

# Logic Riddles

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This is a compilation of our favorite brain teasers. Some are written by us, but the majority are ones we have encountered from various sources. We claim no credit for these. Feel free to email us if you are stuck and would like a hint, want to check a solution, or know of any good riddles we might want to add.

**Note.** For each of the following riddles, all people are assumed to be perfect mathematicians with infinite computing power and memory.

Many riddles require mathematical intuition. Those which require a small amount of mathematical background are marked with (+). Those which are trivial with mathematical knowledge but may be challenging without it are marked with (-). In all problems, the full power of ZFC is at your disposal.

Some settings involve a group of mathematicians imprisoned by a particularly whimsical, riddle-loving warden. The warden sets a challenge for the mathematicians to solve in order to gain their freedom. The mathematicians may confer and agree on a strategy in advance, but the warden listens in on their plans. If they fail, the warden keeps them imprisoned forever.

## Symbol Key.

- (+) Requires some basic (undergraduate-level) mathematics
- (-) More interesting for non-mathematicians
- (n) Subjective difficulty rating out of 5

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## 1 Ants on a Log

### 1.1 Falling off (2)

You have  $n$  ants evenly spaced on a very thin 1-meter log. Each ant moves either left or right at a speed of 1 m/s. When two ants collide, they instantly reverse direction and continue moving at the same speed. What is the longest possible time before all ants fall off the log?

### 1.2 Teleporting (3)

Now suppose that when an ant reaches the end of the log, it teleports to the opposite end. How long does it take to guarantee that all ants have simultaneously returned to their original positions, moving in their original directions?

## 2 Light Switches

### 2.1 Two switches (3)

There are  $n$  mathematicians, each locked in an individual room and placed in a coma so that they do not experience the passage of time. Each day, the warden randomly selects one mathematician, wakes them, and brings them to a room containing two light switches. The mathematician must flip exactly one switch, after which they are returned to their room and placed back into a coma. All communication is eliminated except for the state of the switches.

Find a method by which one mathematician can determine with absolute certainty that every mathematician has been awakened at least once.

**Hint.** What if there were only one light switch and each mathematician could choose whether or not to flip it?

### 2.2 A three-colored switch (4)

Replace the two switches with a single switch that cycles between three states. Every mathematician must change the state of the switch when they enter the room.

## 3 Ropes (3)

You are given several ropes, each of which takes exactly one hour to burn from one end to the other. The ropes burn at irregular rates, and the burning behavior of each rope is independent of the others.

Find a method to measure exactly 45 minutes.

## 4 Too Many Hats

### 4.1 100 mathematicians and two hat colors (3)

One hundred mathematicians are lined up facing the same direction. Each mathematician is given a hat that is either blue or red. Each mathematician can see the hats of everyone in front of them, but not their own hat or the hats of anyone behind them. Starting from the back of the line, the warden asks each mathematician to announce their hat color. They may only press a blue or red button, which announces the color in a flat voice to the entire room.

The goal is for at most one mathematician to guess their hat color incorrectly.

### 4.2 Countably infinitely many mathematicians (2)(+)

Now suppose there are countably infinitely many mathematicians, arranged in a line of order type  $\omega$  (think  $\mathbb{N}$ ), facing toward the infinite direction.

The goal is for at most finitely many mathematicians to guess their hat color incorrectly.

### 4.3 All the hats! (3)(+)

Assume there are countably infinitely many mathematicians and countably many possible hat colors. The mathematicians are lined up as before, with each able to see the hat colors of all mathematicians ahead of them. Find a strategy such that at most one mathematician gives an incorrect response.

## 5 More Hats (5)

There are  $n$  mathematicians and  $n$  hat colors, possibly with repetition. Each mathematician can see everyone else's hat color, but not their own. The warden asks all mathematicians to announce the color of their own hat simultaneously. Find a strategy guaranteeing that at least one mathematician guesses correctly.

## 6 Nickels on a Table (4)(+)

You have 10 dots on a table. Can you always cover all of the dots with non-overlapping nickels?

**Hint.** A calculation you might use is  $\frac{\pi\sqrt{3}}{6} \approx 0.907$ .

## 7 Cards

### 7.1 Standard deck (3)

You and a friend perform a card trick. You draw five cards at random from a standard deck. You keep one card and give the remaining four to your friend,

who must then correctly guess the card you kept. What strategy can you agree on in advance to guarantee success?

## 7.2 Nonstandard deck (5)(+)

For how large a deck is this strategy possible?

# 8 Number Machine

## 8.1 Two values (2)

You are given a machine that takes an integer input and outputs an integer. The machine computes an unknown polynomial with nonnegative integer coefficients. The machine has enough battery power for only two inputs. How can you determine the polynomial?

