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Rotation by arbitrary axis

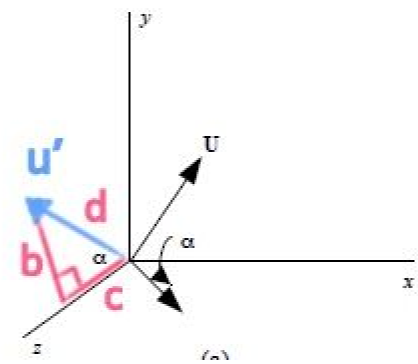
Step1

Change the start point's position of rotation axis to the origin.

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step2

u is a unit vector of rotation axis. And u' is projection of u on yz plane.



Then rotate u' in x axis to xz plane with **α** angle.

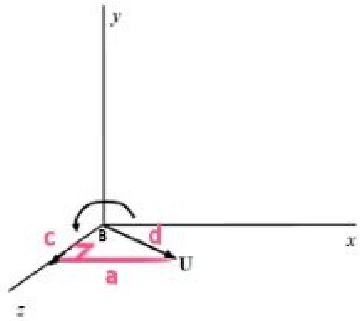
$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Find sin(**α**) and cos(**α**) from pythagoras triangle.

sin(**α**) = $\frac{b}{d}$ and cos(**α**) = $\frac{c}{d}$

Step3

Rotate u’ again with β angel in counter clockwise direction to make u’ lie on z axis.



Rotate in y axis with -β angle.

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Find sin(-β) and cos(-β) from pythagoras triangle.

sin(-β) = $\frac{b}{d}$ and cos(-β) = $\frac{c}{d}$

<u>Step4</u>

Now the rotation axis is lie on z axis so we can rotate it in z axis with any angle.

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

<u>Step5</u>

Rotate back in x and y axis to make it be on the old axis.

There for, formula for rotate in arbitrary axis for 3D is

$$R(\theta) = T^{-1} \cdot R_x^{-1}(\alpha) \cdot R_y^{-1}(\beta) \cdot R_z(\theta) \cdot R_y(\beta) \cdot R_x(\alpha) \cdot T$$

<u>Implement</u> Rotation for 45 degree axis for draw a hexagon

```
RotateX[theta_, {x_, y_, z_}] := Module[{xp, yp, zp},
  xp = x;
  yp = y*Cos[theta] - z*Sin[theta];
  zp = y*Sin[theta] + z*Cos[theta];
  Return[{N[xp], N[yp], N[zp]}];
 ]
```

Module for Rotate any point in x-axis with any angle.

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

```
RotateY[theta_, {x_, y_, z_}] := Module[{xp, yp, zp},
  xp = x*Cos[theta] - z*Sin[theta];
  yp = y;
  zp = z*Sin[theta] + x*Cos[theta];
  Return[{N[xp], N[yp], N[zp]}];
 ]
```

Module for Rotate any point in y-axis with any angle.

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

```
RotateZ[theta_, {x_, y_, z_}] := Module[{xp, yp, zp},
  xp = x*Cos[theta] - y*Sin[theta];
  yp = x*Sin[theta] + y*Cos[theta];
  zp = z;
  Return[{N[xp], N[yp], N[zp]}];
 ]
```

Module for Rotate any point in z-axis with any angle.

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

```
Rotate45axis[theta_, {x_, y_, z_}] := Module[{ans},
  (*Rx(alfa)^-1*Ry(beta)^-1*Rz(theta)*Ry(beta)*Rx(alfa)*)
  ans = RotateX[-45 Degree,
    RotateY[45 Degree,
     RotateZ[theta,
      RotateY[-45 Degree,
       RotateX[45 Degree, {x, y, z}]
      ]
     ]
    ]
   ];
  Return[ans];
 ];
```

Module for Rotate any point in 45 Degree-axis with any angle.

$$R(\theta) = T^{-1} \cdot R_x^{-1}(\alpha) \cdot R_y^{-1}(\beta) \cdot R_z(\theta) \cdot R_y(\beta) \cdot R_x(\alpha) \cdot T$$

For 45 degree axis the rotation axis is start from origin point so it's no need to translate.

First rotate in x-axis with 45 degree angle. And then rotate in y-axis with -45 degree angle.
Then the rotation axis will lies on z-axis so rotate it in z-axis with any angle.
And finally rotate it back,rotation in y-axis with 45 degree angle and rotate in x-axis with -45 degree angle.

```
p0 = {5, 0, 5}
p1 = Rotate45axis[60 Degree, point]
p2 = Rotate45axis[120 Degree, point]
p3 = Rotate45axis[180 Degree, point]
p4 = Rotate45axis[240 Degree, point]
p5 = Rotate45axis[300 Degree, point]
```

Drawing a hexagon.
The first point is (5, 0, 5). And for the next 5 point we have to rotate in each 60 degree. So, rotate with 60, 120, 180, 240, 300 degree in 45 degree axis.

```
(*plot*)
pol = Polygon[{p0, p1, p2, p3, p4, p5}];

{Graphics3D[{Blue, pol}]}
```

Plot polygon from point

<u>Output</u>

Hexagon vertices are

{5, 0, 5}

{3.56671, 6.00358, 1.11159}

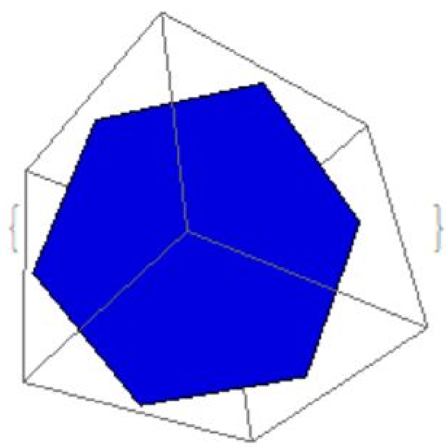
{-0.701053, 5.48581, -4.40618}

{-5., 4.44089×10^{-16}, -5.}

{-5.03118, -4.96804, -0.076053}

{-0.763413, -4.45028, 5.44171}

Polygon graph



Why Know that this graph is hexagon that place about 45 degree axis?
Because it's hexagon when we look from 45 degree axis direction.