Phase retrieval in computer generation of schlieren images

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ABSTRACT

The schlieren technique involves the manipulation of knife edges to limit the field of view in the image formation process. Practical implementation of this technique is not easy due to difficulties in positioning the knife edge in the optical system. A related problem concerns the reproducibility of an event to generate a series of schlieren images for different knife edge positions. A particularly successful method to overcome this problem is the use of the computer to generate such images from single pictures of the event. Computer generation of schlieren images involves the inverse Fourier transformation of the modified complex-valued diffraction pattern (magnitude and phase) of the event. Recording media in general respond only to light intensity and no difficulty is encountered in recording the intensity, and therefore the magnitude. The phase is either unobservable directly or cannot be determined anywhere nearly as accurately as the intensity. The Gerchberg and Saxton iterative algorithm is used to recover the phase from records of intensity (magnitude) taken from the image and Fourier domains of the optical system. The knowledge of magnitude and phase in the Fourier domain (diffraction pattern) will enable us to modify it through a computer knife edge and generate the corresponding schlieren images.

Keywords: schlieren technique; Fourier transform; phase retrieval; iterative algorithm.

1. INTRODUCTION

Many problems of science and engineering involve substances that are transparent and non luminous, so that their observation by direct visual or photographic methods is difficult. The phenomena that are of interest frequently involve changes of the refractive index across the field to be investigated, which can then be visualized by using optical techniques that depend on the effects of the refractive index change on the transmission of the light. Several techniques are available but we are specially concerned with the schlieren technique.

The schlieren technique depends on the deflection of a ray of light from its undisturbed path when it passes through a medium in which there is a component of the gradient of refractive index normal to the ray. It may be shown[1] that the deviation angle of the ray is proportional to the refractive index gradient in the direction normal to the ray.

A typical schlieren apparatus is sketched in Fig. 1. It uses a HeNe laser, a spatial filter SP and a lens L1 to produce a plane parallel beam of light. This beam is used to illuminate an object disturbance placed in the object domain (OD). The object will deflect some of the incident light due to refractive index changes. All the light passing through and deviated by the object is collected by the lens L2 which has a large aperture. The light intensity changes are imaged in the image domain (ID). The Fourier domain (FD) created by the lens L2 contains the diffraction pattern of the object.

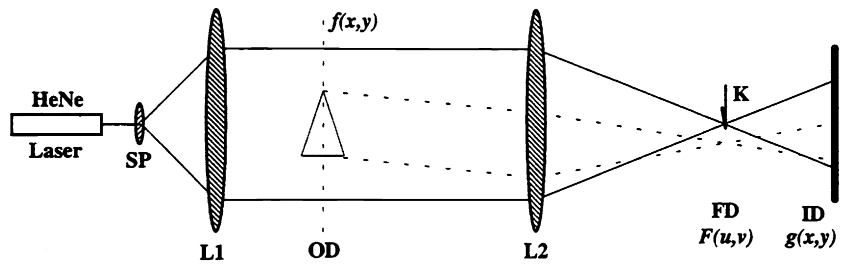


Fig.1 - The arrangement of a typical schlieren apparatus.

The knife edge K can be manipulated to limit the field of view in the image formation process. In this way, a set of schlieren images can be obtained in which the gradual intensity change of the light corresponds to the change in the angle of the deviated light in the object disturbance.

In order to quantitatively characterize an object disturbance the position of the knife edge is extremely important. Since the light can be deviated in any direction, the evaluation of the deviation angle in a specific direction should be performed with the knife edge positioned perpendicularly to that direction. This analysis is not always easy and a large set of schlieren images is needed.

An alternative to the knife edge is the use of a circular stop where the deviated light in all the directions has a contribution for the schlieren image. Now, the problem concerns the size of the circular stop instead of its position. If the object disturbance is stable in time and space there is no problem to generate such schlieren images. But, if the object disturbance is not stable the reproducibility of such an event will be impossible.

