Ludwig-Maximilians-Universität
München
Department Biologie II Computational Neuroscience

Graduate School of
Systemic Neurosciences
LMU Munich

Report

Computational Simulation of Time Perception: Model Description and Implementation

Katharina Bracher
Student ID: 11754625

Supervision: Dr. Kay Thurley

Contents

1 Behavioral Effects in Magnitude Estimation

Magnitude estimation is subject to noise that arises from external sources i.e. the statistics of the environment and internal sources i.e. neural representation of the input and the behavior. Across sensory modalities, characteristic behavioral effects are identified (Petzschner et al. 2015). The most prominent observation is a regression to the mean of the stimulus range, i.e. small stimuli are overestimated whereas large stimuli are underestimated (*regression effect*). This effect intensifies for ranges with larger stimuli (*range effect*) For larger stimuli the standard deviation of estimates increases monotonically (*scalar variability*). Finally, the recent history of stimuli presentations influences the current stimuli estimation (*sequential effect*). All effects mentioned above are displayed in Fig. 1.

Modality-independence of these effects suggests the existence of a common underlying principle or processing mechanisms, that would explain e.g. a optimal strategy for unreliable judgments due to noise (in stimuli and estimates).

During time perception and time reproduction experiments, neural activity displays characteristic trajectories in a low-dimensional space (Meirhaeghe et al. 2021, Wang et al. 2018, Henke et al. 2021). The neural trajectories are consistently influenced by prior beliefs. Flexible motor timing can be achieved by controlling the speed of neural dynamics (Sohn et al. 2019, Wang et al. 2018). Further, it has been found that neural activity in anticipation of a delayed response reaches a fixed threshold with rate inversely proportional to delay period

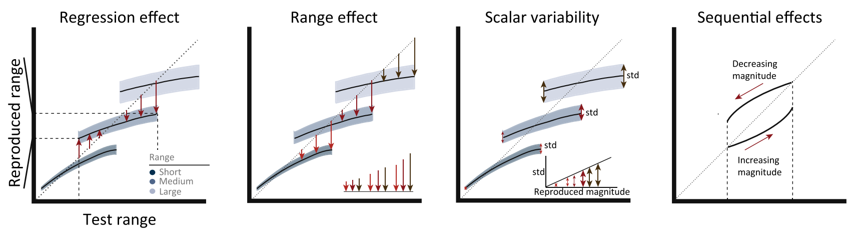

Figure 1: **Behavioral Effects** adapted from Petzschner et al. 2015.

Murakami et al. 2014, Mita et al. 2009). Wang et al. 2018 proposed a potential neural mechanism for speed control and based on that Egger et al. 2020 developed a neural circuit model for sensorimotor timing.

2 Model Description

2.1 Basic Circuit

Flexible speed control can be achieved by a simple model consisting of three units, u, v, y that represent population activity. Two units, u and v receive symmetric input I ($W_{uI} = W_{vI} = 6$) and have symmetric mutual inhibitory projections onto each other ($W_{uv} = W_{vu} = 6$). The inputs to u and v is governed by a sigmoidal activation function $\theta(x) = \frac{1}{1+exp(-x)}$ and all three units have a time constant $\tau = 100$. y is the output unit and receives excitatory input from u and inhibitory input from v ($W_{yu} = W_{yv} = 1$)which results in ramp-like behavior. Stochastic synaptic inputs are modeled as independent white noise η_u, η_v, η_y. The dynamics of u, v, and y are defined as follows:

$$\begin{aligned}
\tau\frac{du}{dt} &= -u + \theta(W_{uI}I - W_{uv}v + \eta_u) \\
\tau\frac{dv}{dt} &= -v + \theta(W_{vI}I - W_{vu}v + \eta_v) \\
\tau\frac{dy}{dt} &= -y + W_{yu}u - W_{yv}v + \eta_y
\end{aligned} \tag{1}$$

The speed at which the output y evolves can be controlled by the input to u and v (Fig. 2)c) and determines the interval after which y reaches a fixed threshold y_0. Reaching the threshold y_0 can be understood as the movement initiation time in time reproduction experiments and adjusting I means producing longer or shorter intervals. Depending on the input I, parameter and initial conditions, the system shows different dynamics. For low levels of I (0<I<0.5) the system has three fixed points (2 stable, 1 unstable at u=v) and y ramps up faster the higher the input I. For intermediate values of I (0.5<I<1) the system still shows three FP of the same sort and y ramps up with a slope that is inversely proportional to the input I (y ramps up slower the higher the input I, see schematic in Fig. 2a, b). For high I (1<I) the system has one stable fixed point (at u=v) and y ramps down faster for higher I. In this report, the intermediate input regime is explored. In this regime, higher a higher input I results in a smaller slope of y, such that the threshold y_0 is reached after a longer interval. Thus, input is controlling the speed of the dynamic. Initial conditions of u, v and y have been optimized for in Egger et al. 2020 and are set to $u_0 = 0.7, v_0 = 0.2, y_0 = 0.5$.

