How to close the skill gap?

Parental Background and Children's Skill Development in Indonesia

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Abstract

Differences in children's skills manifest early in life, and understanding how these translate through childhood into adult skills is crucial for designing suitable policies to close the skill gap and decrease inequality. In this paper, I build a dynamic structural model for parents' investment decisions and children's skill production functions for each childhood period. Using panel data from Indonesia, I estimate how parental investment inputs like nutrition and schooling and early life skill differences by socioeconomic background translate into cognitive skills in primary school, high school, and adulthood. I find nutrition and schooling to be complements, with higher complementarity in high school than in primary school. A large part of the skill gap is driven by productivity differences between parents with low and high education. The second biggest contributor is income. Simulations show that an unconditional cash transfer of 20% of mean income closes the adult skill gap of 0.35 standard deviations by 29% and a nutrition price subsidy of 20% by 20%. Combining nutrition subsidy and cash transfer leads to closing the skill gap by 43%.[1]

Keywords: *Child development, Human capital, Poverty, Inequality, Cognitive skills, Human capital production functions, Development, Cash transfers, Indonesia*
JEL Codes: *D13, I24, I25, I38, J13, J24, O15*

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1 Introduction

Early in life, children from poor backgrounds fall behind in human capital development compared to their wealthier peers. These early differences convert to a persistent skill gap in adulthood, which hinders these children from escaping poverty (Attanasio et al., 2020b). They cannot profit from growth in emerging economies such as Indonesia, leading to decreasing intergenerational mobility and increasing inequality. While the initial skill gap between children is large, also parental investments differ by socioeconomic status from early life onwards. In Indonesia, parents with high school education invest on average 15% more in nutrition diversity and more than triple the amount in schooling expenditure in their child than their peers with no schooling.[2] How much of the skill gap is driven by these investment differences compared to initial skill levels? Where do the investment differences come from? Do parents invest differently in the course of childhood? Understanding how early life differences in cognitive skills translate from childhood into adult skills is crucial to avoid the fallback and close the emerging skill gap with effective policy interventions. Policies like cash transfer or price subsidies can be implemented at different stages of childhood and targeted at specific groups. Consequently, predicting adult outcomes based on early childhood measures and characteristics allows for comparing these different policies and assessing their long-term impacts when only short-term outcomes are measured. However, knowledge of transition mechanisms of skills in interaction with parents' investments through childhood, especially in the lower-middle income country context, is limited (Almond et al., 2018).

To extend this knowledge, in this paper, I investigate how cognitive skill differences transmit from childhood to adult outcomes, like math and language tests. I evaluate which role parental investments and parents' responses to potential policy changes play in Indonesia. Regarding parental choices, I focus on nutrition and schooling as investment inputs. I analyze how much these input choices are driven by household income and education. Using this setting, I then quantify how increases in skills during childhood transform into adult skills. This facilitates comparing the long-term impact of different policies, e.g., cash transfers, home interventions, or nutrition subsidies if implemented separately or together. Thus, I can simulate the impacts of

[2] Author's calculations with data from the Indonesian Family Life Survey (IFLS), supplied by the RAND cooperation. For details, see Frankenberg and Karoly (1995), Frankenberg and Thomas (2000), Strauss et al. (2004), Strauss et al. (2009) and Strauss et al. (2016)

single interventions and complementarities between them.

To do so, I build a dynamic structural model featuring endogenous parental investment decisions on nutrition diversity and schooling expenditure. I estimate children's skill production functions for each childhood period to model next period skills given current parental investments and the child's skill level. By exploiting a rich panel data set, the Indonesian Family Life Survey (IFLS), I analyze how parental investment and skill differences by socioeconomic background influence cognitive skills in primary school, high school, and adulthood. I allow for flexible specifications of the skill production functions. Parental education influences choices via income, differences in preferences for children's skills, and differential investment productivities. For the latter, I distinguish two types, the productivity of investment inputs influencing investment quantity and the productivity of skill formation influencing following period skills. The latter means that a higher educated parent might be more productive in producing skills, for instance, by supplying a quiet home environment where the child can develop well. The first allows higher educated parents to be more productive in schooling investments, leading to higher productivity of inputs. They might help their children with homework, leading to better school outcomes for the same money invested. The productivity of schooling, I also allow for varying by other characteristics (rural area, number of siblings, religion, gender). Further, parents vary by parenting skill type. Thus, they may vary in, e.g., how they interact with the child, which influences how the child does in school. In this model setting, I then identify the substitutability of investment inputs, nutrition, and schooling, using variation in food prices. I use the estimated parameters of the model in simulations to perform policy experiments.

My findings suggest that nutrition and schooling are complements, with a higher complementarity in high school than in primary school. If schooling increases, nutrition should also increase and vice versa. Therefore, when food prices increase, not only demand for nutrition decreases but also schooling. With higher complementarity in high school, food price changes impact investments more in high school than in primary school as parents react stronger in decreasing both investments. While some parenting types and parental education increase the productivity of schooling inputs, the impact of these differences on future skills is limited. Mostly, higher educated parents have greater total factor productivity of skill production. Accordingly, they yield higher results in future skills with the same level of investments and current skills compared to their peers. Mothers with high school education increase their children's future skills by 20-25% each period; all other factors hold fixed compared to mothers with no schooling. Father's education impact ranges around 10% increase. These results on investment complementarities show that food price subsidies might affect investments and skills, especially in high school. However, regarding the results on productivity, increasing parental education will also substantially affect their children's skills.

Regarding period dynamics, investments matter most in early childhood, while the self-productivity of skills increases later in life. For instance, an average effect size of education interventions is 0.1 standard deviations (SD) (see Evans and Yuan (2022)). My results show that an increase of current skills by this magnitude in early childhood leads to an 0.0004 SD increase in adult skills. The same increase in primary school translates to 0.0041 SD higher adult skills, and in high school to 0.0216 SD. This indicates a low persistence of skills through childhood and illustrates how early interventions can fade out regarding cognitive skills. However, these calculations hold all other factors fixed, so I use simulations to extend the analysis.

These simulations show which parental background and environmental factors influence investment choices and the skill gap through childhood. Investment choices are influenced strongly by income contributing to 0.23 SD of the adult skill gap between children of parents with no schooling versus high school. However, in terms of skill development, most of the gap (0.35 SD) relates to productivity differences between parents with different educational backgrounds. Parental preferences varying by education do not contribute to the skill gap but close it by 0.12 SD. Parents with lower education value their children's skills more than their peers. Following these results, income constraints play a substantial role in skill formation in Indonesia, which is illustrated by my policy experiments. An unconditional cash transfer of 20% of mean income targeting parents with no schooling closes the gap by 0.10 SD (29%). A nutrition price subsidy reducing food prices by 20% reduces the gap by 0.07 SD (20%). Combining nutrition subsidy and cash transfer leads to closing the skill gap by 0.15 SD (43%).

The intervention targeting nutrition by reducing food prices comes close to a cash transfer effect size. This result highlights the role of nutrition when growing up. In lower-middle-income countries such as Indonesia, nutrition is especially relevant as resources are scarce, and food insecurity plays a prominent role in child development (Aurino et al. (2019), Galasso et al. (2019)). Nutrition interventions like food stamps in childhood increase adult human capital, e.g., see Bailey et al. (2020). Nutrition supplementation and cash transfers can reduce stunting and, via lower stunting, increase cognitive skills (Galasso et al. (2019), Carneiro et al. (2021)). In the Philippines, Kandpal et al. (2016) and Filmer et al. (2021) find lower stunting for higher protein intake and worse outcomes for ineligible children in highly saturated and remote areas where food prices increased. Protein prices tend to be high in developing countries, which is associated with lower protein intake and higher stunting (Headey et al., 2018). I contribute to the literature by modeling nutrition diversity as a separate investment input. By estimating production functions, I complement the findings of this literature with a framework to quantify nutrition effects in each childhood period on cognitive development.

Additionally, I extend the framework by the possibility of comparing policies such as food price subsidies with cash transfers taking into account parental responses. Hence, I contribute to the literature on long-run policy evaluations in developing countries. In general, Molina Millán et al. (2019) conclude that long-term evidence on adult outcomes is mixed or that data collection is too short to evaluate these effects. Bouguen et al. (2019) find positive effects for direct investments in health, cognitive stimulation in early childhood, scholarships, and in some cases, conditional cash transfers. In terms of unconditional cash transfers, the long-term evidence is scarce due to fewer trials available (exceptions are Araújo et al. (2018) and Baird et al. (2019)). For Indonesia, Cahyadi et al. (2020) find long-term effects on schooling by a cash transfer program. In my model, I can combine some of these policies and simulate the synergies they might create together or at different points in childhood. By this, I further add to the literature on the use of structural models in policy evaluations (Todd and Wolpin (2006), Kaboski and Townsend (2011), Duflo (2012), Daruich (2018), Bobba et al. (2021)). Further, I estimate skill production functions for each childhood period up to adult outcomes. This feature allows me to model the 'missing middle years' of childhood, primary education (Almond et al., 2018). How skill changes by policies translate into middle childhood and how these indicators predict adult outcomes would help compare early life interventions with adolescent ones. I can compare different policies by simulating their outcomes and the effect sizes of existing studies.

Moreover, I use data from a lower middle-income country to estimate child skill production functions. Most of the existing literature on the estimation of skill production functions uses data from high-income countries (Todd and Wolpin (2007), Bernal (2008), Cunha and Heckman (2008), Cunha et al.

(2010), Del Boca et al. (2014), Lee and Seshadri (2019), Caucutt et al. (2020)). Exceptions are, Villa (2017) for the Philippines, Attanasio et al. (2020b) for India and Attanasio et al. (2020a) for Colombia. However, these studies pool investments and do not model inputs like nutrition separately. Thus, parental choices are not modeled explicitly, and their response to policies cannot be simulated. By modeling nutrition and schooling decisions, I can account for parents' responses to policy changes in the simulations and quantify the impact of nutrition diversity on child development in a low- and middle-income country context. Methodologically related to my work are the papers of Del Boca et al. (2014) and Caucutt et al. (2020), as I also explicitly model investment choices. While I use similar methods to estimate parameters, I deviate from their framework by using a different investment input (nutrition), modeling outcomes including adult skills, and using data from a lower-middle income country.

A limitation of the paper is that because of data constraints, I cannot, like, e.g., Del Boca et al. (2014) or Caucutt et al. (2020), model the time parents spend with their children well. Del Boca et al. (2014) find time to matter, especially in early childhood, and money becomes more important in later periods. Thus, I can complement the latter by adding results on other investment inputs like nutrition and schooling and the transformation to the adult skill outcomes. I abstract from modeling intra-household allocation and investment trade-offs between siblings due to data constraints and complexity, although household poverty might be shared unequally (Calvi (2020)). To limit the impact on the results, I control for household size and amount of siblings in the estimation and calibration. I use food diversity, not quantities, which might be more impacted by unequal sharing. Another limitation is that my model does not feature differences in knowledge of the production function by parental education and the child's current skill level. Dizon-Ross (2019) and Cunha et al. (2020) find that parents with lower education overestimate their children's skills and the impact of their investments compared to their peers. They also tend to underestimate the importance of early life investments driven by the persistence of current skills. In my model's context, these differences would lead to underestimating preferences for lower-educated parents. Therefore, I treat these parameters as a lower bound. As I estimate lower-educated parents to value skills higher than higher-educated peers, this gap might be even bigger with knowledge differences. In the result section, I further discuss the relevance of this and other shortcomings.

The rest of the paper is organized as follows. In the section 2, I discuss the data used and present facts on the skill gradient in Indonesia. Afterwards, I introduce the theoretical model and describe the estimation procedure in sections 3 and 4. In section 5, I discuss the results and limitations of the paper. The results are used in sections 6 and 7 to quantify the different contributors to the skill gap and to simulate policy experiments. I summarize remarks on results, their interpretation, and ideas for future research in section 8.

2 Data and evidence on socio-economic background and skills

To motivate model assumptions and the empirical analysis, I start by documenting the skill gap by children's socioeconomic background in Indonesia in subsection 2.2. Using data, I will explore the potential drivers of this gap. However, before discussing the facts in detail, I shortly describe the data I use in section 2.1. For further details on the data, see section A.1.

2.1 Data

As the main data source, I use the Indonesian Family Life Survey (IFLS)[3]. This survey is a panel dataset from 1993 to 2014, allowing me to observe children from childhood to adulthood. Survey waves are 1993, 1997, 2000, 2007 and 2014. The survey area covered represents 83% of the Indonesian population, which gives me regional variation to exploit. Mostly not covered are the Eastern provinces, which are very remote and poor. It allows me to model choices in a setting where investment choices occur as markets are available and schooling options are not strongly limited by availability.

As I model the skill gap between children from different socioeconomic backgrounds, detailed information on the household and investments in children and their skills is necessary. The data set provides information about investments like schooling and nutrition. It follows children long enough to measure materialized skills in adulthood (low attrition rates around 90% to 95% depending on the survey wave). I use survey waves 1997, 2000, 2007 and 2014. I do not use 1993 due to the lack of availability of food prices.

[3]IFLS data was supplied by the RAND cooperation, for details see: Frankenberg and Karoly (1995), Frankenberg and Thomas (2000), Strauss et al. (2004), Strauss et al. (2009), Strauss et al. (2016) and https://www.rand.org/well-being/social-and-behavioral-policy/data/FLS/IFLS.html

Unfortunately, the gaps between waves do not allow me to model the skill process yearly but only in childhood periods (for details, see 3 and 4). As the information on income and investment in between survey years is scarce, I abstract from assumptions to fill in these years. However, for surveyed years, the panel entails rich information on the household and its members. The household head is the source of the primary data. Interviews also occur with the spouse; more detailed information is collected on 2-3 randomly selected children in the household. My sample for the analysis consists of children for whom information on investments and skills is available. Additionally, they need to have sufficient information on their parents' characteristics. For the estimation, this gives me around 5,941 children in early childhood, 3,964 in primary school and 4,338 in high school. On 672 of these, I have complete information, thus in each childhood period and with adult skill data with all required information on investments, skills and prices.Investments used are education investments, like schooling fees, exam fees, books, and health investments. For the latter, I take nutrition diversity as a proxy. Food prices vary by municipality level (kabupaten). In the next paragraphs, I will shortly describe the procedure of constructing price and investment data for each investment input. For further details see section A.1.

For nutrition investments, I use the food consumption information of the household. With that, I can measure which food groups the family consumes. I assume the child to eat from all parts recorded in household consumption. Following Attanasio et al. (2020b), this serves as a proxy for the parents' decisions to invest in the child's health. The food groups counted are vegetables, fruits, dairy, proteins and carbohydrates. Regarding the price of investment for nutrition, I use price data derived from market surveys of the community questionnaires. I use spending reported on schooling fees and materials bought as schooling investments. The price for education, I assume, is one so that the total expenditure on education enters the investments. I only observe schooling investments for primary and high school.

