Optimal time interval reproduction in a neural circuit model

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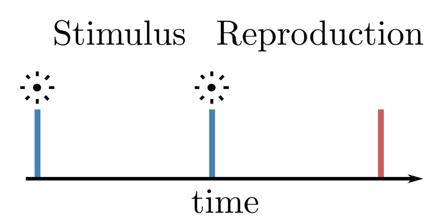
Introduction

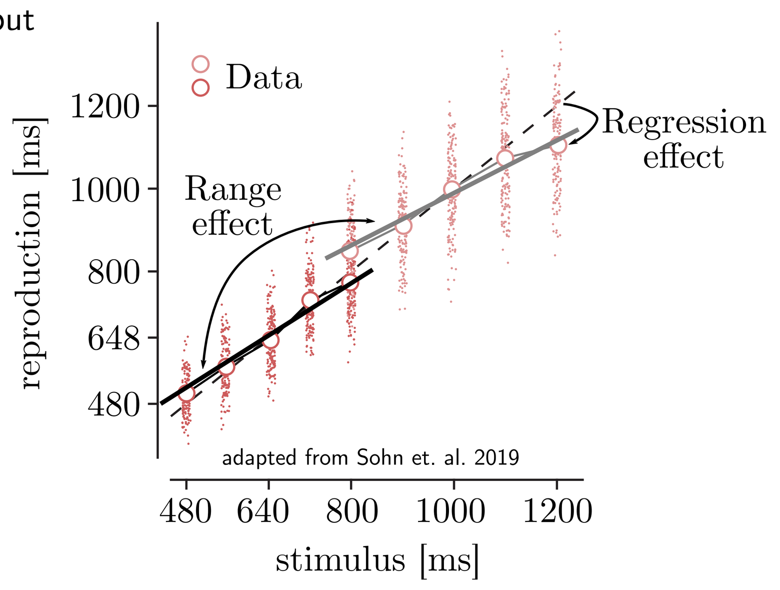
Sensory information is combined with **expectations** (based on prior knowledge) to drive behaviors. The interaction of current sensory input and expectations is likely subject to **error minimization**.

Time reproduction is one of the behavioral methods to investigate error minimization and related **optimal behavioral strategies**.



Psychophysical characteristics of magnitude estimation

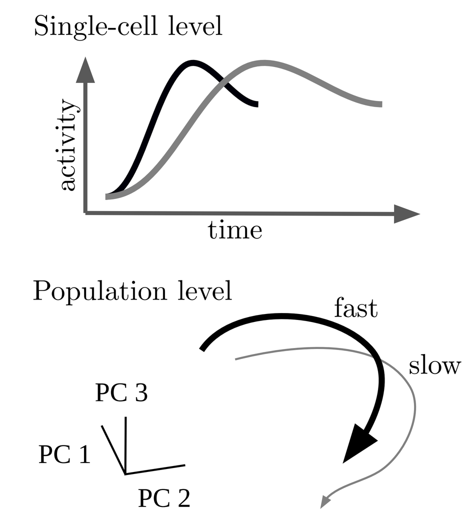
- **Regression effect**: estimates tend towards the mean - the fitted slope is smaller than 1
- **Range effect**: regression effect scales with the range of stimuli
- **Scalar variability**: errors monotonically increase with stimulus size



Timing by temporal scaling

Recordings in the medial frontal cortex (MFC) show:

- **Firing rate** profiles are **temporally scaled** to match the produced intervals.
- **Population activity** evolves along **similar neural trajectory** at **different speeds**.
- Controling timing of future movements by adjusting an internal speed command.
- Speed command is updated after stimulus presentation based on the **error between prediction and the actual stimulus duration**.



