Matrix-chain Multiplication

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Matrix-chain Multiplication

Suppose we have a sequence or chain A1, A2, …, An of n matrices to be multiplied

That is, we want to compute the product $A1 \cdot A2 \cdots An$

There are many possible ways (parenthesizations) to compute the product

Matrix-chain Multiplication

Example: Consider the chain A1, A2, A3, A4 of 4 matrices

Let us compute the product

$$A1 \cdot A2 \cdot A3 \cdot A4$$

There are 5 possible ways:

$$(A1 \cdot (A2 \cdot (A3 \cdot A4)))$$
$$(A1 \cdot ((A2 \cdot A3) \cdot A4))$$
$$((A1 \cdot A2) \cdot (A3 \cdot A4))$$
$$((A1 \cdot (A2 \cdot A3)) \cdot A4)$$
$$(((A1 \cdot A2) \cdot A3) \cdot A4)$$

Matrix-chain Multiplication

To compute the number of scalar multiplications necessary, we must know:

- Algorithm to multiply two matrices

- Matrix dimensions

Algorithm to Multiply 2 Matrices

Input: Matrices $A_{p \times q}$ and $B_{q \times r}$ (with dimensions $p \times q$ and $q \times r$)
Result: Matrix $C_{p \times r}$ resulting from the product $A \cdot B$

MATRIX-MULTIPLY($A_{p \times q}$, $B_{q \times r}$)

```
for i ← 1 to p
        for j ← 1 to r
                C[i, j] ← 0
                for k ← 1 to q
                        C[i, j] ← C[i, j] + A[i, k] · B[k, j]
return C
```

Matrix-chain Multiplication

Example: Consider three matrices

$$A_{10\times100},\ B_{100\times5},\ \text{and}\ C_{5\times50}$$

There are 2 ways to parenthesize

$((AB)C) = D_{10\times5} \cdot C_{5\times50}$

$$\left.\begin{array}{l} AB \Rightarrow 10\cdot100\cdot5=5{,}000 \text{ scalar multiplications} \\ DC \Rightarrow 10\cdot5\cdot50 =2{,}500 \text{ scalar multiplications} \end{array}\right\} \text{Total:}7{,}500$$

$(A(BC)) = A_{10\times100} \cdot E_{100\times50}$

$$\left.\begin{array}{l} BC \Rightarrow 100\cdot5\cdot50=25{,}000 \text{ scalar multiplications} \\ AE \Rightarrow 10\cdot100\cdot50 =50{,}000 \text{ scalar multiplications} \end{array}\right\} \text{Total: } 75{,}000$$

Matrix-chain Multiplication

Given a chain $A_1, A_2, \ldots, A_n$ of n matrices, where for i=1, 2, …, n, matrix A_i has dimension $p_{i-1} \times p_i$

Parenthesize the product $A_1 A_2 \ldots A_n$ such that the total number of **scalar multiplications is minimized**

Matrix-chain Multiplication

```
MATRIX-CHAIN-ORDER(p[ ], n)
    for i ← 1 to n
        m[i, i] ← 0
    for l ← 2 to n
        for i ← 1 to n-l+1
            j ← i+l-1
            m[i, j] ← ∞
            for k ← i to j-1
                q ← m[i, k] + m[k+1, j] + p[i-1] p[k] p[j]
                if q < m[i, j]
                    m[i, j] ← q
                    s[i, j] ← k
  return m and s
```

Matrix-chain Multiplication

- Our algorithm computes the minimum-cost table *m* and the split table *s*

- The optimal solution can be constructed from the split table *s*

- Each entry $s[i, j\ ]=k$ shows where to split the product $A_i\ A_{i+1}\ \ldots\ A_j$ for the minimum cost

Matrix-chain Multiplication

Matrix	Dimension
A_1	30×35
A_2	35×15
A_3	15×5
A_4	5×10
A_5	10×20
A_6	20×25

Matrix-chain Multiplication

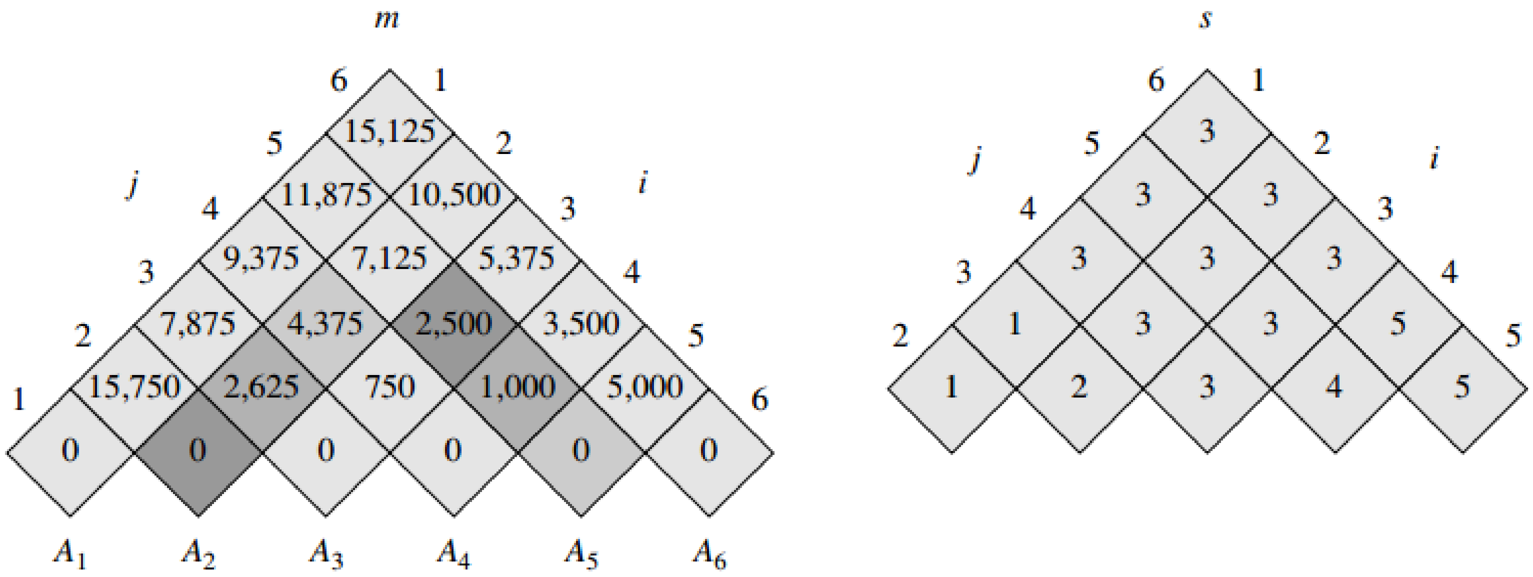

Figure 15.3 The m and s tables computed by MATRIX-CHAIN-ORDER for $n = 6$ and the following matrix dimensions:

matrix	dimension
A_1	30×35
A_2	35×15
A_3	15×5
A_4	5×10
A_5	10×20
A_6	20×25

Thank you