

# CSE12 - Lecture 11 - B00

Monday, October 17, 2022 9:00 AM

PA3 due tomorrow  
Exam 1 → Friday

## Measuring Runtime

Count how many times each line executes, then say which  $\Theta()$  statement(s) is(are) true.

```
int maxDifference(int[] arr){
    max = 0;
    for (int i=0; i<arr.length; i++) {
        for (int j=0; j<arr.length; j++) {
            if (arr[i] - arr[j] > max)
                max = arr[i] - arr[j];
        }
    }
    return max;
}
```

$N^2$   $N$   $1 + (N+1) + N$   $N$   $1 + (N+1) + N$   $0$  or  $N$

$2n + 4 + n(2 + 4n)$

$P(n) = 4n^2 + 4n + 4$

$4n^2 + 4n^2 + 4 \rightarrow 8n^2 + 4$

$C = 8$   $g(n) = n^2$

$N_0 = 4$   $\Theta(n^2)$

best case worst case

Assume  $n = \text{arr.length}$

- A.  $f(n) = \Theta(2^n)$
- B.  $f(n) = \Theta(n^2)$
- C.  $f(n) = \Theta(n)$
- D.  $f(n) = \Theta(n^3)$
- E. Other/none/more



Count how many times each line executes, then say which  $\Theta()$  statement(s) is(are) true.

```
int sumTheMiddle(int[] arr){
    int range = 100;
    int start = arr.length/2 - range/2;
    int sum = 0;
    for (int i=start; i<start+range; i++)
        sum += arr[i];
    return sum;
}
```

$1 + (100+1) + 100$

$6 + 300$

$306$

Assume  $n = \text{arr.length}$

- A.  $f(n) = \Theta(2^n)$
- B.  $f(n) = \Theta(n^2)$
- C.  $f(n) = \Theta(n)$
- D.  $f(n) = \Theta(1)$
- E. None of these

$f(n) = 306$

$C = 306$   $g(n) = 1$

$N_0 = 0$

$\Theta(1)$

constant time

Big 0 Upper bound

$f(n) = \underline{0}(g(n))$ ,  $f(n) \leq c * g(n)$   
for all  $n \geq n_0$

Big  $\Omega$  omega Lower bound

$f(n) = \underline{\Omega}(g(n))$ ,  $f(n) \geq c * g(n)$   
for all  $n \geq n_0$

Big  $\Theta$  theta Tight bound

$f(n) = \underline{\Theta}(g(n))$ ,  $f(n) = c * g(n)$   
for all  $n \geq n_0$

For each function in the list below, it is related to the function below it by  $O$ , and the reverse is not true. That is,  $n$  is  $O(n^2)$  but  $n^2$  is not  $O(n)$ .

- $f(n) = 1/(n^2)$
- $f(n) = 1/n$
- $f(n) = 1$
- $f(n) = \log(n)$
- $f(n) = \text{sqrt}(n)$
- $f(n) = n$
- $f(n) = n^2$
- $f(n) = n^3$
- $f(n) = n^4$
- ... and so on for constant polynomials ...
- $f(n) = 2^n$
- $f(n) = n!$
- $f(n) = n^n$

range = 100  
start =  $\frac{N}{2} - 50$

start + range =  $\frac{N}{2} + 50$

$N = 100$

start =  $\frac{100}{2} - 50 = 0$

start + range =  $\frac{100}{2} + 50 = 100$

$\rightarrow 100$

$N = 10000$

start =  $\frac{10000}{2} - 50 = 4950$

start + range =  $\frac{10000}{2} + 50 = 5050$

$\rightarrow 100$

```
void printAllItemsTwice(int arr[], int size)
```

```
{
  for (int i = 0; i < size; i++) {
    printf("%d\n", arr[i]);
  }
  for (int i = 0; i < size; i++) {
    printf("%d\n", arr[i]);
  }
}
```

$2N$

$1 + (N+1) + N$

$3N+2$

+

$1 + (N+1) + N$

$3N+2$

$6N+4$

$C=6 \rightarrow g(N)=N$

$N_0=4$

$\Theta(N)$

$2N$

What is the tight bound?

