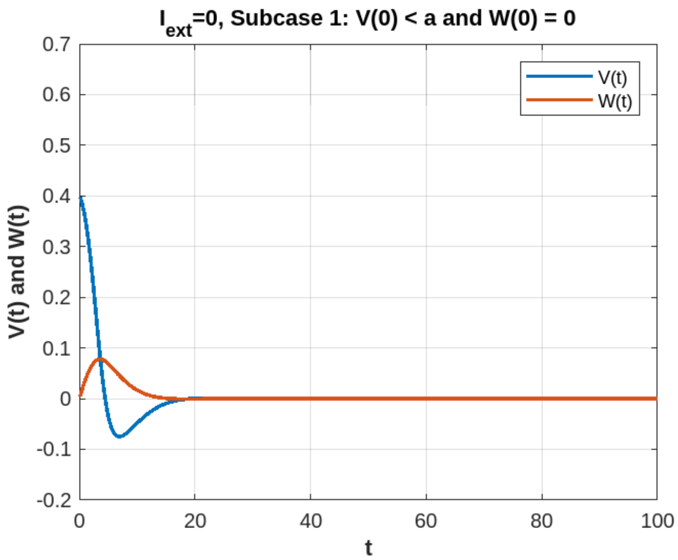


Figure 2: V(t); W(t) across t, when V (0) $<$ a. With sub-threshold pulse injections, no action potentials are observed.

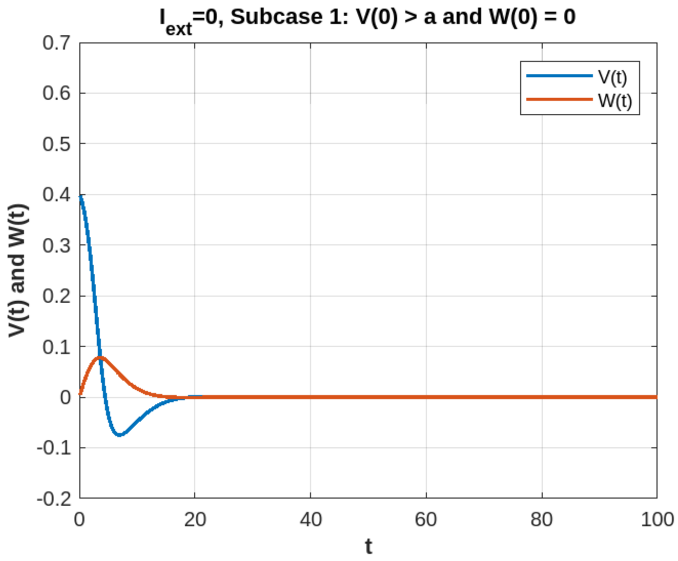


Figure 3: V (t); W(t) across t, when V (0) > a. With sub-threshold pulse injections, no action potentials are observed.

## 1.2 Case 2: $I_1 < I_{ext} < I_2$

### 1.2.1 Phaseplot and Stability Analysis

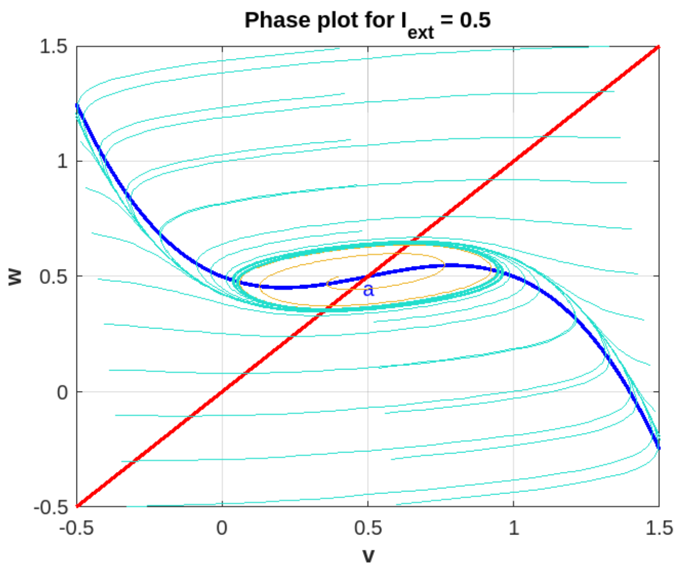


Figure 4: Phase Plot of the system when $I_{ext} = 0.5$

The trajectories were analyzed by using initial points - v=0.4; w = 0. We can see that at the point of intersection of the nullclines, there are circulating fields around the unstable stationary point. Additionally, we also see a limit cycle enclosing the stationary point for $I_1 = 0.313 < I_{ext} < I_2 = 0.650$

