

# Physics 570 Homework 1

Fall 2024

IUPUI Department of Physics

August 15, 2024

*Note: HW due by 11:59 pm on September 30, 2024*

1. Construct the following 2x2 matrix:

$$H = \begin{bmatrix} i\gamma & -1 \\ -1 & -i\gamma \end{bmatrix}$$

2. Find its eigenvalues and eigenvectors for  $\gamma = 0.1$  and  $1.01$ . Are the eigenvectors for both values of  $\gamma$  orthogonal? Find  $H^{-1}$ .
3. We will use  $H$  above to solve the equation,  $id\psi(t)/dt = H\psi(t)$ , which is a linear decay equation. Take the initial condition as  $\psi(0) = (1, 0)^T$  and  $\gamma = 0.5$ .
  - (a) Construct a matrix  $G = e^{-iHt}$  where  $t$  is time. Find  $\psi(t) = G\psi(t = 0)$ , and then  $I(t) = |\psi(t)|^2$ . Plot  $I_1(t)$  and  $I_2(t)$  on same graph.
  - (b) Now construct a matrix  $U = e^{-iDt}$  where  $D$  is a diagonal matrix wherein the diagonal elements are the eigenvalues of  $H$  and the off-diagonal elements are 0. Construct  $\psi(t) = VUV^{-1}\psi(t = 0)$ . Then calculate  $I(t) = |\psi(t)|^2$  and compare with  $I_{1,2}(t)$  of part (a).
4. Solve Problem 3 analytically and compare with the numerical results.