A Project Report

On

The Percolation Model of Stock Markets

BY

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Under the supervision of

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Birla Institute of Technology and Science-Pilani,

Hyderabad Campus

CERTIFICATE

This is to certify that the project report entitled "The Percolation Model of Stock Markets" submitted by Karthik Ramanathan (2014B5PS0934H) in fulfilment of the requirements of the course PHY F366, Laboratory Oriented Project Course, embodies the work done by him under my supervision and guidance.

DATE:

(Prof. P. K. Thiruvikraman)
Head Physics Department

ABSTRACT

Most financial theories, such as Black-Scholes Model (1973), assume the normal distribution of stock price returns. However, stock prices fluctuate by greater magnitudes and with greater frequency than what the Gaussian model predicts. The NIFTY 50 Index has shown daily percentage returns beyond -5% and +5%, 51 times since January 2000. The Gaussian model would need a considerably larger period to witness such extreme fluctuations.

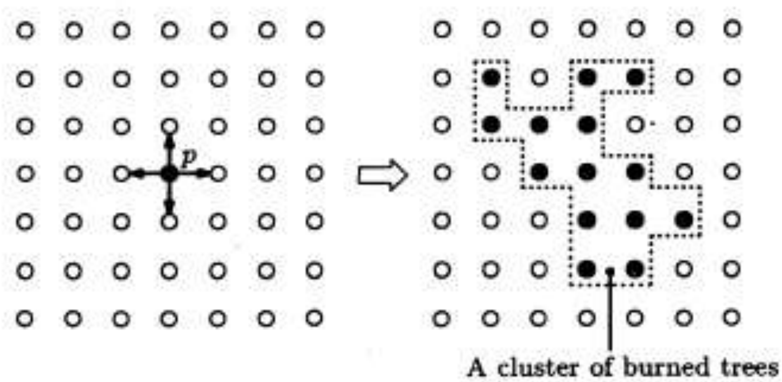
Percolation theory, a branch of statistical physics, is used to model the local interactions between traders in a market. Stauffer and Penna (1998) also incorporated percolation theory to model financial markets. Their model was based on the herding effect, which assumes that traders follow the market trends without the analysis of the economic data available to them. Stauffer and Penna assumed that people form a cluster sharing the same information by going to a neighbourhood bank or broker for investment opinions.

The objective of this paper is to show that the market returns are not normally distributed and check if the fat tails can be explained by local interactions between traders in stock markets. The hypothesis that has been made is that the traders are interested only on the price fluctuations of their own stocks and have no interest in those they do not hold. Since the price variation affects only those traders who have invested in the said stock, the result is a localised interaction between traders with their actions affecting the others involved.

CONTENTS

PERCOLATION THEORY AND THE STAUFFER-PENNA MODEL

In 1957, Broadbent and Hammersley introduced the percolation theory to design and manufacture gas masks. The gist of the idea can be put across by the following example. Suppose a fire breaks out in a large orchard, with trees planted at the vertices of a square mesh. Trees in the neighbourhood of a burning one has a probability p Є [0,1] of catching fire. This probability is also called the ***influence rate.***



The fire results in a cluster of burnt trees. Let C be a cluster, with |C| as the size of the cluster.

The ***percolation probability*** θ(p) is the probability that the origin of the fire (or any other dependent change) belongs to an infinite cluster for p Є [0,1], and defined by:

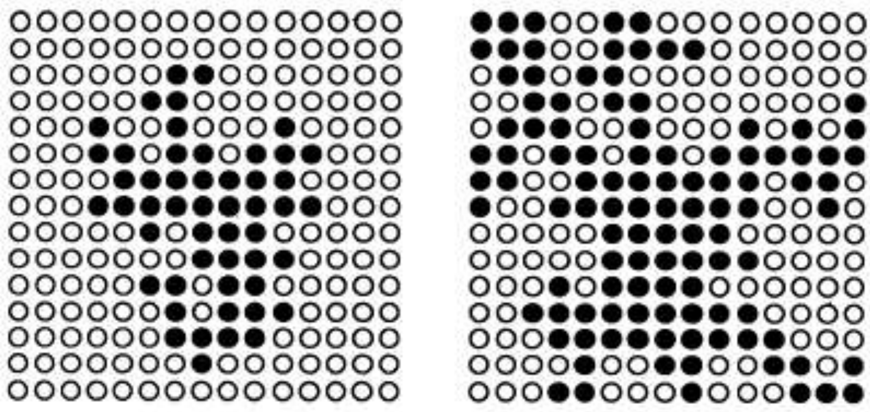
$$\theta(p) = P_p(|C| = \infty) = 1 - \sum_{n=1}^{\infty} P_p(|C| = n)$$

