

PA3 due Tuesday  
Exam 1 → next Friday  
↳ 2D search / BFS / DFS

### Categorizing Runtimes

Let  $f(n) = 100$

Which of the following is **NOT** a correct bound?

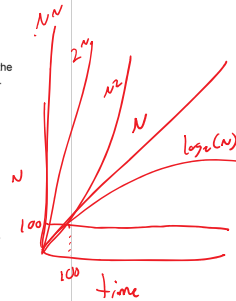
- ☒ A  $f(n)$  is  $O(2n^2)$
- ☒ B  $f(n)$  is  $O(n^2)$
- ☒ C  $f(n)$  is  $O(n)$
- ☒ D  $f(n)$  is  $O(100n^{100})$
- ☐ E None of these

Big-O

$$f(n) = O(g(n)) \iff f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0$$

For each function in the list below, it is related to the function below it by O, and the reverse is not true. That is,  $n$  is  $O(n^2)$  but  $n^2$  is not  $O(n)$ .

- $f(n) = 10n^2$
- $f(n) = 1/n$
- $f(n) = 1$
- $f(n) = \log(n)$
- $f(n) = \sqrt{n}$
- $f(n) = n$
- $f(n) = n^2$
- $f(n) = n^3$
- $f(n) = n^4$
- ... and so on for constant polynomials ...
- $f(n) = 2n$
- $f(n) = n!$
- $f(n) = n^n$



Let  $f(n) = 3n^3 + 2n + 7$

Which of the following is a correct bound?

- ☒ A  $f(n)$  is  $O(\log(n))$
- ☒ B  $f(n)$  is  $O(n^2)$
- ☒ C  $f(n)$  is  $O(n)$
- ☐ D  $f(n)$  is  $O(n^3)$
- ☐ E None of these

$$f(n) \leq 12n^3$$

$$f(n) \leq C + g(n)$$

$$3n^3 + 2n + 7 \rightarrow 12n^3 \quad O(n^3)$$

$$C = 12 \quad g(n) = n^3$$

$$n_0 = 0$$

$$3n^3 + 2n + 7 \rightarrow n^3$$

$$C = 5 \quad g(n) = n^3$$

$$n_0 = 7$$

```
void printAllElementOfArray(int[] arr) {
    for (int i = 0; i < arr.length; i++) {
        printf("%d\n", arr[i]);
    }
}
```

Which of the following is a correct bound?

- ☒ A  $f(n)$  is  $O(\log(n))$
- ☒ B  $f(n)$  is  $O(n^2)$
- ☒ C  $f(n)$  is  $O(n)$
- ☒ D  $f(n)$  is  $O(n^3)$
- ☐ E None of these

$$3n + 2$$

$$3n + 2n \rightarrow 5n$$

$$C = 5$$

$$n_0 = 0$$

$$g(n) = n$$

$$O(n)$$

Which of the following is a correct bound?

- A.  $f(n)$  is  $O(\log(n))$
- B.  $f(n)$  is  $O(n^2)$
- C.  $f(n)$  is  $O(n)$
- D.  $f(n)$  is  $O(n^3)$
- E. None of these

```
void printAllPossibleOrderedPairs(int arr[]) {
    for (int i = 0; i < arr.length; i++) {
        for (int j = 0; j < arr.length; j++) {
            printf("%d = %d\n", arr[i], arr[j]);
        }
    }
}
```

$$1 + (n+1) + n + \dots + n = \frac{n(n+1)}{2} + n = \frac{n^2 + 3n + 2}{2}$$

Which of the following is a correct bound?

- A.  $f(n)$  is  $O(\log(n))$
- B.  $f(n)$  is  $O(n^2)$
- C.  $f(n)$  is  $O(n)$
- D.  $f(n)$  is  $O(n^3)$
- E. None of these

$$3n^2 + 4n + 2$$

$$3n^2 + 4n^2 + 2n^2 = 9n^2$$

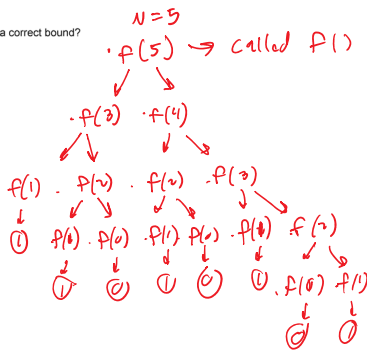
$$c=9, g(n)=n^2$$

$$b=0, O(n^2)$$

```
int fibonacci(int num) {
    if (num <= 1) return num;
    return fibonacci(num - 2) + fibonacci(num - 1);
}
```

Which of the following is a correct bound?