A particularly successful method to overcome these problems is the use of the computer and the Gerchberg and Saxton algorithm[2] (GSA) to generate such images from single intensity records of the event. This method with simulated and real results is presented and discussed in the following sections.

2. PRELIMINARIES

From Fig. 1 we can define the optical wave leaving the object disturbance as a complex-valued function, $f(x,y)$, denoted by

$$f(x,y) = |f(x,y)|\exp[j\theta(x,y)], \tag{1}$$

where (x,y) is the two-dimensional pair of spatial co-ordinates, $|f(x,y)|$ is the magnitude and $\theta(x,y)$ is the phase in the OD. Since OD and ID are conjugate domains, we can define for the ID a wave function, $g(x,y)$, denoted by

$$g(x,y) = f(x,y)^* = |f(x,y)|\exp[-j\theta(x,y)], \tag{2}$$

where $f(x,y)^*$ represents the complex conjugate of Eq. 1. The object wave function, $f(x,y)$, is related to its direct Fourier transform (DFT), $F(u,v)$, by

$$F(u,v) = \iint\limits_{-\infty}^{+\infty} f(x,y) \exp[-j2\pi(xu+yv)]\, dxdy, \tag{3}$$

where (u,v) is the two-dimensional pair of spatial frequency co-ordinates. $F(u,v)$ is always a complex-valued function, called the Fraunhofer diffraction pattern, and is denoted by

$$F(u,v) = |F(u,v)| \exp[j\psi(u,v)], \tag{4}$$

where $|F(u,v)|$ is the spectral magnitude and $\psi(x,y)$ is the spectral phase in the FD. On the other hand, the diffraction wave function, $F(u,v)$, is related to the object wave function, $f(x,y)$, by its inverse Fourier transform (IFT) as

$$f(x,y) = \iint\limits_{-\infty}^{+\infty} F(u,v) \exp[j2\pi(xu+yv)]\, dudv. \tag{5}$$

The complete definition of these wave functions implies the knowledge of both magnitude and phase in the considered domains. In general, recording media respond only to light intensities and no difficulty is encountered in recording the intensity, and therefore the magnitude functions can be calculated because they are the square root of the corresponding intensity functions. The phase functions are either unobservable directly or cannot be determined anywhere nearly as accurately as the intensity functions.

In practice; object, image and Fourier intensity functions can be discretized into a sequence of values by taking *N*x*N* equally spaced samples. The importance of this discretization lies in the fact that it allows the use of the computer for information storage and for the development of practical numerical algorithms to process the stored data. The discrete Fourier transform pair that applies to sampled functions[3,4] is given by

$$F(u,v) = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} |f(x,y)| \exp\left[-j\left(\frac{2\pi}{N}(xu+yv) - \theta(x,y)\right)\right], \tag{6}$$

for $u = 0,1,2,\ldots,(N-1)$; $v = 0,1,2,\ldots,(N-1)$, and

$$f(x,y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} |F(u,v)| \exp\left[j\left(\frac{2\pi}{N}(xu+yv) + \psi(u,v)\right)\right], \tag{7}$$

for $x = 0,1,2,\ldots,(N-1)$; $y = 0,1,2,\ldots,(N-1)$, where $|f(x,y)|$ and $|F(u,v)|$ are the magnitude functions calculated by taking the square root of the corresponding sampled intensity and the functions $\theta(x,y)$ and

$\psi(u,v)$ are the unknown phase functions, that contain most of the information about the object disturbance, in both the ID and FD.

We wish to recover the unknown phase function, $\theta(x,y)$, and equivalently recover the unknown spectral phase function, $\psi(u,v)$ in order to completely define both the complex-valued functions associated with an object disturbance in the ID and FD. This problem can be solved using the method described in the next section, which uses the GSA to retrieve the phase distribution from single intensity records taken in both the ID and FD.

3. PRINCIPLE OF THE METHOD

An experimental arrangement of a typical schlieren apparatus, as shown in Fig. 1, has been set up in the laboratory to apply the proposed method for computer generation of schlieren images. Fig. 2 depicts the modifications that have been done to allow the acquisition and recording of light intensity information on the computer.