where $P_p(|C|=n)$ represents the probability that the cluster size is equal to n when the influence rate p is known.

There uniquely exists p^c Є (0,1) such that:

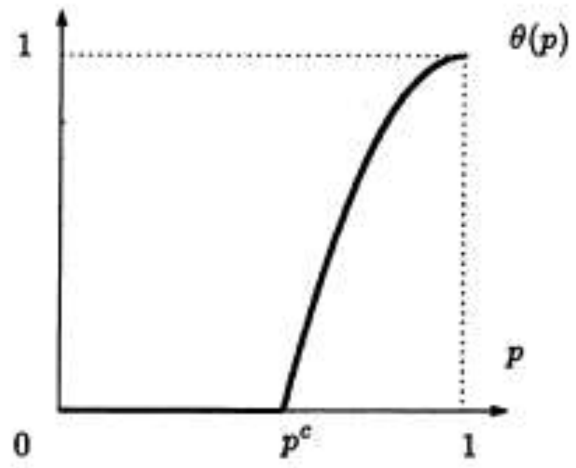
$$\theta_p \begin{cases} = 0 \ (p < p^c) \\ > 0 \ (p > p^c) \end{cases}$$

This implies that there will exist a finite cluster with probability 1 if $p<p^c$, but it will tend to become infinite in size with positive probability if $p>p^c$.this critical value p^c is called the ***critical probability.*** The forest fire model is known as the **site percolation,** and in this case p^c=0.592745.

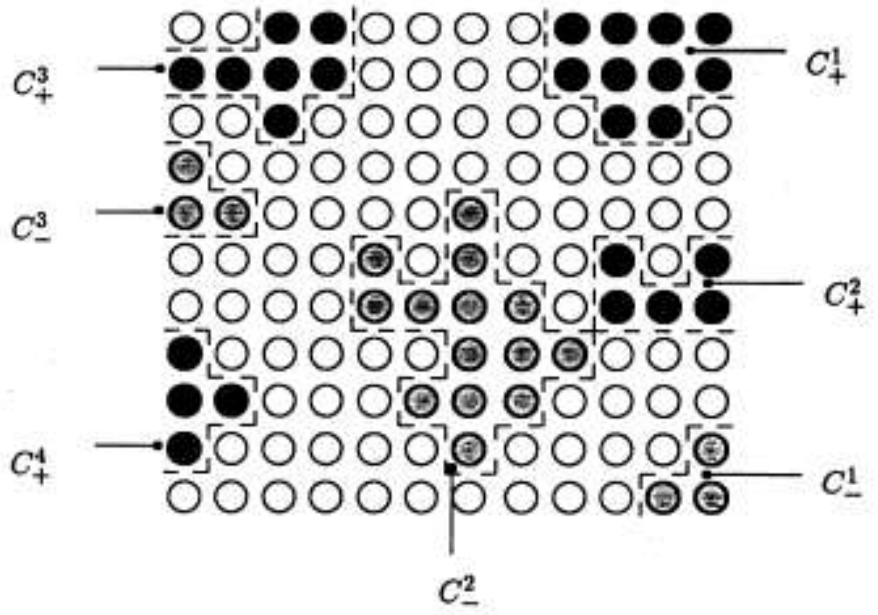


(i) A Finite Cluster: $p < p^c$ (ii) An Infinite Cluster: $p > p^c$

θ(p) is a non-decreasing function of the p, the influence rate, while satisfying both θ(0)=0 and θ(1)=1. This can be represented through the image below.



