

學習歷程及程式 DEMO

一、ytm 計算

計算現值的公式為：

$$PV = \sum_{i=1}^n \frac{C}{\left(1 + \frac{r}{m}\right)^i} + \frac{F}{\left(1 + \frac{r}{m}\right)^n} = C \frac{1 - \left(1 + \frac{r}{m}\right)^{-n}}{\frac{r}{m}} + \frac{F}{\left(1 + \frac{r}{m}\right)^n},$$

公式實作：

```
# 計算 Present Value
def PV(ParValue, Coupon, Period, CouponRate):
    PresentValue = 0
    for i in range(1, Period + 1):
        PresentValue += Coupon / math.pow((1 + CouponRate), i)
    PresentValue += ParValue / math.pow((1 + CouponRate), Period)
    return PresentValue
```

由於我們想要計算 ytm，因此我們透過反覆試驗法（Trial-and-Error）來趨近 ytm 的值。

```
# 利用反覆試驗來趨近 ytm 的值
while flag:
    if CurrentPrice < ParValue:
        ytm += 0.00001
    else:
        ytm -= 0.00001

    PresentValue = PV(ParValue, Coupon/Payment, Period, ytm/Payment)

    if CurrentPrice < ParValue:
        flag = PresentValue > CurrentPrice
    else:
        flag = PresentValue < CurrentPrice
```

將「現值小於票面價值」和「現值大於票面價值」的兩種情況分開處理。

以「現值小於票面價值（Current Bond Price < Par Value）」做舉例：

- (1) 每次將 ytm 減少 0.0001
- (2) 計算當前 Present Value
- (3) 若 Present Value 仍然大於 Current Bond Price，則繼續執行 while 迴圈（flag = True）
- (4) 若 Present Value 小於 Current Bond Price 即跳出迴圈，得到 ytm 值

二、Spot Rate 計算

Spot Rate 解釋：

Spot rate is the yield on a zero-coupon bond. It can be calculated from the equation of value for a unit zero-coupon bond (bond with nominal value). If y_t is the yield, then the equation can be written as follows:

$$P_t = \frac{1}{(1 + y_t)^t}$$

$$\Rightarrow P_t^{-\frac{1}{t}} = (1 + y_t)$$

$$\Rightarrow y_t = P_t^{-\frac{1}{t}} - 1$$

利用最後一個公式實作 Spot Rate：

```
# 計算 Spot Rate
SpotRate = math.pow(CurrentPrice/ParValue, -1/Period) - 1
```

三、Forward Rate 計算

Forward Rate 解釋：

Consider two individual investments, one made now for a period of t years and another made at the end of t years for a period of r years. The overall yield would be equal to the yield on another investment made now, for a period of $t+r$ years.

This could be mathematically written as follows:

$$(1 + y_t)^t (1 + f_{t,r})^r = (1 + y_{t+r})^{t+r}$$

Where,

y_t is the spot rate on the t -year investment made now.

y_{t+r} is the spot rate on the $t+r$ year investment made now.

$f_{t,r}$ is the forward rate(yield) on the investment made after t years for a period of r years.

The above equation can be rewritten as follows:

$$(1 + f_{t,r})^r = \frac{(1 + y_{t+r})^{t+r}}{(1 + y_t)^t} = \frac{P_t}{P_{t+r}}$$

Forward Rate 實作：

```
# 計算 Forward Rate
AllCurrentPrice = []
AllCurrentPrice.append(CurrentPrice)
for i in range(1, Period+1):
    print("Please enter the current price of year", i, ":")
    AllCurrentPrice.append(int(input('-> ')))
print(AllCurrentPrice)

for i in range(1, Period+2):
    for j in range(i, Period+2):
        if i == j or i == (Period+2) or j == (Period+2):
            continue
        NearSpotRate = math.pow(AllCurrentPrice[i-1]/ParValue, -1/i) - 1
        FarSpotRate = math.pow(AllCurrentPrice[j-1]/ParValue, -1/j) - 1
        ForwardRate = (math.pow((1 + FarSpotRate), j)/math.pow((1 + NearSpotRate), i))**(1/(j-i))-1
        print("The forward rate from", i, "th year to", j, "th year is", ForwardRate)
        ForwardTable[i-1][j-1] = '%.3f' % ForwardRate
```

