



Stay-at-home orders and second waves: a graphical exposition

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Received: 2 July 2020 / Accepted: 27 August 2020
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Abstract

Integrated epidemiological-economics models have recently appeared to study optimal government policy, especially stay-at-home orders (mass “quarantines”). But these models are challenging to interpret due to the lack of closed-form solutions. This note provides an intuitive and graphical explanation of optimal quarantine policy. To be optimal, a quarantine requires “the cavalry” (e.g., mass testing, strong therapeutics, or a vaccine) to arrive just in time, not too early or too late. The graphical explanation accommodates numerous extensions, including hospital constraints, sick worker, age differentiation, and learning. The effect of uncertainty about the arrival time of “the cavalry” is also discussed.

Keywords Virus · Epidemiology · Economics · Quarantine

JEL Classification H0 · I0

1 Introduction

Integrated epidemiological-economics models have recently appeared (Alvarez et al. 2020; Barro et al. 2020; Dewatripont et al. 2020; Eichenbaum et al. 2020; Hall et al. 2020; Jones et al. 2020; Piguillem and Shi 2020). But they are challenging to interpret due to the lack of closed-form solutions for optimal policy decisions, including the most common focus of the literature: stay-at-home orders (mass “quarantines” herein) and full opening.

The classic infectious SIR disease model dates back to Kermack and McKendrick (1927), where agents move through various stages (“compartments”): Susceptible, Infectious, and Recovery. The SIR model has been modified over the year to include an Exposed stage (SEIR) as well as two mutually exclusive infections stages

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(asymptomatic and symptomatic) stages (SEIIR). Stock (2020) provides an overview and discusses related estimation challenges.

The SIR model and its derivatives have several virus-specific key parameters. The case fatality rate μ equals total deaths (not recovered) divided by the total number of cases (infections).¹ It places an effective upper-bound on the number of deaths as a percent of the (Susceptible) population.² For COVID-19, the value of μ has been estimated to be as low as 0.2% to as high as 2%; both estimates likely reflect a fair degree of selection bias. The virus' reproduction number (R , sometimes called R_0) equals the number of people that an infected person in turn infects, on average. A value $R > 1$ implies that the number of cases increases geometrically over a *limited* range. In the pre-aware population (before voluntary social distancing), for example, COVID-19's value of R was estimated to be around 3 in China and the United States, although lower in Scandinavian countries.³

Public policy generally aims to drive the value of R below 1 (Budish 2020). Stay-at-home (mass quarantines) policies are a common device that dates back hundreds of years, either by social contrivance (e.g., The Black Death) or by conscription (e.g., The Spanish Flu and, more recently, COVID-19). For COVID-19, mass quarantines have been argued to be effective in "flattening the curve" (i.e., reducing new cases or deaths), with economics damages taken as a mostly secondary consideration, at least initially. Over time, economic considerations appear to become more relevant and the economic losses might be especially regressive, as lower-income workers are less able to work from home.

Figure 1 shows two plots of total projected deaths per 10,000 people over 365 days from a SIR type of model, calibrated with parameters consistent with the United States in early March 2020, including $\mu = 1\%$, $R = 2.04$, and the Susceptible population equal to 80% of the total population.⁴ Total deaths, therefore, are capped at 1% of the Susceptible population, 80 people. The leftmost plot projects total deaths per 10,000 people before any quarantine. The rightmost plot projects deaths with a quarantine equal to 56 days (8 weeks) in length, assuming, quite heroically, that a quarantine perfectly stabilized the number of infected people, that is, without any additional transmission. For now, assume a quarantine equal to 56 days produces the most economic damage that society will tolerate, that is, even if almost everyone would otherwise become infected.

There is no compelling reason to believe that the replication factor R should change *after* the quarantine date, B (discussed more below). The quarantine, therefore, produces a parallel horizontal shift equal to 56 days. However, geometry has the final say. By day 220, total deaths are nearly indistinguishable between the two

¹ In practice, popular press often refers to "cases" as "confirmed cases" consistent with a viral or antibody test. The SIR model itself does not make that distinction and so researchers often attempt to estimate infections based on confirmed cases (Stock 2020).

² Technically, this statement requires that anyone who Recovers either (i) is not re-infected a second time with the virus (or a mutation), or, if they are, (ii) they will recover again.

³ See HortaÇsu et al. (2020), Manski and Molinari (2020), and literature cited therein.

