這次作業實作的是 Black-Scholes Formula，以下為 Black-Scholes Formula 會參考到的一些值：

1. The Black-Scholes formula depends on S, X, r, τ, and σ.
2. r is usually taken as the risk-free interest rate on an investment maturing in τ periods, at the expiration of the option contract.
3. The only parameter that is not directly observable is the stock's volatility, σ.
4. This can be estimated using historical data on stock prices or stock returns.

以下為利用 Black-Scholes Formula 計算 Call Price 和 Put price 的公式：

Consider the value of the European calls and puts when S becomes large. As $S \to \infty$, $d_1 \to \infty$ and $d_2 \to \infty$. Therefore $N(d_1) \to 1$ and $N(d_2) \to 1$, but $N(-d_1) \to 0$ and $N(-d_2) \to 0$. Hence, as $S \to \infty$, for European options:

$$\text{Call:} \quad \lim_{S\to\infty} [S\,N(d_1) - Xe^{-r\tau}N(d_2)] = \infty,$$

$$\text{Put:} \quad \lim_{S\to\infty} [Xe^{-r\tau}N(-d_2) - S\,N(-d_1)] = 0$$

另外，老師也提供了我們一個完整的演算流程：

$$D = \$1\,e^{-0.06\,(\frac{1}{12})} + \$1\,e^{-0.06\,(\frac{4}{12})} \cong \$1.9752.$$

Thus, $\hat{S} = S - D \cong \$75 - \$1.9752 \cong \$73.02.$

$$\begin{aligned} d_1 &= \frac{\ln(\hat{S}/X) + (r + \frac{1}{2}\sigma^2)\,\tau}{\sigma\sqrt{\tau}} \\ &\cong \frac{\ln(73.02/65.00) + (0.06 + \frac{1}{2}0.35^2)\,\frac{6}{12}}{0.35\sqrt{\frac{6}{12}}} \\ &\cong 0.715 \\ d_2 &= d_1 - \sigma\sqrt{\tau} \\ &\cong 0.715 - 0.35\sqrt{\frac{6}{12}} \\ &\cong 0.468. \end{aligned}$$

$N(-0.715) \cong 0.2373$ and $N(-0.468) \cong 0.3199$, so

$$\begin{aligned} p &= Xe^{-r\tau}N(-d_2) - \widehat{S}N(-d_1) \\ &\cong \$65\,e^{-0.06\left(\frac{6}{12}\right)}(0.3199) - \$73.02\,(0.2373) \\ &\cong \$20.179 - \$17.328 \\ &\cong \$2.85. \end{aligned}$$

我們跟著這個演算流程一步一步實作：

(1) 首先，計算 Dividend

```
for i in range(n):
    Dividend += D * (math.exp(-r * (D_month[i]/12)))
```

(2) 計算 S_head

```
S_head = Stock - Dividend
```

(3) 計算 d_1 和 d_2

```
d1 = (math.log(S_head / Strike) + ((r + (1/2) * (Sigma ** 2)) * (M / 12))) / (Sigma * (M/12) ** (1/2))
d2 = d1 - Sigma * (M/12) ** (1/2)
```

(4) 計算 Put price

```
p = Strike * math.exp(-r * M/12) * stats.norm.cdf(-d2) - S_head * stats.norm.cdf(-d1)
```

(5) 計算 Call pricce

```
c = S_head * stats.norm.cdf(d1) - Strike * math.exp(-r * M/12) * stats.norm.cdf(d2)
```

最終結果：

```
Put Price:  2.86
CallStock Price:  12.806
```