Stauffer and Penna, in 1998, applied the percolation theory to stock markets to explain fat tails of stock returns. Their assumption was that the traders had a tendency to follow the market trend without analysing the economic data. Traders are distributed on a square lattice with density p, and when they consult the same local financial institution, they form a cluster following the same advice and act accordingly. Each cluster randomly decides to buy (*with probability α/2),* to sell (*with probability α/2),* or to not perform any trade (*with probability 1-α)* during the current time period.



Let C_+^i(t) be any buying cluster and C_-^j(t) be a selling cluster at a time t. Then the return ΔS/S of the stock S at any time t is proportional to the excess demand:

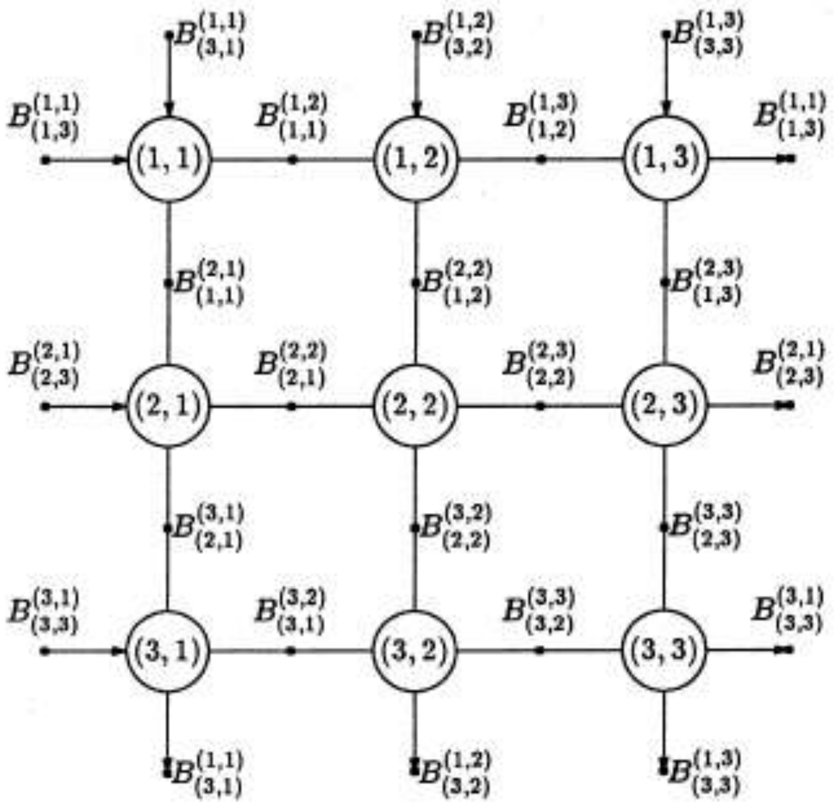
$$\frac{\Delta S_t}{S_t} = \rho(\sum_i C_+^i(t) - \sum_j C_-^j(t))$$

where $\Delta S_t = S_{t+1} - S_t$. Stauffer and Penna investigated how the market model reacted to a change in the ***activity parameter α*** from small values to unity, and showed that the generated distribution had fat tails when $0<\alpha<<1$, but resembles a Gaussian when $\alpha \approx 1$.

Their model successfully generated fat-tails, however, their assumption seems somewhat unrealistic. This is because their model may be valid for the 19th Century where the information trickled down to the masses. In the current modern market, everybody on Earth has access to the same information within minutes. This means that the assumption that people are dependent on financial institutions and hence form clusters is flawed. Moreover their model assumes a constant influence rate, whereas the model in this paper assumes that the influence rate is dependent on the stock price.