### 1.2.2 V (t); W(t) across t

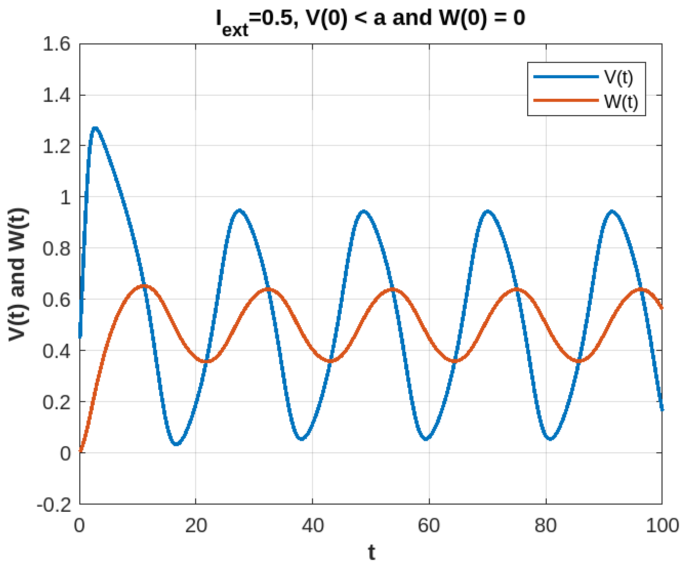


Figure 5: V (t); W(t) across t, when V (0) $<$ a. Sustained oscillations are observed

## 1.3 Case 3: $I_2 < I_{ext}$

### 1.3.1 Phaseplot and Stability Analysis

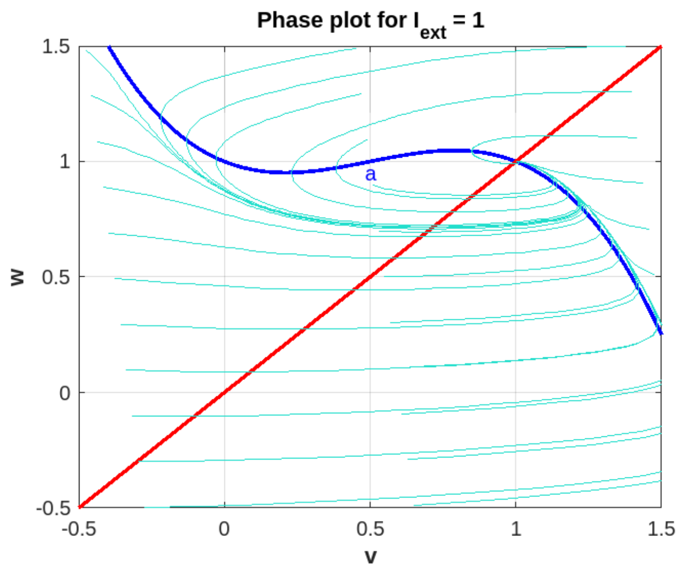


Figure 6: Phase Plot of the system when $I_{ext}$ = 1

The trajectories were analyzed by using initial points - v=0.4; w = 0 We can see that even for large perturbations in the initial start point, we approach the equilibrium point at (1,1). Hence, the point (1,1) is a stable fixed point.

### 1.3.2 V (t); W(t) across t

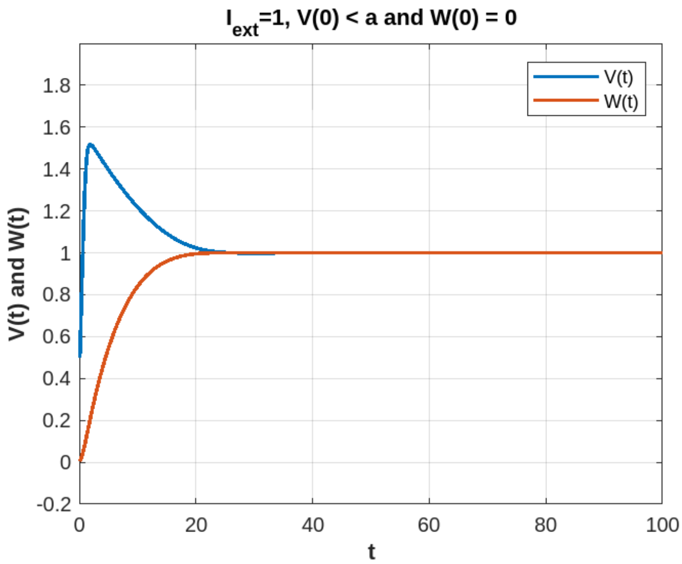


Figure 7: V (t); W(t) across t, when V (0) $<$ a. Depolarisation of Action Potential is seen

## 1.4 Case 4: Bistability

The parameter values used to simulate this case are: b = 0.01; r = 0.1. Hence, b/r = 0.1.