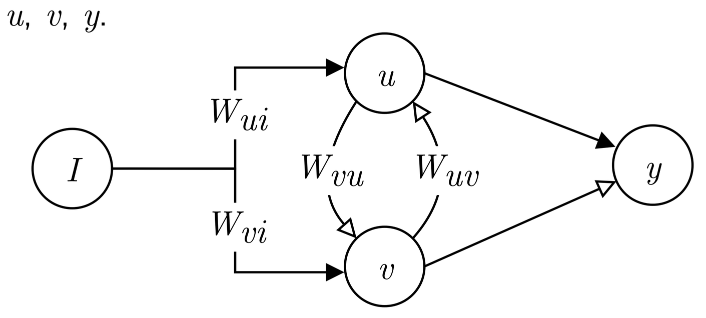
Circuit model to reproduce behavioral effects

- The circuit model proposed by Egger et. al. 2020 is based on the **scaling phenomenon** found in brain data.
- The model is used in simulations of **time reproduction experiments** to **reproduce behavioral effects**.

Circuit model

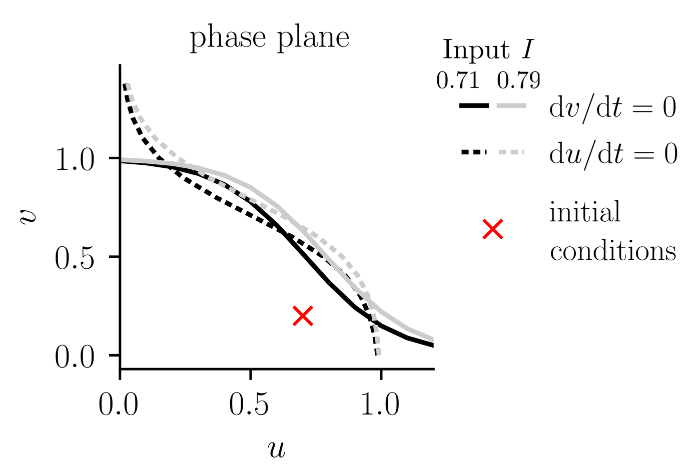
Basic circuit

Speed control can be achieved by a simple model consisting of three units u, v, y.



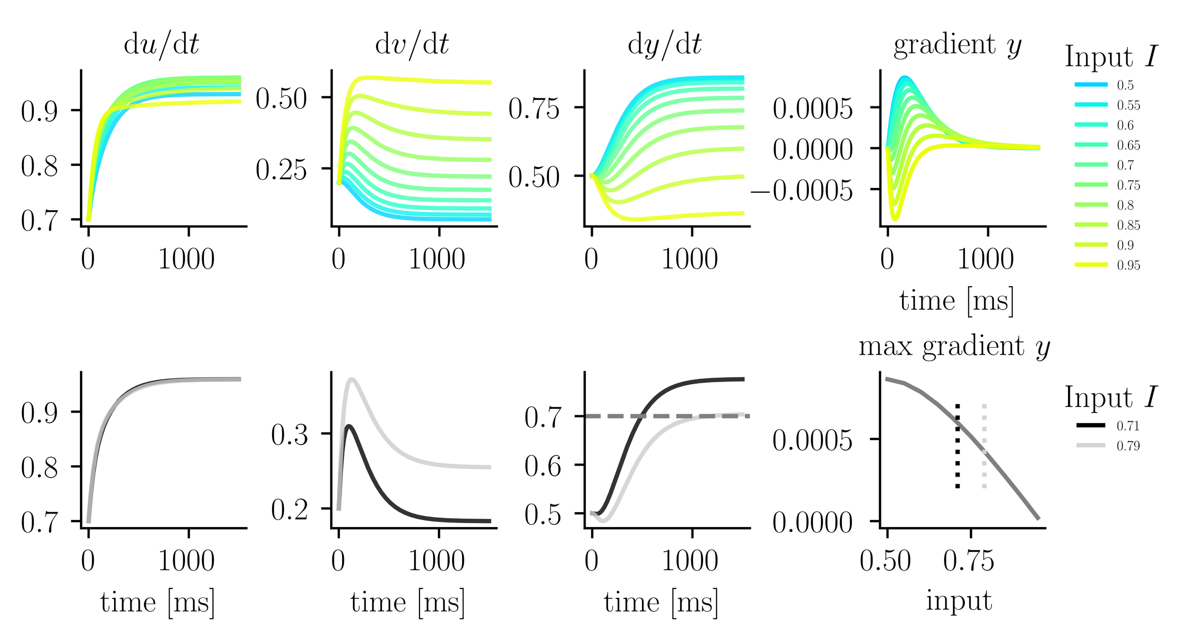
$$\tau \frac{\mathrm{d}u}{\mathrm{d}t} = -u + \theta(W_{uI} I - W_{uv} v + \eta_u)$$
$$\tau \frac{\mathrm{d}v}{\mathrm{d}t} = -v + \theta(W_{vI} I - W_{vu} u + \eta_v)$$
$$\tau \frac{\mathrm{d}y}{\mathrm{d}t} = -y + W_{yu} u - W_{yv} v + \eta_y$$

