Purity Preservation Engineering by Two-Step Floquet-Lindblad in Driven Qubit Systems

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Abstract

We investigate how purity can be preserved or restored in a single driven qubit by employing a piecewise-constant two-step Floquet Lindblad protocol. The qubit undergoes continuous coherent evolution under a constant Hamiltonian, while dissipation is modulated periodically between two distinct channels and also we are finding the way periodic modulation of dissipation enables interconversion between unital and non-unital quantum channels in a single driven qubit. We characterize the eigenstructure of the stroboscopic Floquet map and identify the emergence of **exceptional points** non-Hermitian degeneracies where eigenvectors coalesce as a central mechanism governing transitions between purity-preserving and mixing regimes. Our work maps out precise contours in parameter space where purity is stabilized and provides experimentally viable strategies for quantum control using only minimal periodic modulation of dissipation.

Model and Two-Step Protocol

Lindblad Dynamics

The qubit obeys

$$\dot{\rho} = -i[H_S,\rho] + \sum_{\mu} \tfrac{\gamma_\mu}{2}\mathcal{D}[L_\mu]\rho, \quad \mathcal{D}[L_\mu]\rho = 2L_\mu \rho L_\mu^\dagger - \{L_\mu^\dagger L_\mu, \rho\}.$$

Here H_S generates coherent evolution, while $\mathcal{D}[L_\mu]$ represents dissipative channels.

Hamiltonian

Coherent drive is fixed: $H_0 = J\sigma_x$, producing continuous Rabi oscillations.

Two-Step Protocol

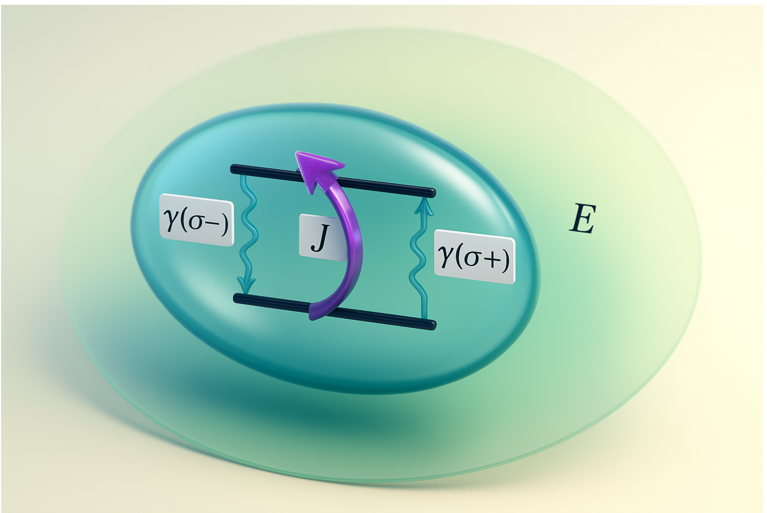
Each period $T = 2\pi/\Omega$ has two halves of duration $\tau = \pi/\Omega$:

$$\mathcal{L}_1 = \mathcal{L}_0 + \gamma\mathcal{L}_{\mu_1}, \qquad \mathcal{L}_2 = \mathcal{L}_0 + \gamma\mathcal{L}_{\mu_2}.$$

The stroboscopic Floquet map

$$G(T) = \mathcal{T}\exp\left(\int_0^T \mathcal{L}(t')\,dt'\right) \equiv e^{T\mathcal{L}_\mathrm{F}},$$

defines the effective superoperator governing purity and entropy evolution.



Protocol Types

Non-unital: $\{\sigma_+,\sigma_-\}$ or $\{\sigma_-,\sigma_+\}$ — irreversible population transfer.
Unital: $\{\sigma_z,\sigma_x\}$ or $\{\sigma_x,\sigma_z\}$ — preserve mixedness.
Hybrid: $\{\sigma_z,\sigma_\pm\}$ or $\{\sigma_x,\sigma_\pm\}$ — allow purity recovery at intermediate $(\gamma/J, \Omega/J)$.
Mechanism: Non-unital drives bias toward pure states; unital drives average populations; hybrids exploit interference when the unital axis anticommutes with H_0.