⁴ Simulations are run using the model provided by Alison Hill available at <https://alhill.shinyapps.io/COVID19seir/>. Additional parameter settings are discussed therein.



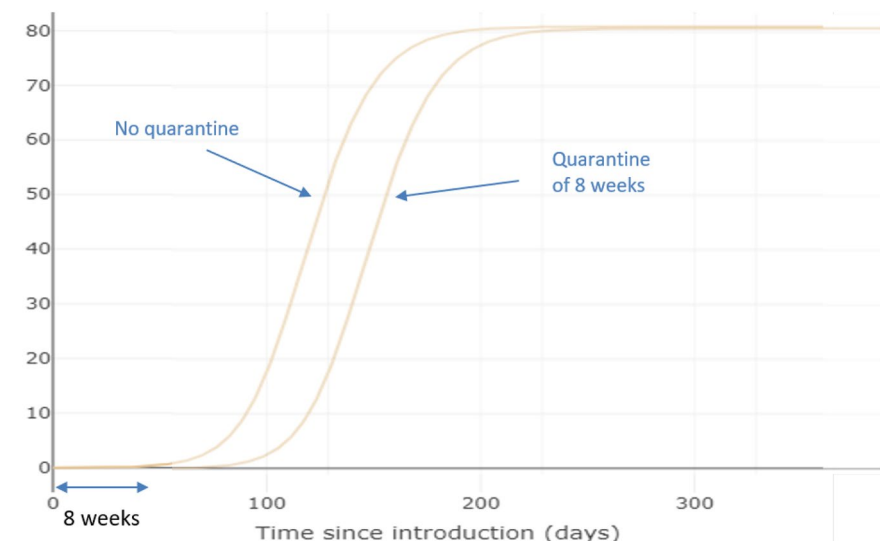


Fig. 1 Total deaths per 10,000 people, with and without quarantine

plots, as the second plot produces a “second wave” that quickly catches up to where a “first wave” would have landed. So, unless “the cavalry” (e.g., testing, strong therapeutics, or vaccines) arrives “just in time,” a mass quarantine of 56 days might result in a similar amount of total deaths but with a substantially more damaged economy.

2 A simple bang–bang model

The standard SIR model is an optimal control problem that can be fairly easily augmented with an economics model.⁵ Without much loss in generality, consider a reduced-form economy where $f(D)$ represents the cumulative total cost (say, lost cumulative Gross Domestic Product) from a quarantine of D days, where, naturally, $f(D \downarrow 0) > 0$, $f' > 0$, and $f'' > 0$.⁶ As in Alvarez et al. (2020), suppose that policymakers have access to only a simple policy tool B : quarantine, $B > 0$, or no quarantine, $B = 0$ (bang–bang control). Let $N(B)$ equal the cumulative number of lives (or life years) saved for a quarantine of B days, with a per-life (or life year) value of v . The policymakers’ objective function is then to choose B as

⁵ The SIR model is typically stated as continuous time rather than discrete, a distinction that is immaterial for our purposes.

⁶ In words, there are fixed costs to stopping the economy for any length and marginal costs rise in D . The assumption $f' > 0$ might not be true if a highly infected population without a quarantine is less productive than the quarantined population. We return to this point below.



$$B = \operatorname{argmax}_D [v \cdot N(D) - f(D)]. \quad (1)$$

Naturally, $N' > 0$, i.e., a longer quarantine saves more lives. Further define:

- L is the minimum plausibly desirable quarantine length, given fixed costs $f(D \downarrow 0) > 0$. Formally, $L = \sup_X [X \mid vN(D) < f(X), \forall D < X]$. Hence, if $B < L$ then the quarantine is clearly inefficient.⁷
- H is the maximum plausibly desirable quarantine length, given the convexity of costs, $f' > 0$ and $f'' > 0$. Formally, $H = \inf_X [X \mid vN(D) < f(X), \forall D > X]$. Hence, if $B > H$ then the quarantine is clearly inefficient.
- C = the number of days before “the cavalry” arrives, after which virus-related deaths cease. We assume that the value of C is exogenous, or, at best, dependent on government research support that has no direct impact on their choice of B .⁸

Clearly, the value of B must equal 0 or lie on the support $[L > 0, H]$. However, as written, equation (1) is deceptively simple since the value of C plays a key role in determining the shape of $N(D)$, i.e., $N(D) = N(D|C)$. Consider three cases:

Case 1 $C < L$, Fig. 2. “The cavalry” date C is too early for a quarantine of any length to cover the fixed costs of lost GDP, indicated by the positive intercept $f(D \downarrow 0) > 0$ on the dotted line representing lost GDP at any positive value of B . Hence, $B = 0$.

