Temperature and velocity measurement fields of fluids using a schlieren system

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This paper proposes a combined method for two-dimensional temperature and velocity measurements in liquid and gas flow using a schlieren system. Temperature measurements are made by relating the intensity level of each pixel in a schlieren image to the corresponding knife-edge position measured at the exit focal plane of the schlieren system. The same schlieren images were also used to measure the velocity of the fluid flow. The measurement is made by using particle image velocimetry (PIV). The PIV software used in this work analyzes motion between consecutive schlieren frames to obtain velocity fields. The proposed technique was applied to measure the temperature and velocity fields in the natural convection of water provoked by a heated rectangular plate. © 2012 Optical Society of America

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1. Introduction

The measurement of temperature is an important task in several areas of science. Full-field optical techniques, such as laser-induced fluorescence, holographic interferometry, laser speckle photography, schlieren, or a combination of these, are used for that purpose.

The schlieren technique has been widely used to quantify temperature in fluid flows. A number of well-known practical techniques, including color schlieren [1–3], background schlieren [4–5], and calibration schlieren [6–8], can be employed. The success of the schlieren technique to measure temperature is due to its relatively easy implementation, low cost, use of conventional light sources, and because of its high and variable sensitivity.

To visualize the path in a fluid flow for the unit of measurement, it is necessary to add tracers to the medium. The tracer material and its size needed depend on the characteristics of the fluid (density, viscosity), dimension of the investigation area, and the required resolution. The particle image velocimetry (PIV) method [9–12] is a contact-free optical capture and analysis of velocity arrays or pattern of intensity in fluids and in granular material. In contrast to the particle tracking velocimetry method where the track of a single particle is recorded, the premise of the PIV method is to record a group of particles that are affected by fluid flow. By analyzing the alteration of the position of a particle group within a defined interval, a value of the velocity is obtained. It provides an accurate, quantitative measurement of fluid velocity vectors at a very large number of points simultaneously and instantaneously in a planar cross section within the flow.

Today, the PIV technique is widely used and has differentiated into many distinct applications, from

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micro flows over combustion to supersonic flows for both industrial needs and research [12].

Some industrial applications require that instantaneous and simultaneous measurements of more than one variable are obtained, and in aerodynamic applications, temperature measurements alone are insufficient; thus, the schlieren-PIV is an appealing approach. The combined or simultaneous measurement of temperature and velocity is valuable for understanding fluid dynamics and for validating numerical models.

The measurement of velocity of a fluid flow using a schlieren system was first attempted by Townsend [13]. Since then, other authors have pursued the same aim for different flows and using different schlieren image analyses; for example, to measure the convective velocity of a supersonic shear layer [14], by using a special image analysis intended to extract velocity from a sequences of schlieren images [15], and to measure the velocity of a convective plume [16] and of a jet issuing from a circular nozzle [17]. Nonsimultaneous measurement of density and velocity of internal gravity waves is presented in [18]. This technique works by extracting quantitative information from alternate frames with a textured background for density measurements and uniform background for velocimetry applications [18]. More recently, the same authors have improved their approach in order to obtain simultaneous measurements of temperature and velocity [19–21]. However, in this approach, two cameras and two sources of light are necessary.

The objective of this work is to use the schlieren technique to simultaneously measure temperature and velocity of the natural convection of a heated water container. In contrast to the previous approaches, we use only one source of light and one camera for both the schlieren and PIV techniques, allowing for a true simultaneous measurement. The primary aim is not a comprehensive study of a flow, but rather to establish a procedure to obtain temperature and velocity in fluid flow. This kind of fluid flow was selected because of the simplicity of the problem. However, the procedure can be applied to different kinds of fluid flows and is the aim of further research on this subject.

2. Theoretical Development

A. Schlieren System Mathematical Background

It is well known that when a light ray passes through an inhomogeneous medium, it suffers a deviation in its trajectory by a certain angle [22–24]. This angle depends on the refractive index and thickness of the medium under test. The equation of the ray path through an inhomogeneous medium is expressed as

$$\frac{d}{ds}\left(n_m \frac{d\vec{r}}{ds}\right) = \nabla n_m, \tag{1}$$

where $n_m = n_m(x, y, z)$ is the refractive index of the medium, ds is arc length, and $\vec{r}$ is a position vector.