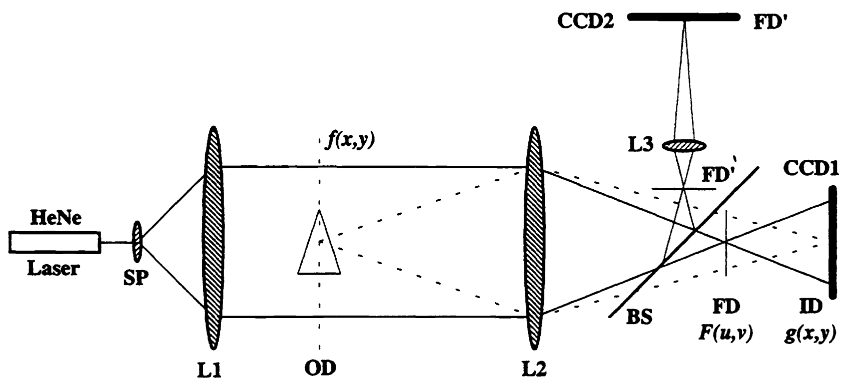


Fig. 2 - Experimental arrangement for recording the wave intensity functions in both the image and Fourier domains.

Basically, a charge coupled device (CCD1) was placed in the ID for acquisition and recording purposes. The FD created by the lens L2 was mirrored to FD' through the beam-splitter BS and the light intensity on it is imaged in another charge couple device (CCD2) using the microscope objective lens L3. Both the CCD sensors are connected to a frame grabber, which is installed in a computer, to record the desired sampled intensity functions for both the ID and FD. An application computer program was written to control the acquisition and recording process. It also performs all the necessary calculations on the recorded data in order to recover the phase information in one of the selected domains through the GSA.

The algorithm is an iterative procedure involving discrete Fourier transformation back and forth between the two domains and application of the recorded sampled intensities (magnitudes) in each domain. The DFT and IFT are carried out by means of the fast Fourier transform algorithm[5] on the computer.

The operation of the algorithm is schematically described on the flow chart shown in Fig 3. The input data to the algorithm are the magnitudes calculated from the sampled intensity in both ID and FD. These two sets of data are accessed once per complete iteration. To begin, a uniform random number generator is used to generate an array of numbers between $-\pi$ and π, which serves as the initial estimate of the phase in the FD. This array is then combined with the corresponding Fourier magnitude data and the IFT of this synthesized complex function is evaluated. The phase of the resulting complex image function is calculated and combined with the corresponding image magnitude data. This new function is then transformed by DFT. The phase of the resulting complex Fourier function is computed and combined with the Fourier magnitude data to form a new estimate. This process should be repeated in order to obtain a better estimate for the complex diffraction pattern function. The convergence of the algorithm is monitored by computing the normalized root mean squared error (NRMSE) in each iteration.

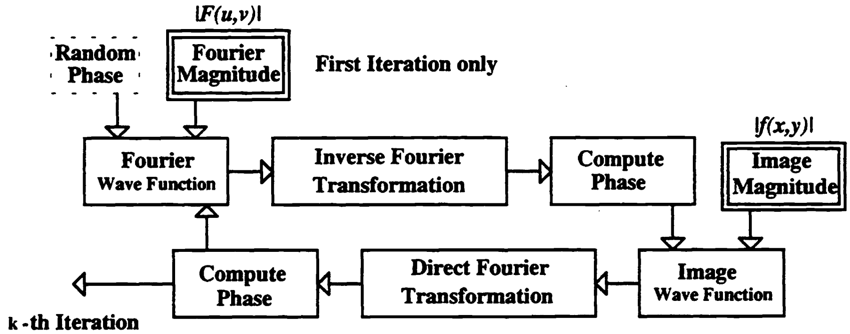


Fig. 3 - The Gerchberg and Saxton algorithm.