Analytical Results of Exceptional Point: $\{\sigma_-,\sigma_z\}$

Define the damped rotation rates and set $\tau = \pi/\Omega$:

$$\Omega_z = \sqrt{4J^2-\gamma^2}, \quad \Omega_- = \sqrt{4J^2 - \frac{\gamma^2}{16}}.$$

The yz-block trace factor is

$$T(\gamma,\Omega) = 2\cos(\Omega_z\tau)\cos(\Omega_-\tau) - \left(\frac{\gamma^2}{2}+8J^2\right)\frac{\sin(\Omega_z\tau)\sin(\Omega_-\tau)}{\Omega_z\Omega_-}.$$

Exceptional points occur when

$$\boxed{T(\gamma,\Omega) = \pm 2.}$$

Small-γ anchor:

$$\boxed{\Omega_n = \frac{4J}{n}, \quad n = 1,3,5,\ldots}$$

Only odd n touch $\gamma = 0$; even n require finite γ to activate off-diagonal mixing.

Exceptional Point Diagnostic

Exceptional points are located by monitoring **eigenmatrix overlap** of the Floquet superoperator:

$$\Pi(\gamma,\Omega) = \max_{m>n} |\langle \rho_m|\rho_n\rangle|,$$

where ρ_m are right eigenmatrices normalized by $\langle\rho_m|\rho_m\rangle = 1$. A value $\Pi \approx 1$ indicates that two (or more) eigenmodes coalesce in Liouville space.

