Q1.1

Two dimensional tracking with a pure translation warp function:

$$W(x;p) = x + p\ ,\ x = \begin{bmatrix} X \\ Y \end{bmatrix},\ p = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \rightarrow\ W(x;p) = \begin{bmatrix} X + px \\ Y + py \end{bmatrix}$$

$$\frac{\partial W(x;p)}{\partial p^T} = \frac{\partial \begin{bmatrix} X+px \\ Y+py \end{bmatrix}}{\partial p^T} = \begin{bmatrix} \frac{\partial (X+px)}{\partial px} & \frac{\partial (X+px)}{\partial py} \\ \frac{\partial (Y+py)}{\partial px} & \frac{\partial (Y+py)}{\partial py} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E = \sum_{x \in N} \left(I_{t+1}(W(x;p) + \Delta p) - I_t(x) \right)^2$$

$$= \sum_{x \in N} (I_{t+1}\left(W(x;p)\right) + \frac{\partial I_{t+1}\left(W(x;p)\right)}{\partial W(x;p)^T} \frac{\partial W(x;p)}{\partial p^T} \Delta p - I_t(x))^2$$

$$= \sum_{x \in N} \left(\frac{\partial I_{t+1}\left(W(x;p)\right)}{\partial W(x;p)^T} \frac{\partial W(x;p)}{\partial p^T} \Delta p + I_{t+1}\left(W(x;p)\right) - I_t(x) \right)^2$$

$$= ||A\Delta p - b||^2$$

$$A = \frac{\partial I_{t+1}\left(W(x;p)\right)}{\partial W(x;p)^T} \frac{\partial W(x;p)}{\partial p^T}$$

$$b = -I_{t+1}\left(W(x;p)\right) + I_t(x)$$

$$\frac{\partial E}{\partial \Delta p} = 0\ \rightarrow\ A^T(A\Delta p - b) = 0\ \rightarrow A^T A \Delta p - A^T b = 0 \rightarrow\ \Delta p = (A^T A)^{-1} A^T b$$

In order to find a unique solution for Δp , the matrix $(A^T A)^{-1}$ should exist which means that $(A^T A)$ needs to have an inverse and its determinant needs not to be zero in other words $(A^T A)$ needs not to be singular.

Q1.3 Lucas-Kanade Tracking with One Single Template

Q1.4 Lucas-Kanade Tracking with Template Correction

Q2.1

$$I_{t+1}(x) = I_t(x) + \sum_{k=1}^{k} w_k B_k(x)$$

$$\min_{p,w} = \sum_x ||I_{t+1}(x+p) - I_t(x) - \sum_{k=1}^{k} w_k B_k(x)||_2^2$$

Treating the image as vector and rewriting the equation:

$$\sum_x [I_{t+1}(x+p) - I_t(x) - \sum_{k=1}^{k} w_k B_k(x)]^2$$

$$= \left\| I_{t+1}(x+p) - I_t(x) - \sum_{k=1}^{k} w_k B_k(x) \right\|^2$$

We can write the above equation in the linear subspace spanned by a collection of vectors B_k by span(B_k) and its orthogonal complement by span($B_k{}^{\perp}$

$$\left\| I_{t+1}(x+p) - I_t(x) - \sum_{k=1}^{k} w_k B_k(x) \right\|_{span(B_k)}^2$$

$$+ \left\| I_{t+1}(x+p) - I_t(x) - \sum_{k=1}^{k} w_k B_k(x) \right\|_{span(B_k)^{\perp}}^2$$

The equation above can be simplified since the norm in the second term only considers the orthogonal components of span (B_k):

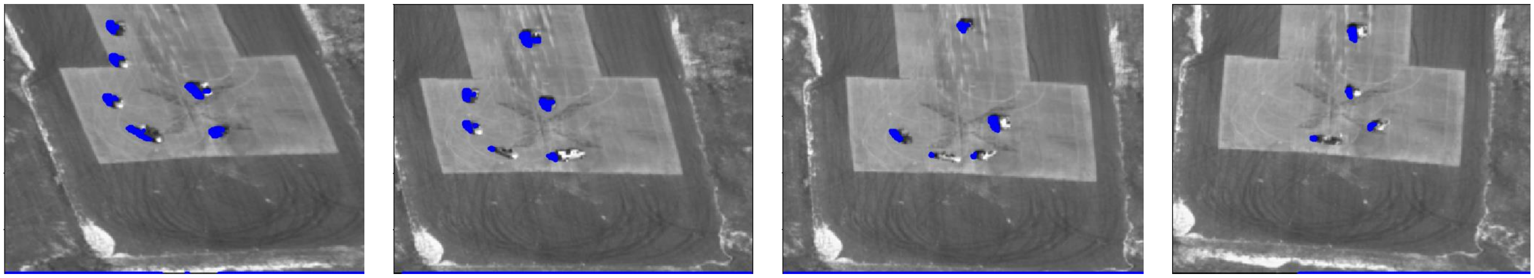
$$\left\| I_{t+1}(x+p) - I_t(x) - \sum_{k=1}^{k} w_k B_k(x) \right\|_{span(B_k)}^2 + \|I_{t+1}(x+p) - I_t(x)\|_{span(B_k)^{\perp}}^2$$

The second term is not dependent on w_k so to minimize the equation we can first minimize the second term with respect to p and then substitute the derived p's in term one and calculate for w_k. Thus by minimization of the first term we can derive the w_k based on the below equation: (This is based on the assumption that the appearance variation vectors B_k are orthogonal)

$$w_k = \sum_x B_k(x).\,[\,I_{t+1}(x+p) - I_t(x)]$$

Q2.3 Lucas-Kanade Tracking with Appearance Basis

Q3.3 Lucas-Kanade Tracking with Motion Detection



Q4.1

We can precompute the Heisian Matrix and reuse it in every iteration and avoid reevaluating the Heissian in every iteration.