MSRI UP 2023 - HW 1 - DAY 3

Due: Friday 6/16 by 7:00am. Submit your assignment to your personal dropbox.

Important: You don't have to complete the entire assignment, but do as much of it as possible. Make sure to present your (partial) solutions in a neat and organized manner. Discuss the problems with at least two other people, but write your solutions on your own.

(1) Let K be the simplicial complex shown in Figure 1 below, let $\mathbb{F}$ be a field and let $\gamma = \{0,1\} + \{1,2\} - \{0,2\} \in C_1(K;\mathbb{F})$.

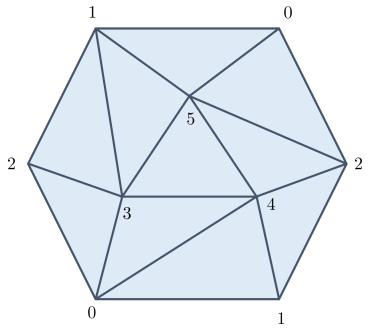

FIGURE 1. Simplicial complex K triangulating the projective plane $\mathbb{R}\mathbf{P}^2$

(a) Show that for any field $\mathbb{F}$, $\gamma \in Z_1(K;\mathbb{F})$.
(b) Show that if $\gamma' \in Z_1(K;\mathbb{Z}/2\mathbb{Z})$, then $\gamma - \gamma' \in B_1(K;\mathbb{Z}/2\mathbb{Z})$. Use this to conclude that $H_1(K;\mathbb{Z}/2\mathbb{Z}) = \{0,[\gamma]\} \cong \mathbb{Z}/2\mathbb{Z}$.
(c) Show that $\gamma \in B_1(K;\mathbb{Z}/3\mathbb{Z})$.

(2) Compute the persistent homology of the mystery data sets from Day 2. You can find them at https://github.com/joperea/MSRI-UP2023/tree/main/juPyter%20notebooks/data
What spaces are they sampling? Feel free to include data plots, persistence diagrams, etc, to argue your point.