- Two mutually inhibitory units u, v receive shared tonic input I.
- Inputs to u, v are governed by a sigmoidal activation function θ.
- The readout unit y receives excitatory and inhibitory inputs from u and v.
- This results in ramp-like activity in y.
- Stochastic synaptic inputs are modeled as independent white noise η with standard deviation σ.



Input dependent speed control

- Increasing the input I to u and v corresponds to moving their nullclines in the phase plane.
- The input to the circuit controls at which speed the readout unit increases its activity.



Inverse relation of speed and input:

- Higher inputs I correspond to smaller gradients in y.
- Lower inputs I correspond to larger gradients in y.

The model uses a fixed threshold y_{th} for reproducing intervals:

- For higher I, the threshold is reached after a longer time interval.
- For lower I, the threshold is reached after a shorter time interval.

Update and reset mechanism

In interval reproduction experiments, a stimulus interval is presented and has to be reproduced.

Reaching a threshold y_{th} can be understood as movement initiation time.

Update mechanism that flexibly adjusts I:

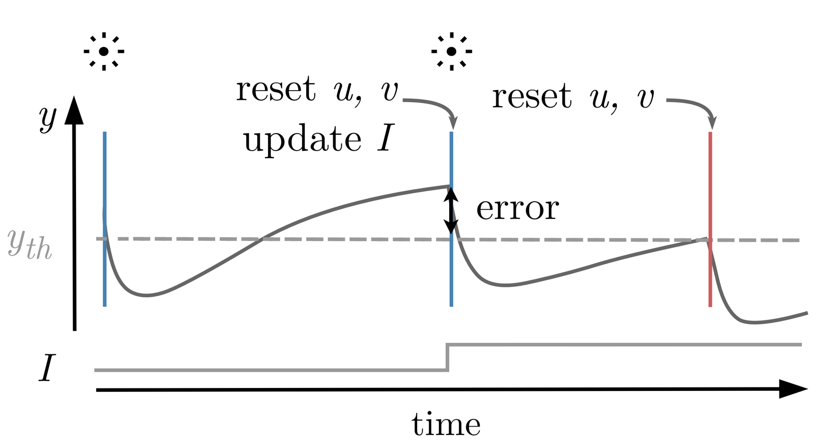
- Adjusting I based on an error signal controlls the reporduced time interval.
- The error signal is based on the difference between the level y reached after the stimulus presentation and the threshold y_{th}.
- The error is weighted by an update parameter K.

$$\tau \frac{\mathrm{d}I}{\mathrm{d}t} = sK(y - y_{\mathrm{th}})$$

Reset mechanism:

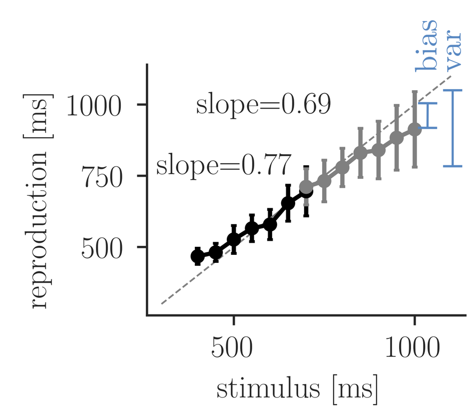
- After stimulus presentation u and v receive a transient input I_r to reset the dynamics for the time reproduction.