## 8.2 One value (1)(+)

Instead, you may input a single real number, which consumes all remaining battery power. Can you determine the polynomial this way?

**Hint.** Can you measure 30 minutes?

# 9 Guessing Jelly Beans (2)(-)

You are locked in an empty room. Behind a curtain are several jars, each containing some number of jelly beans. You do not know how many jars there are or how many jelly beans are in each jar, and you cannot see the jars. The door unlocks only when you correctly list the number of jelly beans in each jar. Can you devise a strategy that eventually guarantees your escape?

# 10 Names in Boxes (4)

There are 100 mathematicians and 100 boxes, each containing the name of one mathematician. Each mathematician enters the room alone and may open at most 50 boxes, trying to find the box containing their own name. The mathematicians must agree on a strategy so that everyone succeeds. The twist is that the first mathematician may open all boxes and swap exactly one pair of names.

# 11 Fortunately, Unfortunately (1)

Unfortunately, you contract a deadly illness. Fortunately, there is a cure. Unfortunately, the cure is expensive. Fortunately, you can afford exactly the number

of pills required: 100 pills of type A and 100 pills of type B. The pills look identical, and you must take exactly one of each per day.

Unfortunately, on the first day, you accidentally place one pill A and two pills B into your hand. Taking all three would cause an overdose. Without wasting any pills, how can you still take exactly one of each type?

## 12 Mailing a Ring (2)(-)

Dan and Maria have fallen in love online. Dan wishes to mail Maria a ring, but they live in Kleptopia, where anything sent through the mail is stolen unless enclosed in a padlocked box. Dan and Maria each have many padlocks, but none to which the other has a key. How can Dan safely send the ring to Maria?

## 13 Four Coins on a Rotating Table (4)

Charlie has a rotating table with four coins, each hidden under a cup. On each turn, Charlie selects two cups to peek under and may flip zero, one, or both coins. The cups are replaced, and the table is randomly rotated while Charlie closes their eyes. None of the cups are distinguishable. The game ends when all four coins show the same side.

Can Charlie guarantee ending the game in at most four moves?

## 14 Scales (2)

You have seven balls of equal weight and one ball that is slightly heavier. You have a balance scale that can be used at most twice. Identify the heavier ball.

## 15 More Scales (2)

You have twelve balls, one of which has a different weight (either heavier or lighter). You may use a balance scale at most three times. Identify the odd ball and determine whether it is heavier or lighter.

## 16 Truthful and Lying Statues

### 16.1 As in *Labyrinth* (2)

You encounter two indistinguishable statues guarding two indistinguishable doors. One door leads to riches; the other leads to doom. One statue always tells the truth, and the other always lies. You may ask one yes-or-no question to a single statue. How do you determine the safe door?

## 16.2 A whimsical statue (3)

Now add a third statue, which randomly chooses whether to lie or tell the truth each time it answers a question. You may ask two questions total, to any statues. The statues refuse to answer paradoxical questions. Can you still determine the safe door?

## 16.3 Ja vs. Da (5)

The statues now answer only with the words ‘ja’ or ‘da,’ meaning yes and no (in some unknown order). One statue tells the truth, one lies, and one is whimsical. The statues refuse to answer paradoxical questions, and they do not know in advance how the whimsical statue will respond.

With three questions, can you identify which statue is which? With two questions, can you identify the safe door?

## 16.4 Imploding Heads (5)

The setting is as above, except that if a statue is asked a paradoxical question, it explodes instead of refusing to answer. The statues will now answer paradox-inducing questions. With two questions, can you determine which statue is which?

# 17 Plus Signs on a Blackboard (3)

An antagonistic professor writes a (very) long positive integer on the blackboard, as a string of decimal digits.

You may perform the following procedure:

1. Insert plus signs “+” between any adjacent digits you like (possibly none, possibly between every pair), thereby partitioning the digit string into blocks.
2. Interpret each block as a integer and take the sum of these integers.
3. Write the resulting sum on the next line.

You may iterate this procedure on the newly written number, choosing the plus signs anew each time.

**Question.** Does there exist an absolute number of steps  $N$  such that, regardless of the initial number the professor writes, you can always choose plus signs so that after at most  $N$  iterations the process reaches a single digit?

## 18 Prime-ish Polynomials (2)(+)

Call a polynomial *prime-ish* if every exponent appearing in it is a prime number. In other words,  $f(x) = \sum_{i=1}^k a_i x^{e_i}$  is prime-ish if each  $e_i$  is prime.

**Riddle.** Show that every polynomial  $p(x) \in \mathbb{Z}[x]$  has a nonzero multiple that is prime-ish.