After the k-th iteration, the GSA gives an estimate of the diffraction pattern and the corresponding phase $\psi(u,v)$ for the FD, or by inversion of the transformation the phase $\theta(x,y)$ for the ID can be calculated. The recovered phase changes for the ID are related to light deflection[6] due to refractive index changes introduced by an object in the field, and measurements of the refractive index changes can be made. The corresponding phase changes in the FD together with the associated magnitude enables us to generate on the computer a discrete complex function that represents the diffraction pattern of the object. The manipulation of a computer knife edge to set to zero some of the frequency components on this function and by inverting the transformation results in the generation of an schlieren image of such object.

The application of this method follows these simple steps:

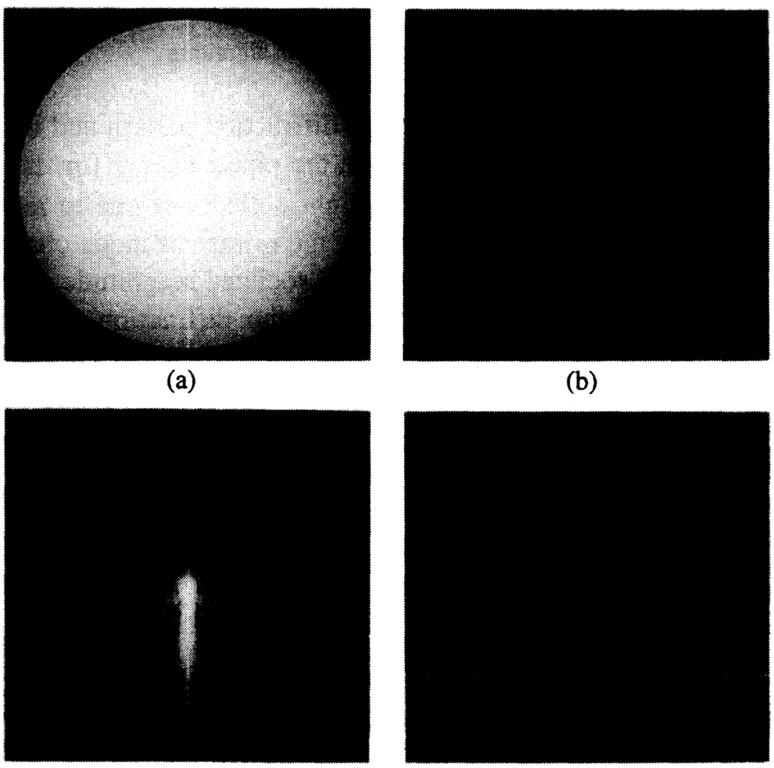
- Record the sampled intensity functions in the ID, $I_f(x,y)$, and in the FD, $I_F(u,v)$;
- Compute the corresponding magnitude functions as $|f(x,y)| = \sqrt{I_f(x,y)}$, and $|F(u,v)| = \sqrt{I_F(u,v)}$ respectively;
- Combining $|F(u,v)|$ with a random phase function, $\psi_0(u,v)$, uniformly distributed between π and $-\pi$, to generate an estimate of the complex diffraction pattern function in the FD;
- Use the GSA to obtain after the k-th iteration an estimate for the Fourier phase, $\psi_k(u,v)$;
- Recombining $|F(u,v)|$ with $\psi_k(u,v)$ and by inversion of the Fourier transformation we can get $|f(x,y)|$ and $\theta_k(x,y)$ in the ID;
- Now, from $\theta_k(x,y)$ one can get the refractive index changes introduced by the object disturbance;
- And from $(|F(u,v)|, \psi_k(u,v))$ we can generate schlieren images on the computer.

In the next sections, this method will be used to recover the phase function from a computer simulated object and to recover the phase for the generation of a schlieren image associated with a real object disturbance.