2.2 Update Mechanism and Experiment simulation

For simulating time reproduction experiments, the relation of the input I to u and v with the slope of y in combination with a fixed threshold for y is used. By flexible adjusting I based on an error signal, the threshold crossing of y can be delayed or moved to earlier times. Similar to classical time reproduction experiments, a simulated trial has two epochs. A measurement epoch that has the duration of the stimulus interval and a reproduction epoch that starts immediately after the measurement epoch. The reproduction ends, when y reaches the fixed threshold y_0. The time from the end of the measurement epoch and the threshold-crossing of y yields the reproduced interval that is aimed to equal the stimulus interval that was given during the measurement epoch.

The following update mechanism of I is based on the intermediate regime of I, that shows an inverse relation of I to the slope in y. The error signal is determined at the end of the measurement epoch, by difference of y t o the threshold y_0. If the threshold y_0 is not reached during the measurement epoch, the slope has to be adjusted, such that y ramps up faster in order to reach the threshold at exactly the time of the stimulus interval. For a steeper slope, I needs to be reduced. If y crossed the threshold before the measurement epoch ends, so is above y_0 by the end of the stimulus interval, the slope needs to be reduced in order to reach the threshold at a later time in the reproduction. For a shallower slope, I needs to be increased. Adjusting I is done by the difference of

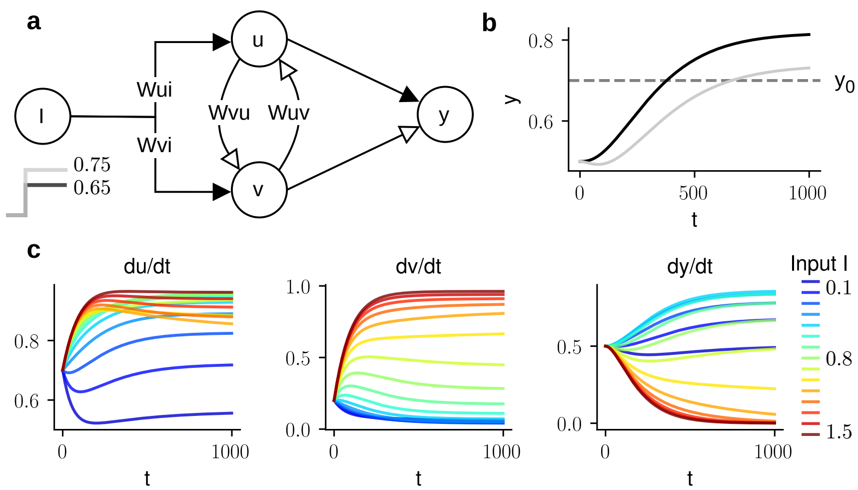

Figure 2: **Basic Circuit and Input Regimes (a)** u and v share a common input I. Dynamics for example input I=0.75 in gray and I=0.65 in black plotted in (b). The input is governed by weights W_{uI} and W_{vI}. The two units have reciprocal inhibitory connections with weights W_{uv} and W_{vu} that determine the inhibitory strength and project to the output unit y with an excitatory connection from u and an inhibitory connection from v. Excitatory and inhibitory connections are shown by filled and open arrows, respectively. **(b)** Dynamics of y for intermediate regime with input I=0.75 in gray and I=0.65 black. There is an inverse relation of input strength and slope. With higher input, the threshold at 0.7 (dashed line) is reached after a longer time interval. **(c)** Dynamics of u, v, y for inputs from 0.1 to 1.5 are shown. Initial conditions are set to $u_0 = 0.7, v_0 = 0.2, y_0 = 0.5$. With these initial conditions and values of I>=0.5, the relation of steady state activity of y (and slope to reach steady state) is inverse to I (intermediate and high I regime). For I<0.5 the steady state (slope) is smaller the smaller I (low I regime).

$(y - y_0)$, weighted by a memory parameter K right at the end of the measurement epoch. u and v receive a transient input $I - R$ to reset the dynamics (see Eq. 2).

$$
\begin{aligned}
\tau\frac{du}{dt} &= -u + \theta(W_{uI}I - W_{uv}v + \eta_u - I_r) \\
\tau\frac{dv}{dt} &= -v + \theta(W_{vI}I - W_{vu}v + \eta_v + I_r) \\
\tau\frac{dy}{dt} &= -y + W_{yu}u - W_{yv}v + \eta_y \\
\tau\frac{dI}{dt} &= sK(y - y_0)
\end{aligned}
\tag{2}
$$

Updating I based on feedback to adjust rate in reproduction stages: measurement, update and reset, reproduction until threshold update: delta y-th, weighted parameter: memory parameter K, reset, initial conditions, threshold timeouts

3 Implementation of Model

Definition of timeouts

3.1 Modules

Euler Implementation to Solve Differential Equation parallel Simulation experiment simulation update mechanism

3.2 Structure of Code

parallel simulations, experiment simulation

4 Results and Outlook

experiment simulation plot behavioral plot parameter search, extending units, neural trajectories

limitations and explorations regimes