By considering the approximation of small angles of deviation, ε, the measured deviation at the observation plane (the exit focal plane) is $\delta x = f_2 \tan \varepsilon \approx f_2 \varepsilon$, where f_2 is the focal length of the focusing mirror in a two-mirror schlieren system (in this type of system, the first mirror serves to collimate the entering light beam). Then, Eq. (1) can be written as [22–23]

$$\varepsilon_\xi = \int_{\zeta_2}^{\zeta_1} \frac{\partial n_m}{\partial \xi} \mathrm{d}z, \tag{2}$$

where ξ can be x or y, depending on the direction in which the knife blocks out the light. In this work, the analysis is done for the x direction. Combining the Gladstone-Dale equation, $(n_m - 1) = K\rho$, where K is the Gladstone-Dale constant, and Eq. (2), we get the following equation:

$$\rho_x = \frac{\partial \rho}{\partial x} = \frac{\delta x}{f_2 h k}, \tag{3}$$

where ρ is the fluid density and h is the thickness of the inhomogeneous medium under test, in the ray propagation direction. The Gladstone-Dale constant K depends on the medium composition and the wavelength of the light beam. Once ρ has been determined from Eq. (3), it can be substituted into Eq. (4) to obtain the corresponding temperature by [25]

$$T = \frac{\rho_0}{\rho}, \qquad T_0 = \frac{n_0 - 1}{n_m - 1} T_0. \tag{4}$$

In Eq. (4), n_o and ρ_o are the refraction index and density, respectively, at reference temperature T_0, and T is the value of the temperature of interest.

1. *Method to Measure Temperature*

In a schlieren system (see Fig. 1), the blockage of light by a knife edge placed at the exit focal plane is due to the deviation of light by an inhomogeneous medium. This blockage can be similarly obtained in a homogeneous medium by a knife edge which is allowed to be translated laterally a quantity Δx [7–8]. By considering this, we can establish a relationship between the light levels at the exit focal plane to the corresponding transverse knife-edge position. This transverse position may cover the conditions from no cutoff of light (maximum intensity) to full cutoff of light (minimum intensity).

An algorithm for measuring temperature that takes into account the above-mentioned relationship is described next. First, a calibration procedure that relates knife position and intensity level is as follows: let $I^{\xi}_{(n,m)}$ be a schlieren image recorded at the observation plane with the condition of no flow, where

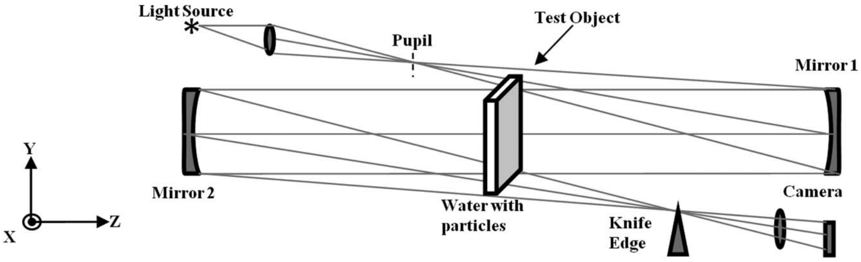

Fig. 1. Schematic of a parallel-light z-type schlieren system used for calibrated schlieren and particle image velocimetry.

$n = 0, 1, \ldots N$, and $m = 0, 1, \ldots M$, with N and M denoting the image pixel size; ξ indicates the x-knife position Δx at which the schlieren image was recorded (i.e., ξ can take on values from $-k\Delta x_0$ to $k\Delta x_0$ with $k = 0, 1, 2, \ldots, l$, with Δx_0 denoting the spacing between consecutive lateral displacements of the knife); $\xi = 0$ represents the condition when the knife edge is in its reference position, which in this analysis corresponds to a knife position such as the light intensity at the observation plane presents an intermediate intensity value between cutoff and no cutoff (i.e., when the intensity value is about 50% of the light observed for the condition of no-cutoff). We used this value because experimentally, it yielded the largest dynamic range for Δx holding a quasi-linear relationship with the schlieren intensities. In Fig. 2, we show a curve representing light intensity levels for different knife-edge positions for a single pixel (n, m). Notice that the curve is presented in terms of $(I^{\xi}_{(n,m)} - I^{0}_{(n,m)})$ versus Δx (i.e., the light intensity is expressed as a deviation from the value obtained when the knife is at its reference position). This is equivalent to making an image mathematical subtraction: the intensity of a schlieren image in presence of flow from a schlieren image with no flow or with undisturbed conditions. Curves relating light intensity to transverse knife position may be used for calibration purposes as described below. Because a calibration curve can be constructed for each pixel of a schlieren image, a total of $N \times M$ calibration curves may be necessary.