4. COMPUTER SIMULATION RESULTS

In this section the simulation will try to be as close as possible to the real experiment, presented in the next section, where a thin soap film will be used as an object to produce the complex wave functions. The inputs to the algorithm are two arrays of 512x512 pixel elements simulating the magnitude functions for both the ID and FD. The information stored in these arrays are normalized real-valued functions in the interval [0.0,1.0]. The minimum value 0.0 will appear on the displayed picture as a black level and the maximum value 1.0 as a white level.

Fig. 4(a) shows the simulated magnitude function, $|f(x,y)|$, which is a centred circular aperture filled with a truncated Gaussian distribution of amplitudes with vertical profile shown in Fig. 4(b). The simulated magnitude of the diffraction pattern function, $|F(u,v)|$, is shown in Fig. 4(c) with the corresponding vertical profile in Fig. 4(d).



(c) (d)

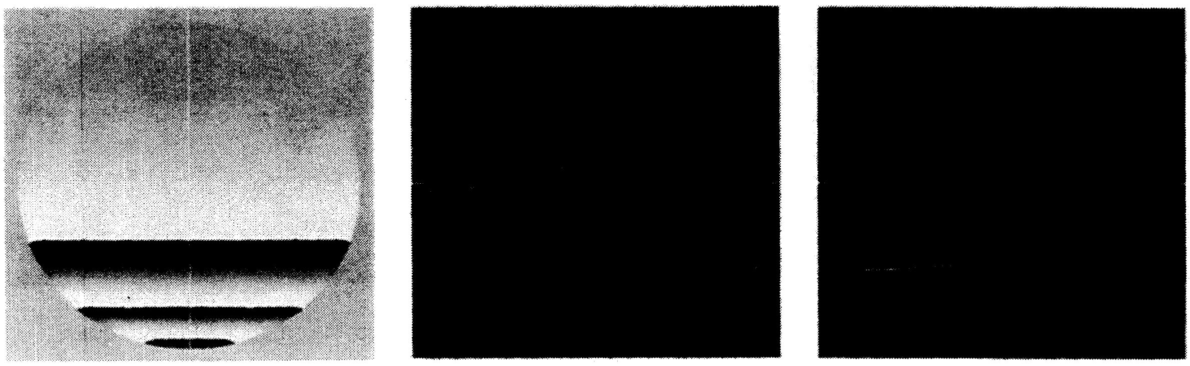
Fig. 4 - Simulated object magnitude functions. (a)- normalized ID magnitude; (b)- profile marked in (a); (c)- normalized FD magnitude; (d)- profile marked in (c).

A set of procedures has been executed to retrieve the phase distribution in the ID for the simulated object wave front. This set was composed by repeating a 300x iteration process several times feeding the algorithm with the stored magnitude information.

The results obtained were all consistent with the original phase function in that it was an equivalent circular aperture filled with an exponential distribution defined in the interval $[-3\pi, 3\pi]$ along the vertical direction. Fig. 5(a) depicts the recovered phase distribution when the NRMSE was 3.35×10^{-4}.

It must be mentioned that the recovered phase is not the absolute phase but the relative phase distribution. Because of the inverse tangent operation on the computer the phase distribution is wrapped into the range $-\pi$ (black level) and π (white level) and 2π jumps occur for variations of more than 2π. The absolute phase change can be obtained by processing of the relative phase information with unwrapping algorithms[7].

The vertical profiles for the relative or wrapped phase and for the absolute or unwrapped phase are shown in Fig. 5(b) and (c), respectively.



(a) (b) (c)

Fig. 5 - Recovered simulated object phase function. (a)- relative phase distribution; (b)- profile marked in (a); (c)- unwrapped profile marked in (a).

The convergence of the algorithm is guaranteed when magnitude information is used. The success of the phase reconstruction algorithm is more related to the accuracy and resolution of the available information rather than to the number of iterations.