In terms of skill measures, measures for health and cognitive skills are available. For cognitive skills, I use the survey's math, logic or language tests for each child, which I standardize by age and year. In terms of health, I use height and weight, transformed to height/weight-for-age with the help of the WHO Child Growth Standards and WHO Reference 2007 composite data files (Vidmar et al., 2013).

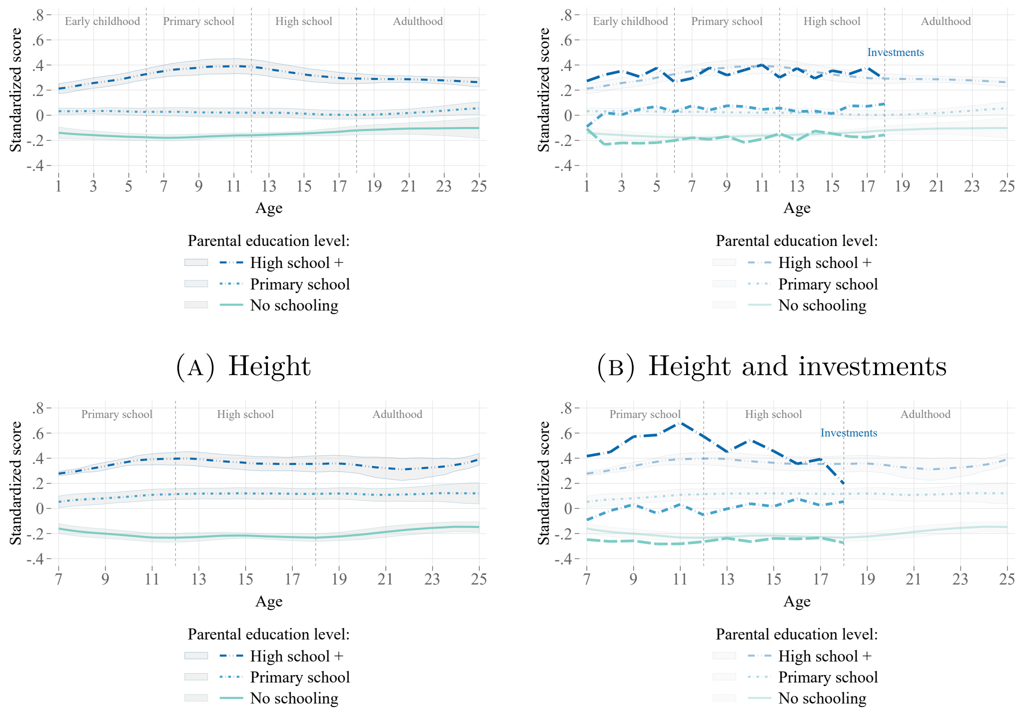
The survey also records other observable characteristics such as the number of siblings, household income, assets and wages. As parental education, I use the parents' education level at the start of the child's life. Thus it does not vary over time. An overview of the descriptives is displayed in table A.1 for children from my sample in early childhood. One can observe that children are between 0 and 5 years old, and a fraction of 0.32 is stunting, while 0.09 is wasting. The fraction of stunting children highlights the food security situation in Indonesia. With the above-mentioned WHO scale for z-scores, children below a height-for-age score of -2 are stunting. Wasting are children below a weight-for-height score of -2 and underweight children below a weight-for-age score of -2. Maternal education is, on average lower than paternal education (years of education). Parents' age varies substantially and is likely not always correctly recorded; however, it does not enter the model except for the household income estimation. A fraction of 0.10 of the sample is not Muslim, and the gap in household income is wide. Average households have around four adults and two children. While a small part of households is extremely large, most households range between 2-6 adults and 1-5 children.

2.2 Empirical evidence on socioeconomic background and skills

Firstly, in this section, I document the size of the skill gap for cognitive skills and health by age in Indonesia. Then, I summarize potential drivers for the skill gap and show how these vary for children from different socioeconomic backgrounds in Indonesia. Last, I will show some descriptive evidence to motivate the need for controlling for unobserved parenting skills.

The skill gap in Indonesia is substantial and opens early in life. To show that, I plot averages of skills by parental education group and age in figure 1. I use standardized test scores for cognitive skills and height to measure health. Visibly, children with lower educated parents show a lower level of health from the start of life (see figure 1a). I only observe test scores from the age of 7, but this initial gap is also large, as shown in figure 1c. For both measures, the gap widens during primary education and closes partly during adolescence. However, it is fairly stable. In adulthood, children from lower educated parents still have substantially lower skills, health and cognitive than their peers. Similar patterns emerge by parental income (see figure 8).

Looking at these differences, the question arises of how this gap interplays with parental investments. To answer this, I plot standardized investments for health; food groups consumed onto the skill gap plot with height in figure



(A) Height

(B) Height and investments

(C) Test score

(D) Test score and investments

FIGURE 1: Children skills over age by parental education

Note: Corresponding skills are fitted with local mean smoothing by age and parental education groups. Parental education groups correspond to the average education of both parents. Confidence intervals displayed are at 95% level. Investments plotted are nutrition investments for height and schooling expenditures for test scores. Scores of skills and investments are standardized by age to have a mean of 0 and SD of 1.

1b. For cognitive investments, I plot standardized schooling expenditure on the graph with test scores (see figure 1d). We can observe a similar gap for cognitive investments. However, the gap widens more in primary school and closes quicker in high schools than the observed skill gap. In contrast, food investment differences are stable over childhood. Thus, parents with higher education mainly increase investments at the end of primary school, while nutrition differences persist over time.

These investment differences are one potential driver for the skill gap and can be driven by several mechanisms via which parental education influences children's skills. Foremost, parents with lower education have fewer resources to invest in their children. As shown in table 2.1, lower educated parents have less income available. By that, they can invest less in children, both for nutritional investments and for schooling. Differences in investments are substantial; parents with high school education spend more than triple on education than their counterparts without education.

TABLE 2.1: Potential sources for the skill gap by maternal education

	Parental education level:					
	None	Primary school	High school	F-test	Mean	Sd
Resources						
HH income	181.02	384.53	522.77	0.00	289.19	479.74
Maternal skill set						
Test score	-0.44	0.24	0.51	0.00	0.00	1.00
Height	-0.15	0.13	0.31	0.00	0.00	1.00
Initial skill levels						
Test score	-0.23	0.21	0.37	0.00	0.00	1.00
Height-for-age	-0.17	0.18	0.41	0.00	0.00	1.00
Childhood investments						
Food groups consumed	3.36	3.71	3.85	0.00	3.57	0.91
Education spending	2.30	5.37	7.53	0.00	5.14	10.50

Note: The last column displays p-values for the null hypothesis that means for none and high school education are equal. Skills are normalized to 0 mean, SD of 1. All values are from period 2 (age 6-11), except initial height. Income and education spending expressed in 100,000 rupees.

Income is not the only potential source of the gap between children's skills. Parents with lower income and education might have lower cognitive skills and worse health. On the one hand, this might lead to different initial skills for the children, which I observe in the data. However, their abilities and health might influence their investment productivity. Parents with higher abilities might be more capable of helping children with homework, which makes their schooling investment more productive.

Apart, parents with higher education seem to invest differently. They spend more significant amounts of money on education. Despite income differences, this might be driven by differences in productivity, similar to the productivity differences by ability mentioned above. Also, parents' preferences might vary with education. Higher educated parents might differ in valuing skills to their peers. For instance, lower-educated parents might wish for their children to do better off than them and invest more. However, resources might constrain them in doing so.

I can only uncover these mechanisms in a structural model, not with the descriptive data available. Therefore, I construct a model where parents decide on different investment inputs, which productivity varies by parental education, among other factors (described in further detail in section 3). These parents face income constraints and value child skills differently by education.

However, controlling only for observable characteristics of the parents might miss an important feature: parenting skills. Some parents could have higher parenting skills, leading them to make better investment decisions due to higher ability. If I omit to control for those that correlate with education, it will lead to biased estimates. To illustrate that they are not aligning with education and income, I plot distribution by parent's income and education groups on figure 2.

As one can see, the distribution in the lower education and income categories is skewed to the left. However, even in these categories, there is substantial heterogeneity, which parenting skills can drive. The impact of these skills might vary by childhood period, similar to the impact of other potential drivers of the skill gap. Resources might play a more critical role during high school than in early childhood since higher investments are needed to affect future skills.

The potential drivers call for a model-set up where investment effects vary across periods and controls for unobserved parenting skills. Also, skills effects on the following period skills need to change over time. Including these dynamics in a theoretical model might allow policy simulations to mimic the potential fade-out of interventions and to see when and why this happens.

3 Model

To capture the empirical facts described in section 2, the theoretical model entails different channels via which socioeconomic background influences skill development. Thus, it captures investment decisions influenced by education and features households' budgets to constrain investment expenditure. Additionally, I will account for parenting skills in the skill production function, and all these influences vary by childhood period.

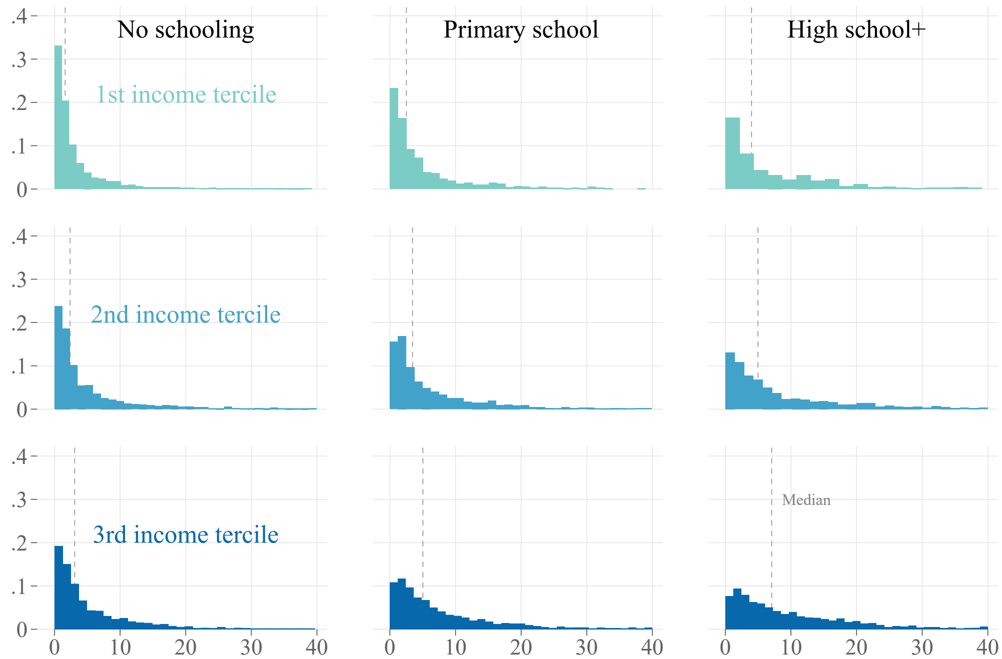


FIGURE 2: Heterogeneity in education spending by parental background

Note: Education spending histograms by parental education level and household income (in terciles). Parental education groups correspond to the average education of both parents. Expenditures are expressed in 100,000 rupees. The grey-dashed line indicates the median value for that category.

Regarding modelling choices and functional form assumptions on the skill production function, I follow Del Boca et al. (2014) and Caucutt et al. (2020). However, in contrast to both, I focus on nutrition and schooling inputs instead of time inputs. Hence, this model will especially capture later childhood periods, where monetary expenditures become more productive and feature the transition of skills in teenage years to adulthood.

Households represent a parent-child pair that decides in each childhood period, early childhood, primary school and high school on investments into the child. In the final period of the model, the child grows up to be an adult, and no further decisions occur. In this period, households only derive utility from the final skills of their children Ψ_{T+1} and assets a_{T+1}. The latter is merely to assure that parents do not deplete assets fully in the high school period to maximize utility in the last period.

In each childhood period, households derive utility from consumption c_t and current child's skills Ψ_t. They can decide to invest their resources into consumption c_t, savings a_{t+1} or investments in the child I_t. Hereby, parents are constrained by their income and their decisions are influenced by the prices of investments. I adjust household income by household size (see A.1 for details). For the moment, I abstain from further modelling the trade-off between siblings, which would be a potential future extension of this model. As the model only contains monetary investments into children, I do not model labor choices for simplicity. They do not influence child development here. Thus the trade-off would only be between consumption and leisure, which is not the model's focus.

Investment decisions are made every period to be able to measure when they matter the most for skill development. Figure 3 illustrates a graphic overview of the timeline. Periods are determined by the child's age, following standard definitions in the literature for an early childhood period, primary education and secondary education. In period $t = 0$, the child is born with an initial skill endowment Ψ_1; then, in early childhood, the household decides on nutrition n_t. In later periods, the parents also choose how much to invest in schooling s_t. In $t = 4$, the child is grown up, and final cognitive skills outcomes realize.

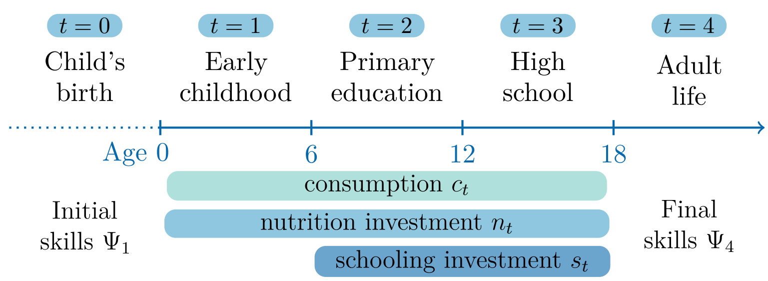


FIGURE 3: Model stages

Formally, each period the household maximization problem looks like the following:

$$
\begin{aligned}
V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, n_t, s_t, a_{t+1}} & \; u(c_t) + \alpha_e v(\Psi_t) \\
& + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\
\text{s.t.} \quad c_t + p_{n,t} n_t + p_{s,t} s_t + a_{t+1} &= (1 + r) a_t + y_t \\
a_{t+1} &\geq a_{min,t}
\end{aligned} \tag{1}
$$

Households maximize utility with respect to consumption c_t, assets a_{t+1} and investment choices. Investments in the child are investment in nutrition n_t and an schooling investment s_t in period 2 and 3 ($s_t = 0$ in period 1). Nutrition investment can be understood as a proxy for health investments and is measured by the number of food groups a child consumes. All investments are associated with their corresponding prices in the budget constraints. The price for nutrition is $p_{n,t}$, and the price for one unit of schooling is $p_{s,t}$. The vector of all prices for investments is denoted by Π_t. The household cannot spend more than their current income y_t and assets a_t. Future utility depends on the evolving state space of future income and prices, as well as future household characteristics Z_{t+1} and future skills Ψ_{t+1}. Households can borrow, but not more than $a_{min,t}$, the maximum amount a household can be in debt.

