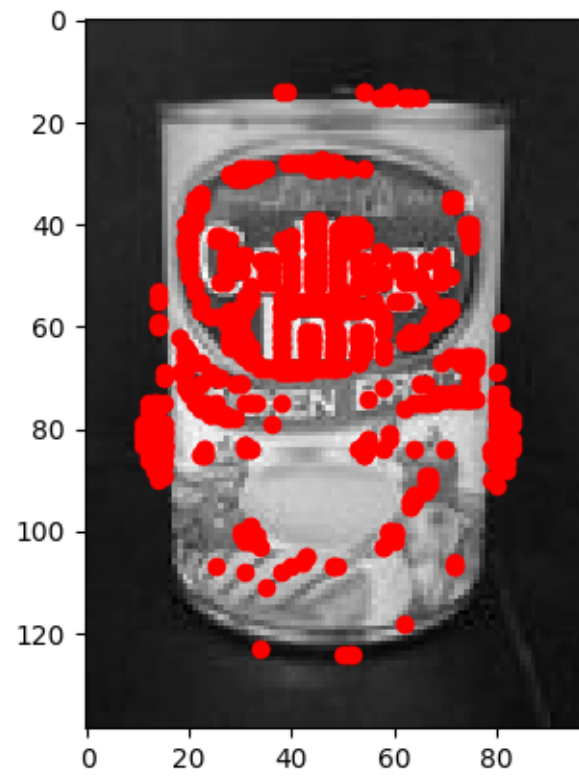
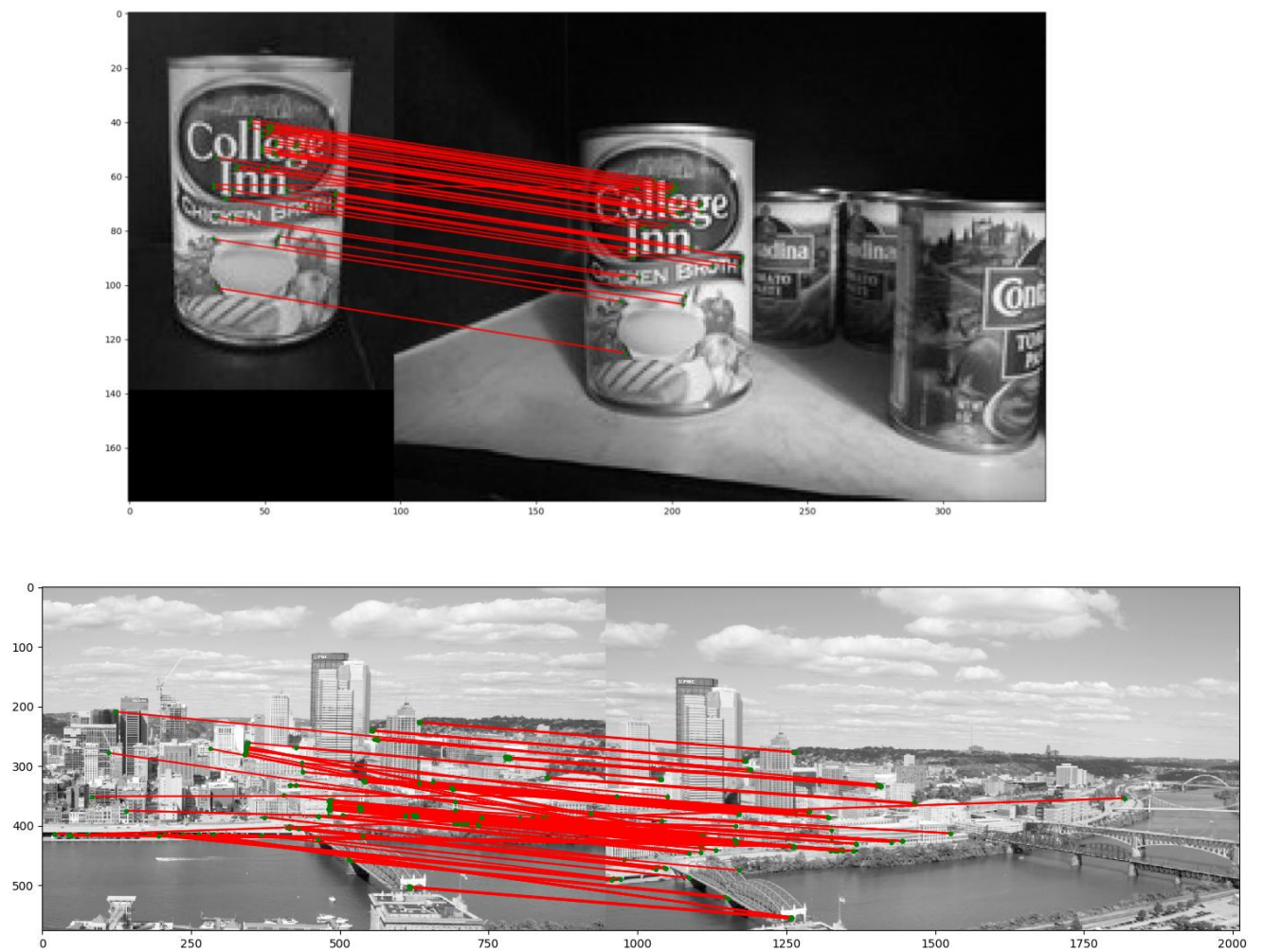
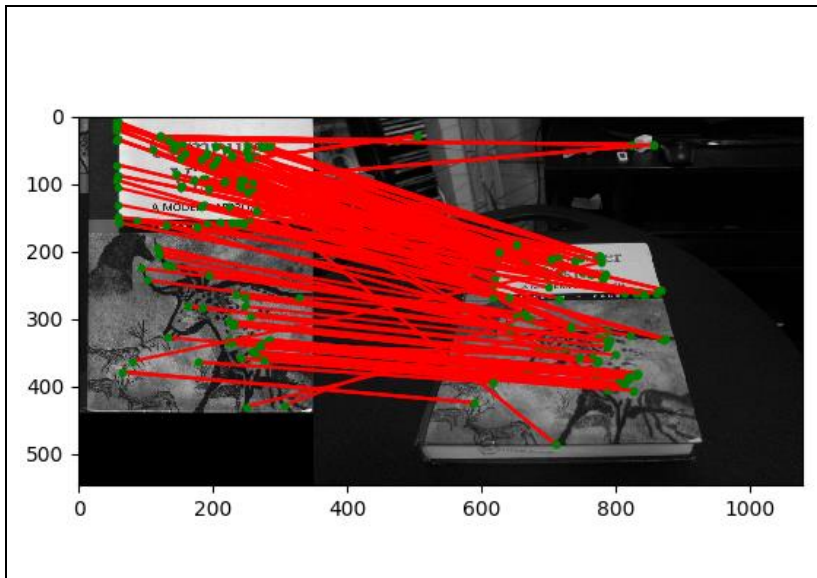