$\Theta(N)$

```
void printFirstItemThenFirstHalfThenSayHi100Times(int arr[], int size)
```

```
{
  printf("First element of array = %d\n", arr[0]);
  for (int i = 0; i < size/2; i++) {
    printf("%d\n", arr[i]);
  }
  for (int i = 0; i < 100; i++) {
    printf("Hi\n");
  }
}
```

$\frac{N}{2}$   $1 + (\frac{N}{2} + 1) + \frac{N}{2}$

$3\frac{N}{2} + 2$

+

$100$   $1 + (100 + 1) + 100$

$300 + 2$

$\frac{3N}{2} + 304$

$\Theta(N)$

$\Theta(N)$

What is the tight bound?

```
void printAllNumbersThenAllPairSums(int arr[], int size)
```

```
{
  for (int i = 0; i < size; i++) {
    printf("%d\n", arr[i]);
  }
  for (int i = 0; i < size; i++) {
    for (int j = 0; j < size; j++) {
      printf("%d\n", arr[i] + arr[j]);
    }
  }
}
```

$1 + (N+1) + N$

$3N+2$

+

$1 + (N+1) + N$

$2N+2$

$N + [1 + (N+1) + N]$

$N + (3N+2)$

$3N^2 + 7N + 4$

$\Theta(N^2)$

$N$   
+  
 $N^2$   
 $N^2+2$   
 $\Theta(N^2)$

What is the tight bound?

## Selection Sort

```
import java.util.Arrays;
public class Sort {
    public static void sortA(int[] arr) {
        for(int i = 0; i < arr.length; i += 1) {
            System.out.print(Arrays.toString(arr) + " -> ");
            int minIndex = i;
            for(int j = i; j < arr.length; j += 1) {
                if(arr[minIndex] > arr[j]) { minIndex = j; }
            }
            int temp = arr[i];
            arr[i] = arr[minIndex];
            arr[minIndex] = temp;
            System.out.println(Arrays.toString(arr));
        }
    }
}
```

Selection Sort – what does it print out?

Sort.sortA(new int[]{ 53, 83, 15, 45, 49 });

[53, 83, 15, 45, 49] ->  $N=5$

15	83	53	45	49	5
15	45	53	83	49	4
15	45	49	83	53	3
15	45	49	53	83	2
15	43	49	53	83	1

Worse case: reverse sorted array  
83 53 49 45 15

best case: sorted array  
15, 45, 49, 53, 83

What is the runtime? Consider the shape of the input array.

Worse case:  $\Theta(N^2)$

Best case:  $\Theta(N^2)$

I → is sorted ( )  
 $\Theta(N)$

## Insertion Sort

```
import java.util.Arrays;
public class Sort {
    public static void sortB(int[] arr) {
        for(int i = 0; i < arr.length; i += 1) {
            System.out.print(Arrays.toString(arr) + " -> ");
            for(int j = i; j > 0; j -= 1) {
                if(arr[j] < arr[j-1]) {
                    int temp = arr[j-1];
                    arr[j-1] = arr[j];
                    arr[j] = temp;
                }
            }
            System.out.println(Arrays.toString(arr));
        }
    }
}
```

$\sim \left[ \frac{N}{2} \right]$

Insertion Sort – what does it print out?

Sort.sortB(new int[] { 53, 83, 15, 45, 49 });

[53, 83, 15, 45, 49] -> 53 83 15 45 49  
 53 83 15 45 49  
 15 53 83 45 49  
 15 45 53 83 49  
 15 45 49 53 83

15 45 49 53 83

What is the runtime? Consider the shape of the input array.

Worse case:  $\mathcal{O}(N^2)$

Best case:  $\mathcal{O}(N)$