- (1) 首先，先輸入每一期的 Price，用以計算 Spot Rate
- (2) 計算期間起點以及期間終點的 Spot Rate
- (3) 再利用以上公式算出 Forward Rate
- (4) 若算出來的 Forward Rate 大於 0，則放入對應的 Forward Table 中

程式 DEMO

一、Ytm

```
C:\Users\user\Desktop>python financial_engineering_hw2.py
Current Bond Price: 900
Bond Par Value: 1000
Bond Coupon Rate (% p.a.): 5
Years to Maturity: 5
Payment (Enter 1 for Annually, 2 for Semi-annually, 4 for quarterly) : 1
Yield to Maturity: 7.470 %
```

與 Calkoo Calculator 網站對照：

Current Bond Price	<input type="text" value="900"/>
Bond Par Value	<input type="text" value="1000"/>
Bond Coupon Rate (% p.a.)	<input type="text" value="5"/> %
Years to Maturity	<input type="text" value="5"/>
Payment	<input checked="" type="checkbox"/> Annually <input type="checkbox"/> Semi-annually <input type="checkbox"/> Quarterly

Result

Yield to Maturity (YTM) 7.4697 %

二、Spot Rate

```
C:\Users\user\Desktop>python financial_engineering_hw2.py
Current Bond Price: 900
Bond Par Value: 1000
Bond Coupon Rate (% p.a.): 5
Years to Maturity: 5
Payment (Enter 1 for Annually, 2 for Semi-annually, 4 for quarterly) : 1
Yield to Maturity: 7.470 %
Spot Rate: 2.130 %
```

與 Calkoo Calculator 網站對照：

Duration of spot rate: (years)

Price of year unit zero-coupon bond:

year spot rate of interest:

三、Forward Table

```
Current Bond Price: 900
Bond Par Value: 1000
Bond Coupon Rate (% p.a.): 5
Years to Maturity: 5
Payment (Enter 1 for Annually, 2 for Semi-annually, 4 for quarterly) : 1
Yield to Maturity: 7.470 %

Spot Rate: 2.130 %
Please enter the current price of year 1 :
-> 800
Please enter the current price of year 2 :
-> 700
Please enter the current price of year 3 :
-> 600
Please enter the current price of year 4 :
-> 500
Please enter the current price of year 5 :
-> 400
[900.0, 800, 700, 600, 500, 400]
The forward rate from 0 th year to 1 th year is 0.12500000000000022
The forward rate from 0 th year to 2 th year is 0.13389341902768193
The forward rate from 0 th year to 3 th year is 0.14471424255333187
The forward rate from 0 th year to 4 th year is 0.15829218528826927
The forward rate from 0 th year to 5 th year is 0.17607902252467356
The forward rate from 1 th year to 2 th year is 0.14285714285714302
The forward rate from 1 th year to 3 th year is 0.15470053837925146
The forward rate from 1 th year to 4 th year is 0.1696070952851465
The forward rate from 1 th year to 5 th year is 0.1892071150027208
The forward rate from 2 th year to 3 th year is 0.16666666666666607
The forward rate from 2 th year to 4 th year is 0.18321595661992318
The forward rate from 2 th year to 5 th year is 0.20507113208761463
The forward rate from 3 th year to 4 th year is 0.20000000000000062
The forward rate from 3 th year to 5 th year is 0.22474487139158894
The forward rate from 4 th year to 5 th year is 0.24999999999999991
```

圖：

	0	1	2	3	4	5
0	0	0.125	0.134	0.145	0.158	0.176
1	-	0	0.143	0.155	0.170	0.189
2	-	-	0	0.167	0.183	0.205
3	-	-	-	0	0.200	0.225
4	-	-	-	-	0	0.250
5	-	-	-	-	-	0