Q1.5

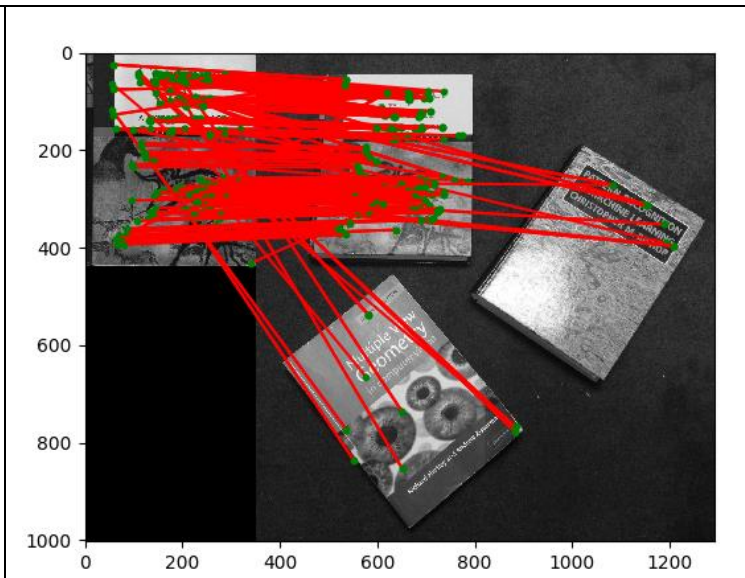


## Q2.4 (Match Points)

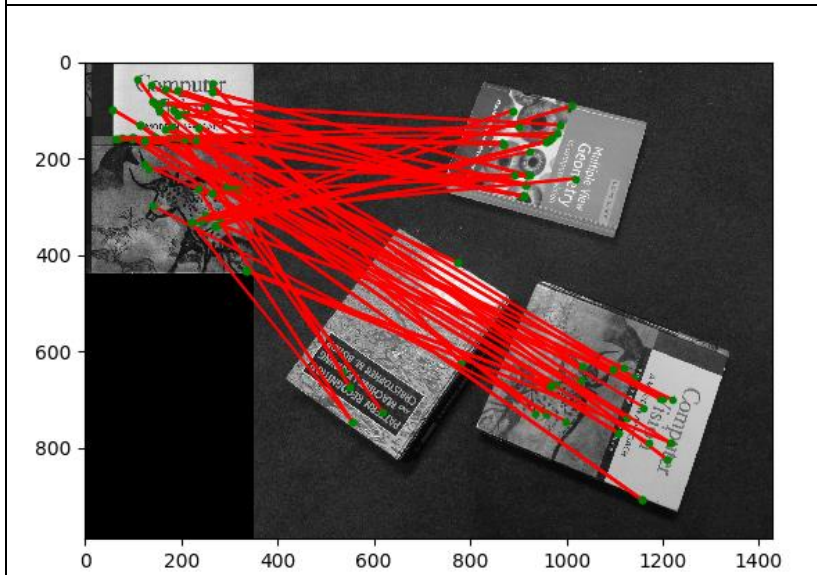




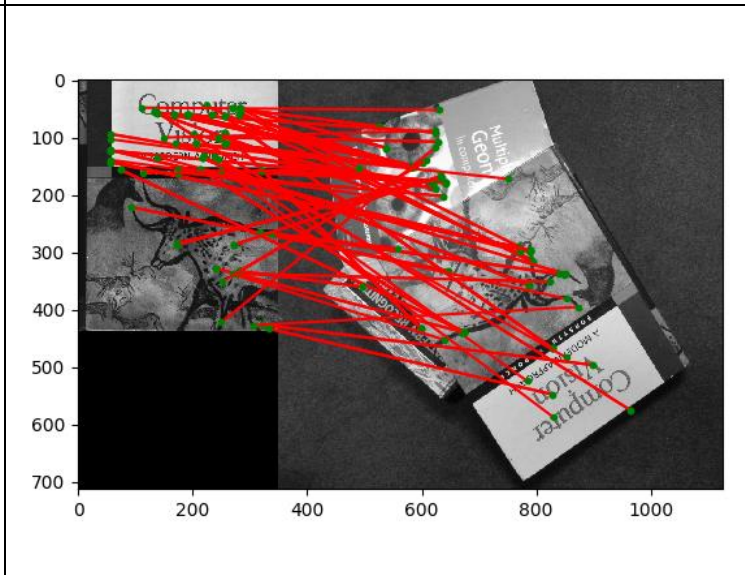
Desk



Floor



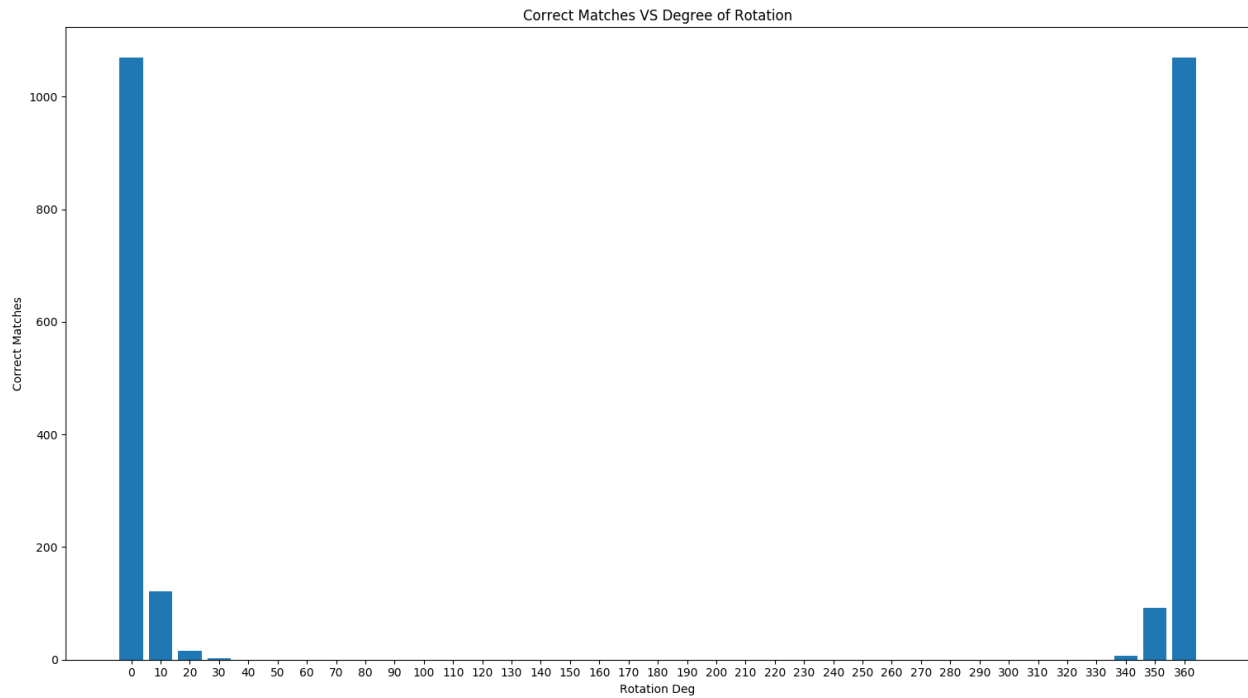
Floor Rotated



Pile

Based on the above images, it can be seen that whenever the orientation of the target Computer Vision book is the same as the reference image on the left, the matching process performs much better than when the book is rotated. Also, as we increase the number of the books in the image, the number of mismatches increases which can be understood by comparing image of the Desk and the Floor with similar book orientation of our target. Also the light condition of the image can affect the results and causes mismatches such as in Floor image. Also the Pile image indicates that complex background casues condusion and can make the matching process more difficult.