The current period's utility depends on consumption and skills. The utility functions take the corresponding forms:

$$u(c_t) = \ln(c_t) \tag{2}$$

$$v(\Psi_t) = \ln(\Psi_t) \tag{3}$$

In the last period of the model, utility exclusively depends on the final skill level of the child Ψ_{T+1} and final assets. By that, a motivation to invest in the child is ensured. Also, not all assets are depleted in the last period:

$$V_{T+1} = u(\Psi_{T+1}) = \alpha_e \gamma_e \ln(\Psi_{T+1}) + \xi \ln(a_{T+1}) \tag{4}$$

Here it is important to note that the altruism factor α_e and γ_e depends on parental education. By this, I allow parents to value their child's skills differently depending on their education. In the adult period, no decisions take place, so the child's skill level is the only variable from which the household derives utility. What is left to specify is how children's skills evolve. Future skills will depend on current investments I_t, current skills Ψ_t and a total factor productivity $\theta_t(Z_{\theta,t})$:

$$\Psi_{t+1} = \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \tag{5}$$

Thus, $\delta_{1,t}$ will describe the impact investments have on future skills, which varies by period. The self-productivity of skills Ψ_t is expressed by $\delta_{2,t}$, also varying by period. I ensure that the estimation is flexible enough to capture that early childhood skills might be less critical for future skills than in high school. Persistence of skills is likely to increase over childhood, and this functional form allows to capture this development flexibly. The total factor productivity depends on observable characteristics $Z_{\theta,t}$. These are parental education and the age of the child.

Total investment are composed of the investment inputs nutrition n_t and schooling s_t:

$$I_t = [n_t^{\rho_t} + a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t}]^{\frac{1}{\rho_t}} \tag{6}$$

I assume a CES investment function, following Caucutt et al. (2020). The parameter ρ_t describes the elasticity of substitution between nutrition and schooling. Schooling investments have a relative productivity of $a_{s,t}$, which depends on observable characteristics. These are parental education e, age, number of siblings and the unobserved parenting skills η. Productivity depends on parental education since one could imagine that the investments have differential by parents' education. Higher-educated parents might buy adequate books for schooling when the child needs them or be able to help the child with homework at later levels of schooling. In a similar spirit, unobserved parenting skills η influence productivity. Controlling for the number of siblings allows either siblings to help with homework or reduce the time parents can spend with the child on homework, thus reducing the productivity of schooling. An assumption is that $a_n = 1$, thus the productivity of nutrition investments is normalized for identification. In early childhood $I_t = n_t$

The elasticity of substitution each period ϵ_t is measured by ρ_t with $\epsilon_t = \frac{1}{1-\rho_t}$. Thus, if $\epsilon_t < 1$ the investments are complements, if $\epsilon_t \geq 1$ they are substitutes. The elasticity will drive price reactions. Suppose goods are substitutes and the price of one rises. In that case, it will be substituted by the other ones to some degree, if they are complements, this substitution will not happen, and overall investment might be decreased depending on the degree of complementarity.

Depending on the productivity of each investment, price rises will have different impacts varying by parental education and other observable factors. For instance, if food prices rise and the goods are substitutes, investments might shift to more schooling expenditure. However, if schooling investments are more productive for high-educated mothers, they might have to buy less quantity to substitute for the loss in nutrition. In terms of complements, the substitution would not take place. However, if schooling is more productive for high-educated parents, changes in food prices might impact them less than low-educated parents. This interplay shows why it is essential to know if investments are substitutes or complements because suitable policies can be designed. In the case of substitutes, a price subsidy on one product might lead to less investment in another. In case of complements, this might lead to an increase in all types of investment.

As Caucutt et al. (2020), Moschini (2019) and Molnar (2018), I exploit the fact that the maximization problem can be separated into an inter-temporal and an intra-temporal problem. The intra-temporal problem minimizes the costs for investments for a given amount of total investments I_t. The inter-temporal will then maximize utility with respect to total investments, leisure and consumption. The minimization problem will minimize costs for investments, given I_t abstracting from maximizing utility. I can derive solutions for each investment input given the total investment level. By doing so, I can reduce the maximization problem to maximizing with respect to I_t, simplifying derivations (see section A.6). The minimization problem takes the following form:

$$\begin{aligned} \min_{n_t, s_t} \quad & p_{n,t} n_t + p_{s,t} s_t \\ \text{s.t.} \quad & n_t \leq 5 \\ & It = [a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} \end{aligned} \tag{7}$$

With having derived equations for the investment inputs n_t and s_t given I_t, the inter-temporal problem can be characterized by:

$$\begin{aligned} V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, I_t, a_{t+1}} & u(c_t) + \alpha_e v(\Psi_t) \\ & + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ \text{s.t.} \quad c_t + \Lambda_t I_t + a_{t+1} = & (1+r) a_t + y_t \\ a_{t+1} \geq & a_{min,t} \end{aligned} \tag{8}$$

Λ_t will describe the price for one unit of total investment, which arises from the results of the cost minimization (see section A.6). Given the results, investment input prices will determine the amount of each investment input and the price for one unit of total investment.

Hence, the model captures investment decisions in children influenced by investment prices and parental preferences, differences in investment productivities and parenting skills. By allowing the interplay of the budget constraint and preference parameters and productivity of skill formation differing by education and observables, I can, with this model, quantify when income and parental education influence children's skill development most. Further, it allows me to distinguish between the influence of nutrition and schooling as inputs and when they have the highest impact on skill development.

4 Estimation and calibration

To estimate the model, the I take the following steps:

1. Estimation of types of parenting skills by k-means algorithm
2. Estimation and prediction of household income by OLS
3. Estimation of skill formation parameters by joint Generalized Method of Moments (GMM) for:
   - Investment parameter: using relative demand ratio moments
   - Human capital parameters: using skill moments and factor loading moments
4. Calibration of preference parameters by simulated methods of moments (SMM)

In the following paragraphs, I describe each step in the listed order in detail. For further details, see appendix A.3. I start the estimation procedure by determining the unobserved parenting skill types. Since all equations depend on the types $k = \{1, ..., K\}$ of unobserved parenting skills η, these need to be estimated first. To do so, I use the k-means algorithm in the spirit of Bonhomme et al. (2022) to control for unobserved heterogeneity. The advantage of this method is that it allows for types whose impacts vary over childhood periods. Additionally, estimating the types outside the model is less computationally intensive, and the strategy uses empirically relevant data to determine the types. For identification, I can exploit the fact that I observe parents over time and across children (siblings) in terms of their investments. Assuming that the impact of parenting skills is the same for each child, I can use this additional data to identify the skill type for each parent pair.

To perform the k-means algorithm, data moments must be chosen, which are influenced by the types. In my case, these are schooling, nutrition, and household income investments. I assume investments to be partly driven by unobserved parenting skills and that these skills can translate into higher productivity in the labor market resulting in higher income. The moments I calculate are lifetime averages of parental investment decisions and income across childhood periods and their children. I calculate lifetime moments because an assumption of the k-means algorithm is that parents of the same type would converge over the life cycle to have the same moments with $T \rightarrow \inf$ (for details, see A.3).

Thus, I can use the variation in lifetime moments in the data to determine types. To do so, the algorithm minimizes the within-cluster (type) variance. The state-space is split into clusters, so that parents within a cluster are as similar as possible:

$$\min_{k \in \{1,..,K\}^N} \sum_{t=1}^{N} \sum_{c=1}^{C} ||\boldsymbol{m_{t,c}} - \boldsymbol{\overline{m}_k}||^2 \tag{9}$$

where $\overline{m}_k$ is the average of moment vector m of parenting skill type k and t stands for each period in the data, while c indexes each child the parents have. The moments are standardized to have mean zero and variance one for the k-means algorithm. To run the minimization, the researcher needs to set the total amount of clusters K. With the help of the elbow and silhouette criteria, I determine the optimal amount of types K, as plotted in figure 9. The optimal number is $K = 4$. A detailed discussion of robustness checks is in the appendix A.3. Then, I can determine for each parent pair the unobserved parenting skill type they have according to the algorithm.

With the parenting skills, household income is estimated with a standard Mincer equation since I abstract for modeling labor choices. Household income depends on parental education, number of household members, rurality, age of the household head, and parenting skills. The parameters for these characteristics will then be used to predict household income for the calibration and simulations. For these predictions, I assume the income shocks to be i.i.d. normally distributed. Thus $\epsilon \overset{i.i.d}{\sim} N(0, \sigma_y)$.

The estimation of the human capital and investment parameters consists of a joint GMM estimation. For this estimation, I derive a set of moments for the investment function parameters in equation 6 and another for the human capital parameters in equation 5.

To do so, for the investment parameter moments, I start by deriving and rearranging the first-order conditions of the cost-minimization problem to formulate the following linear relative demand equations, which I can estimate with OLS for periods 2 and 3 (for derivations, see A.6):

$$\ln\left(\frac{p_{n,t}n_t}{p_{s,t}s_t}\right) = \frac{1}{\rho_t - 1} Z_t'\phi_{s,t} + \frac{\rho_t}{\rho_t - 1}\ln\left(\frac{p_{n,t}}{p_{s,t}}\right) - \frac{1}{1-\rho_t}\eta + \epsilon_{ns,t} \tag{10}$$

The relative demand ratio between nutrition and schooling quantities will depend on observable characteristics $Z_{s,t}$. These form, following Caucutt et al. (2020) assumptions, the relative schooling productivity $a_{s,t}(Z_{s,t}, \eta) = exp(Z_{s,t}'\phi_s + \eta)$. Note, as mentioned in section 3, I normalize $a_{n,t}(Z_{n,t}) = 1, \phi_{n,t} = 0$ to identify all parameters. Thus, I will only be able to have results on the relative magnitude in terms of their impact on the productivity of investments. The characteristics $Z_{s,t}$ include paternal and maternal education and other observable characteristics such as religion, age of the child, rural area, siblings in the household, and gender. Additionally, the productivity will depend on η, the unobserved parenting skills, as one can see in equation 10. $Z_{s,t}$ here is a matrix of variables as parental education. As one can see ρ_t, the substitution parameter for nutrition and schooling is identified with the price ratio of these inputs. As schooling prices are assumed to be 1, this parameter will be identified by variations in the food price.

As instruments $Z_{t,ns}$ for the GMM moments displayed in equation 10, I use the observable characteristics $Z_{s,t}$, the price of inputs and parenting skill types k. Thus I assume the moments to be orthogonal:

$$\mathrm{E}\left(\left[\ln\left(\frac{p_{n,t}n_t}{p_{s,t}s_t}\right) - \frac{1}{\rho_t - 1} Z_t'\phi_{s,t} + \frac{\rho_t}{\rho_t - 1}\ln\left(\frac{p_{n,t}}{p_{s,t}}\right) - \frac{1}{1-\rho_t}\eta\right] Z_{t,ns}\right) = 0 \tag{11}$$

For this equation to be accurate, I need to assume that the measurement error in equation 10 is independently distributed across individuals, and no variables in the error terms influence the demand ratio and instruments used for the moment equations. For this not to be true, a variable would need to influence schooling and nutrition inputs differently, as influences of the same magnitude factor out by the ratio. For example, not controlling for parenting skills η might bias the results as it could influence schooling differently from nutrition but be correlated with parental education. It might be driven by ability which influences education and via parenting skills, also the ratio of investment incomes.

To control for this potential bias, I use the estimated types. As these estimated types do not correlate strongly with education, I assume that education is not working solely through parenting skills in influencing the ratio of nutrition versus schooling parents spend. I understand the influence of education, for example, knowing to help your child with homework. In contrast, unobserved parenting skills capture, e.g., parents' empathy to react to their children's problems at school and spend more time with them, which then increases their productivity at school as it might mitigate behaviors that hinder learning.

Note that the identification of the substitution parameter ρ_t depends on food prices, whose variation I assume to be exogenous. Parents' choices might influence food prices or schooling fees, which would break this assumption. For instance Bold et al. (2015) find that providing free public primary education shifted parents demands to private education and increased prices for these schools in Kenya. I do not model differences in public and private education provision and the supply side for simplicity, a caveat to keep in mind for interpreting the results. Regarding food prices, Filmer et al. (2021) find cash transfers to lead to higher food prices for proteins by increased demands of recipients having negative effects on ineligible children. However, these results only hold in remote areas or when a large proportion of the village received treatment. In this context, this is unlikely to be the case, as I look at only a subpopulation, relatively urban areas, and not extremely remote villages. Thus, for simplicity, I abstract from modeling prices, but this could be a future extension of the model. Nonetheless, it is vital to keep this simplification in mind when evaluating the outcomes of the policy experiments.

Turning to the human capital parameter moments, I will mainly use equation 5. However, one must consider how skills are measured in this context before estimating these parameters. I use raven and math test scores in the later periods of the model for cognitive skills and height and weight in early childhood as a proxy. These measures, however, only proxy the latent skills and are measured with error. To account for this, I follow Cunha et al. (2010) and assume a measurement system for the latent skills Ψ_t. The system looks like the following:

$$S_{ts_1,t} = \lambda_{ts_1,t} \ln(\Psi_t) + \epsilon_{ts_1,t} \tag{12}$$

and:

$$S_{ts_2,t} = \lambda_{ts_2,t} \ln(\Psi_t) + \epsilon_{ts_2,t} \tag{13}$$

where ts stands for test scores I use in the corresponding period, as mentioned above. Following Caucutt et al. (2020), I normalize one factor loading $\lambda_{ts_1} = 1$ each period.

Combining the measurement system equations 12 and 13 with equation 5 for the skill formation process,I derive additional moments for the GMM estimation (for details see A.6):

$$\frac{1}{\lambda_{ts,t+1}} S_{ts,t+1} = \phi_{\theta,t} Z_{\theta,t} + \delta_{1,t} \ln(I_t) + \delta_{2,t} \frac{1}{\lambda_{ts,t}} S_{ts} + \epsilon_{\Psi,t} \tag{14}$$

Moreover, to identify the factor shares:

$$0 = \mathrm{E}[(S_{ts_1,t+1} - \lambda_{ts_2,t+1} S_{ts_2,t+1}) S_{ts_1,t}] \tag{15}$$

and:

$$0 = \mathrm{E}[(S_{ts_1,t} S - \lambda_{ts_2,t} S_{ts_2,t}) S_{ts_1,t+1}] \tag{16}$$

In this context, $Z_{\theta,t}$ entails parental education and the child's age. Again, I assume these factors to map into the total factor productivity $\theta_t(Z_{\theta,t})$. As instruments Z_{t,Ψ_t} for the skill moments I use the characteristics in $Z_{\theta,t}$ and investment inputs schooling s_t and n_t. Thus:

$$\mathrm{E}\left(\left[\frac{1}{\lambda_{ts,t+1}} S_{ts,t+1} - \phi_{\theta,t} Z_{\theta,t} + \delta_{1,t} \ln(I_t) + \delta_{2,t} \frac{1}{\lambda_{ts,t}} S_{ts}\right] Z_{t,\Psi_t}\right) = 0 \tag{17}$$

I abstract for modeling investments between the points of time I observe the children in the data, as I do not have enough information on investments or income to impute those. Another shortcoming is that while I control for measurement error in skills, I do not do so for investments, which could lead to biased results, and therefore the results have to be taken with caution. However, as I do instead treat investments in nutrition as a proxy for health investments and schooling for education investments, these inputs are not supposed to be understood as precisely modeled. In general, measurement error in investments is likely to decrease the coefficient of investments, thus underestimating the impact (Cunha et al., 2021).

