

$$\textcircled{I} \int_0^1 x dx \rightarrow \max \quad |\dot{x}| \leq 2 \quad x(0) = x(1) = 0$$

$$\int_0^1 x_1 dx_1 \rightarrow \max$$

$$g = g_1 x_1(0) + g_2 x_2(1)$$

$$x_1(0) = x_2(1) = 0$$

$$|u| \leq 2$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u$$

$$H = x_1 + \psi_1 x_2 + \psi_2 u \rightarrow \max_{|u| \leq 2} \Leftrightarrow \tilde{H} = \psi_2 u \rightarrow \max_{|u| \leq 2}$$

$$\psi_1(0) = \frac{\partial g}{\partial x_1(0)} = g_1 \quad \psi_2(0) = 0$$

$$\psi_1(1) = \frac{\partial g}{\partial x_1(1)} = 0 \quad \psi_2(1) = g_2$$

$$\dot{\psi}_1 = 1$$

$$\psi_1 = t + C_1$$

$$\psi_1(1) = 0 \Rightarrow C_1 = -1$$

$$\dot{\psi}_2 = \psi_1$$

$$\psi_2 = \frac{1}{2} t^2 - t + C_2$$

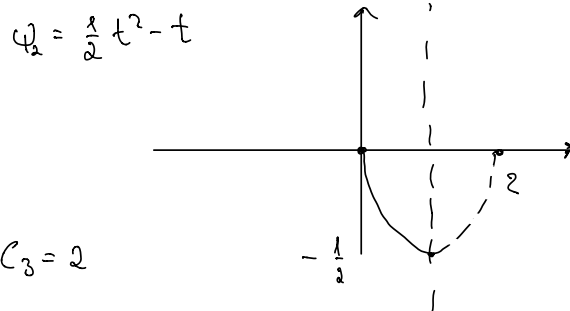
$$\psi_2(0) = 0 \Rightarrow C_2 = 0$$

$$\psi_1 > 0 \Rightarrow u = 2 \quad t \in \emptyset$$

$$\psi_2 < 0 \Rightarrow u = -2 \quad t \in [0; 1]$$

$$u = -2 \Rightarrow x_2 = -2t + C_3 \quad x_2(1) = 0 \Rightarrow C_3 = 2$$

$$x_1 = -t^2 + 2t + C_4 \quad x_1(0) = 0 \Rightarrow C_4 = 0$$



$$\text{Answer: } x_1 = -t^2 + 2t$$

$$\text{II} \quad \int_0^1 x \, dx \rightarrow \min \quad \begin{array}{l} |\ddot{x}| \leq 2 \\ x(0) = \dot{x}(1) = 0 \end{array}$$

$$\begin{cases} y_1 = x \\ y_1' = y_2 \\ y_2' = u \end{cases} \quad \begin{cases} y_1(0) = 0 \\ y_2(1) = 0 \\ |u| \leq 2 \end{cases} \quad - \int_0^1 y_1 \, dy_1 \rightarrow \max$$

$$g = p_1 y_1(0) + p_2 y_2(1)$$

$$H = -y_1 + \psi_1 y_2 + \psi_2 u \xrightarrow{|u| \leq 2} \max \Rightarrow \max_{|u| \leq 2} (\psi_2 u)$$

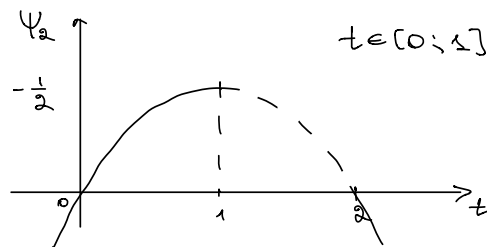
$$\begin{cases} \psi_1(0) = \frac{dg}{dy_1(0)} = p_1 \\ \psi_1(1) = \frac{dg}{dy_1(1)} = 0 \end{cases} \quad \begin{cases} \psi_2(0) = 0 \\ \psi_2(1) = p_2 \end{cases}$$

$$\begin{cases} \dot{\psi}_1 = \frac{\partial H}{\partial y_1} = -1 \\ \dot{\psi}_2 = \psi_1 \end{cases} \Rightarrow \begin{cases} \psi_1 = -t + C_1 \\ \psi_2 = -\frac{1}{2}t^2 + C_1 t + C_2 \end{cases} \Rightarrow \begin{cases} \psi_1 = -t + 1 \\ \psi_2 = -\frac{1}{2}t^2 + t \end{cases}$$

$$1) \quad \psi_2 < 0 \Rightarrow u = -2$$

$$2) \quad \psi_2 > 0 \Rightarrow u = 2$$

$$\Rightarrow \psi_2 > 0 \Rightarrow u = 2$$



$$\begin{cases} y_2 = 2t + A \\ y_2(1) = 0 \end{cases} \Rightarrow y_2 = 2t - 2$$

$$\begin{cases} y_1 = t^2 - 2t + B \\ y_1(0) = 0 \end{cases} \Rightarrow B = 0$$

$$\text{Daher: } x = t^2 - 2t + B$$



функция цели:  $f(x): \mathbb{R}^n \rightarrow \mathbb{R}$   $f(x) \rightarrow \min$

$f(x)$  — функция относительной пригодности

Особь  
(хромосома)

$(x_1, x_2, \dots, x_n) \in \mathbb{R}^n$   
└ ГЕНЫ ─┘

МУТАЦИЯ  $(x_1 + \Delta x_1, x_2 + \Delta x_2, \dots, x_n + \Delta x_n)$

СКРЕЩИВАНИЕ  $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow z$

РЕПРОДУКЦИЯ = МУТАЦИЯ + СКРЕЩИВАНИЕ

ПОКОЛЕНИЕ

популяция

$$\left\{ \begin{array}{l} (x_1, x_2, \dots, x_n) \\ (y_1, y_2, \dots, y_n) \\ \vdots \\ (z_1, z_2, \dots, z_n) \end{array} \right\}^{(1)} \left\{ \begin{array}{l} (x_1, x_2, \dots, x_n) \\ (y_1, y_2, \dots, y_n) \\ \vdots \\ (z_1, z_2, \dots, z_n) \end{array} \right\}^{(2)} \cdot \cdot \cdot \left\{ \begin{array}{l} (x_1, x_2, \dots, x_n) \\ (y_1, y_2, \dots, y_n) \\ \vdots \\ (z_1, z_2, \dots, z_n) \end{array} \right\}^{(k)}$$



**ОПЕРАТОР СКРЕЩИВАНИЯ**  
(ПЛОСКИЙ КРОССОВЕР)

$$X_i^{\text{child}} \leftarrow x \in [X_i^{(1)}, X_i^{(2)}]$$

ХРОМОСОМЫ  
из  $\mathbb{R}^n$

**ОПЕРАТОР МУТАЦИИ**

$$X_i' = X_i + F(A_i - B_i)$$

СЛУЧАЙНЫЕ  
ОСОБИ