№4

$$f = \|X_2 - (X_1 R - 1\bar{\mu}^T)\|_F = tr\left([X_2 - (X_1 R - 1\bar{\mu}^T)]^T [X_2 - (X_1 R - 1\bar{\mu}^T)]\right) =$$

$$= tr\left([X_2^T - (R^T X_1^T - \mu^T 1^T)][X_2 - (X_1 R - 1\bar{\mu}^T)]\right) = tr\left(X_2^T X_2 - X_2^T (X_1 R - 1\bar{\mu}^T) - R^T X_1^T - \bar{\mu} 1^T) X_2 + \right.$$

$$\left. + R^T X_1^T X_1 R - \bar{\mu} 1^T X_1 R - R^T X_1^T 1\bar{\mu}^T + \bar{\mu} 1^T 1 \bar{\mu}^T\right) =$$

* $tr(A+B) = tr(A^T) + tr(B) = tr(A+B)^*$

$$= tr(X_2^T X - 2X_2^T X_1 R + 2X_2^T 1\bar{\mu}^T - 2\bar{\mu} 1^T X_1 R + \bar{\mu} 1^T 1 \bar{\mu}^T +$$

$$+ R^T X_1^T X_1 R = tr(X_2^T X_2 + X_1^T X_1 - 2(X_2^T X_1) R +$$

$$+ 2 X_2^T 1\bar{\mu}^T - 2R^T X_1^T 1\bar{\mu}^T + \bar{\mu}\bar{\mu}^T 1 1^T)$$

$$\frac{\partial}{\partial \bar{\mu}^T} f = \frac{\partial}{\partial \bar{\mu}} tr\left[2X_2^T 1\bar{\mu}^T\right] - 2\frac{\partial}{\partial \bar{\mu}^T} tr\left[R^T X_1^T 1\bar{\mu}\right] +$$

$$+ \frac{\partial}{\partial \bar{\mu}} n\, tr(\bar{\mu}\bar{\mu}^T)$$

$$R = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$X_1^T X_2 = S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

$$tr(X_1^T X_2 R) = \cos\alpha (S_{11} + S_{22}) + \sin\alpha (S_{21} - S_{12})$$

$$\frac{\partial}{\partial \alpha} tr(S^T R) = \sin\alpha (S_{11} + S_{22}) + \cos\alpha (S_{21} - S_{12}) =$$

→ (1)

$$S^T R = \begin{pmatrix} s_{11}\cos\alpha + s_{12}\sin\alpha & s_{21}\cos\alpha - s_{11}\sin\alpha \\ s_{12}\cos\alpha + s_{12}\sin\alpha & s_{22}\cos\alpha - s_{12}\cos\alpha \end{pmatrix}$$

$$R^T S = \begin{pmatrix} s_{11}\cos\alpha + s_{12}\sin\alpha & s_{21}\cos\alpha - s_{11}\sin\alpha \\ s_{12}\cos\alpha + s_{12}\sin\alpha & s_{22}\cos\alpha - s_{12}\cos\alpha \end{pmatrix}$$

$S^T R = (S^T R)^T \Rightarrow s_{21}\cos\alpha - s_{11}\sin\alpha =$

$= s_{12}\cos\alpha + s_{22}\sin\alpha \Rightarrow$ (1)

$(U\Lambda V^T)^T R = R^T (U\Lambda V^T) \Rightarrow V\Lambda U^T U V^T = V U^T U \Lambda V^T$

$V\Lambda V^T = V\Lambda V^T$ – получили тождество ■

№ 5

1) Рассмотр. $v^T B B^T v$ – скаляр произв в метрике BB^T

Возьмем $a = B^T v \hookrightarrow v^T B B^T v = a^T a = \|a\|_2^2 \geq 0$

– есть евклид. норма $\Leftrightarrow$ B – положит полуопределена