Q1.1

Considering P_1 as a 2D point on the left image and P_2 on the right image we have:

$$P_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ P_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},F = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}$$

$$P_1^T F P_2 = 0 \rightarrow [0 \quad 0 \quad 1] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \rightarrow [F_{31} \quad F_{32} \quad F_{33}] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \rightarrow F_{33} = 0$$

Q1.2

We only have pure translation parallel to x axis so $t_3$and t_2 are zero and cross product matrix of translation vector can be computed as below:

$$t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} \rightarrow t = \begin{bmatrix} t_1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \bar{T} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$$

Since we have pure translation the rotation matrix would be identity.

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The essential Matrix could be calculated as below:

$$E = \bar{T}R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$$

If $p1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ is a point on left image and $p2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ is a 2d point on the right image, then the epipolar line on the right image can be calculated as below:

$$l_2 = E \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_1 \\ t_1 y_1 \end{bmatrix}$$

$$[x_2 \quad y_2 \quad 1] \begin{bmatrix} 0 \\ -t_1 \\ t_1 y_1 \end{bmatrix} = 0 \rightarrow t_1 y_1 - t_1 y_2 = 0$$

The calculation for the epipolar line on the left image would be the same. Based on the calculated line above, we can see that the epipolar lines would be parallel to x axis

Q1.3

If we consider $p_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$ as a 2d point on the image and $P = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ as its 3D corresponding point in the world coordinate, we have:

$$p = K[R|t]P \rightarrow \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = K[R_i|t_i]\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Considering $p_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ on image one and $p_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ on image two we have:

$$1)\ \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K[R_1|t_1]\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$2)\ \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = K[R_2|t_2]\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = K[R_2|t_2]\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = KR_2\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + Kt_2 \rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = (KR_2)^{-1}(\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - Kt_2)$$

Substituting $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$ in equation 1.

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = KR_1\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + Kt_1 = KR_1\left[(KR_2)^{-1}\left(\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - Kt_2\right)\right] + Kt_1 \rightarrow$$

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = KR_1R_2^{-1}K^{-1}\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - KR_1R_2^{-1}K^{-1}Kt_2 + Kt_1 = KR_1R_2^{-1}K^{-1}\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - KR_1R_2^{-1}t_2 + Kt_1$$

$$\rightarrow t_{rel} = -KR_1R_2^{-1}t_2 + Kt_1$$

$$\rightarrow R_{rel} = KR_1R_2^{-1}K^{-1}$$

Essential and Fundamental Matrix are calculated as below. $\bar{t}_{rel}$ is the cross product matrix of t_{rel}

$$E = \bar{t}_{rel}R_{rel}$$

$$F = K^{-1}EK = K^{-1}\bar{t}_{rel}R_{rel}K$$

Q1.4

We consider the real world coordinates of the image as P and P' as its reflection.

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ P' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

P and P' are just different by pure translation. By having rotation matrix R and the translation matrix as T we have:

1) $P' = RP + T$

If we consider K as intrinsic matrix for the camera, the corresponding points on the images would be:

2) $p = KP\ ,\ p' = KP'$

Substituting equations 2 in equation 1 :

$$K^{-1}p' = RK^{-1}p + T$$

Having $\overline{T}$ as the cross product with T

$$K^{-1}\overline{T}p' = RK^{-1}\overline{T}p + T\overline{T}$$

Since the angle between them is zero we have $T\overline{T} = 0$ so the equation above would be:

$$K^{-1}\overline{T}p' = RK^{-1}\overline{T}p$$

By dot producting $K^{-1}p'$ on both side we would have:

$$(K^{-1}p')^T K^{-1}\overline{T}p' = \ (K^{-1}p')^T RK^{-1}\overline{T}p$$

$$K^{-T}p'^T K^{-1}\overline{T}p' = \ K^{-T}p'^T RK^{-1}\overline{T}p$$

$$K^{-T}p'^T RK^{-1}\overline{T}p = 0$$

$$p'^T K^{-T}RK^{-1}\overline{T}p = 0$$

$$p'^T(K^{-T}RK^{-1}\overline{T})p = 0$$

$$p'^T Fp = 0$$

There exists a pure translation so the rotation matrix R is identity.

$$F = K^{-T}RK^{-1}\overline{T}$$

$$F = K^{-T}K^{-1}\overline{T}$$

The intrinsic matrix K would not affect the skew symmetricity of Fundamental Matrix F so we only consider $\overline{T}$. If the translation matrix is $T = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$

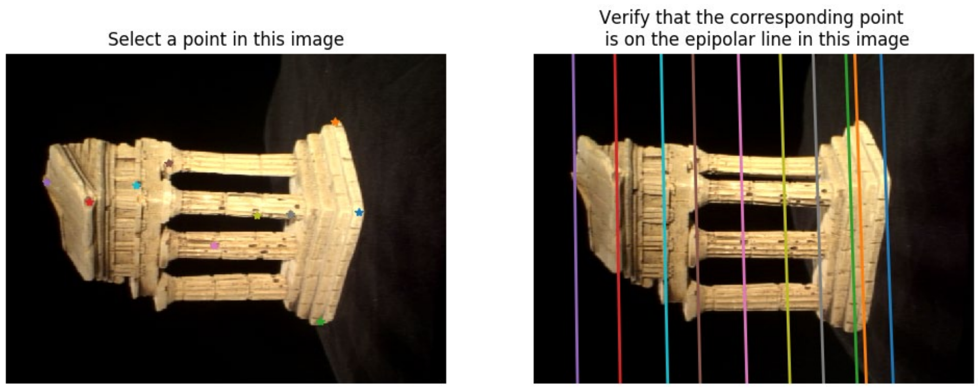
$$\overline{T} = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} \rightarrow -\overline{T}^T = \overline{T}$$

The above equation indicated that $\overline{T}$ is a skew symmetric matrix which based on the calculated equation for F, it would result in F to be also a skew symmetric matrix.