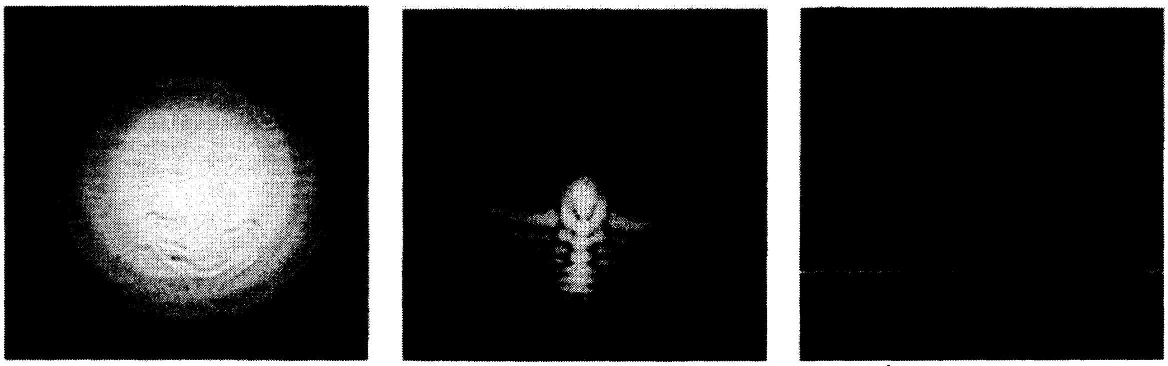
5. EXPERIMENTAL RESULTS

As pointed out in the previous section, we should measure and record the intensity functions as accurately as possible in order to obtain from the calculations the corresponding magnitude information without loss of information in its high frequency components, associated normally with low intensities of light.

By coincidence, the CCD sensors that we are using have the capability to perform electrically an operation called the *gamma correction*, where γ is the electrical analogue of the response of a photographic emulsion. A switch, that is available in each of the sensors, can be set to $\gamma = 1$ and a linear transformation between light intensity and output voltage is performed or it can be set to $\gamma = 0.45$ and therefore a non-linear transformation is performed. This transformation gives an increased output voltage for low intensities of light based on the intensity raised to the power of γ. Thus, the measuring and recording of low intensity information is possible.

Since the magnitude is the square root of the measured intensity or in other words, the magnitude is the measured intensity raised to the power of 0.5, we can state that: if the sensor CCD2 is set to perform the gamma correction for $\gamma = 0.45$ the information available on the output of this sensor is the *quasi* magnitude information of the diffraction pattern. In this way, we can use this capability to record the quasi magnitude of the diffraction pattern and then by processing the recorded data on the computer, correct it to obtain a more accurate measurement.

Fig. 6 depicts the recorded intensity functions taken from the ID and FD on the experimental arrangement when a thin soap film was used as an object. In particular, the real object intensity function taken from CCD1 is shown in Fig. 6(a). The sensor CCD1 was set to perform linear transformation.

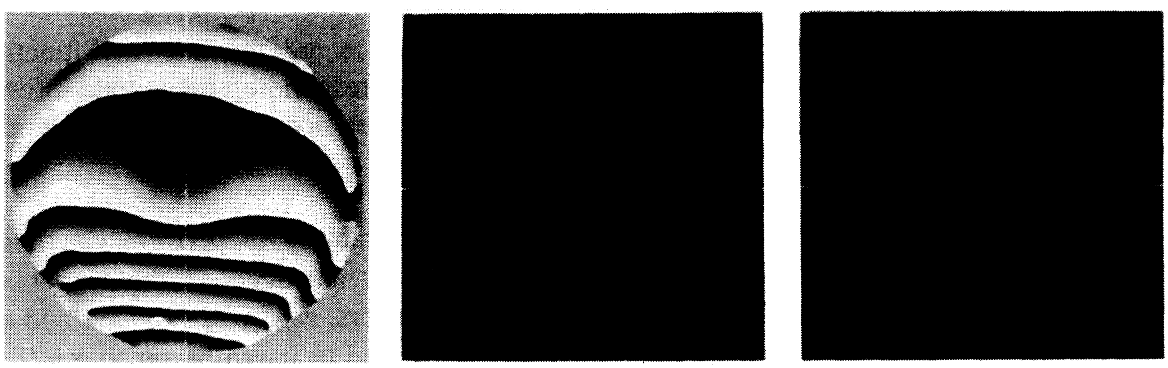


(a) (b) (c)

Fig. 6 - Real object intensity functions. (a)- normalized image intensity; (b)- normalized Fourier quasi magnitude; (c)- profile marked in (b).