## Q 2.5



Based on the above graph it can be seen that by rotating the second image, the number of correct matches decreases. This is because of the fact that the Brief Descriptor is calculated using same (x,y) pairs in the 9\*9 patch. This means that by rotating the images, the derived key points are going to be the same but the descriptor is going to use the same information in the CompaeX and CompareY vectors that results in selection of different pixels for calculating the Descriptor. Thus, the calculated descriptors are not oing to be the same when the image is rotated and less similarities and correct matches are derived.

### 3. Planar Homographies

a)

$$\lambda \tilde{x} = H \tilde{u}$$

$$\lambda \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$x_2 = \frac{H_{11}u_1 + H_{12}v_1 + H_{13}}{H_{31}u_1 + H_{32}v_1 + H_{33}}$$

$$y_2 = \frac{H_{21}u_1 + H_{22}v_1 + H_{23}}{H_{31}u_1 + H_{32}v_1 + H_{33}}$$

$$x_2(H_{31}u_1 + H_{32}v_1 + H_{33}) = H_{11}u_1 + H_{12}v_1 + H_{13}$$

$$y_2(H_{31}u_1 + H_{32}v_1 + H_{33}) = H_{21}u_1 + H_{22}v_1 + H_{23}$$

$$\begin{bmatrix} -u_1 & -v_1 & -1 & 0 & 0 & 0 & u_1x_2 & v_1x_2 & x_2 \\ 0 & 0 & 0 & -u_1 & -v_1 & -1 & u_1y_2 & v_1y_2 & y_2 \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

We need 8 points(4 correspondence points) to solve this equation.

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} \begin{bmatrix} u_3 \\ v_3 \end{bmatrix} \begin{bmatrix} u_4 \\ v_4 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} \begin{bmatrix} x_4 \\ y_4 \end{bmatrix}$$

$$\begin{bmatrix} -u_1 & -v_1 & -1 & 0 & 0 & 0 & x_1u_1 & x_1v_1 & x_1 \\ 0 & 0 & 0 & -u_1 & -v_1 & -1 & y_1u_1 & y_1v_1 & y_1 \\ -u_2 & -v_2 & -1 & 0 & 0 & 0 & x_2u_2 & x_2v_2 & x_2 \\ 0 & 0 & 0 & -u_2 & -v_2 & -1 & y_2u_2 & y_2v_2 & y_2 \\ -u_3 & -v_3 & -1 & 0 & 0 & 0 & x_3u_3 & x_3v_3 & x_3 \\ 0 & 0 & 0 & -u_3 & -v_3 & -1 & y_3u_3 & y_3v_3 & y_3 \\ -u_4 & -v_4 & -1 & 0 & 0 & 0 & x_4u_4 & x_4v_4 & x_4 \\ 0 & 0 & 0 & -u_4 & -v_4 & -1 & y_4u_4 & y_4v_4 & y_4 \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow Ah = 0$$

b) A is  $8 \times 9$  matrix and h is  $9 \times 1$ . There are 9 elements in matrix h.

c) 4 pairs is required to solve this system. Although h has 9 elements but it has 8 degrees of freedom because of the property of homogenous coordinates. Each point correspondence can be used to derive 2 degrees of freedom of h matrix so we will need 4 pairs to solve the 8 freedom degree system.

d) In order to derive the non-trivial solution of  $h=0$ , a constraint is added.

$$\|h\| = 1$$

So our problem is going to be  $\min \|Ah\|_2$  subject to  $\|h\| = 1$

Based on Lagrange Multipliers:

$$L(h, \lambda) = \|Ah\|^2 + \lambda(1 - \|h\|^2) = h^T A^T A h + \lambda(1 - h^T h)$$

$$\frac{\delta L}{\delta h} = 0 \rightarrow 2A^T A h - 2\lambda h = 0 \rightarrow 2(A^T A - \lambda I)h = 0$$

Based on the above equation we can see that h is an Eigen vector of  $A^T A$  and  $\lambda$  is an eigenvalue.

Least square error:

$$A^T A h = \lambda h \rightarrow e = h^T A^T A h = h^T \lambda h \rightarrow \text{error is minimal for } \lambda = \min \lambda_i$$

The eigenvector corresponding to the smallest eigenvalue would be solution for h.



**Q 6.1**

**Second Image Warped**



**Panorama Clipped**



**Q 6.2**



**Q 6.3**

Final panorama view.