- A.  $f(n)$  is  $O(2n)$
- B.  $f(n)$  is  $O(n^2)$
- C.  $f(n)$  is  $O(n)$
- D.  $f(n)$  is  $O(n^3)$
- E. None of these



$n=4$ , 9 times

$n=3$ , 5 times

$2^N \rightarrow$

$2^5 \rightarrow 32$

$2^4 \rightarrow 16$

$2^3 \rightarrow 8$

sorted

```
// A recursive binary search function. It returns location
// of x in given array arr[l..r] is present, otherwise -1
int binarySearch(int arr[], int l, int r, int x)
{
    if (r >= l) {
        int mid = l + (r - l) / 2;

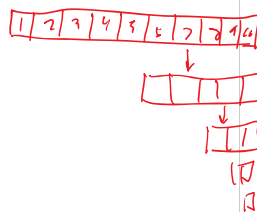
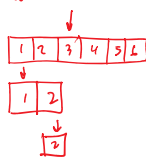
        // If the element is present at the middle
        // itself
        if (arr[mid] == x)
            return mid;

        // If element is smaller than mid, then
        // it can only be present in left subarray
        if (arr[mid] > x)
            return binarySearch(arr, l, mid - 1, x);

        // Else the element can only be present
        // in right subarray
        return binarySearch(arr, mid + 1, r, x);
    }

    // We reach here when element is not
    // present in array
    return -1;
}
```

worst case



What are some correct bounds for binarySearch? What is the smallest correct bound?

$O(N), O(N^2), \dots, O(\log_2(N))$

```
boolean isPrimeAll(int num) {
    // Check for divisors of num
    for (int i = 0; i < num; i += 1) {
        if (num % i == 0) {
            // Any divisor other than 1 or num means num is not prime
            return false;
        }
    }
    // No other divisors found means num is prime
    return true;
}
```

What is the smallest correct bound?

$1 + (n+1) + n + \dots + n = \frac{n(n+1)}{2} + n = \frac{n^2 + 3n + 2}{2}$

$3n + 3$

$c=3, N_0=3$

$g(n)=n$

```
boolean isPrimeHalf(int num) {
    // Check for divisors of num
    for (int i = 0; i < num / 2; i += 1) {
        if (num % i == 0) {
            // Any divisor other than 1 or num means num is not prime
            return false;
        }
    }
    // No other divisors found means num is prime
    return true;
}
```

What is the smallest correct bound?

$1 + (\frac{n}{2} + 1) + \frac{n}{2} = \frac{3n}{2} + 3$

$c=2, N_0=3$

$g(n)=n$

```
    // No other divisors found means num is prime
    return true;
}
```

1

✓  
N<sub>1</sub> = 3 (90, 1, 10)

```
void printAllItemsTwice(int arr[], int size)
{
    for (int i = 0; i < size; i++) {
        printf("%d\n", arr[i]);
    }

    for (int i = 0; i < size; i++) {
        printf("%d\n", arr[i]);
    }
}
```

What is the smallest correct bound?

```
void printFirstItemThenFirstHalfThenSayHi100Times(int arr[], int size)
{
    printf("First element of array = %d\n", arr[0]);

    for (int i = 0; i < size/2; i++) {
        printf("%d\n", arr[i]);
    }

    for (int i = 0; i < 100; i++) {
        printf("Hi\n");
    }
}
```

What is the smallest correct bound?

```
void printAllNumbersThenAllPairSums(int arr[], int size)
{
    for (int i = 0; i < size; i++) {
        printf("%d\n", arr[i]);
    }

    for (int i = 0; i < size; i++) {
        for (int j = 0; j < size; j++) {
            printf("%d\n", arr[i] + arr[j]);
        }
    }
}
```

What is the smallest correct bound?