THE MODEL

The model given in this paper uses a two dimensional square lattice network. Each node is assumed to be trader and every node is connected to all four of its adjacent nodes. Assuming there are NxN traders in the market, each trader (i, j), for $1 \leq i$ and $j \leq N-1$, holds 4 kinds of stocks $B_{(i,j)}^{(i+1,j)}$, $B_{(i,j)}^{(i,j+1)}$, $B_{(i,j)}^{(i-1,j)}$ and $B_{(i,j)}^{(i,j-1)}$. Trader (i, N) is assumed to hold $B_{(i.N)}^{(i,1)}$ instead of $B_{(i,N)}^{(i,N+1)}$, and trader (N, j) holds $B_{(N,j)}^{(1,j)}$ instead of $B_{(N,j)}^{(N+1,j)}$. The complete market can be represented using the image given below:



Once the market structure is in place, at every period t, traders behave as follows:

- ***STEP 1***-
  At the beginning of every period t, a trader (i, j) is chosen at random and he becomes bull (with probability 0.5) or bear (with probability 0.5). If he becomes bull, he sends a buying order to increase his

holdings. If he becomes bear then he tries to sell his holdings. The reasons why trader (i, j) wants to buy or sell may be rational (e.g. he has received news regarding the fundamentals of the company), irrational (e.g. fads, trends) or exogenous (e.g. he needs money for marriage).

- ***STEP 2-***

  By looking at the actions of trader (i, j), the traders sharing stocks with him have a tendency to go bull or bear themselves.

  If they see that trader i is trying to buy the stocks, each of them can go bull with a probability p^u, or remain neutral with probability $1-p^u$. Traders influenced by trader i act accordingly in the market.

  If they see that trader i is trying to buy the stocks, each of them can become bear with a probability p^d, or remain neutral with probability $1-p^d$. Traders influenced by trader i act accordingly in the market.

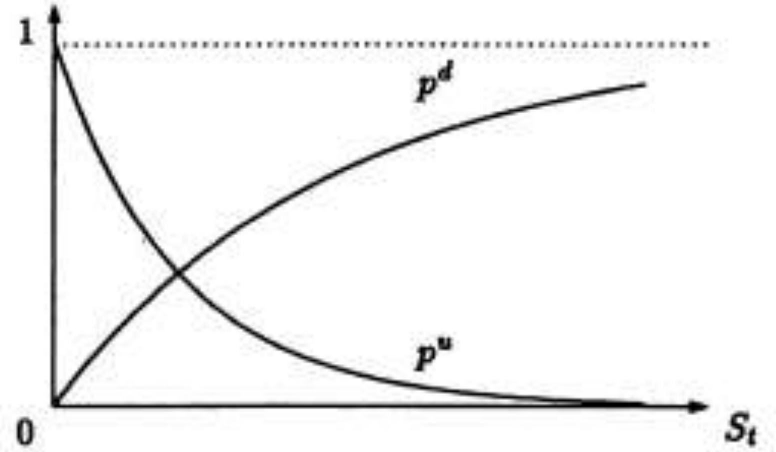
  The influence rates were assumed to be functions of the stock price index i.e. p^u is a decreasing function of S_t and p^d is an increasing function of S_t.

$$p^u(S) = e^{-\alpha S}$$

$$p^d(S) = 1 - e^{-\beta S}$$

  The sum of p^u and p^d is always less than unity. The reason being that there will be individuals who as into long term investments and will not react to the information.

$$e^{-\alpha S} + 1 - e^{-\beta S} < 1$$
$$\Rightarrow e^{-\alpha S} < e^{-\beta S}$$
$$\Rightarrow -\alpha < -\beta$$
$$\Rightarrow \alpha > \beta$$



- ***STEP 3-***

  The return $\frac{\Delta S_t}{S_t}$ of the stock price index S_t is proportional to the change in the bull vs. bear composition of the mesh network.