Now, let $I^{p}_{(n,m)}$ be a schlieren image recorded at the observation plane in presence of flow with the knife at the reference position. Subtracting both $I^{\xi}_{(n,m)}$ and $I^{p}_{(n,m)}$ from the image captured at $\xi = 0$ with the condition of no flow (i.e., $I^{0}_{(n,m)}$), we can write the following expressions:

$$I^{\Delta x}_{(n,m)} = I^{\xi}_{(n,m)} - I^{0}_{(n,m)}, \tag{5}$$

$$I^{\delta x}_{(n,m)} = I^{p}_{(n,m)} - I^{0}_{(n,m)}. \tag{6}$$

Here, $I^{\Delta x}_{(n,m)}$ gives the relationship between image intensity variations and corresponding knife position Δx defined in the calibration procedure, for each pixel (n, m). Conversely, the recorded intensity values in presence of flow, $I^{\delta x}_{(n,m)}$, are used to find the corresponding transverse knife position by comparing with the closest intensity values produced by Eq. (5) and represented in Fig. 2, for example, this process is done for each pixel. Thus, the corresponding calibration transverse knife position for pixel (n, m) of the schlieren image in presence of flow can be obtained by using the following criterion:

$$\delta x_{(n,m)} = \Delta x, \quad \text{where } \Delta x \text{ corresponds to the condition} \quad \min |I^{\delta x}_{(n,m)} - I^{\Delta x}_{(n,m)}|. \tag{7}$$

To find the minimum of this equation, we need to make reference to calibration curves of the type shown in Fig. 2 for each pixel. When the match between intensity values with flow and those from the calibration curve is not perfect, interpolation is applied.

When the transverse knife position Δx from the calibration curves has been assigned to its corresponding intensity value for all pixels of a schlieren image, in presence of flow, the ρ_x value can be determined via Eq. (3). The density value can be stated as [23]

$$\rho(x) = \rho_0 + \frac{1}{f_2 h K} \int_{\zeta_1}^{\zeta_2} \delta x \mathrm{d}x. \tag{8}$$

The trapezoidal numerical integration algorithm is used to integrate Eq. (8). Reference [8] includes a detailed description of the procedure used in this work to determine temperature measurements.

B. Velocity Fields

The same schlieren images used to calculate temperature field are utilized to compute velocity fields

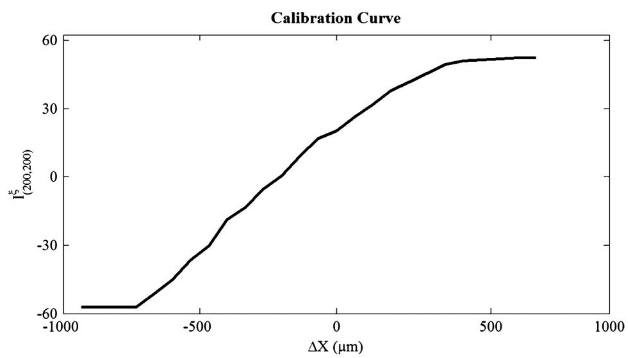

Fig. 2. Calibration curves of measured intensity across pixels (n, m).

of the fluid flow. We used commercial PIV software provided by IDT Inc. to analyze the schlieren images obtained in our experiments. The software includes several functions that facilitate the analysis of our schlieren—PIV images. The velocity is computed using a mesh-free, second-order accurate algorithm. The algorithm was designed to improve the accuracy and spatial resolution of the conventional cross-correlation schemes. The displacement between image pairs is found by means of cross correlation, and a velocity vector is assigned at the mid-distance between the image pairs. Therefore, each particle pair contributes to a second-order approximation of the velocity. However, in contrast to the traditional approach, which uses structured grids, these velocities are evaluated in an unstructured grid. The flow field at any point is described by an analytical function obtained using a least-squares-fitting algorithm.

