$$\frac{\partial^2 T}{\partial x^2} - \frac{\partial T}{\partial \tau} = 0$$

Граничные условия:

1) $-\frac{\partial T}{\partial x} + h(T - T_0)\Big|_{x=0} = 0$ 3) $T\Big|_{\tau=0} = 0$ $\frac{\partial T}{\partial x}(0) = h(T(0,\tau) - T_0)$

2) $T\Big|_{x=a} = T_1$

Вспомогательная задача:

$$X'' - \lambda X = 0$$

4) $-X' + hX\Big|_{x=0} = 0$

5) $X\Big|_{x=a} = 0$

$$\lambda_n = \frac{\gamma_n^2}{a^2}$$

$$X_n = \sin\left(\frac{\gamma_n(a-x)}{a}\right)$$

$$X'_n = \cos(\cdots)\left(-\frac{\gamma_n}{a}\right)$$

$$T = \sum_{n=1}^{\infty} C_n(\tau) X_n(x) \qquad C_n = \frac{(X_n(x), T)}{\|X_n\|^2}$$

$$\|X_n\|^2 = \int_0^a \sin^2\left(\frac{\gamma_n(a-x)}{a}\right) = \frac{1}{2}\left(a - \sin\left(\frac{\gamma_n(a-x)}{a}\right)\left(-\frac{\gamma_n}{a}\right)\Big|_0^a\right) = \frac{1}{2}a - \sin(\gamma_n)\left(\frac{\gamma_n}{a}\right)$$

$$(X_n(x), T) = \int_0^a T X_n dx$$

$$* \int_0^a \left(\frac{\partial^2 T}{\partial x^2} - \frac{\partial T}{\partial \tau}\right) X_n dx = 0$$

$$\int_0^a \frac{\partial^2 T}{\partial x^2} X_n dx - \frac{\partial \tilde{T}}{\partial \tau} = 0$$

$$\frac{\partial T}{\partial x} \overset{5)\ 0}{\cancel{X_n}}\Big|_0^a - \int_0^a \frac{\partial T}{\partial x} X'_n dx - \frac{\partial \tilde{T}}{\partial \tau} = 0$$

$$-h(T(0,\tau) - T_0)\sin(\gamma_n) - T X'_n\Big|_0^a + \int_0^a T X''_n dx - \frac{\partial \tilde{T}}{\partial \tau} = 0$$

$$\|$$

$$\left[\begin{array}{l} -T_1 \cos\left(\frac{\gamma_n(a-a)}{a}\right)\left(-\frac{\gamma_n}{a}\right) + T(0,\tau)\cos(\gamma_n)\left(-\frac{\gamma_n}{a}\right) \\ T_1\left(\frac{\gamma_n}{a}\right) - T(0,\tau)\cos(\gamma_n)\left(\frac{\gamma_n}{a}\right) \end{array}\right]$$

$$-h(T(0,\tau) - T_0)\sin(\gamma_n) + T_1\left(\frac{\gamma_n}{a}\right) - T(0,\tau)\cos(\gamma_n)\left(\frac{\gamma_n}{a}\right) - \lambda_n \tilde{T} - \frac{\partial \tilde{T}}{\partial \tau} = 0$$

$$\operatorname{tg}(\gamma_n) = \frac{\sin(\gamma_n)}{\cos(\gamma_n)} = -\frac{\gamma_n}{ha} \Rightarrow \cos = -\frac{ha}{\gamma_n}\sin(\gamma_n)$$

$$-hT(0,\tau)\sin(\delta_n) - T(0,\tau)\left(\frac{\delta_n}{a}\right)\left(-\frac{ha}{\delta_n}\sin(\delta_n)\right) = 0$$

$$+hT_0\sin(\delta_n) + T_1\cdot\frac{\delta_n}{a} - \lambda_n\tilde{T} - \frac{\partial\tilde{T}}{\partial\tau} = 0$$

$$\lambda_n\tilde{T} + \frac{\partial\tilde{T}}{\partial\tau} = \underbrace{hT_0\sin(\delta_n) + T_1\cdot\frac{\delta_n}{a}}_{A}$$

$$\tilde{T}_{n.r.} = \frac{A}{\lambda_n} \qquad \tilde{T}_{n.o.} = C_1 e^{-\lambda_n\tau}$$

$$T\Big|_{\tau=0} = 0 \Rightarrow \tilde{T}\Big|_{\tau=0} = 0$$

$$\frac{A}{\lambda_n} + C_1 = 0 \Rightarrow C_1 = -\frac{A}{\lambda_n}$$

$$\tilde{T}_n = \frac{A}{\lambda_n} - \frac{A}{\lambda_n}e^{-\lambda_n\tau} = \left(h\cdot T_0\cdot\frac{a^2}{\delta_n^2}\sin(\delta_n) + T_1\frac{a}{\delta_n}\right)\left(1 - e^{-\lambda_n\tau}\right)$$