VE230 RC slides Week 4

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Overview

Content

- Electric Statistics
- Capacitance and Capacitors
- Boundary Conditions

Electric Displacement

- **electric flux density/electric displacement,** $\vec{D}$:

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad (C/m^2)$$

- $$\nabla \cdot \vec{D} = \rho \quad (C/m^3)$$

  , where ρ is the volume density of free charges.
- Another form of **Gauss's law**:

$$\oint_S \vec{D} \cdot d\vec{s} = Q \quad (\vec{C})$$

  , the total outward flux of the electric displacement (the total outward electric flux) over any closed surface is equal to the total free charge enclosed in the surface.

Electric Displacement

- If the dielectric of the medium is **linear and isotropic**,

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \epsilon_0 (1 + \chi_e) \vec{E} = \epsilon_0 \epsilon_r \vec{E} = \epsilon \vec{E}$$

, where χ_e is a dimensionless quantity called electric susceptibility,
ϵ_r is a dimensionless quantity called as relative permittivity/ electric constant of the medium,
ϵ is the absolute permittivity/permittivity of the medium (F/m).

Boundary Conditions for Electrostatic Fields

- the tangential component of an $\vec{E}$ field is continuous across an interface.

$$E_{1t} = E_{2t} \quad (V/m)$$

, or

$$\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$$

- The normal component of $\vec{D}$ field is discontinous across an interface where a surface charge exists - the amount of discontinuity being equal to the surface charge density.

$$\vec{a_{n2}} \cdot (\vec{D_1} - \vec{D_2}) = \rho_s$$

, or

$$D_{1n} - D_{2n} = \rho_s \quad (C/m^2)$$

Capacitance and Capacitors

Definition

$$C = \frac{Q}{V}.$$

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Series and Parallel

Series:

$$C = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

Parallel:

$$C = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

Calculating the Capacitance

You can calculate the capacitance by the following steps:

1. Choose a proper coordinate system.
2. Assume $+$Q, -Q on the conductors.
3. From **E** from Q.
4. Find $V_{12} = \int_2^1 \mathbf{E} \cdot d\mathbf{l}$
5. $C = Q/V_{12}$

Capacitor System

Isolated Conductor System:

$$Q_0 + Q_1 + ... + Q_N = 0.$$

Self energy:

$$W = Q_2 V_2 = Q_2 \frac{Q_1}{4\pi\epsilon_0 R_{12}}.$$

Mutual energy:

$$W_e = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k.$$

Boundary Conditions

Poisson's Equation:

$$\nabla^2 V = -\frac{\rho}{\epsilon}.$$

In Cartesian System:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Laplace Equation:

$$\nabla^2 V = 0.$$

Uniqueness Theorem: A solution of Poisson's Equation or Laplace's Equation that satisfies the given boundary conditions is a unique solution.

Solution of Laplace Equations

There are three kinds of solutions for the equations: (discuss only about 2D situation) (The method to get the solution form can be learned in Vv557, taught by Horst)

- $X = Ae^{-k_1 x} + Be^{k_1 x}, \quad Y = Ce^{-k_2 x} + De^{k_2 x}$
- $X = A\sin(k_1 x) + B\cos(k_1 x), \quad Y = C\sin(k_2 x) + D\cos(k_2 x)$
- $X = a + bx, \quad Y = c + dx$

Method of Images: The method of images is just a method that try to satisfy the boundary conditions. If you are interested in it, you can ask me for 557 slides. :)