$$\frac{\Delta S_t}{S_t} = \rho . sgn(C_t) . |C_t|$$

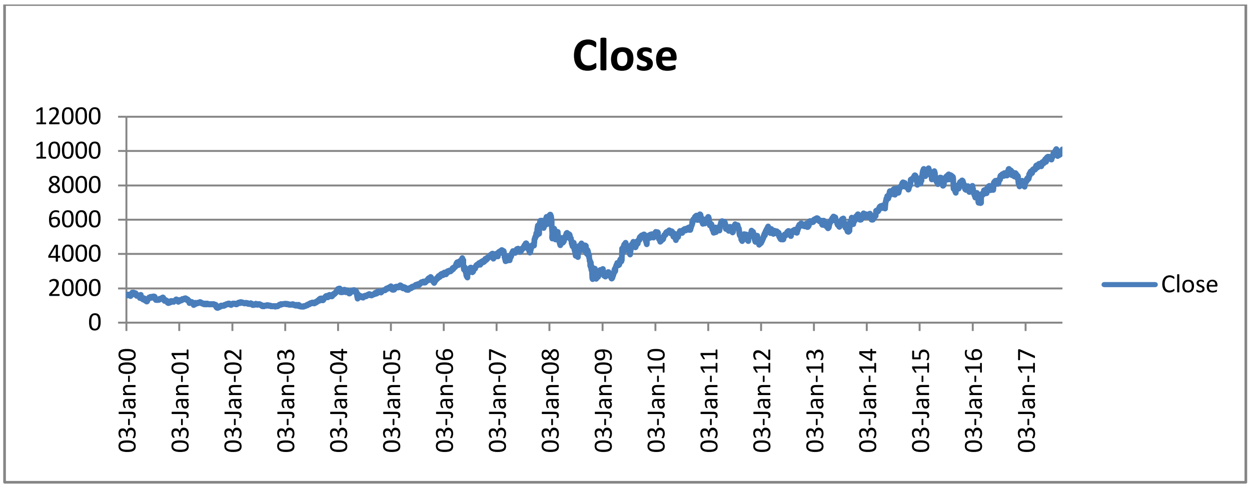
where ρ is a positive constant and $|C_t|$ is the size of the cluster of the final composition of the mesh network.

$$sgn(C_t) = \begin{cases} +1 \ (if\ the\ net\ composition\ is\ bull) \\ -1 \ (if\ the\ net\ composition\ is\ bear) \end{cases}$$

By varying the value of the three constants α, β and ρ we can obtain the ΔS_t values and obtain the Gaussian distribution for the daily returns.

THE NIFTY 50 MARKET INDEX

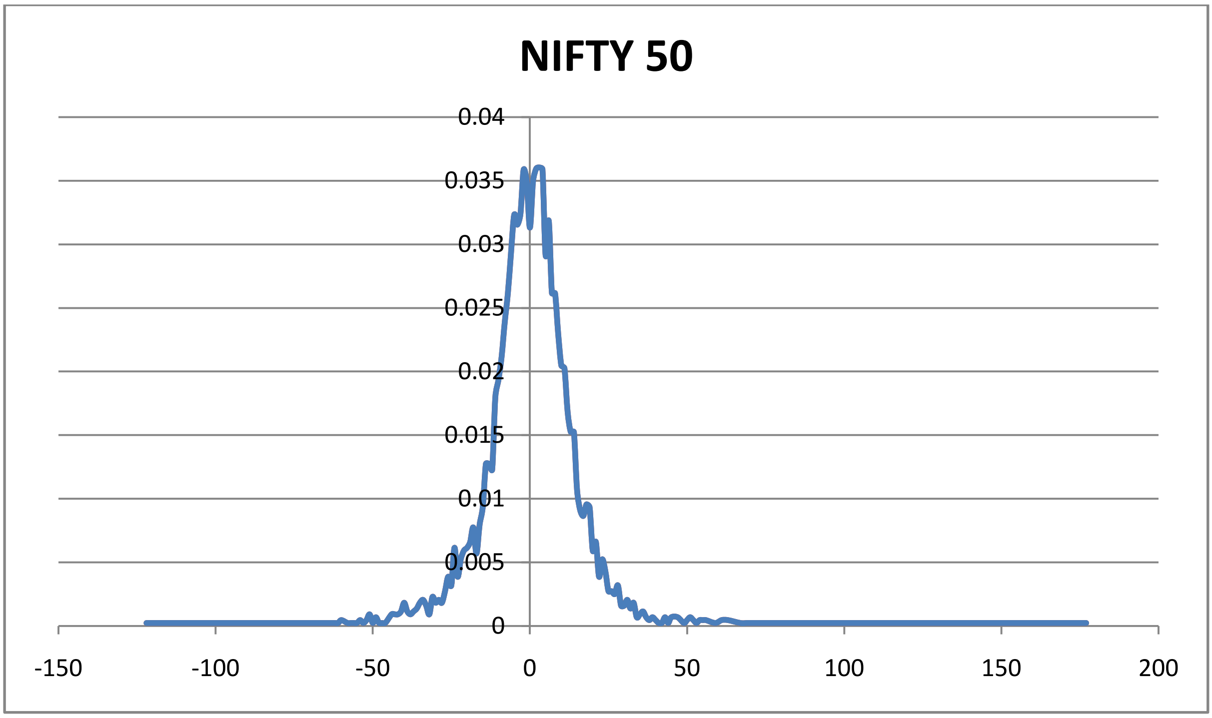
The NIFTY 50 Index daily return data, from 3rd January 2000 to 13th September 2017, has been studied in this paper. The Percolation Model of the Financial Market will try to mimic the Index. The average daily return and the standard deviation of the daily returns of the model will be made to approach the value of the real market index by varying the constants. This ensures that the model represents the real market and can be used as a basis for drawing conclusions regarding the flat tails.



