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Report

Computational Simulation of Time Perception: Model Description and Implementation

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1 Behavioral Effects in Magnitude Estimation

Magnitude estimation is subject to noise that arises from external sources i.e. the statistics of the environment and internal sources i.e. neural representation of the input and the behavior. Across sensory modalities, characteristic behavioral effects are identified (Petzschner et al. 2015). The most prominent observation is a regression to the mean of the stimulus range, i.e. small stimuli are overestimated whereas large stimuli are underestimated (*regression effect*). This effect intensifies for ranges with larger stimuli (*range effect*) For larger stimuli the standard deviation of estimates increases monotonically (*scalar variability*). Finally, the recent history of stimuli presentations influences the current stimuli estimation (*sequential effect*). All effects mentioned above are displayed in Fig. 1.

Modality-independence of these effects suggests the existence of a common underlying principle or processing mechanisms, that would explain e.g. a optimal strategy for unreliable judgments due to noise (in stimuli and estimates).

During time perception and time reproduction experiments, neural activity displays characteristic trajectories in a low-dimensional space (Meirhaeghe et al. 2021, Wang et al. 2018, Henke et al. 2021). The neural trajectories are consistently influenced by prior beliefs. Flexible motor timing can be achieved by controlling the speed of neural dynamics (Sohn et al. 2019, Wang et al. 2018).

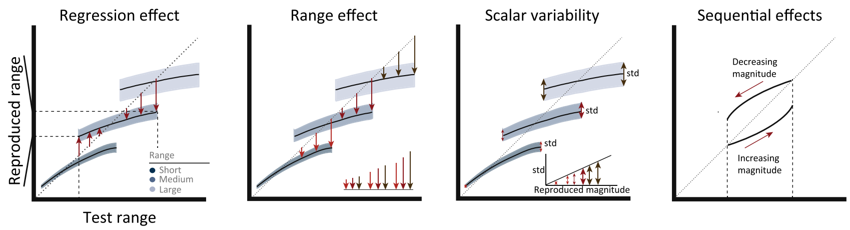

Figure 1: **Behavioral Effects** adapted from Petzschner et al. 2015.

Further, it has been found that neural activity in anticipation of a delayed response reaches a fixed threshold with rate inversely proportional to delay period Murakami et al. 2014, Mita et al. 2009). Wang et al. 2018 proposed a potential neural mechanism for speed control and based on that Egger et al. 2020 developed a neural circuit model for sensorimotor timing.

2 Model Description

2.1 Basic Circuit

Flexible speed control can be achieved by a simple model consisting of three units, u, v, y that represent population activity. Two units, u and v receive symmetric input I ($W_{uI} = W_{vI} = 6$) and have symmetric mutual inhibitory projections onto each other ($W_{uv} = W_{vu} = 6$). The inputs to u and v is governed by a sigmoidal activation function $\theta(x) = \frac{1}{1+exp(-x)}$ and all three units have a time constant $\tau = 100$. y is the output unit and receives excitatory input from u and inhibitory input from v ($W_{yu} = W_{yv} = 1$)which results in ramp-like behavior. Stochastic synaptic inputs are modeled as independent white noise η_u, η_v, η_y with standard deviation σ. The dynamics of u, v, and y are defined as follows:

$$\begin{aligned}\tau\frac{du}{dt} &= -u + \theta(W_{uI}I - W_{uv}v + \eta_u) \\ \tau\frac{dv}{dt} &= -v + \theta(W_{vI}I - W_{vu}v + \eta_v) \\ \tau\frac{dy}{dt} &= -y + W_{yu}u - W_{yv}v + \eta_y\end{aligned} \tag{1}$$