$$\tau \frac{\mathrm{d}u}{\mathrm{d}t} = -u + \theta(W_{uI} I - W_{uv} v + \eta_u - I_r)$$
$$\tau \frac{\mathrm{d}v}{\mathrm{d}t} = -v + \theta(W_{vI} I - W_{vu} u + \eta_v + I_r)$$



Reproducing behavioral effects

Behavior of simulation



Experiment simulation:

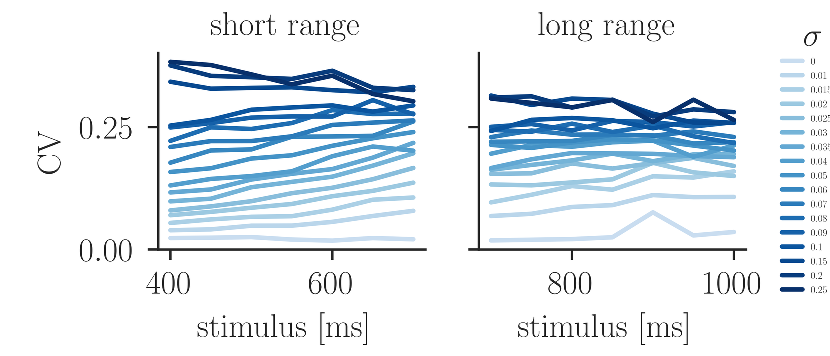
- large circles show average reproduction with standard deviation of stimulus interval for both short (black) and long (grey) range.
- Experiment was contucted with a time constant $\tau = 140$ ms.
- The update parameter was chosen as: $K_{\mathrm{short}} = 15, K_{\mathrm{long}} = 10.5$

Coefficient of variation

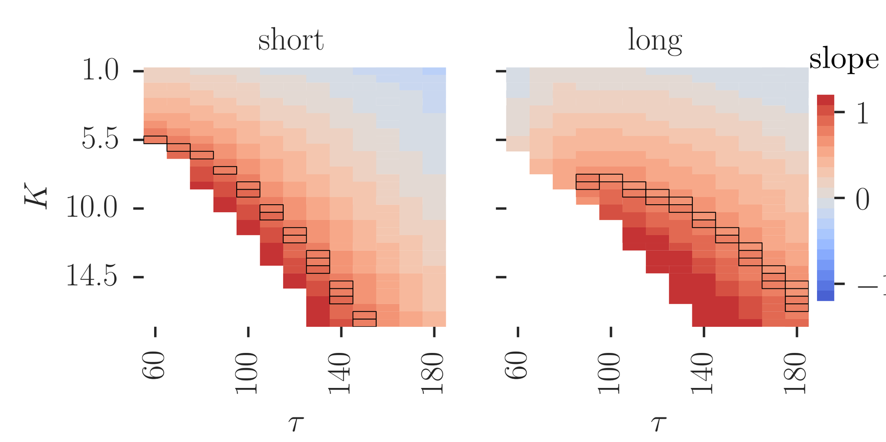
The CV is a measure for variability.

$$\mathrm{CV} = \frac{\sigma_{\text{productions}}}{\text{stimulus}}$$

- In data, the CV ranges between 0.1 to 0.4.
- We chose the noise with standard deviation 0.02, to get a CV of 0.1 in the experiment.