After this estimation procedure, left to calibrate are the preference parameters γ_e, ξ and α_e. To calibrate them, I use the optional solution for total investments (see section A.6 for details) and simulated methods of moments. I set the discount factor β to 0.98, following calibrations in the literature on Indonesia (Dutu, 2016). I match mean investments by childhood periods and parental education level to their data counterparts (see section A.3 for details). For the calibration and simulations, for now, I assume assets $a_t = 0$, an assumption to be relaxed in future research. Additionally, in terms of transition of state variables, wages and prices change over time, while for simplicity, I assume all other household characteristics to be fixed. Thus, households do not move from rural to urban, and the number of siblings does not change. This process could also be enriched in future research.

5 Results

Regarding parenting skill types determined by the k-means algorithm, the type distribution suggests that there are four types. In the upper graph of figure 4 I show these types' distribution and their characteristics in terms of income and investments (see table A.2 for further details). The two most often occurring parenting skill types, 0 and 1, have low income and schooling investments compared to the other types. Additionally, type 1 also has low food investments. In contrast, type 2 has higher income but also very high education expenditure. Type 3 seems to have mainly very high income and modestly increased investments. As shown in the downer part of the figure, the types partly correlate with education. The share of mothers with no schooling is higher for the low-income and low-investment types 0 and 1, while the share of high school mothers is higher for types 2 and 3. The share of mothers with primary education is similar for all types. Hence, while there is some correlation between education and types, there is still some variation regarding unobserved parenting skills within education groups.

Turning to the results on household income, one can observe that parenting skill types matter. In table A.3, one can see that types 1 and 2, which are associated with higher income (table A.2), also tend to have higher productivity of income in the household income estimation. Especially type 3 has high productivity, which is the one with the highest observed income, while type 1, the lowest, is associated with a negative coefficient. In terms of magnitude, being of type 2 corresponds to an increase in household income of having a mother with high school education. Furthermore, being of type 3 exceeds this by influencing a third more than both parents' high school education. Apart from that, the coefficients from the household income estimation all show the expected signs and magnitudes; education and age

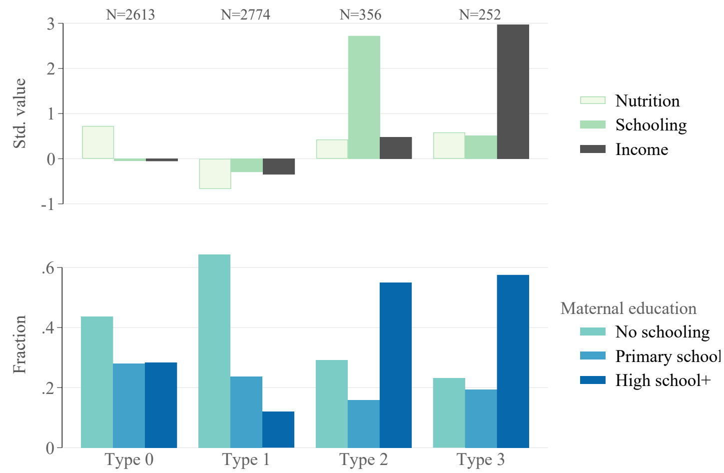


FIGURE 4: Characteristics of parenting types η (investments/resources and education)

Note: Nutrition is food groups consumed, schooling expenditure, and income annual household income (lifetime averages by parenting pair).

increase income, while living in a rural area decreases it.

The estimation results for investment parameters using equation 10 reveal the degree of complementarity for investment inputs and their productivity by period (see table 5.1 and for further parameters A.4). Nutrition is complementary to schooling in both periods, primary and high school. Worth noting is that complementarity increases in high school. Consequently, if prices for nutrition increase, this would induce a decrease in investing in schooling, especially in later childhood periods. Investments would not be reallocated to the now relatively cheaper input. Reallocation does not happen because strong complementarity means that if both investment inputs increase simultaneously, this yields the highest total investment. Increasing only one is not efficient. Considering policies, this is an essential result since decreasing nutrition prices might increase food diversity and schooling expenditure. However, this depends on how parents react to price changes (e.g., if they reallocate money to another input or spend the money for consumption). For this question, policy counterfactuals are necessary.

Additionally, schooling productivity differences might affect how parents react to price changes. Regarding productivities, table 5.1 shows results. Relative productivity of schooling increases with maternal education, especially in the last childhood period. Thus, schooling is more productive for

Table 5.1: Estimation results for investment parameters

	Early childhood	Primary school		High school	
Investment elasticity:					
ρ_t		-3.75	(0.86)***	-11.38	(5.11)**
Implied elasticity		0.21		0.08	
Schooling investment productivity $\phi_{s,t}$*:*					
Constant		-3.68	(0.51)***	-42.17	(16.55)**
Mother primary		1.10	(0.25)***	3.06	(1.32)**
Mother high		1.87	(0.39)***	5.04	(2.15)**
Father primary		0.09	(0.16)	0.63	(0.47)
Father high		-0.08	(0.19)	0.51	(0.50)
Parenting type 1		-0.24	(0.14)*	0.06	(0.34)
Parenting type 2		4.74	(0.97)***	9.62	(4.10)**
Parenting type 3		1.64	(0.50)***	2.47	(1.29)*
N	27,366				

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

children with mothers with high school education. Similarly, parenting types 2 and 3 are more productive in schooling. Living in a rural area decreases the productivity of schooling, especially in high school. In terms of magnitude, this offsets the productivity increase of having a mother with high school education. Having siblings negatively influences schooling productivity, more so in high school, while not being Muslim increases productivity. By the same magnitude, productivity increases for female children, both are only significant in the high school period. Other estimation and calibration results are needed to interpret the results on productivities for policy implications because these enter several spots in total investment prices and investment choices.

The human capital parameters, δ_1 and δ_2 vary by period. Investments have a higher impact early in life and similar impacts in primary and high school. Thus, early childhood investments are most impactful on skills. Skill persistence increases over life. In the first period, the current skills have a lower impact on future skills. However, in the first period, I only use a proxy for cognitive skills, which are health measurements. These parameters are not directly comparable and just indicative in their compared magnitudes.

Table 5.2: Estimation results for human capital parameters

	Early childhood		Primary school		High school	
Human capital parameters:						
$\delta_{1,t}$ (investment)	0.28	(0.06)***	0.16	(0.05)***	0.18	(0.03)***
$\delta_{2,t}$ (skills)	0.10	(0.02)***	0.19	(0.02)***	0.22	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.73	(0.08)***	-0.02	(0.12)	-0.22	(0.09)**
Mother primary	0.02	(0.04)	0.06	(0.04)	0.05	(0.02)**
Mother high	0.22	(0.03)***	0.25	(0.04)***	0.16	(0.03)***
Father primary	0.02	(0.04)	0.13	(0.04)***	0.03	(0.03)
Father high	0.11	(0.03)***	0.07	(0.04)*	0.11	(0.03)***
N	27,366					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

In figure 5 I illustrate what these coefficient sizes mean if one calculates the change in percent of adding 1 unit of investment versus 1 unit of skills each period. This change varies depending on the starting values. Thus I once compare it with average values and once for a child with a low initial investment. Adding one unit of investments increases future skills by around 9% in period one and around 5% in later childhood periods. In comparison, from a lower level of investments, adding 1 unit induces a change of 20% in the first period and around 12% afterward. In contrast, adding one unit of skills to the current skills in early childhood leads to 6% higher skills in primary skills. Later, the effect of increasing skills by 1 unit is higher than that of investments, increasing to around 12-15%. Thus, investing early to increase current skills in the next period leads to higher adult skills with lower costs.

Total factor productivity (TFP) increases with parental education. In early childhood, only parents with high school education have a higher TFP, whereas in later periods also, parents with primary school education do. While maternal education's impact decreases over childhood, paternal education seems to stay the same in magnitude. The impact of age is negligible. The coefficient sizes translate into percentage differences in next period skills as depicted in figure 5. Having a mother with high school education leads to around 25% higher next-period skills in early childhood and primary school

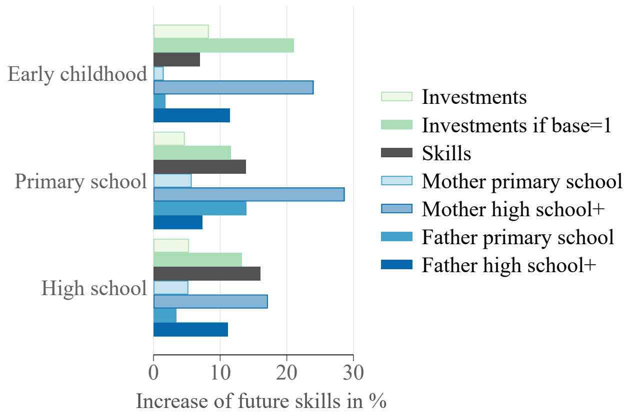


FIGURE 5: Increase of future skills if corresponding characteristic/input increases by 1 unit
Note: Increase in investment and skills each by 1 unit. Sample mean serves as base skills (1.01) and investments (3) if not otherwise indicated.

and 18% in adulthood. Father's education, in contrast, has a lower impact, around 10%. These differences also magnify investment or skill input changes as they multiply with skills and investments in the skill formation equation (see equation 5). A reason for this high magnitudes could be neighbourhood effects, as I only control for rural areas but not more nuanced units (see Chetty and Hendren (2018a), Chetty and Hendren (2018b)). Parents with high school education might live in districts with better amenities or schools. Similarly, they might send their children to different schools. If the qualities of these schools is not reflected in the differences in fees, I do not capture them separately but with the productivity differences by education. Also, as Biasi (2021) shows, differences by district in public school financing influences intergenerational mobility, which is not necessarily reflected in fees.

With this set of parameters, one can now calculate how each period's investment differences translate into differences in adult skills. In A.8, I display the effect differences in investment inputs by parental education have for each period. Thus, if in early childhood parents with no schooling would invest the average nutrition inputs parents with high school education do, their children would have 0.0018 SD higher adult skills. In later childhood, this effect becomes smaller, while offsetting schooling differences leads to more prominent effects. If parents with no schooling invest the average of parents with high school education in schooling expenditure in high school, this increases their children's adult skills by 0.0114 SD. Note that this is assuming all other factors fixed. Thus, e.g., adjusting the other inputs might lead to larger effects. Another metric to look at is how skill differences translate through childhood. For instance, what happens to adult skills if a policy intervention increases current skills in a certain childhood period by 0.1 SD? Evans and Yuan (2022) find this effect size to be the average for education interventions. In early childhood, increasing initial skills by 0.1SD leads to an adult skills increase of 0.0004. In primary school, an increase of current skills of 0.1 leads to 0.0041 higher adult skills. In high school, the same effect leads to 0.0216 SD higher skills. However, skills are more amenable early in life. Thus, in high school, it might be harder/more expensive to reach the same effect sizes as in early childhood.

In terms of preference parameters parents vary by education (see table 5.3). Parents with higher education value skills less than their lower-educated peers compared to consumption. Thus, the reason parents from lower educated backgrounds invest less in their children is not driven by their preferences.

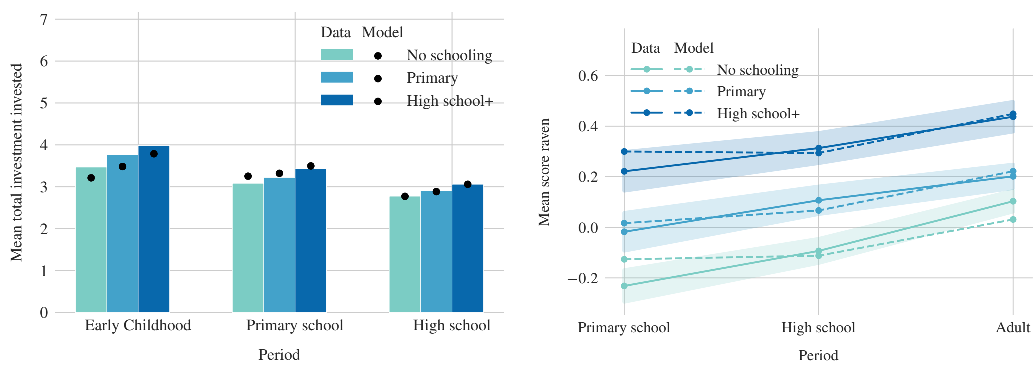
TABLE 5.3: Calibrated preference parameters

	Parental education:		
	No education	Primary school	High school+
For current skills:			
α_e	3.98	2.49	1.48
For final skills:			
γ_e	0.54	0.52	0.62

Note: Calibration method used: simulated methods of moments. Moments targeted were investments by parental education and by childhood period.

If anything, their budget constraint or their productivities keep them from investing more in their children. Higher preferences for skills are the case for their utility from current skills α_e. However, they also, to a lesser extent, derive higher utility for the final skills of their child γ_e. These parameters are derived under the assumption that parents fully know the skill formation process. Dizon-Ross (2019) and Cunha et al. (2020) find that parents with lower education overestimate the impact of their skills and underestimate the persistence of current skills. Thus, they invest less than optimal in this scenario and should invest more. As I do not account for that, in the model, the optimal value is the one observed. Hence, preference parameters are derived for these values indicating the utility derived in contrast to the one from consumption. These parents would invest more without the knowledge barrier, lowering their consumption, and the value for preferences would be even higher. Therefore, the values found here are instead the lower bound of parameters.

Comparing the targeted moments of the model with the data shows that the model does reasonably well (see table A.5). Regarding investments and untargeted moments for skills, the model fits the data well, as shown in figure 6. If anything, total investments in the early and primary school periods seem slightly off in the model simulations.



(A) Investment

(B) Skills

FIGURE 6: Model fit for investment choices and skills by period and parental education

Note: Investment and skill means plotted by parental education and childhood periods.