EP Landscapes: Different Protocols

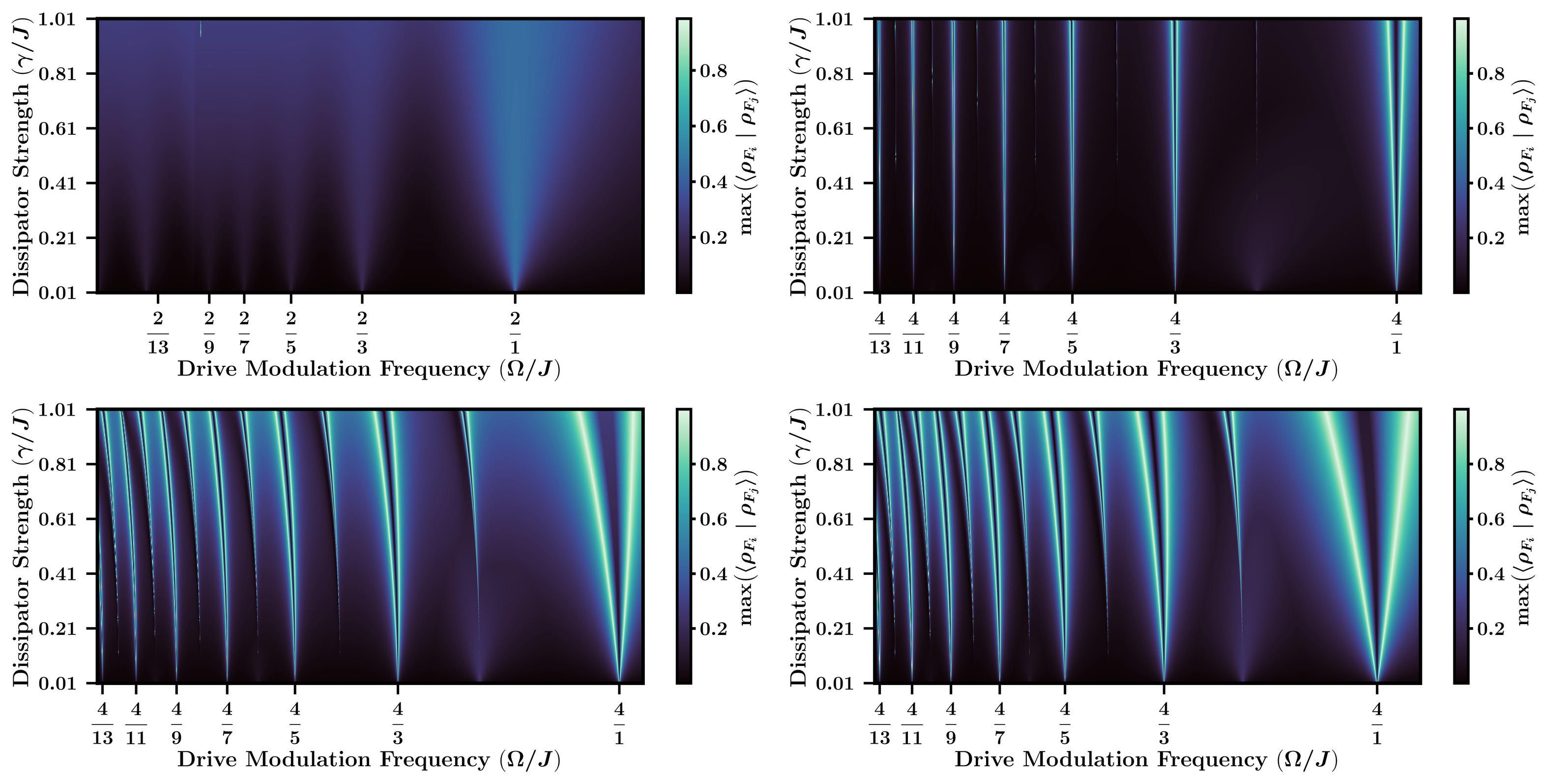


Figure 1. $\Pi(\gamma/J, \Omega/J)$ maps for two-step dissipative protocols. (a) $\{\sigma_-,\sigma_+\}$: non-unital channel with a single relaxation branch. (b) $\{\sigma_-,\sigma_x\}$: commuting hybrid showing an EP ridge near $\Omega/J \simeq 4$. (c) $\{\sigma_-,\sigma_y\}$: anti-commuting hybrid with broad EP domains from strong Floquet mixing. (d) $\{\sigma_-,\sigma_z\}$: partially commuting pair displaying multiple EP contours at $\Omega_n = 4J/n$, marking the oscillatory–relaxation crossover.

Design Rules for Purity Engineering

Linear entropy: $S_L = 1 - \mathrm{Tr}(\rho^2)$ ($S_L = 0$ for pure states).

We analyze how two-step Floquet dissipation protocols influence S_L and identify the conditions under which periodic modulation can preserve or restore purity in open quantum systems.