3. Experimental Procedure

This experiment is aimed at measuring temperature and velocity fields of the convection of water caused by a hot rectangular metal plate (7.3 cm × 12 cm) located under a water container. The rectangular metal plate was set to temperature values of 30 °C, 40 °C, 50 °C, and 60 °C. These metal plate temperatures correspond to temperatures at the middle of the water container of 22 °C, 28 °C, 31 °C, and 34 °C.

A Z-type configuration schlieren system [22–23] is used in this research (see Fig. 1), and it includes two similar spherical mirrors of diameter $D = 0.15$ m, focal distance $f = 1.54$ m, and a knife edge that can be translated in the x or y direction by two-dimensional micrometer with a step size of 1.25 μm. The system also includes a 5 mm diameter white super-bright LED with luminous intensity of 2500 lm and dominant wavelength at 0.31 nm. In this work, for the analysis of the flow a glass tank with dimensions 71 mm × 27 mm × 2 mm were used. The water inside the container was seeded with 50 μm polystyrene latex particles used for fluid flow velocity calculations. The water container was positioned horizontally on an optical table. The stream-wise direction of the fluid flow is the y direction, and the knife edge is positioned parallel to this direction. The schlieren images are acquired with a CDD Lumenera LU175M camera, which provides a frame rate of 15 frames per second and a resolution of 1280 × 1024 pixels; these images were stored in bitmap file format (BMP) and digitized in 8-bit gray levels. The selected spatial and temporal resolutions of the camera allow us to capture the main characteristics of the dynamics of the fluid flow under study.

The first step of the experiment is to recording gray-level schlieren images with no flow for several transverse positions of the knife edge covering the conditions of no cutoff of light to full cutoff of light. These images are used for calibration purposes, as was explained in Subsection 2.A.1.

During the calibration process, the glass tank was filled with tridistilled water containing polystyrene particles. The seeding density was about 5–8 particles within each testing region of 24 pixels× 24 pixels. The tank did not add heat to the calibration images obtained. We acquired a total of 40 schlieren images for the four temperatures under analysis. All measurements were done at a room temperature of 20 °C. The same schlieren images were used for temperature and velocity flow field measurements.

4. Data Analysis

In order to show the feasibility of the technique, we determined the temperature and velocity fields of a convective fluid flow from experimental data as outlined in section 2. For each temperature under test, a total of 40 schlieren images were recorded.

A. Temperature and Velocity Measurements

The value of ρ_x can be determined using Eqs. (3) and (7); integration of this quantity is required to obtain the value of ρ as it is shown in Eq. (8). The resulting density values are substituted in Eq. (4) in order to obtain the temperature of interest.

In Fig. 3, a flow chart of the procedure is depicted. To obtain calibrations curves for each pixel, the knife is laterally displaced from −1.25 mm to 1.25 mm with a step size of $\Delta x = 62.5$ μm, and at each knife position, a schlieren image is recorded. Figure 3 shows some calibration images for different knife-edge positions. All the schlieren images are

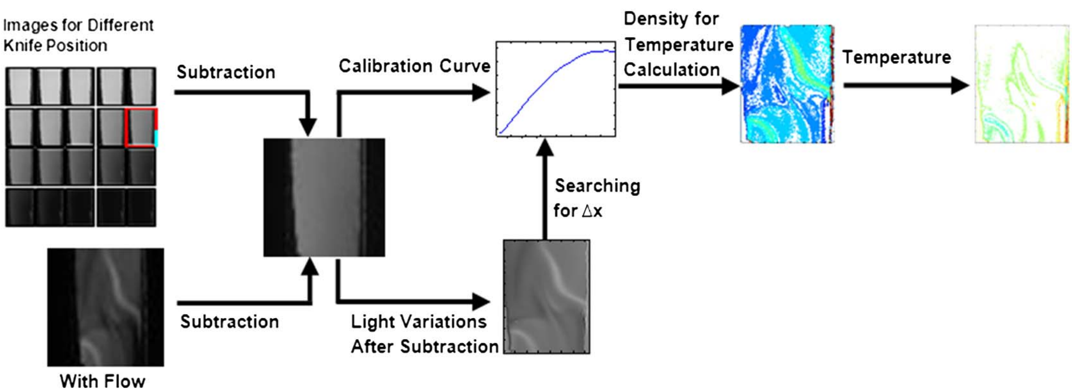

Fig. 3. (Color online) Flow chart for fields calculations in z-type schlieren system.

