Case Study: Optimization of Promotional Campaigns and Orders for Amber India Restaurants.

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PART I

Amber India's owner Vijay Bist has reached out to us, Bay Creative, a consulting agency to evaluate it's ad campaign in order to get the largest possible engagement to customers in a one year period in Northern California. From analyzing Vijay's constraints, we were able to put together our recommendation in maximizing the number of exposures of ads.

In order to understand our analysis we must define our decision variables. The variables being maximized in the first portion of our analysis can be defined as:

- X1 = Each TV Commercial
- X2 = Each Social Media Ad
- X3 = Each Newspaper Ad

Identifying the relevant decision variables enables us to facilitate the analysis as we have clear objective variables of interest that we would like to optimize. After identifying the variables our next step was to evaluate Vijay's constraints. His constraints consisted of a mix of financial budget, market segmentation, as well as coupon constraints. The following constraints were obtained from Vijay's documentation and are reported in millions of dollars ($MM):

I. 0.3X1+0.15X2+0.1X3<= 4
II. X1-5<=0
III. 0.09X1+0.03X2+0.04X3<=1
IV. 1.2X1+0.2X2>=5
V. 0.5X1+0.2X2+0.2X3>=5
VI. 0.04X2+0.12X3=1.49

Constraint I and III factor in Vijay's budget constraint of having $4 million allocated for advertising costs and $1 million for planning costs. Our second constraint, II, factors in that each year major Bay Area TV stations only have 5 spots available. Constraints IV and V segments Vijay's target audience evenly across young people, those individuals whose age range from 18 to 30, and middle-aged people, those individuals whose age range from 30 to 55. Lastly, VI, factors in coupon discounts which can reach a maximum level of $1,490,000. These six constraints must be subject to an objective function which has been given by Vijay as he wishes to maximize total number of engagements. The following function allows us to subject our constraints using a KKT (Karush-Kuhn-Tucker) approach:

- F(X1,X2,X3) = 1.3X1 + 0.6X2 + 0.5ZX3

Using software such as R helps us facilitate the analysis. We first created a function f, which represents F(X1,X2,X3). In the programming language in order to maximize the function we must take -f. Afterwards, we created two variables, Equalities and Inequalities, and initialized both by including the variable h=0. We then inputted the VI into the equalities function. I, II, III, IV, and V were inputted into the inequalities function. In our scratchwork, we then created additional dummy variables in our R code in order to help us identify the product mix, p0, that would allow us to maximize the number of engagements. These dummy variables were tacked onto the end of our inequality functions h[4] through h[8]. The variables x[4], x[5], x[6], x[7], and x[8], were then included in our equalities function because we want to ensure that when maximizing our objective function that these additional dummy variables go to 0 and do not affect the outcome of our result. By minimizing x[4] through x[8] we arrive at our profit maximizing mix of 2.32, 17.68, and 6.52. These values correspond to X1, X2, and X3 respectively. This results in **-16.88409** million with relation to maximum number of impressions which include first time impressions;

-F(X1,X2,X3) =- (1.3X1 + 0.6X2 + 0.5X3)

= -[1.3(2.31) + 0.6(17.68) + 0.5(6.52)]

= -16.88409MM maximum number of impressions

Multiplying by 1,000,000; actual becomes 16,884,090 maximum number of impressions

PART II

While these values above maximizes the number of exposures of ads, they do not necessarily maximize Amber India's total profit. To determine the maximized total profit, we need to find the number of first-time customer visits from the number of ads from each medium. Using the relationship equations between the number of ads from each medium and the number of first-time customer visits, we can find the number of customers from each medium. Given that Amber India profits $5 per customer before advertising and planning costs from each medium, we must then subtract the total advertising and planning costs from each medium to find the total profit. The constraints from Part 1 apply to Part 2. Running the R code will find the optimal values for each medium. The optimal profit value is $39,848,320 and the optimal values for TV Ads are 3.6, Social Media Ads are 10.73, and Newspaper Ads are 8.83. The optimal values for each medium maximizes Amber India's total profit while satisfying all the constraints listed in Part 1. While the optimal values in Part 1 maximize exposure, they are not accurate and do not maximize profit.

f=function(x) -(5*(-0.1*X1]^2 + 1.13*X1 - 0.04)) + (5*(-0.002*X2^2 + 0.124*X2 + 0.14)) + (5*(-0.0321*X3^2 + 0.706*X3] - 0.09)) - (0.3*X1 - 0.15*X2 - 0.1*X3) - (0.09*X1 - 0.03*X2 - 0.04*X3))