### 1.4.1 Phaseplot and Stability Analysis

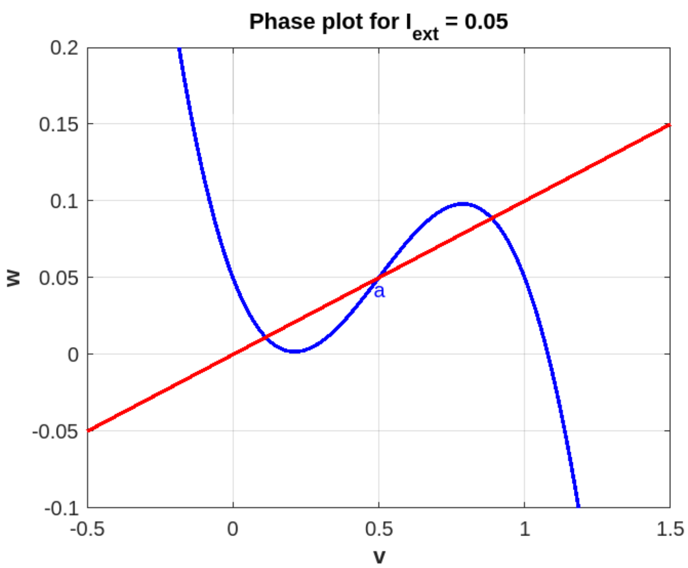


Figure 8: Phase Plot of the system when $I_{ext}$ = 0.05

The trajectories were analyzed by using initial points - v=0.4; w = 0. The stationary points are P1, P2, and P3, in that order. In the case of P1 and P3 - small and intermediate perturbations lead back to P1 and P3 respectively. Hence P1 is a stable point. In case of P2, small perturbations along one axis leads to large change in final point. Hence, P2 is a saddle node.

#### 1.4.2 V (t); W(t) across t

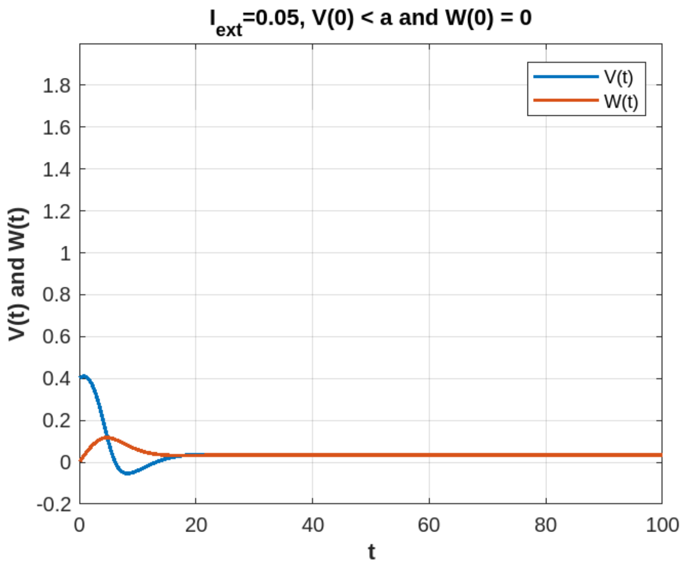


Figure 9: V (t); W(t) across t, when V (0) < a. The neuron exists in a tonically down state.