The speed at which the output y evolves can be controlled by the input to u and v (Fig. 2)c) and determines the interval after which y reaches a fixed threshold y_0. Reaching the threshold y_0 can be understood as the movement initiation time in time reproduction experiments and adjusting I means producing longer or shorter intervals. Depending on the input I, parameter and initial conditions, the system shows different dynamics. For low levels of I (0<I<0.5) the system has three fixed points (2 stable, 1 unstable at u=v) and y ramps up faster the higher the input I. For intermediate values of I (0.5<I<1) the system still shows three FP of the same sort and y ramps up with a slope that is inversely proportional to the input I (y ramps up slower the higher the input I, see schematic in Fig. 2a, b). For high I (1<I) the system has one stable fixed point (at u=v) and y ramps down faster for higher I. In this report, the intermediate input regime is explored. In this regime, higher a higher input I results in a smaller slope of y, such that the threshold y_0 is reached after a longer interval. Thus, input is controlling the speed of the dynamic. Initial conditions of u, v and y have been optimized for in Egger et al. 2020 and are set to $u_0 = 0.7, v_0 = 0.2, y_0 = 0.5$.

2.2 Update Mechanism and Experiment Simulation

For simulating time reproduction experiments, the relation of the input I with the slope of y in combination with a fixed threshold for y is used. By flexible adjusting I based on an error signal, the threshold crossing of y can be delayed or moved to earlier times. Similar to classical time reproduction experiments, a simulated trial has two epochs. A measurement epoch that has the duration of the stimulus interval and a reproduction epoch that starts immediately after the measurement epoch. The reproduction ends, when y reaches the fixed threshold y_0 from below. The time from the end of the measurement epoch and the threshold-crossing of y yields the reproduced interval that is aimed to equal the stimulus interval that was given during the measurement epoch (Fig. 4a). Here, reproducing the interval is done predicatively, by adjusting the slope of the ramp such that the output reaches the threshold after intended time.

The following update mechanism of I is based on the intermediate regime of I, that shows an inverse relation of I to the slope in y (Fig. 2b). The error signal is determined at the end of the measurement epoch, by difference of y to the threshold y_0. If the threshold y_0 is not reached during the measurement epoch, the slope has to be adjusted, such that y ramps up faster to reach the threshold at exactly the time of the stimulus interval. For a steeper slope, I is

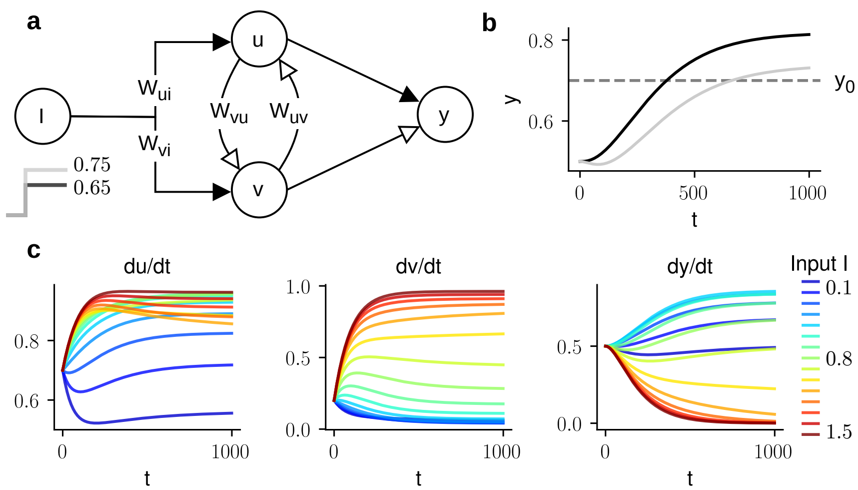

Figure 2: **Basic Circuit and Input Regimes (a)** u and v share a common input I. Dynamics for example input I=0.75 in gray and I=0.65 in black plotted in (b). The input is governed by weights W_{uI} and W_{vI}. The two units have reciprocal inhibitory connections with weights W_{uv} and W_{vu} that determine the inhibitory strength and project to the output unit y with an excitatory connection from u and an inhibitory connection from v. Excitatory and inhibitory connections are shown by filled and open arrows, respectively. **(b)** Dynamics of y for intermediate regime with input I=0.75 in gray and I=0.65 black. There is an inverse relation of input strength and slope. With higher input, the threshold at 0.7 (dashed line) is reached after a longer time interval. **(c)** Dynamics of u, v, y for inputs from 0.1 to 1.5 are shown. Initial conditions are set to $u_0 = 0.7, v_0 = 0.2, y_0 = 0.5$. With these initial conditions and values of I>=0.5, the relation of steady state activity of y (and slope to reach steady state) is inverse to I (intermediate and high I regime). For I<0.5 the steady state (slope) is smaller the smaller I (low I regime).

