Assignment 1: Error Analysis

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1. (a) Measured height $h = 5.03 \pm 0.04$ m
   (b) Measured time $t = 1.5 \pm 1$ s
   (c) Measured charge $q = -3.21x10^{-19} \pm 0.27x10^{-19}$ C
   (d) Measured distance $d = 5.6x10^{-7} \pm 0.7x10^{-7}$ m
   (e) Measured momentum $p = 3,267 \pm 42$ gcm/s

2. (a) Given a set of 5 measurements of a liquid's density 1.8, 2.0, 2.0, 1.9, and 1.8, the average density is

$$d_{ave} = \frac{1.8+2.0+2.0+1.9+1.8}{5}.$$

   To find the uncertainty from such a small data set, I found the standard deviation

$$\Delta d = \frac{\sqrt{\sum_{i=1}^{N} |d_i^2 - d_{ave}^2|}}{N-1}$$

   Substituting in the values from our given data set, this yields

$$\Delta d = \frac{\sqrt{|(3.24-3.61)|+|(4.0-3.61)|+|(4.0-3.61)|+|(3.61-3.61)|+|(3.24-3.61)|}}{4}$$

$$\Delta d = \frac{\sqrt{1.52}}{4} = 0.31$$

   This gives our result for the liquid's density and uncertainty

$$d = 1.9 \pm 0.31 \frac{g}{cm^3}$$

   (b) The student's estimate was within one standard deviation of the accepted value. The percent error in their calculation was

$$\% \text{ error} = \frac{accepted - measured}{accepted} * 100 = \frac{1.85-1.9}{1.85} * 100 = 2.7\%$$

3. The equation for the volume of a sphere of radius r is

$$v = \frac{4}{3}\pi r^3$$

   To find the uncertainty for a sphere of radius $r = 2.0 \pm 0.1m$, we find

$$\Delta v = \sqrt{(\frac{\partial v}{\partial r}\Delta r)^2}$$

   We take the appropriate partial derivative of the volume function to get

$$\Delta v = \sqrt{(4\pi r^2 \Delta r)^2}$$

Which for our given r, $\Delta v = 5.0m^3$. Thus, our sphere has volume

$$V_{sphere} = \frac{4}{3}\pi r^3 \pm \Delta v \ m^3 = 33.5 \pm 5.0m^3$$

4. Given the equation $q = xy + \frac{x^2}{y}$ with $x = 6.0 \pm 0.2$ and $y = 3.0 \pm 0.1$, we can find our uncertainty

$$\Delta q = \sqrt{(\frac{\partial q}{\partial x}\Delta x)^2 + (\frac{\partial q}{\partial y}\Delta y)^2}$$
$$\frac{\partial q}{\partial x} = y + 2xy^{-1} \text{ and } \frac{\partial q}{\partial y} = x - xy^{-2}$$
$$\Delta q = \sqrt{(y + 2xy^{-1}\Delta x)^2 + (x - xy^{-2}\Delta y)^2}$$
$$\Delta q = \sqrt{2.21} = 1.49$$

We can now solve for q with our uncertainty

$$q = 30.0 \pm 1.5$$

5. (a) Figure 1 shows the python generated plot of the square of the velocity of a vertically thrown stone as a function of its height.

   The plot shows a linear relationship between the height and square of the velocity, which is consistent with $v^2 \propto h$.

   (b) The above plot created through python has a slope of $m = 18.3 \pm 0.6$. As $2g = 19.6$, this is fairly close, but doesn't fall within the uncertainty given by the code. It might be that for such a small sample set, a different method of calculating the uncertainty might have been more appropriate.

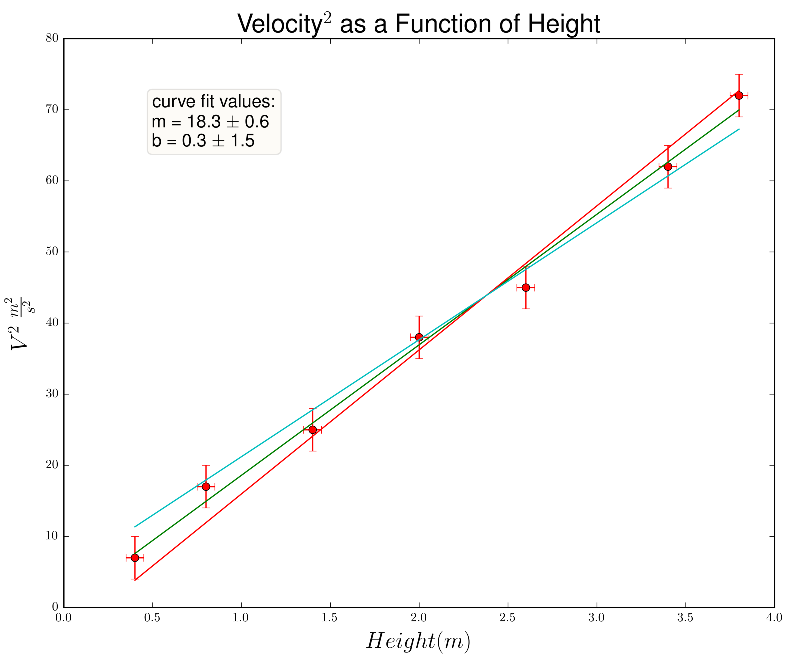

Figure 1: Plot of $velocity^2$ vs $height$