Case 2A $C > L$, Fig. 3. “The cavalry” arrives just in time, and so there is a value of $B > 0$ such that $0 < L < B < H$ where $v \cdot N(B|C) > f(B)$, where we normalize $v = 1$ without loss in generality.

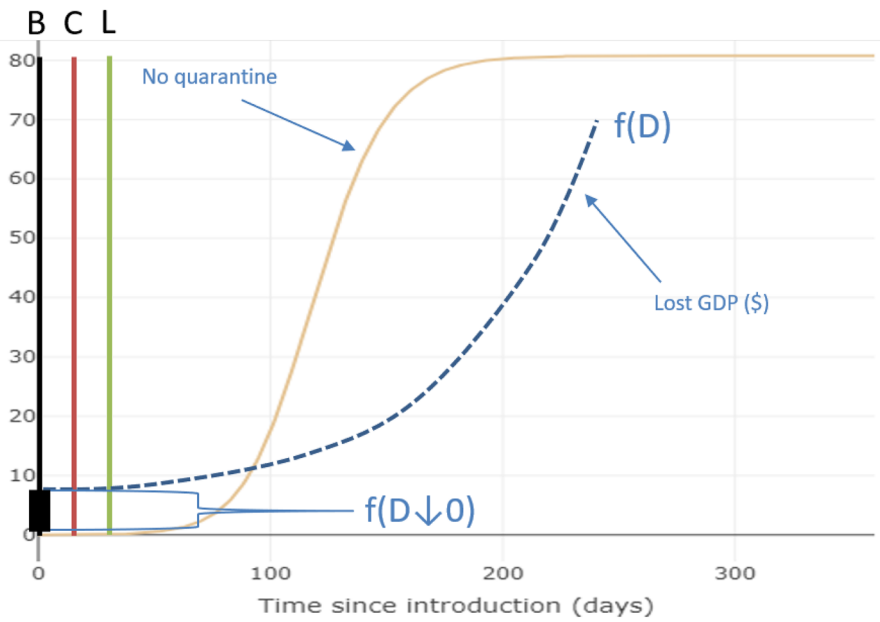
Case 2B $C > L$ with capacity constraints, Fig. 4. This case adds a capacity constraint (e.g., ICU beds, ventilator machines) that, once hit, creates an inflection point in the rate of deaths. Notice that the presence of capacity constraints increases the value of $v \cdot N(B|C)$ relative $f(B)$ even more, thereby increasing the date range in which “the cavalry” can arrive where $B > 0$ is optimal.

Case 3 $C \gg L$, Fig. 5. The “the cavalry” arrives too late, and so $v \cdot N(B|C) < f(D \downarrow 0)$ for all values $B \in [L, H]$.

⁷ In practice, a quarantine is announced with sufficient notice to, for example, allow workers who can work remotely to access their business offices in order to relocate their required business tools, such as computer. Given this assumption, it is not optimal in the model herein for a quarantine to start with an additional delay since the fixed costs would not be further reduced.

⁸ In principle, a government could choose to fund more research in exchange for a smaller value of B . I have not seen any evidence that this trade-off is considered in actual policy-making.





Source: Authors' calculations using Hill (2020).

Fig. 2 Case 1, $B = 0$ Source Authors' calculations using Hill (2020)