Purity Dynamics Across Two-Step Protocols

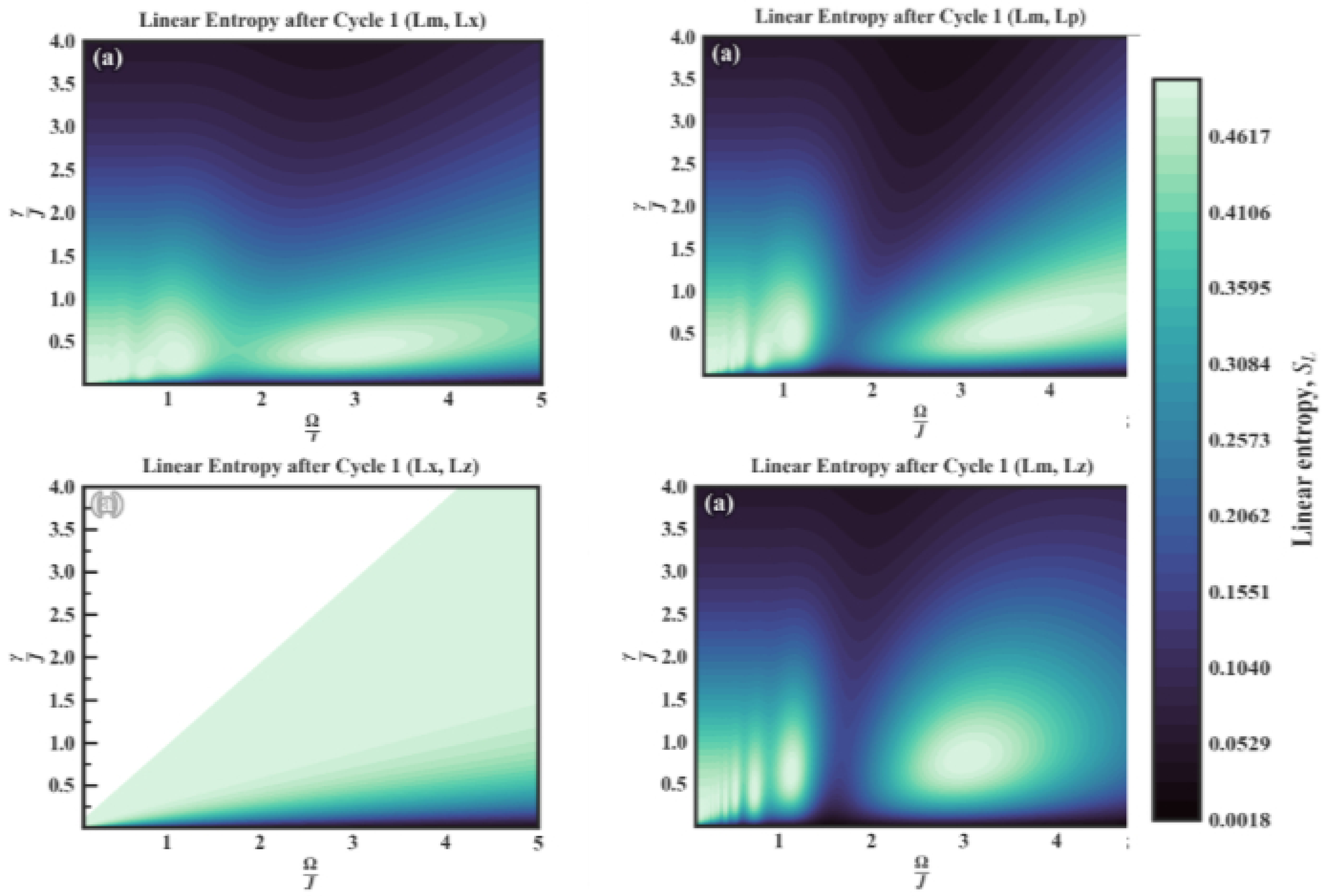
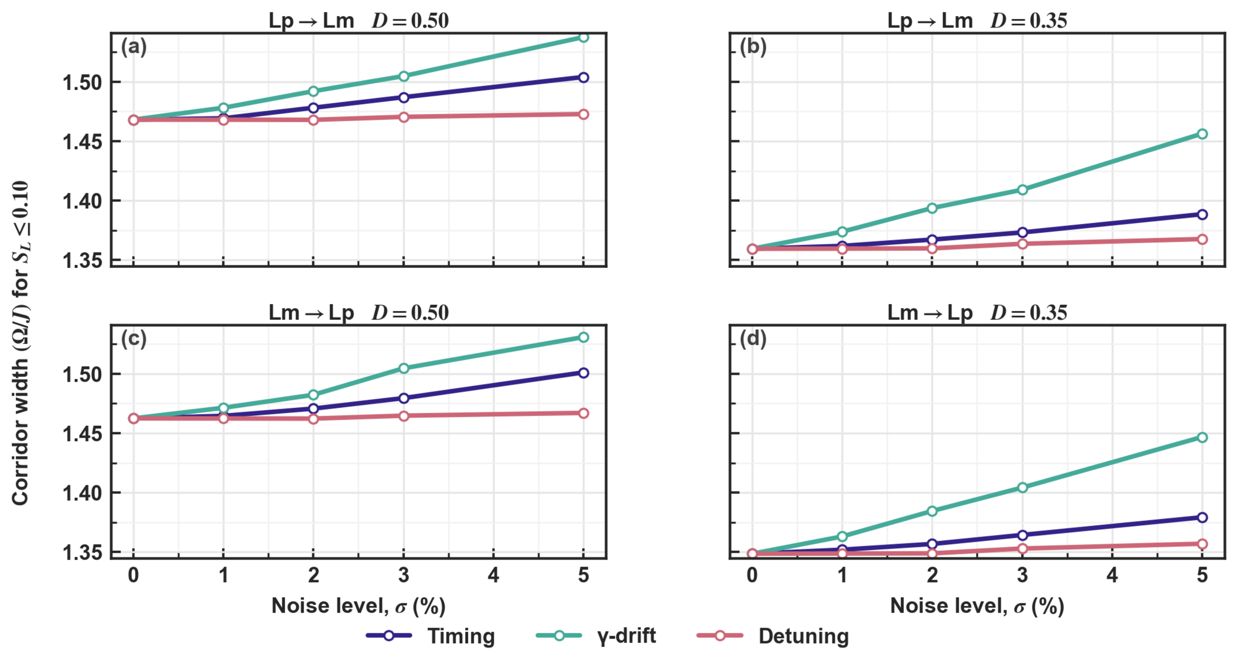