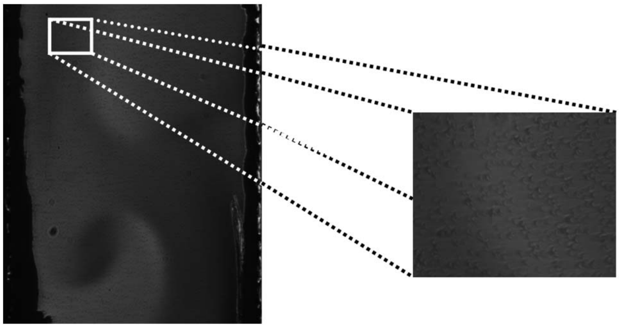

Fig. 4. Image of the experimental flow for simultaneous schlieren and PIV measurement. A shape distortion of the 50 μm polystyrene latex particles with a knife edge at a cutoff of approximately 60%. One part of the particles is suppressed, making PIV analysis difficult.

subtracted from the image with the knife edge in its reference position at $\xi = 0$ (upper path in the flow chart). In the flow chart and in Fig. 3, the schlieren image for the reference knife-edge position is indicated by a white square.

On the lower path of the flow chart, a particular schlieren image in presence of flow, with the knife edge at its reference position, is shown. This image is subtracted from a schlieren image with no flow and knife edge at its reference position. The light deviation intensity for each image pixel is directly related to its corresponding transverse knife position from the calibration curves. In order to increase the resolution in Δx, each calibration curve is interpolated from 40 points to 250 points by using a piecewise cubic Hermite interpolating polynomial provided by MatLab. This algorithm does not modify the shape of the curve.

A zoomed section of the images obtained from one of the experiments is shown in Fig. 4, where an instantaneous schlieren image is seen with an inset of particles images present inside the water container. The temperature and velocity of this kind of schlieren images are calculated by using the procedure outlined in Subsections 2.A and 2.B, respectively.

In Fig. 5, we show schlieren images and temperature contour plots of the four cases under study. The schlieren images show fluid flow patterns characteristic of this kind of flow. An averaging filter was applied to the temperature data to reduce high-frequency noise due to the particles presents in the fluid flow. We found that the temperature close to the middle of water container is consistent with the temperature measured by using thermocouples.

B. Velocity Measurement

From the experimental analysis, it is apparent that considering the quality of the images as a key issue in applying PIV analysis to schlieren images is required. For PIV, the image scale and resolution are determined by three main factors: the maximum resolution and pixel size of the CCD camera sensor, the focal distance of the lens employed, and the scale of the flow under study. In addition, schlieren images have additional image quality issues arising from the sensitivity of the optics to the refractive disturbances in the flow under investigation. Particle