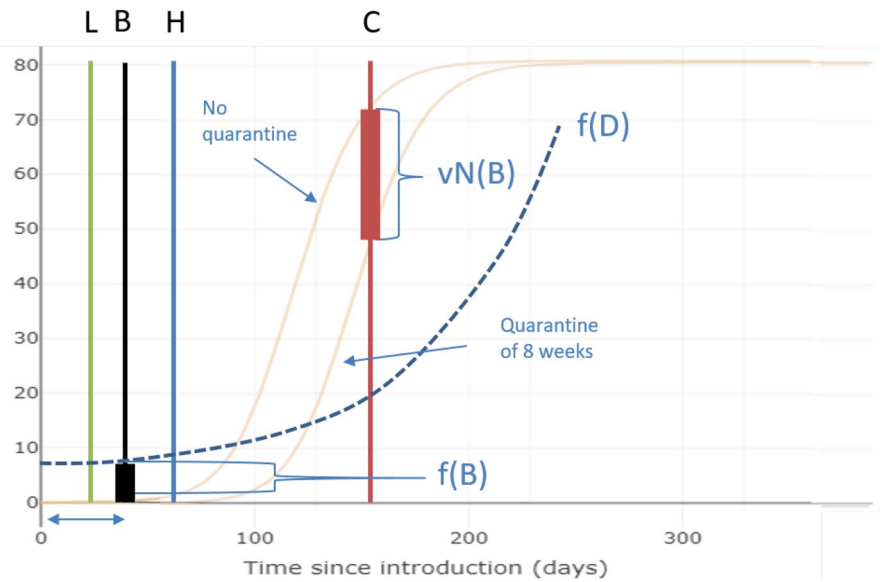


Fig. 3 Case 2A, $B > 0$



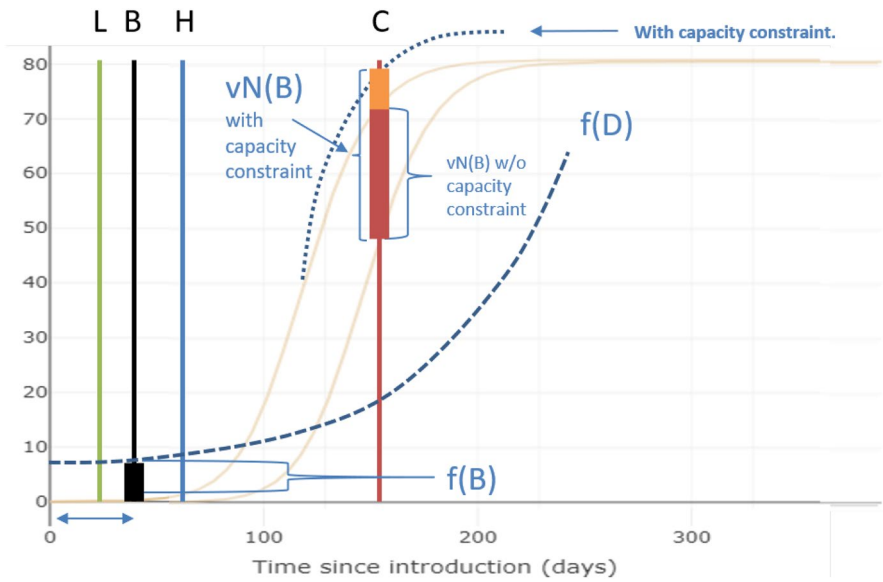


Fig. 4 Case 2B, capacity constraints, $B > 0$

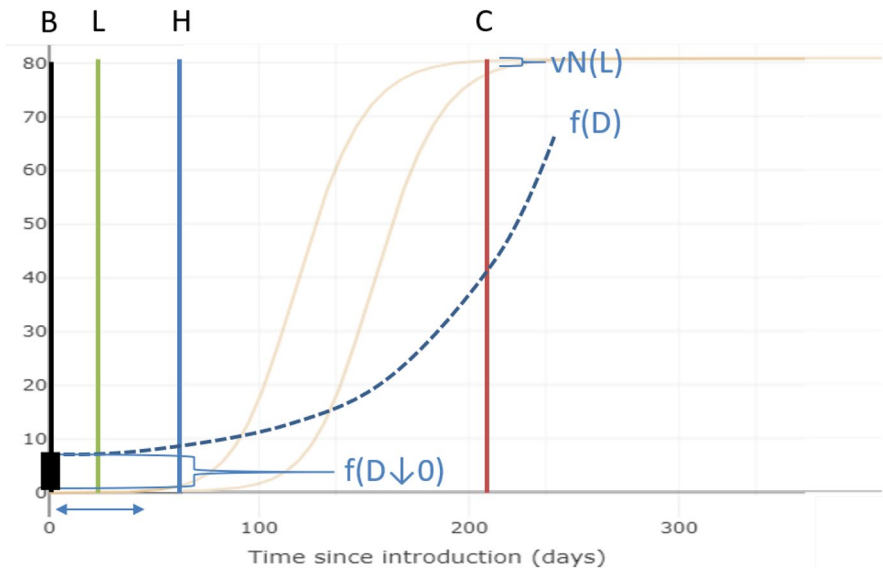


Fig. 5 Case 3, Do Nothing Again

3 Discussion

Notice that quarantines are optimal only if “the cavalry” arrives just in time, not too soon and not too late. Since quarantines are used sparingly in practice in