The above graph shows the fluctuation in the NIFTY 50 Index over the 17 years. The average daily return for this period comes out to be **1.0691** and the standard deviation of the daily returns from the mean was calculated to be **14.5003**.

ANALYSIS

To keep the Financial Market Model accurate and as close to the NIFTY 50 index the three constants α, β and ρ were varied to get the mean of the generated returns close to **1.0691** and the standard deviation close to **14.5003**.

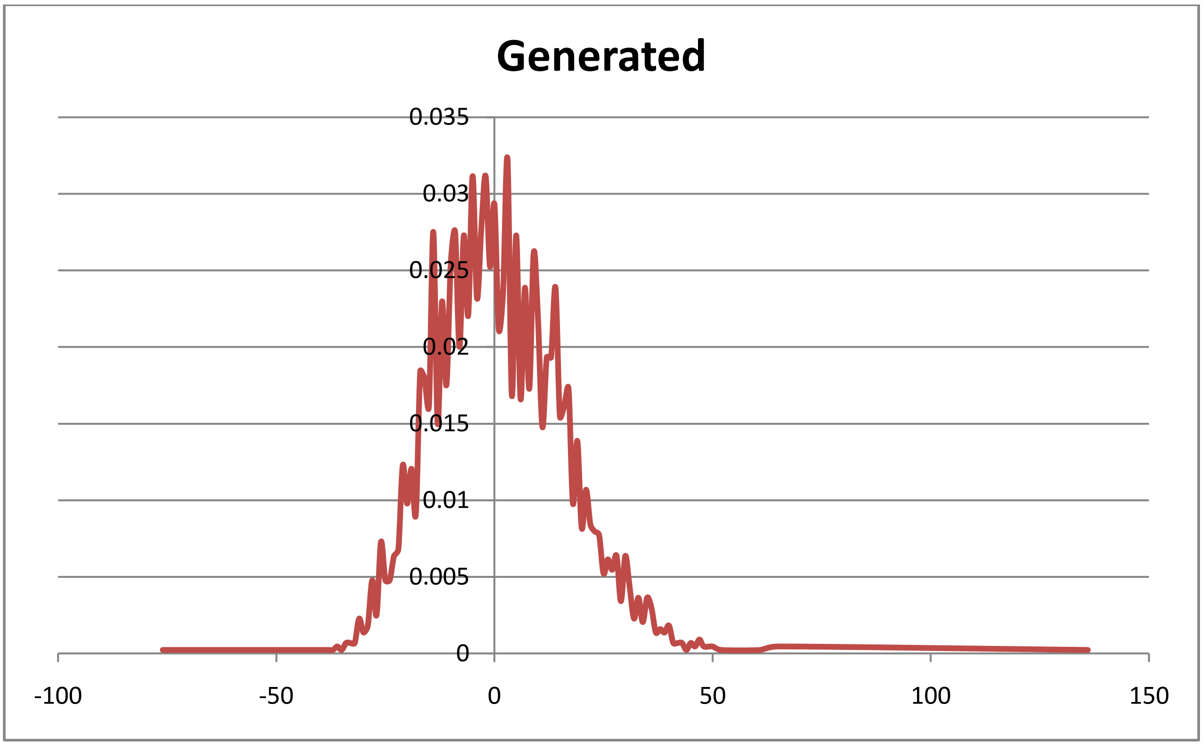


Applying trial and error method, the mean of the generated returns came to **1.085** and the standard deviation to **14.9077** when the constants were set at:

