Coursework 3

This coursework is about the leaky integrate and fire neuron. It is intended to make clear the particular way neuronal connectivity works: a neuron has synapses, when it fires the spikes go to its synapses and they in turn cause a change in conductivity for the post-synaptic neuron.

Question 1

Simulate a leaky integrate-and-fire model with the following parameters for 1 second:

$$\begin{aligned} \tau_m &= 10 \text{ ms}, & E_L &= -70 \text{ mV}, & V_r &= -70 \text{ mV}, \\ V_t &= -40 \text{ mV}, & R_m &= 10 \text{ M}\Omega, & I_e &= 3.1 \text{ nA}. \end{aligned} \tag{1}$$

Use Euler's (forward) method with timestep $\Delta t = 1$ ms. Here E_L is the leak potential, V_r is the reset voltage, V_t is the threshold, R_m is the membrane resistance, that is one over the conductance, and τ_m is the membrane time constant. Plot the voltage as a function of time.

For simplicity assume that the neuron does not have a refractory period after producing a spike. You do not need to plot spikes - once membrane potential exceeds threshold, simply set the membrane potential to V_r.

As a sanity check, you may wish to re-write the voltage update to use C_m, the membrane capacitance in nanofarads, whose value you can retrieve from the above given (τ_m, R_m) parameters using $\tau_m = R_m C_m$.

Q1 Optional Bonus Material (Exponential Euler)

1. Calculate $\bar{v}$, the steady-state membrane voltage assuming the input I_e remains constant and ignoring the spiking dynamics.
2. Implement the forward exponential Euler method by solving analytically for the decay of $v(t)$ toward $\bar{v}$ on each Δt width time-step.
3. Is the resulting solution the same as forward Euler? How small do you need to make Δt to get the same number of spikes from both approaches?

4. Analytically obtain the inter-spike-interval by solving for the earliest $t > 0$ such that $v(t) \geq V_t$, assuming we start at the reset voltage $v(0) = V_r$. Does this result match the inter-spike interval of the forward exponential Euler solution at $\Delta t = 1$ ms resolution? If not, what accounts for the discrepancy?

Question 2

Simulate two neurons that have synaptic connections between each other, that is the first neuron projects to the second, and the second neuron projects to the first. Both model neurons should have the same parameters:

$$\begin{aligned} &E_L = -70 \text{ mV}, \qquad &V_r = -80 \text{ mV}, \qquad &V_t = -54 \text{ mV}, \\ &\tau_m = 20 \text{ ms}, \qquad &R_m = 10 \text{ M}\Omega, \qquad &I_e = 1.8 \text{ nA}, \end{aligned} \tag{2}$$

and their synapses should also have the same parameters:

$$P = 0.5, \qquad \tau_s = 10 \text{ ms}, \qquad R_m\bar{g}_s = 0.15; \tag{3}$$

don't get confused by being given $R_m\bar{g}_s$ rather than $\bar{g}_s$ on its own, to get τ_m rather than the capacitance on the left hand side of the integrate and fire equation everything is multiplied by R_m.

For simplicity take the synaptic conductance

$$g_s = \bar{g}_s s \tag{4}$$

to satisfy

$$\tau_s \dot{s} = -s \tag{5}$$

with a spike arriving causing s to increase by P. This is equivalent to the simple synapse model in the lectures.

Simulate two cases:

a) assuming that the synapses are excitatory with $E_s = 0$ mV, and

b) assuming that the synapses are inhibitory with $E_s = -80$ mV.

For each simulation set the initial membrane potentials of the neurons V to different values chosen randomly from between V_r and V_t and simulate 1 s of activity. For each case plot the voltages of the two neurons on the same graph (with different colours).

Q2 Optional Bonus Material

This question uses parameters from Q1 above.

1. Compute analytically the minimum current I_e required for the neuron with the above parameters to produce an action potential.
2. Simulate the neuron for 1 s for the input current with amplitude I_e which is 0.1 [nA] lower than the minimum current computed above, and plot the voltage as a functions of time.
3. Simulate the neuron for 1 s for currents ranging from 1 nA to 5 nA in steps of 0.1 nA. For each amplitude of current count the number of spikes produced, that is the firing rate. Plot the firing rate as the function of the input current.