Q2.1

Fundamental Matrix calculated using the eight-point algorithm.

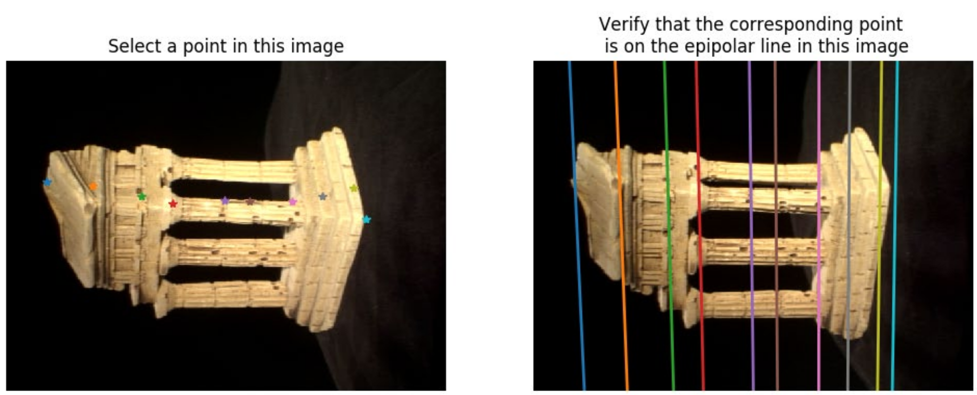
$$F = \begin{bmatrix} 9.7883e-10 & -1.3213e-07 & 1.1258e-03 \\ -5.7384e-08 & 2.9680e-09 & -1.1761e-05 \\ -1.0826e-03 & 3.0484e-05 & -4.4703e-03 \end{bmatrix}$$



Q2.2

Calculated Fundamental Matrix using the seven-point algorithm. In the code in order to choose appropriate seven points for calculation of Fundamental matrix, Ransac algorithm has been implemented in the function named “BastFSevenPoints” which also calculates the best F out of calculated Farray list.

$$F = \begin{bmatrix} -2.7126e-08 & -1.9599e-07 & 7.9684e-04 \\ 9.8196e-08 & 4.7882e-09 & -4.1126e-05 \\ -7.5952e-04 & 4.2409e-05 & -2.4471e-03 \end{bmatrix}$$



Q3.1

The Essential Matrix calculated by using the Fundamental Matrix from the eight-point algorithm.

$$E = \begin{bmatrix} 2.26268e-03 & -3.0655e-01 & 1.6626e+00 \\ -1.3313e-01 & 6.9106e-03 & -4.3300e-02 \\ -1.6672e+00 & -1.3321e-0.2 & -6.7218e-04 \end{bmatrix}$$

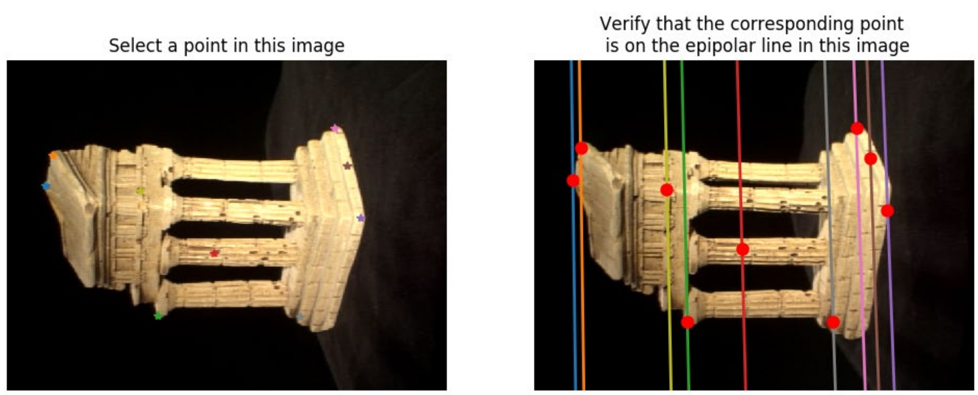
Q3.2

Matrix A used for triangulation of set of 2D coordinates in the images to a set of 3D points.

$$A = \begin{bmatrix} x_1 C1_3 - C1_1 \\ y_1 C1_3 - C1_2 \\ x_2 C2_3 - C2_1 \\ y_2 C2_3 - C2_2 \end{bmatrix}$$

Q4.1

Corresponding points in two images,



Q4.2

3D reconstruction plots