As these parameters are modeling the skill formation process well, I can now use them to simulate the skill gap by socio-economic status and for policy experiments. For these, it is vital to keep in mind that they will only use the estimated parameters, thus not capturing behavioral responses not modeled. Thus, I will not be able to account for differences in, e.g., school quality or network effects, other than the parts captured by parental education productivities or parenting skill types.

Because of data limitations, I also cannot model time investments, the time parents spend actively with their children, well. In general, Del Boca et al. (2014) find parental time to matter most in early childhood and monetary investments in later childhood. Hence, later periods should be less impacted by this modeling choice. My focus lies on modeling the whole childhood period and not only early childhood. Thus, more insights on monetary investments can complement the literature by adding results on other investment inputs and the transformation to the adult skill outcomes.

Also, I do not observe children in between periods and do not impose assumptions on the inputs in between periods. Therefore, I only model skill development by period and control for the age I observe the child. I abstract from modeling intra-household allocation and investment trade-offs between siblings due to data constraints and complexity, although household poverty might be shared unequally (Calvi (2020)). I account for the number of siblings in the schooling productivity and for number of children and adult household member in the income estimation. Further, I adjust household consumption with equivalence scales (for details, see appendix A.3). To limit the impact on the results, I control for household size and amount of siblings in the estimation and calibration. Additionally, I only use food diversity as a measure and not quantity, which is more likely to be impacted by disproportionate sharing.

In this context, also investment differences by gender and ethnic group might play a role, as Ashraf et al. (2020) find that Indonesian parents who have the tradition of bride prices to invest more in a girl's education after an education policy. While I control for gender in the investment function estimations, I did not find significant differences for education expenditure by gender for groups with bride prize traditions. This might be driven by the sample size as share of children who grew up in families with bride price tradition is not high with 17%. Nonetheless, future work might extend the model on this notion.

Further, a key limitation is that I so far calibrate the model without assets, which is subject to change for future research. However, for the moment, it is important to highlight this in the context of policy counterfactuals and the further analysis, as people can not borrow against future consumption or income. Thus, e.g., in case of cash transfer, a potential channel of individuals saving transfers for other periods is shut down.

6 Decomposing the skill gap

To see which factors drive the skill gap in Indonesia, I offset each potential factor and compare the original skill and investment gap to the new value simulated with the new parameters. To do so, I compare parents with no schooling to parents with high school education. For the drivers, I will start with the factors which depend on parental education: investment and human capital productivity coefficients, utility parameters, and household income. I do so by allocating parents with no education to the respective productivity parameters of parents for high education. For preferences and income, I do the same. Then I offset the differences by parenting skills for productivity and income. To do so, I apply the distribution of types for high school-educated parents to parents without schooling. I simulate one outcome taking only into account changes in the schooling productivity and one only changing income productivity. Last, I adjust for other income differences and initial skills. The results are displayed in table 6.1.

Table 6.1: Skill gap decomposition

	Investment gap change (%)	Adult skill gap change (std.)
Baseline gap	11.64	0.35
Parental education:		
Income	-24.92	-0.09
Preferences	46.86	0.12
Investment productivities	2.54	0.02
Skill productivities	0.00	-0.35
Parenting skills:		
Income by type	-12.19	-0.04
Type productivities	0.88	0.01
Other:		
Income (rest)	-25.61	-0.10
Initial skills	0.00	-0.00

Note: Gaps indicated are between high school parents and parents with no schooling.

Regarding investment differences over the whole childhood, most impacts have preferences and income. Offsetting the differences in income driven by education reduces the skill gap by 60%. In contrast, if parents with no schooling had the same preferences as parents with high school, the gap would increase by 46%. Investment and parenting skill type differences in productivity do not seem to influence the investment skill gap much; parenting skill types do mostly so by income.

For policy conclusions, it is essential how these investment changes translate into adult skill outcomes. The adult skill gap is 0.35 SD wide for children of parents with no schooling versus children with parents with high school education. Regarding differences in investments, mainly income differences translate into decreasing this gap by 0.23 SD. Again, investment productivities have a limited impact. However, a large part of the skill gap is driven by the total factor productivity differences, which do not influence investments. Closing those would decrease the skill gap by 0.35 SD. If parents with no schooling had the same preferences as parents with high school education, the skill gap would widen further by 0.12 SD. Initial skills do not play a significant role. However, note that these are probably imperfectly proxied by health measures in the first period.

To understand further the dynamics of skill development, figure 7 plots the changes of offsetting some of the drivers by period. I plot average skills by period of children with parents with no schooling and high school education. Thus, this indicates the initial skill gap. Then, I plot the average skills by period for children of parents without schooling, offsetting: preference, initial skill, income by education, and total factor productivity by education differences.

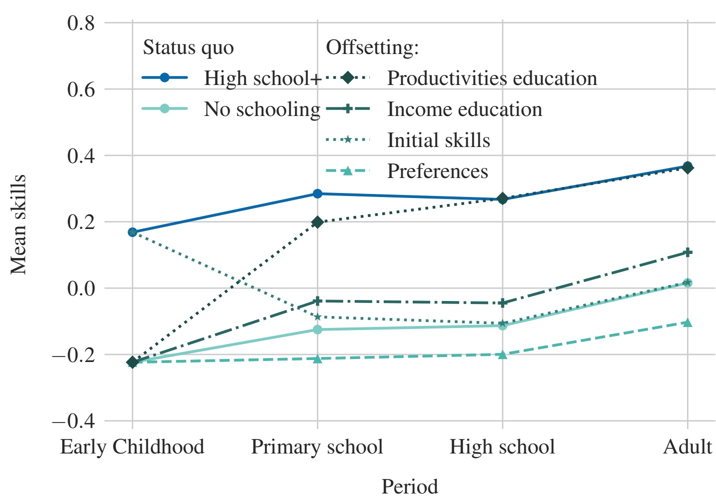


Figure 7: Skill gap decomposition

In early childhood and high school, income has a larger impact on the future skill gap than in primary childhood. In contrast, differences in productivity by education widen the skill gap, especially in primary school. This indicates that the environmental factors related to parental education are more critical during primary school, while budget constraints drive the differences in early and late childhood. To compare this with investment gap changes by period, see table A.7. The importance of income increases over childhood periods to close the investment gap. This could be driven by the fact that monetary investments become more critical with time, and schooling gets more expensive in high school. Preference differences magnify in high school, same for differences driven by investment productivities, although those are small in comparison. In primary school, income seems to contribute the lowest part to the investment gap.

For policies, offsetting income differences would have significant effects, especially in early childhood and primary school. Targeting the total factor productivity is more challenging. Nonetheless, differences in parents' education might be mitigated by changes in the environment these children grow up in, as this parameter captures all this. They do not seem to be driven too much by any productivities related to investments but rather parts that are not explicitly modeled here and call for further extensions of this model. Increasing parents' education would generally lead to a smaller skill gap as it would also mitigate large parts of the income differences.

7 Policy experiments

I simulate three policies, first alone and then in interaction. Using the model to simulate adult outcomes of policies implemented together, I can see how parental responses adjust depending on the policy setting. The first policy experiment is an unconditional cash transfer in each childhood period of 25% of the mean income. This simulation mimics the effect of unconditional cash transfers, a widespread policy tool. However, what would be the outcome if these would be paid throughout the whole childhood, in contrast to only in early childhood, as often the case. Second, I will model the impact of a home intervention that has sustained effects on parenting skills. Thus, I assume the home intervention changes the parenting type of parents without schooling to the most productive one, type 2. A third experiment will introduce a food price subsidy of 20% for all households. I then simulate the effect of the cash transfer once joint with the parenting intervention and once joint with the price subsidy. The simulations' results are summarized in table 7.1.

Table 7.1: Skill gap change by policy experiments

	Investment gap change (%)	Adult skill gap change (std.)
Baseline gap	11.68	0.35
Unconditional cash transfer:		
25% of income	-23.77	-0.10
Home intervention:		
Parenting type 2	-16.92	-0.05
Food price subsidy:		
20% food price decrease	-20.28	-0.07
Combined:		
Cash + home	-34.85	-0.14
Cash + food price	-35.07	-0.15

Note: Gap indicated are between high school parents and parents with no schooling.

As one can see, the cash transfer closes the gap on investments to some extent, which translates into a decrease in the skill gap of 0.10 SD (29%), which indicates a significant long-run effect. In contrast, the home intervention leads to half of the effect, 0.05 SD (14%) smaller skill gap. The nutrition price subsidy leads to a decrease of 0.07 SD (20%). If paid only to parents without schooling, this effect would be even bigger as the subsidy also increases investments by parents with high school education. However, as the skill gap is still smaller, the policy has a larger effect on lower-educated parents.

Combining the policies shows that both interventions have no additional increase in skills when jointly implemented. This is driven by the fact that the investment gap does not close further. The total gap is less than if you run the interventions separately and then calculate the expected gap. This indicates that parents change investment behavior when receiving both interventions simultaneously. Hence there are no significant dynamic complementarities between these two policies when one considers parental responses. Regarding the changes, the home intervention and cash transfer lose less of the original combined effect size than the food price subsidy and the cash transfer.

8 Concluding remarks

This paper documents the skill gap for children from different socioeconomic backgrounds in Indonesia. I quantify which drivers contribute to the skill gap in each childhood period: early childhood, primary school, and high school. To do so, a flexible model of children's skill formation and parental investment decisions on nutrition and schooling is estimated and calibrated. Results show that investments matter, especially in early childhood, and skills become more persistent in later childhood. Nutrition and schooling are complements and more complementary in high school than in primary school.

Regarding drivers of the skill gap, parental income contributes to 0.23 SD of the adult skill gap. Mainly, the skill gap is driven by differences in the total factor productivity of investments by parental education (0.35 SD). These differences are particularly evident in primary school. Regarding preferences, parents with lower education value their children's skills more than parents with high school education in Indonesia. Thus, the differences in skills are not driven by preferences but mainly by income and skill production productivity.

Policies such as unconditional cash transfers and nutrition interventions can partly close the skill gap. Income transfers reduce the skill gap by 0.10 SD and food price subsidies by 0.07 SD. Jointly they reduce the skill gap by 0.15 SD. The combined effect is smaller than the single effects added together, highlighting the importance of considering parents' responses when designing policies. Future research could focus on accounting for information disparities between parents from different socioeconomic statuses and how they influence parents' responses to policies. Further, the interplay between time investments and a more detailed modeled first period of childhood and prenatal investment could lead to additional insights into the skill formation process.

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A Appendix

A.1 Data

A.1.1 Food prices and nutrition investments

To capture nutritional diversity, nutrition investments are proxied by the number of food groups. Food groups in the consumption data are carbons, protein, dairy, vegetables, and fruits. If the household expenditure on one food group is more than 5% of the total expenditure, it is counted as an investment in this food group. Due to data constraints, I cannot identify if household consumption aligns with the child's nutrition. However, I assume that it is a good enough proxy for nutritional diversity since it is unlikely that children receive entirely different food than those bought by the household. Nutrition diversity is expressed by a measure between 1 and 5, with $n_t = 5$ meaning that a child consumes all five food groups and $n_t = 1$ that it consumes only one food group.

For food prices, I rely on the community surveys in the IFLS, which survey food prices in the community markets and shops. I construct unit prices of protein, carbons, and vegetables, which are the most prominent consumption expenditure groups and have the most reliable price data (in terms of units).

Then I build the food price by weighting prices by the median consumption fraction for households in the sample consuming all three groups. This leads to a weight of 0.43 for carbs, 0.14 for vegetables, and 0.43 for meat. These prices are then scaled by the average kilograms consumed by households using equivalence scales for Indonesia estimated by (Olken, 2006) for different ages and household compositions, which are close to the modified OECD scale. To do so, I use equivalence scales and median prices to find the median amount of kg consumed by a household. This I then multiply by the factor an additional child of the corresponding age eats and the median regional food price mentioned above.

A.1.2 Schooling prices and investments

For each household, I have detailed information on what they spend on schooling, e.g., the school fees and books, uniforms, and transport. As investments, I define all registration costs, exam costs, and fees, which the household pays for the child's education. I add the investments into books. I restrain from adding food, uniforms, and transport costs, since I do not assume them to measure the school's quality and influence skill formation. However, this neglects potential budget constraints for these items. The schooling price is assumed to be equal to 1.

A.1.3 Household income

I sum all income reported for the household. This includes business and farm business income, as well as all other income received by any of the household members. Further, this entails non-labor income, the number of transfers, retirement payments, and scholarships received. I adjust household income by the household size for the calibration. For that, I use Olken (2005) equivalence scales derived for Indonesia. As these are derived from aid allocated by the Raskin rice program to different family structures, I assume they will mimic the family's income and how it translates into consumption. Deaton and Zaidi (2002) and Batana et al. (2013) state that the wildly used modified OECD scale or square root scales suit high-income countries. Using the scale for low-income countries might overestimate the degree of the economics of scale, as durables are easier to share than food, a significant fraction of the expenditure in low-income countries. Further, they tend to overestimate the cost of children. Hence, I use Olken's estimated scale, which is higher. Thus the economics of scale are lower. Most scales are convertible in the following:

$$N^{eq} = (n_a + \alpha n_c)^{\theta} \tag{18}$$

where n_a is the number of adults in the household, and n_c is the number of children. α is the cost of children, and θ expresses the economies of scale. In the scare root scale, $\alpha = 1$ and $\theta = 0.5$. In contrast Olken estimates $\alpha = 0.93$ and $\theta = 0.85$, which confirms Deaton and Zaidi (2002)'s claim that the economies of scale are lower, thus θ higher in low-income countries. This also goes with Santaeulàlia-Llopis and Zheng (2017), who estimate scale parameters in Malawi to be higher than the OECD ones.

A.1.4 Skill measures

For health skills, the following measures are used: height and weight. With the help of the WHO Child Growth Standards and WHO Reference 2007 composite data files as the reference data, I build z-scores for children under 20 years old (Vidmar et al., 2013). Hereby the height-for-age, weight-for-age, BMI-for-age and weight-for-height z-scores are computed. BMI is taken as an indicator for older individuals, thus the parents and adults. In period 1, early childhood, the measures used are height-for-age and weight-for-age since no cognitive measures are available.