Figure 2. Linear entropy $S_L = 1 - \mathrm{Tr}(\rho^2)$ after one Floquet cycle as a function of $(\Omega/J, \gamma/J)$ for different two-step protocols. Darker regions indicate lower S_L (higher purity).

Observations. Each panel presents the cycle-1 linear entropy $S_L = 1 - \mathrm{Tr}(\rho^2)$ mapped over $(\Omega/J, \gamma/J)$; darker regions correspond to lower S_L (higher purity). (i) The non-unital protocol (L_m, L_p) exhibits discrete low-S_L pockets at subharmonic modulation frequencies $\Omega_n = 2J/n$, demonstrating that purity can be recovered even at weak dissipation. (ii) The commuting hybrid (L_m, L_x) produces a narrow low-S_L band centered near specific Ω/J values. (iii) The anti-commuting hybrid (L_m, L_z), where the unital axis anticommutes with $H_0 = J\sigma_x$, forms a broad, tunable corridor of high purity for $\gamma/J \lesssim 1$. (iv) The fully unital protocol (L_x, L_z) fails to generate purification at small γ; suppression of S_L appears only at large modulation frequencies.

Interpretation. The dissipator pairing fixes the interference structure of the Floquet map $G(T) = e^{\tau\mathcal{L}_2}e^{\tau\mathcal{L}_1}$. Hybrid protocols, especially (L_m, L_z), place exceptional-point arcs and iso-S_L valleys inside the weak-dissipation regime; scanning Ω drives the system across these arcs, suppressing mixing modes and restoring purity at fixed small γ.

Robustness to Experimental Imperfections



Frequency span vs. noise level for three error types.

Error Type	W (Ω/J)	Change	Slope
Baseline	0.420	—	—
Timing jitter	0.406 ± 0.008	-3.3%	-0.66
γ-drift	$\mathbf{0.344 \pm 0.015}$	$\mathbf{-18.1\%}$	$\mathbf{-3.62}$
Detuning (J)	0.417 ± 0.003	-0.7%	-0.14

We tested how the hybrid $\{\sigma_z,\sigma_+\}$ protocol responds to small experimental errors (5% random noise) in timing, energy scale (J), and dissipation strength (γ). The *corridor width* W measures how wide the region of stable, high-purity operation remains as the modulation frequency Ω is varied. Timing and energy errors cause only minor changes, but when γ drifts, the purity corridor narrows by nearly 20%.
Takeaway: to keep the system stable and pure, the dissipation rate must be controlled within about $\pm 2.5\%$; other imperfections are far less critical.

Future Directions

- **EP-based metrology:** use Floquet-engineered exceptional points to amplify small parameter shifts (e.g., strain or bias field) in nanoscale resonators for precision calibration.
- **On-chip dissipation control:** implement tunable non-Hermitian couplings in superconducting nanodevices to actively stabilize coherence at hardware level.