The intensity function of the real diffraction pattern was taken from CCD2 when the sensor was set to perform gamma correction with $\gamma = 0.45$. Fig. 6(b) shows, for this particular case, the recorded quasi magnitude information and in Fig. 6(c) is shown the corresponding vertical profile.

A set of procedures to retrieve the phase distribution in the ID for this real object has been executed. This set was composed by repeating a 300x iteration process several times, feeding the algorithm with the magnitude data calculated from the recorded quasi magnitude information.



(a) (b) (c)

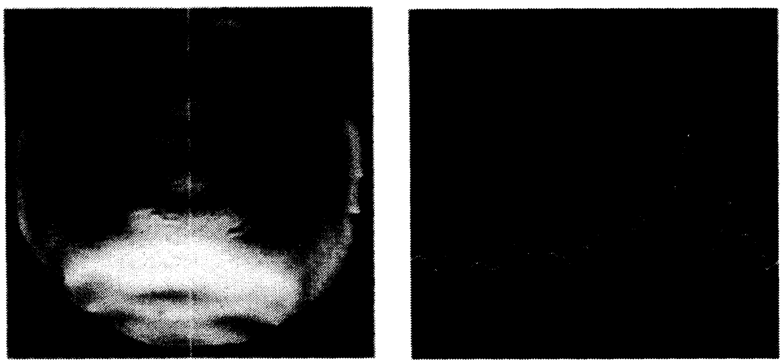
Fig. 7 - Recovered real object phase function. (a)- relative phase distribution; (b)- profile marked in (a); (c)- unwrapped profile marked in (a).

The results obtained were quite consistent and Fig. 7(a) shows one of the recovered phase distributions. The wrapped phase profile is shown in Fig. 7(b) and in Fig. 7(c) the corresponding unwrapped vertical profile. As we expect, the recovered phase has an exponential distribution along the vertical direction with a phase change of approximately 14π and the NRMSE was 3.85×10^{-1}.

We consider this cycle of phase retrieval a success due to the consistency of the retrieved phase distributions in all the iterative processes. The power of this algorithm to reconstruct the important phase information of a real unknown object is now evident.

Knowing both the magnitude and phase information of an object in the FD, it will be possible to recreate on the computer its schlieren image. This new technique for computer generated schlieren images is currently in use and involves the simple manipulation of a computer knife edge to remove specific components (magnitude and phase) in the FD before executing the IFT, which generates the so called schlieren image.

Fig. 8(a) shows the intensity distribution of the computer generated schlieren image when a circular stop was used to remove (set to zero) the low order frequency components in the complex diffraction pattern function of the real object (soap film) before the image formation process.



(a) (b)

Fig. 8 - Generated schlieren image. (a)- normalized image intensity distribution; (b)- profile marked in (a).

Fig. 8(b) shows one vertical profile, where the deviation of the light due to the refractive index gradient associated with the phase change, see Fig. 7(c), is clearly visible.

6. CONCLUSIONS

The Gerchberg and Saxton iterative algorithm has been used to retrieve the phase information, associated with an optical wave front, from intensity measurements made in the image and Fourier domains of an optical system. Simulated and experimental results show that special attention should be given to the Fourier domain, where vital information for the phase reconstruction process can easily be lost due to the low light intensities usually involved. The use of CCD sensors with the capability to perform gamma correction and hence record the quasi magnitude information enables accurate measurements of low light intensities. The retrieval of phase for real simple objects is possible and in turn it can be used to generate schlieren images on the computer screen. Measurements of refractive index changes in the object or image domain can also be made using the retrieved phase. Further research is being carried out with more complex phase objects disturbances.

7. ACKNOWLEDGEMENTS

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