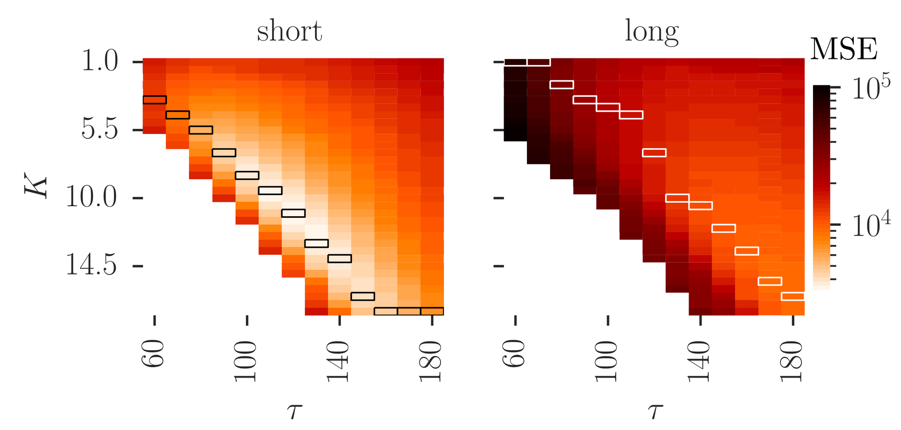


Behaviorally plausible slopes



- In data, slopes are around 0.83 for the short range and around 0.73 for the long range.
- Behaviorally plausible slopes can be achived by the model for different time constants τ and update parameter K (framed black).

Optimal update parameter

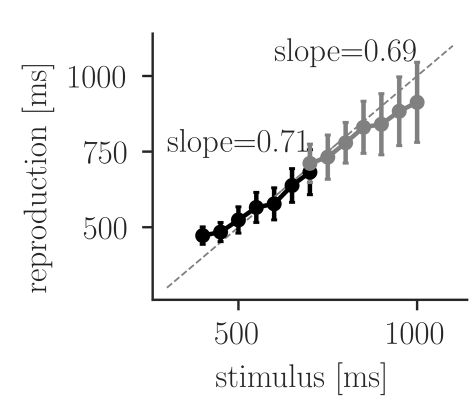


Can plausible slopes be achieved by minimizing the mean squared error (MSE)?

- MSE is defined as the squared bias and variance over the course of the experience.

$$\mathrm{MSE} = \mathrm{BIAS}^2 + \mathrm{VAR}$$

$$\mathrm{BIAS}^2 = \frac{1}{S}\sum_{i=1}^{S}(\bar{t}_{r_i} - t_{s_i})^2 \qquad \mathrm{VAR} = \frac{1}{S}\sum_{i=1}^{S}(\sigma_i^2)$$

