1. $$\begin{cases} x u_x - (2x+y)u_y = 0 \\ u|_{x+y=2} = \varphi(y) \end{cases}$$

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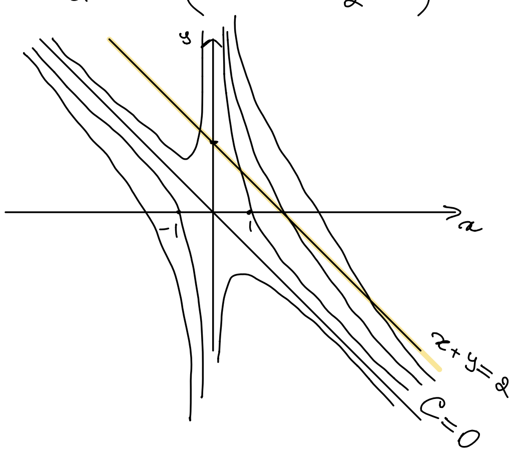
$$\frac{dx}{x} = \frac{dy}{-(2x+y)} \iff x y' = -2x - y \iff y = c\frac{1}{x} - x \iff C = (y+x)x$$

Общее решение $F((y+x)x)$

$$u|_{x+y=2} = F((y+x)x)\Big|_{x+y=2} = \varphi(y)$$

$$\exists\ z = [(y+x)x] = 2(2-y) \Rightarrow y = 2 - \frac{z}{2}$$

$$u = \varphi\left(2 - \frac{(y+x)x}{2}\right)$$



Ответ: $u = \varphi\left(2 - \frac{(y+x)x}{2}\right)$

2. $\begin{cases} 2x u_x + (x+y)u_y = au & a = const \\ u|_{x=1} = \varphi(y) \end{cases}$

$$\begin{cases} \dfrac{dx}{ds} = 2x & x(0)=1 \\ \dfrac{dy}{ds} = x+y & y(0)=\tau \end{cases} \Leftrightarrow \begin{cases} x = e^{2s} \\ \dfrac{dy}{ds} = y + e^{2s}, \; y(0)=\tau \end{cases} \Leftrightarrow \begin{cases} x = e^{2s} \\ y = (\tau-1)e^{s} - e^{2s} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} s = \ln\sqrt{x} \\ \tau = \dfrac{y-x}{\sqrt{x}} + 1 \end{cases}, \; x>0$$

Перейдём к $v(s,\tau) = u(x,y)$

$$\begin{cases} \dfrac{dv}{ds} = av \\ v|_{s=0} = \varphi(\tau) \end{cases} \Rightarrow \quad v(s) = \varphi(\tau)e^{as}$$

$$u(x,y) = \varphi\left(\frac{y-x}{\sqrt{x}} + 1\right)e^{a\ln\sqrt{x}} = \varphi\left(\frac{y-x}{\sqrt{x}} + 1\right)x^{\frac{a}{2}}$$

Ответ: $u(x,y) = \varphi\left(\dfrac{y-x}{\sqrt{x}} + 1\right)\sqrt{x}^{\,a}, \; x>0$

$$\begin{array}{r|l} 111 & 10 \\ -10 & \overline{11} \\ \hline 11 & \\ -10 & \\ \hline 1 & \end{array}$$

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