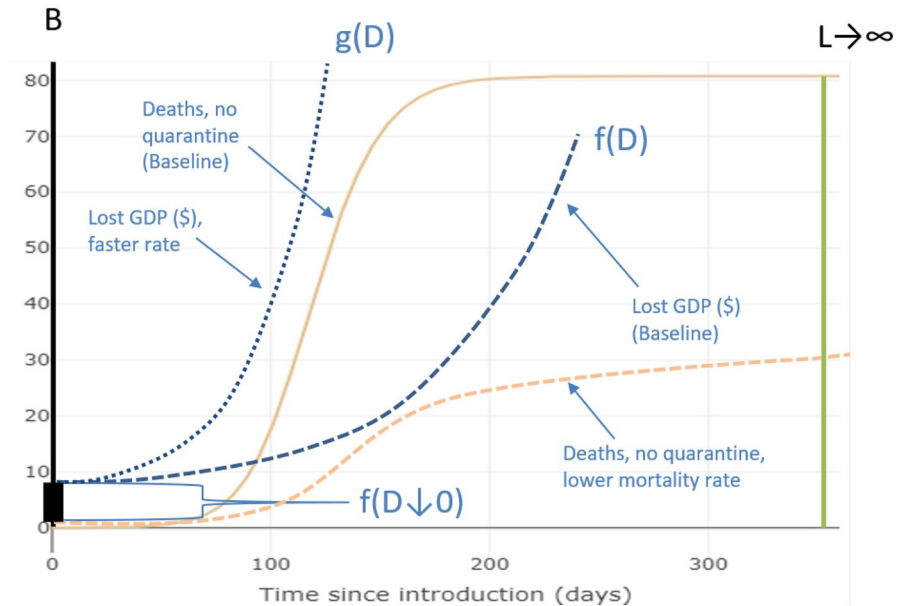


Fig. 6 Case 1, Modified, $B = 0$

response to novel viruses, Case 1 is apparently fairly standard. In fact, as Fig. 6 shows, it is possible that there is no date $D > 0$ where graphs $v \cdot N(D)$ and $f(D)$ cross, and so $L \rightarrow \infty$. That could occur if:

- Costs rise (e.g., GDP falls) even faster in D , as shown by $g(D)$.
- The value of μ is sufficiently small (see the new dotted brown cumulative death line).
- The value of v were sufficiently small (not shown).

While v is often taken as the “statistical value” of human *life*, existing public health policies, including organ transplant selection, tend to value *life years*, which brings up the uncomfortable topic regarding the age bias of any given disease (e.g., H1N1 and COVID-19 have the opposite age biases). Indeed, focusing on lives instead of life years easily produces *reductio ad absurdum* type of arguments in public health (e.g., spending \$8 million per dose on a new drug that extends life expectancy by minutes). If so, the age bias of COVID-19 would tend to reduce the case for mass quarantines.

4 Extensions

The graphical analysis herein can easily accommodate numerous extensions:



4.1 Sick workers

We have not explicitly discussed the case where sick people cannot work (contribute to GDP). It might be easy to embellish the importance of this effect in a discrete time model, as many infected people will be asymptomatic and many symptomatic people recover during normal “paid time off” policies already allotted by many firms (i.e., the time away from production may be used anyway). Of course, death will reduce productivity, although the age bias of COVID-19 reduces this impact as well. Even so, the cost function $f(D)$ could simply be redefined as “excessive cost” of D quarantine days *relative* to a sicker population. It would be highly unlikely—in fact, none of the current integrated models in the literature suggest—that this excess cost would be negative over the range $D > 0$.

4.2 Multi-risk model

We also extend the graphical analysis to a multi-risk model as in Acemoglu et al. (2020) that distinguishes by age cohorts. In this case, the cost function $f(D)$ associated with an *age-targeted quarantine* (e.g., lock down of homes for the aged) would be flattened, thereby opening up the range in which $B > 0$ is optimal. Simply redraw $f(D)$ in Fig. 3 or Fig. 4 to be less convex.