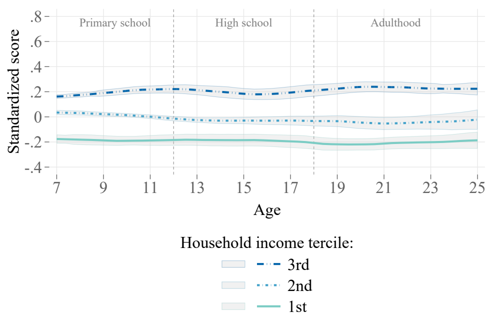
For cognitive skills outcomes, cognitive tests conducted by the survey team are available, which I standardize by age. The IFLS has several test score metrics available: In 1997, a math test with 40 questions was conducted for the following age groups: 7-9, 10-12, and 13-24, and the same was done for a language evaluation. For younger ages, no test scores are available. Therefore, in the early childhood period, only health outcomes can serve as a measure of skills. For 2000, 2007, and 2014 a raven test was conducted with 12 questions, followed by a math test of 5. These were designed in 2 versions, one for age group 7 to 14, the other 15 to 24. In both cases, the number of correct answers is standardized by age and year. Adult respondents answered a cognitive test in 2007 and 2014. The tests ask them to remember ten words for a short period, and a second round asks how many they remember after some minutes. In 2014 additionally, a simple subtraction exercise was asked. Adult test scores are standardized by year to avoid some candidates being counted double. As cognitive measures during childhood, raven or language and math scores are taken, while for adults, an average for word- and math tests is taken.

A.2 Stylized facts and descriptives

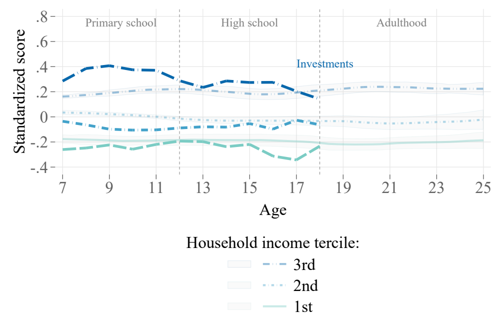
Table A.1: Sample characteristics in early childhood

	Mean	SD	Min	Max
Age	3.22	1.48	0	5
Female	0.50	0.50	0	1
Weight-by-age	-0.97	1.40	-4.97	4.92
Height-by-age	-1.17	1.74	-4.98	4.97
Stunting	0.32	0.47	0	1
Wasting	0.09	0.29	0	1
Year of birth	1997.63	4.48	1991	2008
Mother's years of education	5.92	4.01	0	18
Father's years of education	6.83	4.23	0	18
Mother's age	36.71	7.80	18	55
Father's age	41.85	8.45	20	60
Not Muslim	0.10	0.30	0	1
Household income	218.59	205.4	0	1339.2
Adult household members	3.80	1.98	1	23
Household members <18	1.99	1.50	0	8
N	3,092			

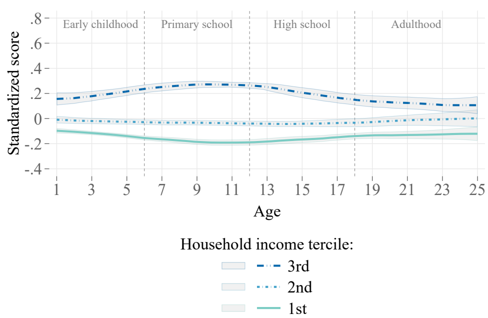
Monetary values are deflated and reported in 100,000 Rupees



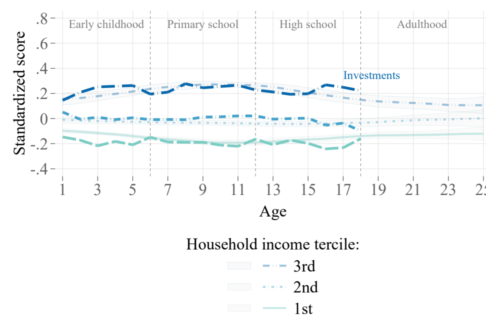
(A) Test score



(B) Test score and investments



(C) Height



(D) Height and investments

FIGURE 8: Children skills over age by parental income

Note: Corresponding skills are fitted with local mean smoothing by age and parental education groups. Parental income corresponds to the household income in terciles. Confidence intervals displayed are at 95% level. Investments plotted are schooling investments for test scores and nutrition investments for height. Scores of skills and investments are standardized by age to have mean 0 and SD of 1.

A.3 Estimation and calibration details

A.3.1 K-means algorithm

I follow Bonhomme et al. (2022) to estimate the unobserved types of parenting skills outside of the model. To do so, I build means over the life-cycle of schooling, nutrition investments, and household income for each parent couple. I then standardize these and run the k-means clustering procedure, which will allocate each household to the cluster whose moments have the least distance to the cluster mean.

To estimate heterogeneity groups using the k-means clustering algorithm, I need to choose the number of clustering groups K. As this is a data-driven approach, they are not known as apriori, but data can be used to determine it. To do so, I use the commonly used Elbow statistic. For a given number of clusters K, the algorithm minimizes the total within-cluster variance:

$$\min_{k\in\{1,..,K\}^N} \sum_{t=1}^{N}\sum_{c=1}^{C} ||\boldsymbol{m_{t,c}} - \overline{\boldsymbol{m}}_{\boldsymbol{k}}||^2 = SSE_k \tag{19}$$

To compare Elbow statistics, the variance SSE_k is calculated for each number of clusters run, $k = 1; ...; K_{max}$. These statistics are then plotted against their corresponding number of clusters, as seen in figure 9a. With an increasing number of clusters, the variance decreases as observations within a cluster become more similar. The optimal number of clusters is at the kink in the plot, i.e. the point where the decrease in SSE changes the most. Adding more clusters than at this king would have limited value in explaining the variation in the data. The silhouette criterion in figure 9b. The higher the criteria value, the more the two clusters are from each other. Thus, the borders between them are well defined.

As shown in figure 9a, the elbow criteria determines the optimal amount of clusters K to be 4. The silhouette criterion is maximized at two but also high at 4. To check if the number of clusters drives the results, I run the GMM estimation for $K \in \{2, 3, 4, 5\}$ clusters. As one can see the results for $K = 2$ in table A.9, $K = 3$ in table A.10, $K = 5$ in table A.11 are comparable to the main results in table A.4 with $K = 4$. Coefficients and standard errors only vary marginally. Thus, the amount of clusters does not drive the results and, if anything, adds explanatory power. More clusters seem to explain more unobserved heterogeneity in investments, as schooling productivity varies by type. However, after $K = 4$, the amount of observations decreases by type, as shown in table A.6. Hence, increasing the computational burden further has little reward. This is confirmed by the fact that these amounts exceed the amount determined to be optimal by the elbow criterion.

A.3.2 Household income

To estimate household income, I regress parental education, number of household members (adults and children), rurality and age of the household head, and parenting skills on household income. Additionally, I include year and province fixed effects. Thus:

$$\ln(y_t) = Z'_{y,t}\gamma_y + \eta'\gamma_\eta + \epsilon_{y,t} \tag{20}$$

Here, $Z_{y,t}$ are the named household characteristics that can vary by period. η is the unobserved parenting skills I assume the household income, as it is likely that characteristics resulting in productive parents also translate at least partly into higher wages. Results can be found in table A.3.

I use the resulting coefficients to predict future household income for the calibrations and simulations. Further, I assume the income shocks to be i.i.d. normally distributed. Thus $\epsilon \overset{i.i.d}{\sim} N(0, \sigma_y)$.

A.3.3 Transition of other household characteristics

I assume all household characteristics to be stable over time, except the year, age, and age of the household head. Since period one observations I use for the calibration start are either in 1997 or 2000 for the transition to the next period, I get either 2000 or 2007 for 1997 or 2007 for 2000 (observed for the first period, as I know next period). Afterward, due to the survey design, all future waves are seven years apart. Thus I apply that to simulate the year in which the child is observed in the next period. Then apply this gap to its age and the father's age.

Knowing the next year then allows me to allocate the correct food price for the given community in that year to the simulated period. Thus, I assume they do not move. For now, I assume the number of household members and other children in the household to be stable across childhood, the same for the location in a rural or urban area. To relax this assumption could be a potential future extension.

A.3.4 Skill formation estimation

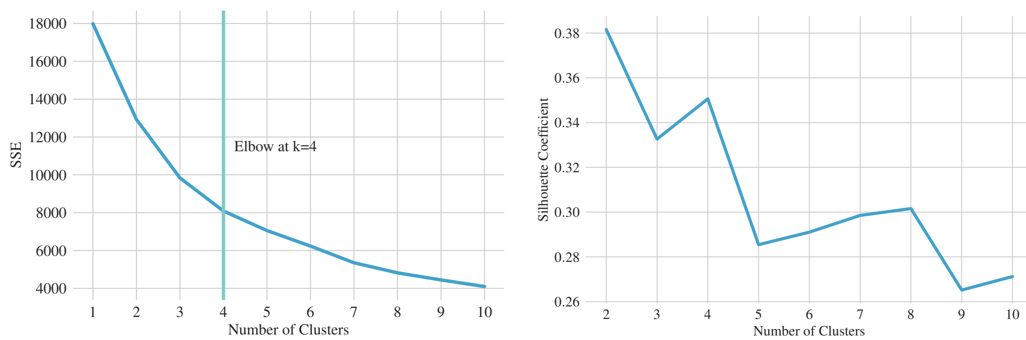
Regarding the GMM estimation, two obstacles driven by data constraints occur. Firstly, only nutrition inputs are available to measure investments in the first period. Thus, there is no stage with relative investment input ratios, which can then be plugged into the human capital parameters. Hence the food groups are directly plugged into this equation. Further, I do not observe cognitive skills in the early stage of childhood. Hence, I use height and weight as a proxy. Therefore, $\delta_{2,1}$, the persistence of skills cannot be directly compared to the parameters in later periods, as it measures the persistence of height and weight on future cognitive skills.

Second, as nutrition is discrete and constraint, the optimal demand ratios for the GMM moments hold only if $n_t < 5$ (see A.6 for details for $n_t = 5$). In the main specifications, I also include $n_t = 5$, assuming that it does not drive the results. As a robustness check, I dropped them and ran the results without using observations with $n_t = 5$ to estimate the relative demand equations (see table A.12). The results are relatively similar, which indicates that this subgroup does not drive the general results. If anything, the estimates are less precise, but this could also come from the smaller sample. However, dropping them introduces selection. Thus the results have to be taken with a grain of salt. Future work should exploit how these constraints bias the estimation results.

Calibration

To calibrate the model, I use the optimal solution for investments in equations 57 and 68. I match simulated investment means by parental education and childhood period to get γ_e and α_e, thus six missing parameters. To calibrate, I use the sample from period one and simulate periods two to four forward with them, to then compare it to the data I observe in those periods in the survey. For now, I assume no saving or borrowing.

A.4 Estimation results



(A) Elbow criterion (B) Silhouette criterion

FIGURE 9: Criterion plots to determine number of clusters for parenting skills

Note: K-means algorithm run for different number of clusters to determine correct number for the following estimation. Plotted are on the right-hand side the within cluster variance, on the left-hand side the Silhouette coefficient by number of clusters used.

Table A.2: Characteristics of parenting skill types η

	Averages for type:			
Variable	0	1	2	3
Food investments	0.73	-0.67	0.43	0.58
Education investment	-0.05	-0.30	2.72	0.51
Household income	-0.06	-0.35	0.48	2.97
Fraction mothers with primary school	0.27	0.24	0.14	0.20
Fraction mothers with high school	0.30	0.14	0.55	0.55
Observations	2,613	2,774	356	252

Note: This table displays summary statistics for each of the four clustering groups resulting from k-means procedure. All variables are life-cycle averages and standardized to have mean 0 and standard deviation 1.

Table A.3: Estimation results for household income

	Log(income)	
Father primary education	0.152***	(0.014)
Father high school+	0.422***	(0.016)
Mother primary education	0.112***	(0.014)
Mother high school+	0.294***	(0.017)
Parenting type 1	-0.375***	(0.012)
Parenting type 2	0.296***	(0.027)
Parenting type 3	1.066***	(0.026)
Father age	0.053***	(0.003)
Father age squared	-0.001***	(0.000)
Rural area	-0.348***	(0.012)
Adult household members	0.104***	(0.003)
Non-adult household members	0.016***	(0.004)
Constant	3.109***	(0.079)
Year fixed effects	Yes	
Province fixed effects	Yes	
N	36,169	

Note: Standard errors in parenthesis, *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table A.4: Estimation results for skill formation parameters

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.75	(0.86)***	-11.38	(5.11)**
Implied elasticity			0.21		0.08	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.68	(0.51)***	-42.17	(16.55)**
Mother primary			1.10	(0.25)***	3.06	(1.32)**
Mother high			1.87	(0.39)***	5.04	(2.15)**
Father primary			0.09	(0.16)	0.63	(0.47)
Father high			-0.08	(0.19)	0.51	(0.50)
Age			-0.05	(0.04)	3.14	(1.30)**
Female			0.05	(0.13)	1.29	(0.61)**
Rural area			-2.64	(0.53)***	-5.19	(2.22)**
No. of siblings			-0.73	(0.14)***	-2.14	(0.88)**
Mother not Islam			0.39	(0.22)*	1.68	(0.85)**
Parenting type 1			-0.24	(0.14)*	0.06	(0.34)
Parenting type 2			4.74	(0.97)***	9.62	(4.10)**
Parenting type 3			1.64	(0.50)***	2.47	(1.29)*
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.06)***	0.16	(0.05)***	0.18	(0.03)***
$\delta_{2,t}$ (skills)	0.10	(0.02)***	0.19	(0.02)***	0.22	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.73	(0.08)***	-0.02	(0.12)	-0.22	(0.09)**
Mother primary	0.02	(0.04)	0.06	(0.04)	0.05	(0.02)**
Mother high	0.22	(0.03)***	0.25	(0.04)***	0.16	(0.03)***
Father primary	0.02	(0.04)	0.13	(0.04)***	0.03	(0.03)
Father high	0.11	(0.03)***	0.07	(0.04)*	0.11	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)**	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	1.00	(0.07)	1.07	(0.01)	1.09	(0.01)
$\lambda_{4,ts}$					1.21	(0.04)
N	27,366					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

A.5 Calibration results

Table A.5: Model fit - targeted moments

	Model	Data	SD	Difference
No education:				
Early childhood	3.21	3.47	0.80	-0.32
Primary school	3.25	3.08	0.94	0.18
High school	2.77	2.78	1.13	-0.00
Primary school:				
Early childhood	3.48	3.76	0.83	-0.34
Primary school	3.32	3.22	0.99	0.10
High school	2.89	2.90	1.16	-0.01
High school+:				
Early childhood	3.79	3.98	0.80	-0.24
Primary school	3.50	3.43	1.08	0.06
High school	3.06	3.06	1.26	0.00

Note: Calibration method used: simulated methods of moments. Differences are expressed in standard deviations. Values are total investments by parental education and childhood period