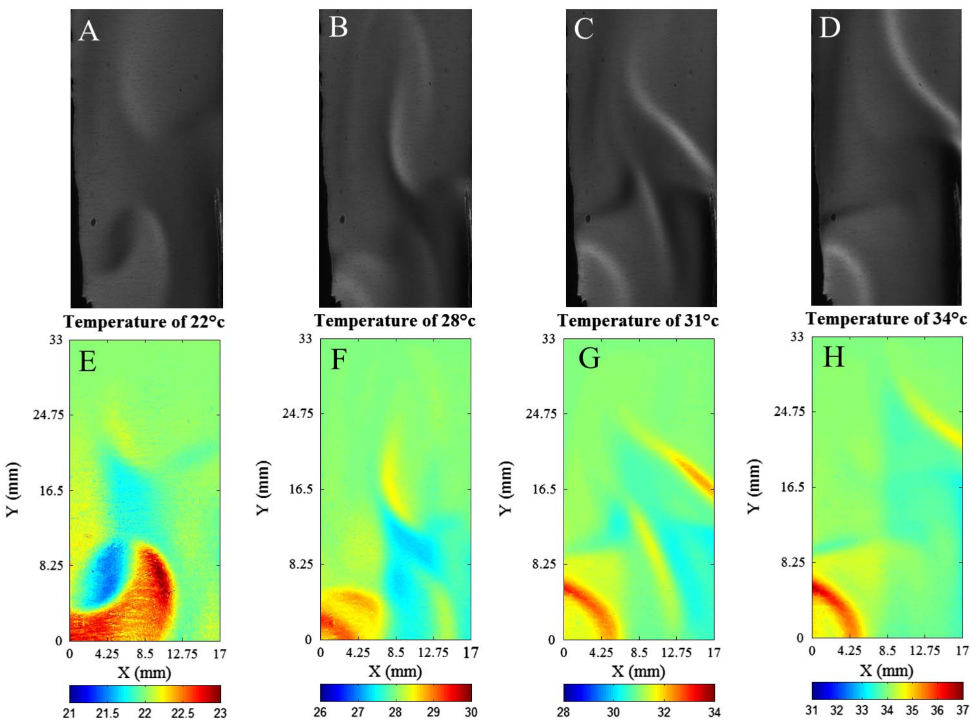

Fig. 5. (Color online) A–D, example schlieren pattern results with different temperatures (22 °C, 28 °C, 32 °C, and 34 °C); E–H, contour plots of temperature fields for the same temperatures as above.

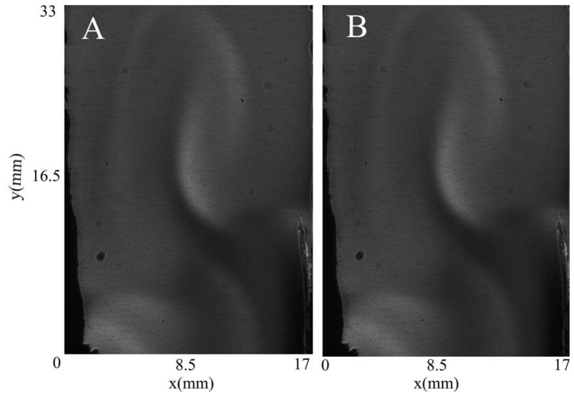

Fig. 6. Typical images of particles for PIV measurement (in this case, obtained in the z-type schlieren system). Images A and B are separated by 1/15 s, which contain particles 50 μm in size.

image velocimetry requires the identification of particle images from the fluid flow in order to get real measurements from all region of interest; the schlieren technique adds shadows in the contour of the gradient variation of density that can affect the velocity measurement. Equilibrium between the sensitivity in schlieren and definition of the particles in PIV is needed, and this equilibrium is defined by the edge knife position. An knife edge position of more than 50% cutoff affects the visualization of the particles because a segment of the image particle is suppressed, as shown in Fig. 4. For our research, the knife edge position used avoids the effect in velocimetry measurements.

In our preliminary experimental results, we considered a convective fluid for the simplicity of the problem. However, the procedure can be applied to different kinds of fluid flows, which is the aim of future work on this subject. The distance of the PIV camera from the target was chosen to maximize resolution while maintaining focus of the particles. The depth of focus was selected according to the width of the container. In the setup, the particles in fluid flow scatter light in the forward direction, which is the most efficient light transmission; the source used in this experiment is adequate enough to get PIV images. The schlieren images obtained previously were analyzed using IDT Inc. software.

In our analysis, an interrogation window of 32 was used, and the velocity information was interpolated in a mesh of 45 points × 60 points. In our experiment, the apparent number of visible particles decreases when the percent of knife-edge cutoff increases more than 50%; schlieren sensitivity increases but PIV resolution decreases with higher percentages. To reduce this effect, we used the contrast parameter that defines whether a mesh point can be considered as a dropout; the value that we use is the minimum accepted for IDT software (0.05).

Figure 6 shows two consecutive schlieren images at different instances of time for PIV purposes. The PIV results for instantaneous schlieren images can be seen in Fig. 7, which shows a vector plot of an instantaneous PIV image of the convective water flow at temperatures under analysis. In these figures, the convective flow pattern characteristic of this kind of flow is observed. A review of the results showed that the velocity distribution of the convective flow obtained by the proposed method captured main features of the fluid flow under analysis. Maximum velocities of 0.0108 m/s, 0.0131 m/s, 0.0136 m/s, and 0.099 m/s were found in the entire fluid flow field analyzed in each case.

It was observed that PIV results are comparable when the fluid flow is analyzed without knife edge and when it is analyzed together with the schlieren measurements.

The new approach of combining schlieren and PIV techniques using one source of light has proven to be a convenient and effective way of obtaining true simultaneous velocity and density measurements in this water flow pattern. Figure 8A show laboratory experiments of the natural convection of water provoked by a heated rectangular plate at 22 °C; Fig. 8B shows a contour map of the temperature distribution obtained by the schlieren method with a cutoff of 50%. The velocity field obtained by the PIV method is plotted in Fig. 8C, and the velocity field superimposed on the temperature distribution is illustrated in Fig. 8D.