4.3 Viral testing

The analysis above alluded to the availability of greater testing over time as one potential way to interpret the arrival of “the cavalry” at the right point in time. We think that is the best interpretation, rather than changing the cost or death curves. Changing the cost or death curves in response to testing risks muddling the distinction between a *mass* quarantine policy, such as indiscriminate stay-at-home orders, from *specific* quarantines, which discriminate on detected infections, and represent business as usual for any detected sickness (i.e., stay home if you are infected).

4.4 Learning/education

One potential argument for mass quarantines reflects a reduction in ambiguity when clinical evidence is early or unreliable, potentially due to selection bias in testing. However, the presence of ambiguity could cut either direction, as it is unclear if the cost of a type-1 (false rejection of the null that new disease is substantially different) or the cost of a type-2 error (failure to reject the false null), when there is a clearly trade-off between the costs of economic misery (which might be especially regressive) versus lost life years. Another argument is that the viral reproduction rate R might be lower after the quarantine is lifted (for $D > B$). If true, the accumulated death curve would no longer just parallel shift. It would also flatten out at a lower value of total deaths (per 10,000 people) than



the original curve, thereby opening up the range where “the cavalry” can successfully arrive and produce $B > 0$. (For example, redraw Fig. 3 or 4 where the second accumulated death curve flattens out at a value 50 instead of around 80, as currently shown.) However, it is unclear why a mass quarantine would be more of a “teachable moment” than a public education campaign without quarantine, or how relaxing the quarantine does not signal of relaxation of concern.

4.5 Uncertainty⁹

In reality, the timing of the arrival of “the cavalry” is subject to considerable uncertainty. The graphical analysis presented herein does not easily accommodate this uncertainty. However, the model itself could be easily adjusted to allow for it. The cost function (lost GDP) f is strictly convex in the number quarantine days D . While mortality is convex across a limited range of D , the *difference* in mortality, which conveys the actual value of the quarantine, is nearly linear. The additive nature of the benefit and cost of a quarantine in the objective function (1) means that the net benefit (benefit less cost) of a quarantine is largely concave (when positive valued) across its length in days D . The third derivative of this net benefit function can be positive or negative, implying that uncertainty over the arrival of the cavalry could raise or lower the optimal length of quarantine B .¹⁰

5 Conclusions

An apparent guiding principle in recent policy-making is that a temporary quarantine would sufficiently “crush the curve” to produce a *permanent* reduction in infections and deaths. Indeed, advocates of the quarantine argued that the short-run costs would be outweighed the long-term benefits in terms of lost lives.

However, the simple model herein shows that this logic is incomplete at best. Society would never tolerate a permanent quarantine, as the costs would be too high. The argument for a quarantine, therefore, is not about short-run costs versus long-term benefits *per se*: it is a pure timing game. And, it is a very specific timing game at that. “The cavalry” (e.g., mass testing, vaccines, therapeutics, etc.) must arrive “just in time”—not too soon or too late—for the benefits of a quarantine to exceed the costs.

At the time of this writing, in fact, “the cavalry” has apparently not arrived. Many countries throughout the world are experiencing the so-called “second waves” in the form of re-emergence of COVID-19 infections after lifting quarantines.

⁹ I appreciate conversations with the Editors, Alex Mürmann and Casey Rothschild, which led to the discussion and specific insights about the role of uncertainty.

¹⁰ Define Γ as the number of deaths at day D so that $N(B) = \Gamma(C) - \Gamma(C - B)$ is the number of lives saved with a quarantine of length B . If C is uncertain, then the first-order condition (FOC) for the optimal quarantine date B at an (interior) point is $E[v\Gamma'(C - B)] = f'(B)$. So, a small amount of uncertainty in C reduces (increases) the left-hand side of the FOC if Γ' is concave (convex), thereby reducing (increasing) B . Notice from Fig. 3, for example, that Γ' is both concave and convex in D .



In theory, extensive viral testing could have been the required cavalry, and so “second waves” are being interpreted as a failure of government policy to make testing more available. Maybe. But probably not.

At the time of this writing, current COVID-19 tests are either too slow, if they are reliable, or too unreliable, if they are not slow. To have any meaningful impact on the reproduction number R , current viral tests would have to be administered much more frequently and across a much larger share of the population—likely testing a majority of the population more than once per month—during a post-quarantine period at pre-pandemic levels of social distancing.

In the end, personal behavior (e.g., wearing masks, proper spacing, small group sizes, and meeting outdoors), rather than direct government policy, might still be the most effective short-term response to pandemics like COVID-19.

Acknowledgements Thanks to the editors and an anonymous referee for very useful comments.

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