Optimal Profit Value: $39,848,320
Optimal Values for:
TV Ads: 3.6
Social Media Ads: 10.73

Newspaper Ads: 8.83

PART III

X = quantity of gallons purchased / decision variable
D = demand of drinks
C(x) = cost of making the drink which is \$75 per gallon; $C(x) = 75x$
R(x) = Revenue generated by selling x drinks
P(x) = Price per drink, which is given as \$12 per glass
Per gallon would be \$12 per glass * 25 glasses per gallon, 12*25 = \$300 per gallon
Price = 12/0.04 per gallon = \$300 / gallon
Profit (x) = R(x) – C(x)
The weekly demand (D) is uniformly distributed from 200 to 500 gallons
$200 < D < 500$
x = quantity of gallons

Revenue depends on the minimum order the restaurant has on hand and what the customer has ordered (supply and demand relation)
Therefore if $X > D$ which means the restaurant will only sell D quantity demanded by customers.
If $X < D$, the company will be selling only what they have in X quantity in inventory which means they will be short, losing profits

$R(x) = 300 \min(x,D)$
Profit function = R(x) – C(x)
Profit function $Pr(xD) = 300 \min(x,D) - 75x$

We know demand is uniformly distributed
Thus $D \sim U[200, 500]$
$f(D) = 1 / (500-200) = 1/300$

Let us assume that the profit function is
g(D), thus:
$E[g(D)] = \int_{200}^{500} g(D) f(D)\, dD$
$E[g(D)] = \int_{200}^{500} 300 \min(x,D) - 75x]*1/300\, dD$
$E[g(D,x)] = 1/300 \int_{200}^{500} 300 \min(x,D) dD - 1/300 \int_{200}^{500} 75\, xd$
Let us solve the integral separately
$E[g(D,x)] = \underbrace{1/300 \int_{200}^{500} 300 \min(x,D) dD}_{\text{Part 1}} \underbrace{- 1/300 \int_{200}^{500} 75\, xd}_{\text{part 2}}$

Part 1
$= 1/300 \int_{200}^{500} 300 \min(x,D) dD$
$= 1/300 \int_{200}^{x} 300 \min(D) dD + = 1/300 \int_{x}^{500} (x) dD$
With these two conditions: if $500 >= D >= X$, $\min(D, X) = x$
If $200 <= D <= x$ $\min(D, x) = x$.

D is the demand from the customer
X is the quantity of drinks
In the integral x acts as a constant and D is the variable. Therefore

$1/300\ [300D^2/2 + C\ |200 \text{ to } x + 300Dx + C\ |\ x \text{ to } 500$
$1/300[150D^2 + C\ |\ 200 \text{ to } x + 300Dx + C\ |\ x \text{ to } 500$
$1/300\ [150D^2 + C - 1500(200)^2 - C + 300*500x + C - x^2 - C$
$X2/2 - 2000 + (5x/3 - x^2/300)*300$
Let us say, $m(x) = x^2/2 - 20000 + 500x$
$M(x)' = - 2x+1000$

Part 2
Cost = 75x

$= 1/300 \int 200 \text{ to } 500 \quad 75\ dD \quad = 75x$
$M(x) + Cost\ (x) \quad -x^2 - 40000 + 1000x - 150x$
$X^2 - 40000 + 850x = g(D,x)$
Getting the derivative,
$M'(x) + cost(x)' = 2x +850 = 0$
$-2x = 850$, therefore $x = 850/2 = 425$ gallons

$E[g(D,x)] \quad -2 < 0$

X = 425 is the quantity of gallons the restaurant needs to sell to optimize the profit.

$H(g(D,x)) = -x^2/2 - 20000 + 500x - 75x$
$-x^2/2 + 425x - 20000$
Getting the derivative, $-2x + 425 = -x + 425$, equating this to zero, results in
$-x + 425 = 0$ therefore $x = 425$ gallons
minimum of D and X (2 parts - find expected value of G)

$H(g(D,x))'' = -2 < 0$

The restaurant needs to sell x = 425 gallons of the drink in order to maximize profit.
Substituting this value in the equation,
$-x^2/2 - 20000 + 425x$ OR rewritten as $-x^2/2 + 425x - 20000$

$h(x) = h(425) =$
$-425^2/2 + 425(425) -20000 = -90312.50 + 180625 - 20000$
The profit will be = **$70,312.50**
given randomness of D, profit is still random.