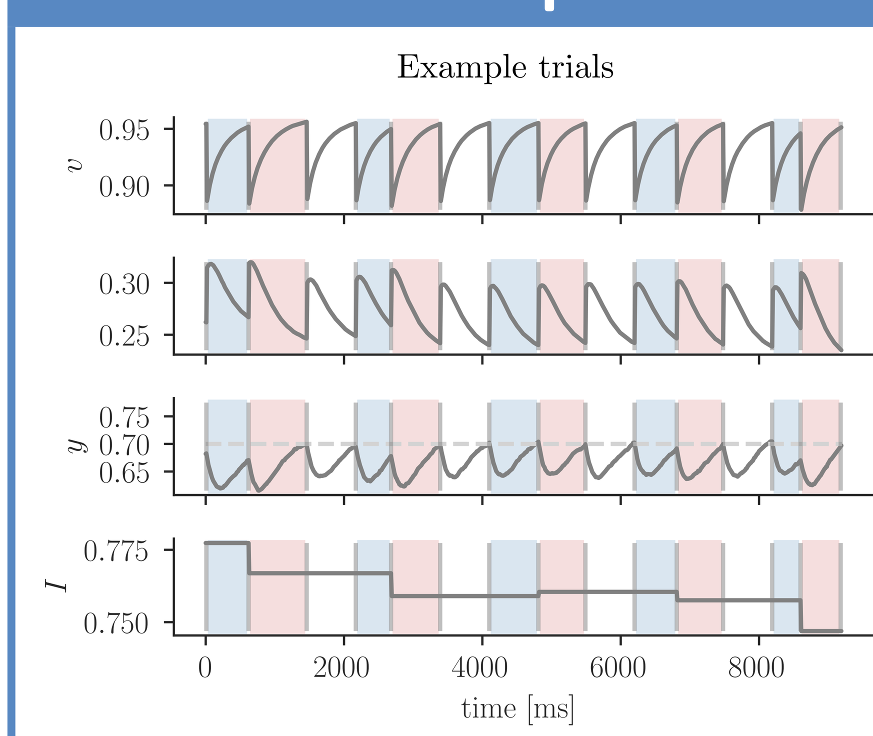


Behavior with optimal update parameter K based on MSE with time constant $\tau = 140$ ms.

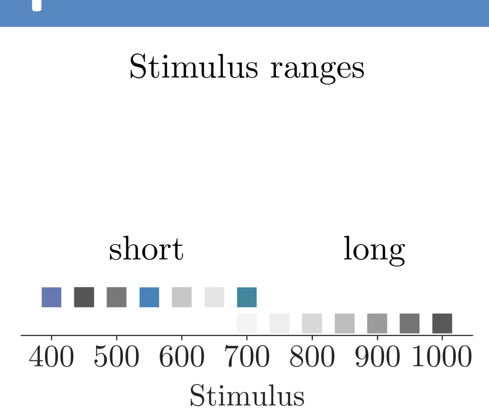
$K_{\mathrm{short}} = 14$
$K_{\mathrm{long}} = 10.5$

- Smaller optimal K for longer range means less influence of current stimulus (error) and more influence of prior on the update of I.

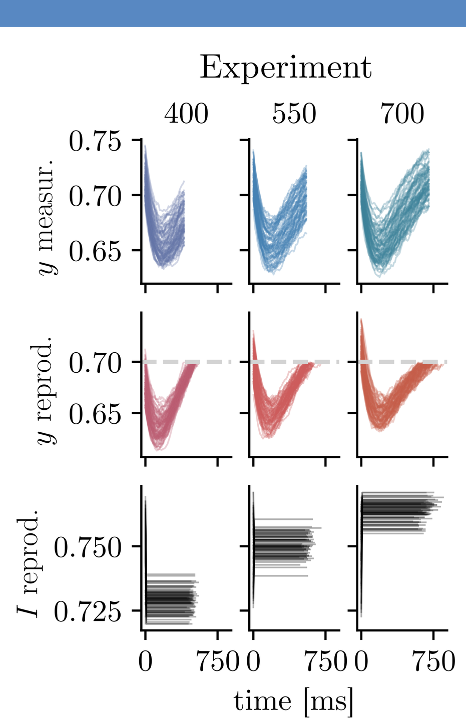
Time interval reproduction experiment



- Five trials with stimulus presentation (blue), its reproduction (red) and a fixed delay between consequitive trials (white).
- I is updated after each stimulus presentation.
- The reproduction is terminated when y reaches the threshold $y_{\mathrm{th}} = 0.7$.



- Simulations with two stimulus ranges: a short range comprising shorter time intervals and a long range comprising longer time intervals.
- The ranges both contained a 700 ms stimulus interval.
- Experiment was simulated with 500 randomly chosen time intervals from one of the stimulus ranges.



- Measurement, reproduction and input over the course of the experiment.
- Trials sorted according to stimulus: shown are trials for 400, 550 and 700 ms of the short range.

Conclusions

- The **update parameter K** was adjusted in accordance with **error minimization**.
- The **regression effect**, the **range effect** and **scalar variability** could be reprocuced.
- **Optimal K** resulted in putting more weight on the **prior experience** for longer stimuli.
- Long stimuli naturally entail **more uncertainty**, and thus putting more **weight on prior expectations** is biological plausible.

Limitations:

- There is a **general underestimation** of longer intervals not just a simple regression effect.
- **Intervals lasting several seconds** cannot be reproduced by the model, since the activity y does not reach the **threshold y_{th}** anymore. This is because the **slope** of the activity becomes **too shallow**.

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