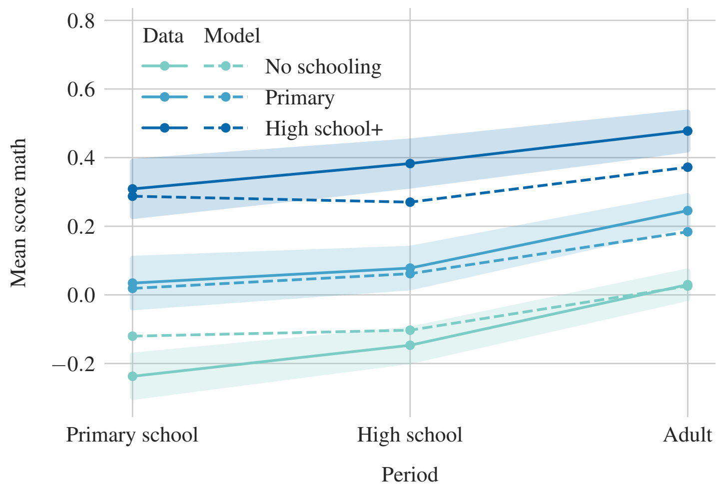
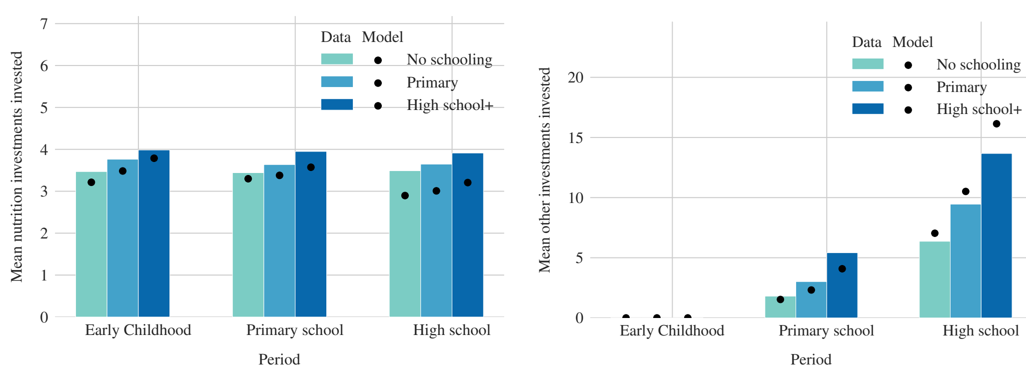


FIGURE 10: Model fit for children's skills by period



(A) Nutrition

(B) Schooling expenditure

FIGURE 11: Data and model moments for investment input choices by period

Note: Investment inputs means plotted by parental education and childhood periods. Black dots are corresponding simulated moments.

A.6 Derivation Formulas:

Inter-temporal solution n_t and s_t and relative demands

To derive the relative demands we take first-order conditions for the minimization problem:

$$\begin{aligned} \min_{n_t, s_t} \quad & p_{n,t} n_t + p_{s,t} s_t \\ \text{s.t.} \quad & n_t \leq 5 \\ & It = [a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} \end{aligned} \tag{21}$$

The Lagrangian looks the following:

$$\mathcal{L} = p_{n,t} n_t + p_{s,t} s_t - \lambda_{1,t}(I_t - [a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}) - \lambda_{2,t}(n_t - 5) \tag{22}$$

Deriving first order conditions in period 2 and 3:

$$\frac{\partial \mathcal{L}}{\partial s_t} = p_{s,t} - \lambda_{1,t}(a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t - 1} \rho_t) \times [a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t} - 1} \frac{1}{\rho_t} = 0 \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = p_{n,t} - \lambda_{1,t}(n_t^{\rho_t - 1} \rho_t) \times [a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t} - 1} \frac{1}{\rho_t} - \lambda_{2,t} = 0 \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{1,t}} = I_t - [a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} = 0 \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{2,t}} = n_t - 5 = 0 \tag{26}$$

Case 1a: Non-binding constraints

If both constraints are not binding, $\lambda_{2,t} = 0$, since $n_t < 5$. Taking ratios $\frac{\frac{\partial \mathcal{L}}{\partial n_t}}{\frac{\partial \mathcal{L}}{\partial s_t}}$ leads:

$$\frac{p_{n,t}}{p_{s,t}} = \frac{n_t^{\rho_t - 1}}{a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t - 1}} \tag{27}$$

which allows to get n_t in terms of s_t:

$$n_t = \left(\frac{p_{n,t}}{p_{s,t}} a_{s,t}(Z_{s,t}, \eta) \right)^{\frac{1}{\rho_t - 1}} s_t = \Phi_1 s_t \tag{28}$$

and vice versa:

$$s_t = \Phi_1^{-1} n_t \tag{29}$$

Case 1b: $n_t \leq 5$ is binding

This means $n_t = 5$ and $I_t = [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}$. If I_t is given, it follows:

$$s_t = \left(\frac{(I_t^{\rho_t} - 5^{\rho_t})}{a_{s,t}(Z_{s,t}, \eta)} \right)^{\frac{1}{\rho_t}} \tag{30}$$

Price for total investments Λ_t and relative demands I_t and I_{t+1}

The price for total investments I_t is supposed to mimic the cost for one unit of investment, thus:

$$\begin{aligned} E_t &= \Lambda_t I_t \\ \Lambda_t &= \frac{E_t}{I_t} \\ \Lambda_t &= \frac{p_{n,t} n_t + p_{s,t} s_t}{[a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}} \end{aligned} \tag{31}$$

To calculate prices we use 28 to get expressions for n_t in terms of s_t:

$$n_t = \left(\frac{p_{n,t}}{p_{s,t}} a_{s,t}(Z_{s,t}, \eta) \right)^{\frac{1}{\rho_t - 1}} s_t = \Phi_1 s_t \tag{32}$$

Replacing n_t in yields in 31 with moving s_t out of E_t:

$$\begin{aligned} \Lambda_t &= \frac{s_t(p_{s,t} + p_{n,t}\Phi_1)}{[a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + (\Phi_1 s_t)^{\rho_t}]^{\frac{1}{\rho_t}}} \\ &= \frac{(p_{s,t} + p_{n,t}\Phi_1)}{[a_{s,t}(Z_{s,t}, \eta) + \Phi_1^{\rho_t}]^{\frac{1}{\rho_t}}} \end{aligned} \tag{33}$$

In case the household is constrained ($n_t = 5$), this price does not apply, as it uses the fact that, s_t can be expressed as a share of n_t given the level of investments. In the case that $n_t = 5$, therefore, the household maximizes differently (see next section). In period 1 $\Lambda_t = p_{n,t}$ as investment input decisions only take place for nutrition.

Intra-temporal solution for I_t

Case 1a: Non-binding constraints
If case 1a holds then, $n_t < 5$, and equation 28 holds, thus we can use the total price of investment equation 31 for the maximization problem to derive solutions for I_t, c_t and a_{t+1} :

$$\begin{aligned} V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, I_t, a_{t+1}} & u(c_t) + \alpha_e v(\Psi_t) \\ & + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ \text{s.t.} \quad c_t + \Lambda_t I_t + a_{t+1} &= (1+r)a_t + y_t \\ a_{t+1} &\geq a_{min,t} \\ \text{with} \quad \Psi_{t+1} &= \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \\ V_{T+1}(\Psi_{T+1}) &= \alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1}) \\ u(c_t) &= \ln(c_t) \\ v(\Psi_t) &= \ln(\Psi_t) \end{aligned} \tag{34}$$

Which gives the Lagrangian:

$$\begin{aligned} \mathcal{L} = u(c_t) + \alpha_e v(\Psi_t) + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ - \lambda_t(c_t + \Lambda_t I_t + a_{t+1} - (1+r)a_t - y_t) - \xi_t(a_{min,t} - a_{t+1}) \end{aligned} \tag{35}$$

T=3 here, because the period 3 is the last one, where the household makes decisions. The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial I_t} = \beta \frac{\partial V_{t+1}}{\partial I_t} - \lambda_t \Lambda_t = 0 \tag{36}$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = u'(c_t) - \lambda_t = 0 \tag{37}$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = \beta \frac{\partial V_{t+1}}{\partial a_{t+1}} - \lambda_t + \xi_t = 0 \tag{38}$$

$$\textit{or} \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = -\lambda_t + \xi_t + \mathbb{1}\{t < T\}(\lambda_{t+1}\beta(1+r)) + \mathbb{1}\{t = T\}\beta \frac{\partial V_{T+1}}{\partial a_{T+1}} \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = c_t + \Lambda_t I_t + a_{t+1} - (1+r)a_t - y_t = 0 \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial \xi_t} = a_{min,t} - a_{t+1} = 0 \tag{42}$$

$$\tag{43}$$

Following these one can derive a solution for I_t. First one needs to derive after I_t, which will vary by period due to the continuation value. In period 3, the continuation value looks the following:

$$\begin{aligned} \beta V_{T+1}(\Psi_{T+1}) =& \beta(\alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1})) \\ \text{with } \Psi_{t+1} =& \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \end{aligned} \tag{44}$$

Plugging it in V_{t+1}:

$$\beta V_{t+1}(\Psi_{t+1}) = \beta(\alpha_e \gamma_e \ln(\theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}}) + \zeta \ln(a_{T+1})) \tag{45}$$

Thus:

$$\beta \frac{\partial V_{t+1}}{\partial I_t} = \frac{\beta \delta_{1,t} \alpha_e \gamma_e}{I_t} = \frac{K_t}{I_t} \tag{46}$$

For period 2:

$$\beta V_{t+1}(\Psi_{t+1}) = \beta(u(c_{t+1}) + \alpha_e v(\Psi_{t+1})) + \beta^2(\alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1})) \tag{47}$$

which is:

$$\begin{aligned} \beta V_{t+1}(\Psi_{t+1}) =& \beta(\ln(c_{t+1}) + \alpha_e \ln(\theta_t(Z_{\theta,t})) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}}) \\ & + \beta^2(\alpha_e \gamma_e \ln(\theta_{t+1}(Z_{\theta,t+1}) I_{t+1}^{\delta_{1,t+1}} \Psi_{t+1}^{\delta_{2,t+1}}) + \zeta \ln(a_{t+2})) \end{aligned} \tag{48}$$

plugging in Ψ_{t+1}:

$$\begin{aligned} \beta V_{t+1}(\Psi_{t+1}) =& \beta(\ln(c_{t+1}) + \alpha_e \ln(\theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}})) \\ & + \beta^2(\alpha_e \gamma_e \ln(\theta_{t+1}(Z_{\theta,t+1}) I_{t+1}^{\delta_{1,t+1}} (\theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}})^{\delta_{2,t+1}}) \\ & + \zeta \ln(a_{t+2})) \end{aligned} \tag{49}$$

Thus:

$$\beta \frac{\partial V_{t+1}}{\partial I_t} = \frac{\beta \delta_{1,t}(\alpha_e + \beta \delta_{2,t+1} \gamma_e \alpha_e)}{I_t} = \frac{K_t}{I_t} \tag{50}$$

For period 1:

$$\begin{aligned} \beta V_{t+1}(\Psi_{t+1}) = \beta(u(c_{t+1}) + \alpha_e v(\Psi_{t+1})) + \beta^2(u(c_{t+2}) + \alpha_e v(\Psi_{t+2})) \\ + \beta^3(\alpha_e \gamma_e \ln(\Psi_{t+3}) + \zeta \ln(a_{t+3})) \end{aligned} \tag{51}$$

Resulting in:

$$
\begin{aligned}
\beta V_{t+1}(\Psi_{t+1}) =& \beta(u(c_{t+1}) + \alpha_e \ln(\theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}})) + \beta^2(u(c_{t+2}) \\
& + \alpha_e \ln(\theta_{t+1}(Z_{\theta,t+1}) I_{t+1}^{\delta_{1,t+1}} (\theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}})^{\delta_{2,t+1}})) \\
& + \beta^3(\alpha_e \gamma_e \ln(Z_{\theta,t+2}) I_{t+2}^{\delta_{1,t+2}} (\theta_t(Z_{\theta,t+1}) I_{t+1}^{\delta_{1,t+1}} (Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}})^{\delta_{2,t+1}})^{\delta_{2,t+2}}) \\
& + \zeta \ln(a_{t+3}))
\end{aligned} \tag{52}
$$

Giving:

$$
\beta \frac{\partial V_{t+1}}{\partial I_t} = \frac{\beta \delta_{1,t}(\alpha_e + \beta \delta_{2,t+1}(\alpha_e + \beta \delta_{2,t+2} \gamma_e \alpha_e))}{I_t} = \frac{K_t}{I_t} \tag{53}
$$

Using the FOCs for c_t and I_t, and the values above for K_t, results in:

$$
\frac{\partial \mathcal{L}}{\partial I_t} = \frac{K_t}{I_t} - u'(c,t)\Lambda_t = 0 \tag{54}
$$

Now to derive an optimal solution for I_t, I use:

$$
c_t = -\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t \tag{55}
$$

plugging in:

$$
\begin{aligned}
\frac{K_t}{I_t} - \frac{\Lambda_t}{-\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t} &= 0 \\
\frac{\Lambda_t}{-\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t} &= \frac{K_t}{I_t} \\
(-\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t)K_t &= \Lambda_t I_t \\
(-a_{t+1} + (1+r)a_t + y_t)K_t &= \Lambda_t I_t + K_t \Lambda_t I_t
\end{aligned} \tag{56}
$$

Thus, the optimal solution for I_t:

$$
I_t = \frac{K_t(-a_{t+1} + (1+r)a_t + y_t)}{\Lambda_t(1+K_t)} \tag{57}
$$

For the borrowing constrained case, $a_{t+1} = a_{min,t}$, for the non-borrowing constrained case, an optimal solution for a_{t+1} is needed.If $a_t = 0$ and there are no assets, the amount of I_t depends apart from the parameters and related characteristics only on household income y_t. This solution can also be used for period 1, as $I_t = n_t$ and $\Lambda_t = p_{n,t}$.