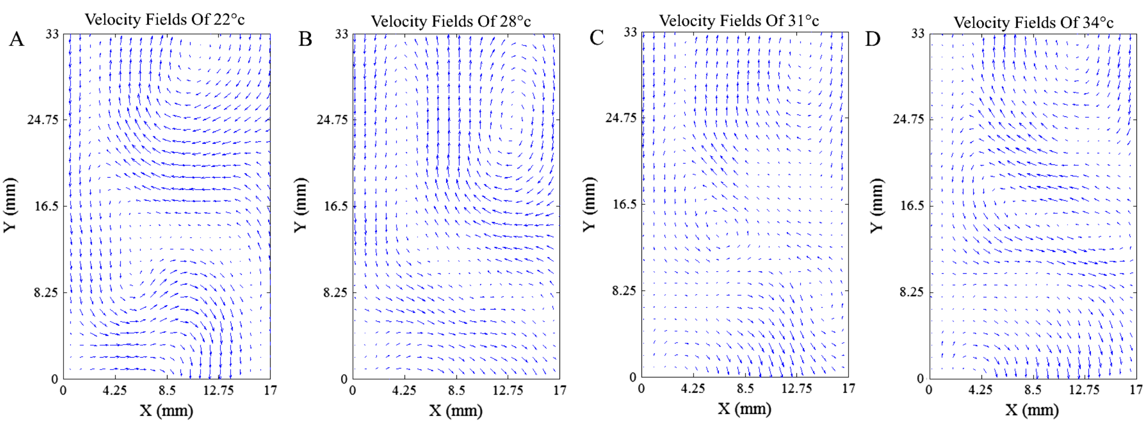

Fig. 7. (Color online) Displacement of particles found by the PIV method. Images obtained from the schlieren system at temperatures of A, 22 °C; B, 28 °C; C, 31 °C; and D, 34 °C; driven by the natural convection of water provoked by a heated rectangular plate.

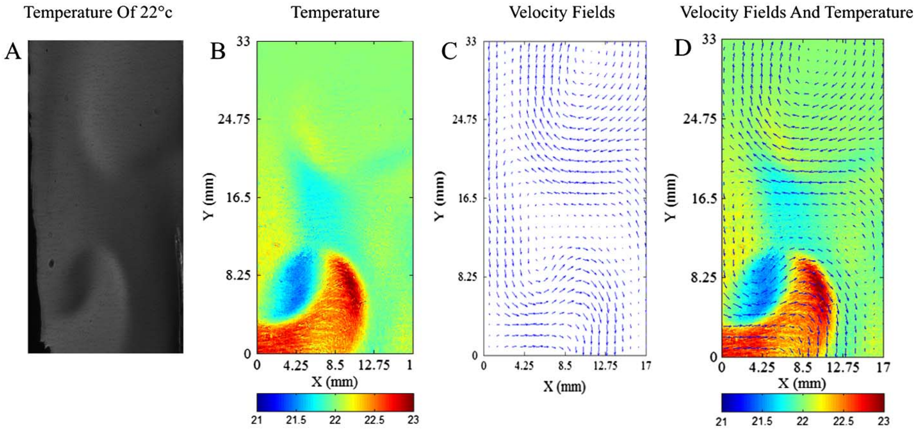

Fig. 8. (Color online) Images of data derived from the analysis of A, the original schlieren image acquired with the system; B, temperature analysis; C, velocimetry field vectors; and D, temperature and velocimetry field vectors of the same region of interest.

5. Conclusion

In this paper, we successfully built a system based on the traditional schlieren system to measure velocity and temperature fields of a water convective fluid flow caused by a heated rectangular metal plate. In the proposed method, the temperature is measured using a calibration procedure that allows us to directly convert intensity level for each pixel in a schlieren image into a corresponding transverse knife position, and the velocity is measured using the PIV technique. Our approach has shown that for the PIV analysis, the position of the knife edge acts as a filter, which is important for obtaining information on velocity. It was shown that a balance between the knife position is needed in order to have suitable temperature and velocity measurements. This study shows that to obtain PIV results, it was sufficient to use a conventional CCD camera and a white light source. For velocimetry measurement, 50 μm polystyrene particles were used. Future work will attempt to use smaller particles, such as those used in gas dynamic applications.

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