reduced. If y crossed the threshold before the measurement epoch ends, so is above y_0 by the end of the stimulus interval, the slope needs to be reduced in order to reach the threshold at a later time in the reproduction. For a shallower slope, I is increased. Adjusting I is done by the difference of $(y - y_0)$, weighted by a memory parameter K right at the end of the measurement epoch. Further, u and v receive a transient input I_r to reset the dynamics (see Eq. 2) for the subsequent reproduction epoch (Fig. 4a).

$$\begin{aligned}
\tau\frac{du}{dt} &= -u + \theta(W_{uI}I - W_{uv}v + \eta_u - I_r) \\
\tau\frac{dv}{dt} &= -v + \theta(W_{vI}I - W_{vu}v + \eta_v + I_r) \\
\tau\frac{dy}{dt} &= -y + W_{yu}u - W_{yv}v + \eta_y \\
\tau\frac{dI}{dt} &= sK(y - y_0)
\end{aligned} \tag{2}$$

Resetting the dynamics after the reproduction epoch enables the model to simulate an arbitrary number of stimulus intervals. Before each trial (measurement and reproduction epoch) there is a delay period and in the beginning of each trial there is a initial interval that starts with the initial condition and an initial Input I_0 (Fig. 3b). Trials that do not reach the threshold in a certain time span (twice the stimulus interval) are classified as timeout trials. If the threshold is crossed particularly early, the trial is also classified as timeout trial (crossing before 0.4 of stimulus interval) (Fig. 4b).

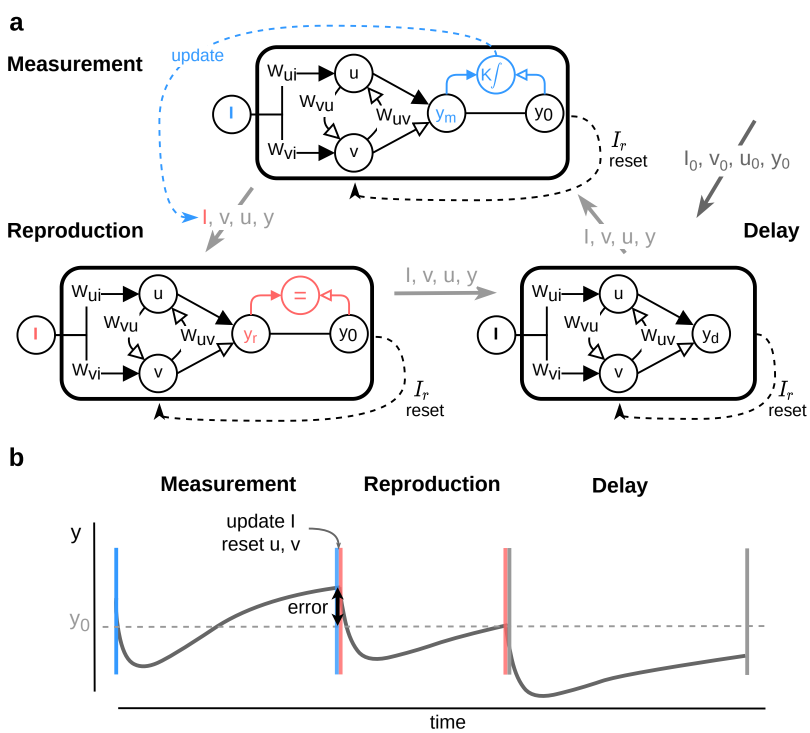

Figure 3: **Extended Circuit for Experiment Simulation (a)** The circuit for measurement, reproduction and delay epoch is displayed. All circuits comprise the same basic structure with different additional elements that are unique for the epoch. Initial values of u, v, y and I are fed into the delay circuit for the initial duration. u and v are reset with I_r before values of u, v, y and I are transferred to the measurement circuit. After the duration of the stimulus interval the difference between y and y_0 is used with the memory parameter K to update I, that is, after another reset of u and v, transferred with the other variables to the reproduction circuit. The reproduction ends when y reaches the threshold y_0 from below. Before another stimulus interval, the reset values are fed into the delay circuit again for a fixed duration. The model is able to simulate an arbitrary number of stimulus intervals. Adapted from Petzschner et al. 2015. **(b)**