### 1.5 Code used for the assignment:

```
close all
clear all

% Defining Variables
a = 0.5;
b = 0.1;
r = 0.1;

% Defining the FitzHugh-Nagumo equations
fv = @(v) v*(a-v)*(v-1);
dv_dt = @(v, w, I) fv(v) - w + I;
dw_dt = @(v, w) b*v - r*w;

v = -0.5 : 0.01 : 1.5;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Case 1 --------------------------------------------------
dt = 0.1;
tt = 100;
Iext = 0;
```

```
[vnc, wnc] = nullClines(v,Iext, b,r,fv,1);

v01 = 0.4;
v02 = 0.6;
w0 = 0;
t = 0.1:dt:tt;
[vhist1,whist1] = eulerInt(v01,w0,Iext,dt,tt, dv_dt,dw_dt);
[vhist2,whist2] = eulerInt(v02,w0,Iext,dt,tt, dv_dt,dw_dt);
plotfw(vhist1,whist1,t,0.7,2,1,Iext)
plotfw(vhist1,whist1,t,0.7,3,2,Iext)

v0 = -0.5:1:1.5;
w0 = -0.5:0.2:1.5;
for i=1:length(v0)
    for j=1:length(w0)
        [vhist3,whist3] = eulerInt(v0(i),w0(j),Iext,dt,tt, dv_dt,dw_dt);
        figure(1)
        plot(vhist3,whist3,'Color','#21DECC')
    end
end
xlim([-0.5 1.5]);

plot(vhist1,whist1,'Color','#6600FF',LineWidth=1.5)
plot(vhist2,whist2,'Color','#FF80ED',LineWidth=1.5)

% Case 2 --------------------------------------------------
dt = 0.1;
tt = 100;
Iext = 0.5;
[vnc, wnc] = nullClines(v,Iext, b,r,fv,4);

v0 = 0.4;
w0 = 0;
t = 0.1:dt:tt;
[vhist1,whist1] = eulerInt(v0,w0,Iext,dt,tt, dv_dt,dw_dt);
plotfw(vhist1,whist1,t,1.6,5,3,Iext)

v01 = 0.4;
w01 = 0.5;
[vhist2,whist2] = eulerInt(v01,w01,Iext,dt,tt, dv_dt,dw_dt);
figure(4)
plot(vhist2,whist2)

v0 = -0.5:1:1.5;
w0 = -0.5:0.2:1.5;
for i=1:length(v0)
    for j=1:length(w0)
        [vhist2,whist2] = eulerInt(v0(i),w0(j),Iext,dt,tt, dv_dt,dw_dt);
        figure(4)
        plot(vhist2,whist2,'Color','#21DECC')
    end
end
xlim([-0.5 1.5]);
```

```
% Case 3 --------------------------------------------------
dt = 0.1;
tt = 100;
Iext = 1;
[vnc, wnc] = nullClines(v,Iext, b,r,fv,6);

v0 = 0.4;
w0 = 0;
t = 0.1:dt:tt;
[vhist1,whist1] = eulerInt(v0,w0,Iext,dt,tt, dv_dt,dw_dt);
plotfw(vhist1,whist1,t,2,7,3,Iext)

v0 = -0.5:1:1.5;
w0 = -0.5:0.2:1.5;
for i=1:length(v0)
    for j=1:length(w0)
        [vhist2,whist2] = eulerInt(v0(i),w0(j),Iext,dt,tt, dv_dt,dw_dt);
        figure(6)
        plot(vhist2,whist2,'Color','#21DECC')
    end
end
xlim([-0.5 1.5]);

% Case 4 --------------------------------------------------
dt = 0.1;
tt = 100;
Iext = 0.05;
b = 0.01;
[vnc, wnc] = nullClines(v,Iext, b,r,fv,8);
ylim([-0.1 0.2]);

v0 = 0.4;
w0 = 0;
t = 0.1:dt:tt;
[vhist1,whist1] = eulerInt(v0,w0,Iext,dt,tt, dv_dt,dw_dt);
plotfw(vhist1,whist1,t,2,9,3,Iext)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Function for plotting nullclines and trajectories
function [vnc, wnc] = nullClines(v,Iext, b,r,fv,fig)
    iter = 1;
    for i = v
        vnc(iter) = fv(i) + Iext;
        wnc(iter) = b*i/r;
        iter = iter + 1;
    end
    figure(fig);
    plot(v,vnc,'b',v,wnc,'r',LineWidth=1.75)
    title(['Phase plot for I_{ext} = ', num2str(Iext)])
    xlabel('v','FontWeight','bold')
    ylabel('w','FontWeight','bold')
    text(0.5,Iext,'a','HorizontalAlignment','center','VerticalAlignment','top','
    Color','b')
```

```
    grid on
    hold on
    ylim([-0.5 1.5]);
end

function [vhist,whist] = eulerInt(v0,w0,Iext,dt,tt, dv_dt,dw_dt)
    niter = tt/dt;
    for i = 1:niter
        v0 = v0 + dv_dt(v0,w0,Iext)*dt;
        w0 = w0 + dw_dt(v0,w0)*dt;
        vhist(i) = v0;
        whist(i) = w0;
    end
end

function [] = plotfw(vhist,whist,t,ymax,fig,subcase,Iext)
    s = {'I_{ext}=%s, Subcase 1: V(0) < a and W(0) = 0','I_{ext}=%s, Subcase 1: V
    (0) > a and W(0) = 0','I_{ext}=%s, V(0) < a and W(0) = 0'};
    figure(fig)
    plot(t,vhist,t,whist,LineWidth=1.75)
    ss = string(s(subcase));
    title(sprintf(ss,num2str(Iext)))
    legend('V(t)','W(t)')
    ylabel('V(t) and W(t)','FontWeight','bold')
    xlabel('t','FontWeight','bold')
    ylim([-0.2,ymax]);
    grid on
end
```

### 1.6 Assumptions Made:

- Step size of the change in voltage was kept low (0.01 $\mu A$) for the calculation of all the plots.
- Step size of the external current was kept (0.001 $\mu A$) while finding I1 and I2 by iterating over the entire range of minima to maxima.
- Initiation of Limit Cycle was found by observational analysis.
- Change in time dt was taken as 0.1 ms.