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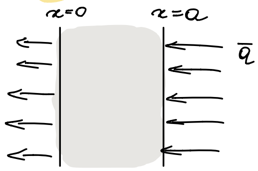

$$\left(-\frac{\partial T}{\partial x} + h(T-T_0)\right)\Big|_{x=0} = 0$$

$$\Delta T - \frac{\partial T}{\partial \tau} = 0$$

Закон Фурье:
$$\bar q = -k\nabla T$$

Из закона Фурье: $\frac{\partial T}{\partial x}\Big|_{x=a} = \frac{q}{k}$ 1)

$$\left(-\frac{\partial T}{\partial x} + hT\right)\Big|_{x=0} = 0 \quad 2)$$

$$\frac{\partial^2 T}{\partial x^2} - \frac{\partial T}{\partial \tau} = 0$$

$T(x,0) = 0$ — нач. усл.

$$T(r,\tau) \to \tilde T(r,p)$$

$$T'_\tau(r,\tau) \to p\tilde T - T_0$$

Условие:

$$\frac{\partial \tilde T}{\partial x}\Big|_{x=a} = \frac{q}{kp}$$

$$\left(-\frac{\partial \tilde T}{\partial x} + h\tilde T\right)\Big|_{x=0} = 0$$

$$\cancel{\tilde T(x,0) = 0}$$

Ур-е:

$$\frac{\partial^2 \tilde T}{\partial x^2} - p\tilde T - \cancel{T_0}^{\,0} = 0$$

$$\tilde T'' - p\tilde T = 0$$

$$\tilde T = \overbrace{C_1 \operatorname{sh}(\sqrt p(a-x))} + C_2 \operatorname{ch}(\sqrt p(a-x))$$

$$\tilde T\big|_{x=0} = C_1 \operatorname{sh}(\sqrt p\, a) + C_2 \operatorname{ch}(\sqrt p\, a)$$

$$\tilde T' = -C_1\sqrt p \operatorname{ch}(\sqrt p(a-x)) - C_2\sqrt p \operatorname{ch}(\sqrt p(a-x))$$

$$\tilde T'\big|_{x=a} = -C_1\sqrt p \overset{1}{\operatorname{ch}(0)} - C_2\sqrt p \overset{0}{\operatorname{sh}(0)} = \frac{q}{kp}$$

$$C_1\sqrt p = -\frac{q}{kp} \Rightarrow C_1 = \frac{-q}{kp\sqrt p}$$

$$\tilde T'\big|_{x=0} = -C_1\sqrt p \operatorname{ch}(\sqrt p\, a) - C_2\sqrt p \operatorname{sh}(\sqrt p\, a) = h\tilde T\big|_{x=0}$$

$$\frac{q}{kp\sqrt p}\sqrt p \operatorname{ch}(\sqrt p\, a) - C_2\sqrt p \operatorname{sh}(\sqrt p\, a) = -\frac{hq}{kp\sqrt p}\operatorname{sh}(\sqrt p\, a) + hC_2 \operatorname{ch}(\sqrt p\, a)$$

$$C_2\left[\sqrt p \operatorname{sh}(\sqrt p\, a) + h\operatorname{ch}(\sqrt p\, a)\right] = \frac{q}{kp\sqrt p}\left[h\operatorname{sh}(\sqrt p\, a) + \sqrt p \operatorname{ch}(\sqrt p\, a)\right]$$

$$C_2 = \frac{q\left[h\operatorname{sh}(\sqrt p\, a) + \sqrt p\operatorname{ch}(\sqrt p\, a)\right]}{kp\sqrt p\left[\sqrt p\operatorname{sh}(\sqrt p\, a) + h\operatorname{ch}(\sqrt p\, a)\right]}$$

$$\tilde T = \underbrace{\frac{-q}{kp\sqrt p}\operatorname{sh}(\sqrt p(a-x))}_{\tilde T_1} + \underbrace{C_2 \operatorname{ch}(\sqrt p(a-x))}_{\tilde T_2}$$

$$T = \sum \text{вычеты}$$

$$T_1 e^{p\tau} = \frac{-a \,\operatorname{sh}(\sqrt{p}(a-x)) e^{p\tau}}{k p \sqrt{p}} = \frac{-a\left(\cancel{\sqrt{p}}(a-x) + \frac{(\sqrt{p}(a-x))^3}{3!} + \dots\right)\left(1 + p\tau + \dots\right)}{k \cancel{\sqrt{p}}\, p \quad p^n (C + f(p))} =$$

$C + f(p)$

$p = 0$ 1 порядка

$$T = \int_{-\infty}^{+\infty} \underbrace{\tilde{T} e^{+p\tau}}\, dp$$

$$= \frac{-\frac{a}{k}\left(a - x + \frac{p(a-x)^3}{3!} + \dots\right)\left(1 + p\tau + \dots\right)}{p}$$

$$\operatorname{res}_{p=0}\left(\tilde{T}_1 e^{p\tau}\right) = -\frac{a}{k}\left(a - x + \cancel{\frac{p(a-x)^3}{3!}}^{\to 0}\right) \cancel{e^{p\tau}}\, 1 = -\frac{a}{k}(a-x)$$

$$T_1 = -\frac{q}{k}(a-x)$$

$$\tilde{T}_2 e^{p\tau} = \frac{q\left[h \operatorname{sh}(\sqrt{p}a) + \sqrt{p}\operatorname{ch}(\sqrt{p}a)\right]\operatorname{ch}(\sqrt{p}(a-x))}{k p \sqrt{p}\left[\sqrt{p}\operatorname{sh}(\sqrt{p}a) + h \operatorname{ch}(\sqrt{p}a)\right]} e^{p\tau}$$