Optimal solution for s_t and n_t, if non-binding

With I_t one can derive n_t and s_t:

$$I_t = [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + (\Phi_1 s_t)^{\rho_t}]^{\frac{1}{\rho_t}} = [a_{s,t}(Z_{s,t},\eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}} s_t \tag{58}$$

using equation 57 for I_t:

$$\frac{K_t(-a_{t+1} + (1+r)a_t + y_t)}{\Lambda_t(1+K_t)} = [a_{s,t}(Z_{s,t},\eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}} s_t \tag{59}$$

$$s_t = \frac{K_t(-a_{t+1} + (1+r)a_t + y_t)}{\Lambda_t(1+K_t)[a_{s,t}(Z_{s,t},\eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}}} \tag{60}$$

With equation 28:

$$n_t = \Phi_1 \frac{K_t(-a_{t+1} + (1+r)a_t + y_t)}{\Lambda_t(1+K_t)[a_{s,t}(Z_{s,t},\eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}}} \tag{61}$$

Case 1b: $n_t < 5$ is binding

This means $n_t = 5$ and $I_t = [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5]^{\frac{1}{\rho_t}}$

$$\begin{aligned}
V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, s_t, a_{t+1}} &\; u(c_t) + \alpha_e v(\Psi_t) \\
&+ \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\
\text{s.t.} \quad c_t + 5p_{n,t} + p_{s,t}s_t + a_{t+1} &= (1+r)a_t + y_t \\
a_{t+1} &\geq a_{min,t} \\
\text{with} \quad \Psi_{t+1} &= \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \\
V_{T+1}(\Psi_{T+1}) &= \alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1}) \\
u(c_t) &= \ln(c_t) \\
v(\Psi_t) &= \ln(\Psi_t) \\
I_t &= [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}
\end{aligned} \tag{62}$$

Then:

$$\frac{\partial \mathcal{L}}{\partial s_t} = \beta \frac{\partial V_{t+1}}{\partial I_t} \frac{\partial I_t}{\partial s_t} - \lambda_t(p_{s,t}) = 0 \tag{63}$$

Drawing from the non-binding case, therefore:

$$\beta \frac{\partial V_{T+1}}{\partial I_t} = \frac{K_t}{I_t} = \frac{K_t}{[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}} \tag{64}$$

which results in:

$$\frac{\partial \mathcal{L}}{\partial s_t} = \frac{K_t}{[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}}(a_{s,t}(Z_{s,t},\eta)s_t^{(\rho_t-1)}[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}-1}) \\ - u'(c,t)p_{s_t} = 0 \quad (65)$$

which yields:

$$u'(c,t)p_{s_t} = \frac{K_t}{[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]}a_{s,t}(Z_{s,t},\eta)s_t^{(\rho_t-1)} \quad (66)$$

Plugging in the budget constraint:

$$\frac{p_{s_t}}{-5p_{n_t} - p_{s_t}s_t - a_{t+1} + (1+r)a_t + y_t} = \\ \frac{K_t}{[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]}a_{s,t}(Z_{s,t},\eta)s_t^{(\rho_t-1)} \quad (67)$$

yields:

$$0 = p_{s_t}[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}] \\ - K_t a_{s,t}(Z_{s,t},\eta)s_t^{(\rho_t-1)}(-5p_{n_t} - p_{s_t}s_t - a_{t+1} + (1+r)a_t + y_t) \quad (68)$$

which can only be solved numerically.

Optimal solution for c_t

If values for I_t, by that s_t and n_t, and a_{t+1} are determined, the optimal c_t simply is:

$$c_t = (1+r)a_t + y_t - p_{n,t}n_t - p_{s,t}s_t - a_{t+1} \quad (69)$$

GMM equations for investment parameters

To derive the relative demand ratios, one goes back to equation 27 and takes logs to get linear equations, using that $a_{s,t}(Z_{s,t},\eta) = exp(\phi_{s,t}Z_{s,t} + \eta)$:

$$\ln(\frac{p_{n,t}}{p_{s,t}}) = -\phi_{s,t}Z_{s,t} + (\rho_t - 1)\ln(\frac{n_t}{s_t}) - \eta$$
$$\ln(\frac{n_t}{s_t}) = \frac{1}{\rho_t - 1}Z'_{s,t}\phi_{s,t} - \frac{1}{1-\rho_t}\ln(\frac{p_{n,t}}{p_{s,t}}) - \frac{1}{1-\rho_t}\eta$$

Adding $\ln(\frac{p_{n,t}}{p_{s,t}})$ to both sides yields:

$$\ln(\frac{p_{n,t}n_t}{p_{s,t}s_t}) = \frac{1}{\rho_t - 1}Z'_{s,t}\phi_{s,t} + \frac{\rho_t}{\rho_t - 1}\ln(\frac{p_{n,t}}{p_{s,t}}) - \frac{1}{1-\rho_t}\eta$$

GMM equations for human capital parameters

$$\Psi_{t+1} = \theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}} \tag{70}$$

Using the human capital formation with $\theta_t(Z_{\theta,t}) = \exp(\phi_{\theta,t}Z_{\theta,t})$, taking logs:

$$\ln(\Psi_{t+1}) = \phi_{\theta,t}Z_{\theta,t} + \delta_{1,t}\ln(I_t) + \delta_{2,t}\ln(\Psi_t) \tag{71}$$

Since Ψ_t are latent skills, I assume the underlying measurement system with $S_{hs,t}$ and $S_{ts,t}$, which are observed height and test scores:

$$S_{ts_1,t} = \lambda_{ts_1,t}\ln(\Psi_t) + \epsilon_{ts_1,t} \tag{72}$$

and:

$$S_{ts_2,t} = \lambda_{ts_2,t}\ln(\Psi_t) + \epsilon_{ts_2,t} \tag{73}$$

Since height is observed in all periods, I can normalize $\lambda_{ts_1} = 1$ to allow for comparability of measures (see Cunha et al. (2010)).

Replacing the latent skills with the measurements leads too:

$$S_{ts_1,t+1} = \phi_{\theta,t}Z_t + \delta_{1,t}\ln(I_t) + \delta_{2,t}S_{ts_1} \tag{74}$$

and:

$$\frac{1}{\lambda_{ts_2,t+1}}S_{ts_2,t+1} = \phi_{\theta,t}Z_t + \delta_{1,t}\ln(I_t) + \delta_{2,t}\frac{1}{\lambda_{ts_2,t}}S_{ts_2} \tag{75}$$

To identify $\lambda_{ts_2,t}$ further equations are needed. To get these I exploit the covariance structure, similar to (Cunha et al., 2010). One can replace Ψ_t in equation 72 with using equation 72:

$$\frac{Cov(S_{ts_1,t}, S_{ts_1,t+1})}{Cov(S_{ts_2,t}, S_{ts_1,t+1})} = \lambda_{ts_2,t} \tag{76}$$

and:

$$\frac{Cov(S_{ts_1,t}, S_{ts_1,t+1})}{Cov(S_{ts_1,t}, S_{ts_2,t+1})} = \lambda_{ts_2,t+1} \tag{77}$$

Using that these measures have mean 0, the covariance can be rearanged to:

$$0 = \mathrm{E}[(S_{ts_1,t+1} - \lambda_{ts_2,t+1}S_{ts_2,t+1})S_{ts_1,t}] \tag{78}$$

and:

$$0 = \mathrm{E}[(S_{ts_1,t}S - \lambda_{ts_2,t}S_{ts_2,t})S_{ts_1,t+1}] \tag{79}$$

A.7 Additional tables

TABLE A.6: Distribution of parenting skill types η by total amount of types

	Observations for type:				
Amount of types	Type 0	Type 1	Type 2	Type 3	Type 4
K=2	2,020	4,417			
K=3	2,992	2,831	614		
K=4	2,813	2,956	391	277	
K=5	2,664	547	2,863	9	354

Note: This table summarizes the amount of observation for each set of types, for different total amount of types secified.

Table A.7: Investment gap decomposition by childhood period

	Investment gap change (%):		
	Early childhood	Primary school	High school
Baseline gap	17.77	7.08	9.37
Parental education:			
Income	-24.56	-22.18	-28.21
Preferences	38.85	46.93	57.57
Investment productivities	0.00	1.09	7.51
Skill productivities	0.00	0.00	0.00
Parenting skills:			
Income by type	-12.15	-10.67	-13.93
Type productivities	0.00	0.68	2.16
Other:			
Income (rest)	-24.91	-22.59	-29.54
Initial skills	0.00	0.00	0.00

Note: Gap indicated are between high school parents and parents with no schooling.

Table A.8: Transmission of effects into adult skills

	Adult skills increase (std.)		
Period of change:	Early childhood	Primary school	High school
Offsetting:			
Nutrition	0.0018	0.0025	0.0114
Schooling	0.0000	0.0005	0.0170
Increase by 0.1SD:			
Current skills	0.0004	0.0041	0.0216

Note: Values calculated for average parents with no schooling. For investment offsetting avergage investments of high school parents are taken.

Table A.9: Estimation results for skill formation parameters for 2 types

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.10	(0.65)***	-10.12	(4.16)**
Implied elasticity			0.24		0.09	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-2.24	(0.39)***	-35.08	(12.33)***
Mother primary			0.88	(0.19)***	2.58	(1.02)**
Mother high			1.51	(0.30)***	4.14	(1.62)**
Father primary			0.01	(0.14)	0.38	(0.38)
Father high			-0.18	(0.17)	0.20	(0.41)
Age			-0.04	(0.04)	2.80	(1.05)***
Female			0.06	(0.11)	1.21	(0.52)**
Rural area			-2.27	(0.41)***	-4.47	(1.74)**
No. of siblings			-0.61	(0.11)***	-1.90	(0.71)***
Mother not Islam			0.32	(0.19)*	1.35	(0.67)**
Parenting type 1			-1.53	(0.29)***	-3.13	(1.23)**
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.07)***	0.14	(0.06)**	0.23	(0.03)***
$\delta_{2,t}$ (skills)	0.08	(0.03)**	0.18	(0.02)***	0.20	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.75	(0.10)***	-0.06	(0.13)	-0.26	(0.10)***
Mother primary	0.06	(0.04)	0.08	(0.04)*	0.05	(0.03)**
Mother high	0.24	(0.04)***	0.26	(0.05)***	0.14	(0.03)***
Father primary	-0.01	(0.05)	0.13	(0.04)***	0.04	(0.03)
Father high	0.08	(0.04)**	0.08	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	0.97	(0.11)	1.06	(0.01)	1.12	(0.01)
$\lambda_{4,ts}$					1.26	(0.04)
N	27,257					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

Table A.10: Estimation results for skill formation parameters for 3 types

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.37	(0.74)***	-10.37	(4.36)**
Implied elasticity			0.23		0.09	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.68	(0.47)***	-38.98	(14.12)***
Mother primary			1.01	(0.22)***	2.84	(1.14)**
Mother high			1.71	(0.34)***	4.54	(1.81)**
Father primary			0.05	(0.15)	0.51	(0.41)
Father high			-0.12	(0.17)	0.37	(0.44)
Age			-0.05	(0.04)	2.89	(1.11)***
Female			0.03	(0.12)	1.19	(0.53)**
Rural area			-2.44	(0.46)***	-4.75	(1.89)**
No. of siblings			-0.67	(0.12)***	-1.98	(0.76)***
Mother not Islam			0.33	(0.20)	1.43	(0.71)**
Parenting type 1			0.14	(0.13)	0.03	(0.31)
Parenting type 2			3.49	(0.68)***	6.45	(2.55)**
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.07)***	0.14	(0.06)**	0.22	(0.03)***
$\delta_{2,t}$ (skills)	0.08	(0.03)**	0.18	(0.02)***	0.20	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.75	(0.10)***	-0.06	(0.13)	-0.25	(0.09)***
Mother primary	0.06	(0.04)	0.08	(0.04)*	0.05	(0.03)**
Mother high	0.24	(0.04)***	0.26	(0.05)***	0.14	(0.03)***
Father primary	-0.01	(0.05)	0.13	(0.04)***	0.04	(0.03)
Father high	0.08	(0.04)**	0.08	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)*	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	0.97	(0.11)	1.06	(0.01)	1.13	(0.01)
$\lambda_{4,ts}$					1.26	(0.04)
N	27,257					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

TABLE A.11: Estimation results for skill formation parameters for 5 types

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.19	(0.68)***	-9.81	(3.92)**
Implied elasticity			0.24		0.09	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.61	(0.44)***	-37.26	(12.76)***
Mother primary			0.98	(0.21)***	2.71	(1.04)***
Mother high			1.61	(0.31)***	4.39	(1.66)***
Father primary			0.06	(0.14)	0.54	(0.40)
Father high			-0.12	(0.17)	0.39	(0.42)
Age			-0.04	(0.04)	2.75	(1.00)***
Female			0.04	(0.11)	1.10	(0.48)**
Rural area			-2.37	(0.43)***	-4.63	(1.74)***
No. of siblings			-0.64	(0.11)***	-1.89	(0.69)***
Mother not Islam			0.36	(0.19)*	1.41	(0.67)**
Parenting type 1			1.52	(0.35)***	2.32	(1.01)**
Parenting type 2			-0.04	(0.12)	0.36	(0.33)
Parenting type 3			-0.04	(2.44)	16.02	(7.23)**
Parenting type 4			4.25	(0.82)***	8.36	(3.17)***
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.07)***	0.14	(0.06)**	0.23	(0.03)***
$\delta_{2,t}$ (skills)	0.07	(0.03)**	0.18	(0.02)***	0.20	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.75	(0.10)***	-0.06	(0.13)	-0.27	(0.09)***
Mother primary	0.06	(0.04)	0.07	(0.04)*	0.06	(0.03)**
Mother high	0.24	(0.04)***	0.26	(0.05)***	0.13	(0.03)***
Father primary	-0.01	(0.05)	0.13	(0.04)***	0.04	(0.03)
Father high	0.08	(0.04)**	0.08	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)*	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	0.98	(0.11)	1.07	(0.01)	1.13	(0.01)
$\lambda_{4,ts}$					1.27	(0.04)
N	27,257					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1.

Table A.12: Robustness check: GMM without constrained individuals

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.33	(0.76)***	-14.60	(8.71)*
Implied elasticity			0.23		0.06	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.42	(0.48)***	-53.02	(28.45)*
Mother primary			1.10	(0.24)***	4.10	(2.34)*
Mother high			1.78	(0.37)***	7.24	(4.12)*
Father primary			0.23	(0.16)	0.79	(0.69)
Father high			0.05	(0.19)	0.25	(0.62)
Age			-0.04	(0.04)	4.05	(2.26)*
Female			0.02	(0.13)	1.53	(0.94)
Rural area			-2.36	(0.46)***	-6.64	(3.80)*
No. of siblings			-0.68	(0.13)***	-2.71	(1.51)*
Mother not Islam			0.21	(0.21)	2.06	(1.34)
Parenting type 1			-0.37	(0.15)**	-0.68	(0.56)
Parenting type 2			4.26	(0.90)***	12.26	(6.99)*
Parenting type 3			1.62	(0.52)***	2.93	(1.99)
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.06)***	0.16	(0.05)***	0.17	(0.03)***
$\delta_{2,t}$ (skills)	0.10	(0.02)***	0.19	(0.02)***	0.22	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.73	(0.08)***	-0.02	(0.12)	-0.21	(0.09)**
Mother primary	0.02	(0.04)	0.06	(0.04)	0.05	(0.02)**
Mother high	0.22	(0.03)***	0.25	(0.04)***	0.16	(0.03)***
Father primary	0.02	(0.04)	0.13	(0.04)***	0.03	(0.03)
Father high	0.11	(0.03)***	0.07	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)**	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	1.00	(0.07)	1.07	(0.01)	1.09	(0.01)
$\lambda_{4,ts}$					1.21	(0.04)
N	27,366					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.