3 Implementation of Model

bin dt for reset and update Definition of timeouts Euler Implementation to Solve Differential Equation fixed reproduction and after determined the crossing and actual length of reproduction epoch

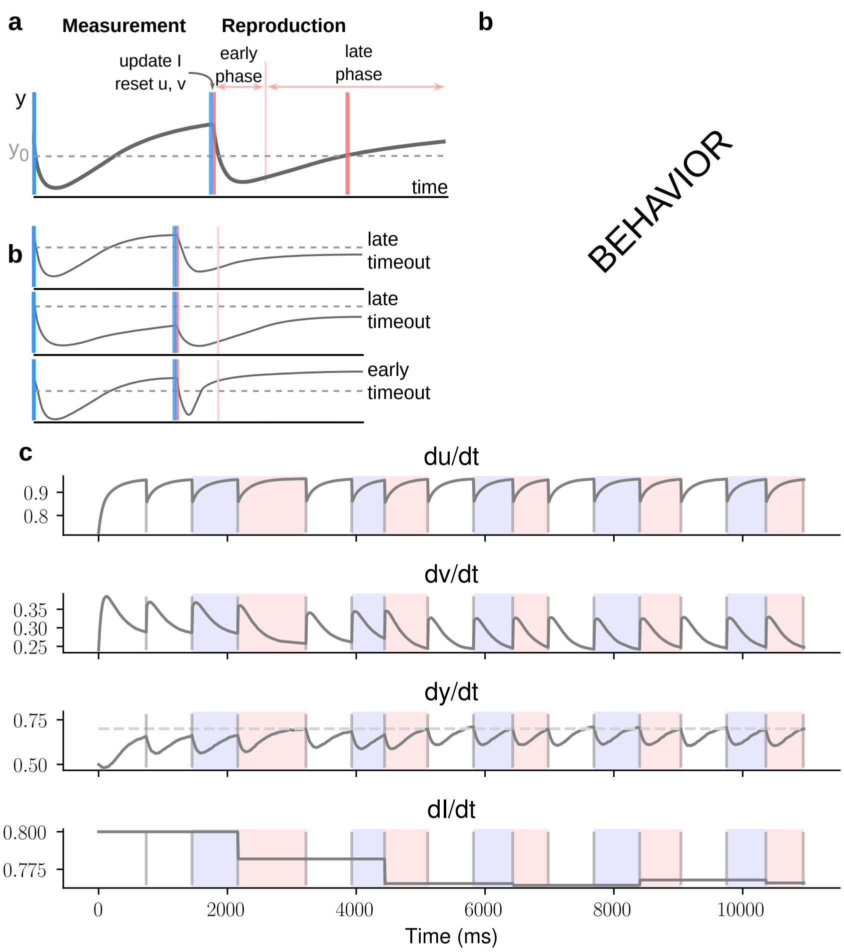

Figure 4: **Timeput and Experiment Simulation (a) (b) (c)** Example trial with 5 stimulus intervals (700, 500, 600, 700, 600 ms). Initial values are set to $u_0 = 0.7, v_0 = 0.2, y_0 = 0.5. I_0 = 0.8$) other parameters are $\sigma = 0.01$, the initial duration of 750 ms and a 700 ms delay period before new stimuli. The first stimulus interval is reproduced poorly as the system needs more trials to adapt I to a suitable range.

4 Results and Outlook

experiment simulation plot behavioral plot parameter search: initial conditions across experiment extending units, neural trajectories

limitations and explorations regimes

5 Supplements

5.1 Structure of Code

parallel simulations, experiment simulation

Figure 5: **Classes**