$p = 0$
1 порядка

$$\frac{q\left(h\left(\cancel{\sqrt{p}}a + \frac{(\sqrt{p}a)^3}{3!} + \dots\right) + \cancel{\sqrt{p}}\left(1 + \frac{(\sqrt{p}a)^2}{2!} + \dots\right)\right)\left(1 + \frac{(\sqrt{p}(a-x))^2}{2!} + \dots\right) e^{p\tau}}{k p \cancel{\sqrt{p}}\left[1 + f(p)\right]}$$

$$\operatorname{res}\left(\frac{\varphi(p)}{\psi(p)},\ p = a\right) = \frac{\varphi(a)}{\psi'(a)}$$

$// \varphi(p)$

$$\operatorname{res}\left(\tilde{T}_2,\ p=0\right) = \left.\frac{\dfrac{q\left[h \operatorname{sh}(\sqrt{p}a) + \sqrt{p}\operatorname{ch}(\sqrt{p}a)\right]\operatorname{ch}(\sqrt{p}(a-x))}{k \sqrt{p}\left[\sqrt{p}\operatorname{sh}(\sqrt{p}a) + h\operatorname{ch}(\sqrt{p}a)\right]} e^{p\tau}}{p = \psi(p)}\right|_{p=0} =$$

$$= \frac{qha + q}{kh}$$

$$p:\ \sqrt{p}\operatorname{sh}(\sqrt{p}a) + h\operatorname{ch}(\sqrt{p}a) = 0 \qquad p_n = \left(\frac{\gamma_n}{a}\right)^2 \text{ — 1 порядка}$$

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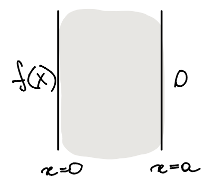

$T(x,0)=0$

$$\Delta T - \frac{\partial T}{\partial \tau} = 0$$

$$\frac{\partial^2 T}{\partial x^2} - \frac{\partial T}{\partial \tau} = 0$$

Условие:

$T(0,\tau) = f(\tau)$

$T(a,\tau) = 0$

$T(x,0) = 0$

$$\tilde{T} = C_1\, sh\left(\sqrt{p}(a-x)\right) + C_2\, ch\left(\sqrt{p}(a-x)\right)$$

$$\tilde{T}\big|_{x=a} = C_2 = 0$$

$$\tilde{T}\big|_{x=0} = C_1\, sh(\sqrt{p}a) = \tilde{f}(p)$$

$$C_1 = \frac{\tilde{f}(p)}{sh(\sqrt{p}a)}$$

$$\tilde{T} = \frac{\tilde{f}(p)}{sh(\sqrt{p}a)}\, sh\left(\sqrt{p}(a-x)\right) = \tilde{f}(p)\, \underbrace{\frac{sh\left(\sqrt{p}(a-x)\right)}{sh(\sqrt{p}a)}}_{\tilde{T}_1(x)}$$

По th о свёртке: $T(x,\tau) = \int\limits_0^{\tau} f(t)\cdot T_1(x,\tau-t)\,dt$

$$T_1(x,\tau) = \frac{1}{2\pi i}\int \tilde{T}_1 e^{p\tau}\,dp$$

$$sh(x) = \frac{e^x - e^{-x}}{2}$$

$$sh(\sqrt{p}a) = 0 \qquad p_n = -\frac{(\pi n)^2}{a^2} \qquad \frac{sh(ix)}{i} = \frac{e^{ix}-e^{-ix}}{2i} = \sin(x)$$

$$ch(ix) = \cos(x) \qquad sh(ix) = i\sin(x)$$

$$i\sin\left(\frac{\sqrt{p}a}{i}\right) = 0$$

$$\frac{\sqrt{p}a}{i} = \pi n$$

$p = p_n$ – 1 порядка

$$p_n = -\left(\frac{\pi n}{a}\right)^2$$

$$\left.\frac{sh\left(\sqrt{p}(a-x)\right)}{\frac{a}{2\sqrt{p}}\, ch(\sqrt{p}a)}\right|_{p_n} = \frac{\cancel{i}\sin\left(\frac{\sqrt{-\left(\frac{\pi n}{a}\right)^2}(a-x)}{i}\right)}{\frac{a}{2\sqrt{-\left(\frac{\pi n}{a}\right)^2}}\, ch\left(\frac{\sqrt{-\left(\frac{\pi n}{a}\right)^2}\,a}{\cancel{i}}\right)}$$

$$\shortparallel$$

$$\frac{\sin\left(\left(\frac{\pi n}{a}\right)(a-x)\right)}{\frac{a}{2\frac{\pi n}{a}}\underbrace{\cos\left(\frac{\pi n}{\cancel{a}}a\right)}_{=(-1)^n}}$$

$$\cancel{x}\left(\frac{\pi n}{a}\right)$$