$$\alpha = 0.002399$$

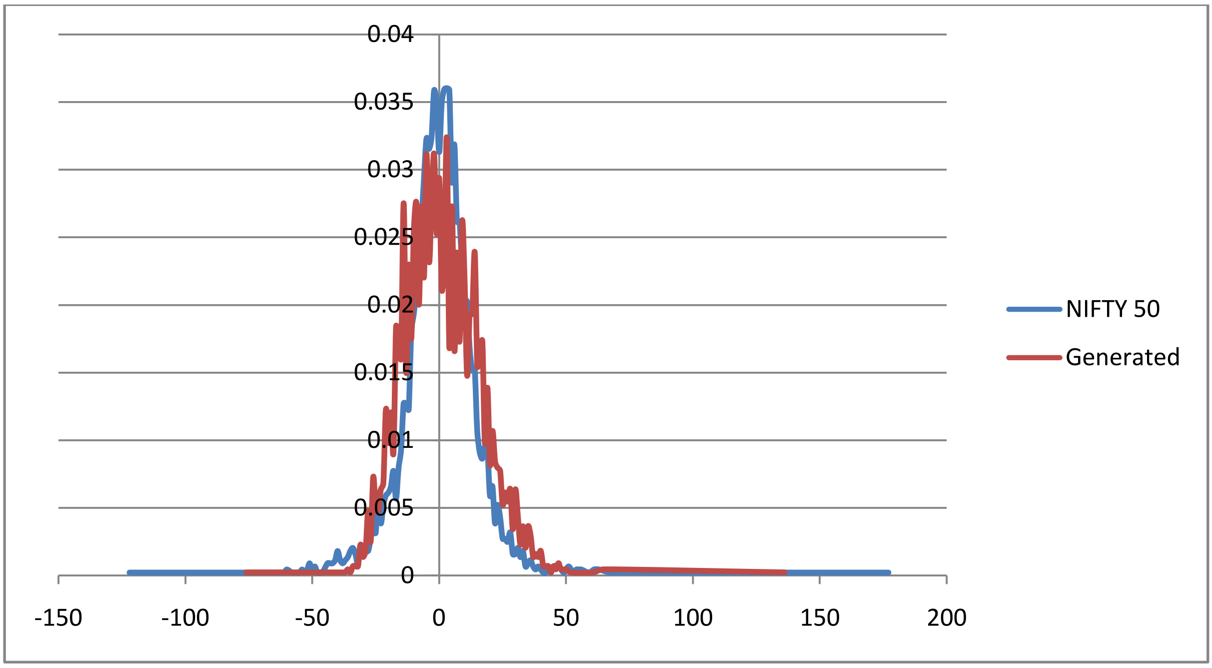
$$\beta = 0.00099$$

$$\rho = 0.001053999$$



Fat- tails can be defined as the increased probability of the extreme scenarios to occur. In stock markets, the daily returns are considered to be normally distributed. However, in the empirical probability distribution of the generated data, the probability of the extreme values should be slightly higher than that of the original market.

The graph given below clearly shows that the probabilities of the daily returns being [-16%, -30%] or +15% and above, is clearly greater in the generated data than in the NIFTY 50 data. This is because the daily returns are not specifically normally distributed and they have inherent fat-tails which are suppressed because of the constraints of normal distribution. When the generated data was empirically distributed it exhibited fat-tails. In our model we assumed that traders have bounded sight that is each trader's private information is unavailable to the general public. The data that was generated using this model exhibited fat-tails. Hence, our assumption stands validated and that fat tails are a consequence of the inclination of the traders to act according to the market trend.



CONCLUSIONS

To conclude, in this project we modelled a stock market as a network of locally interacting traders. The assumption was made that when each trader makes his decision, they are influenced by the actions of their immediate neighbours (family members, friends, advisors .etc.) and this information is private. The daily returns data that was generated using the model exhibited fat-tails which shows that the market returns are not normally distributed and the increased probabilities of extreme returns is due to a tendency of traders to join a bandwagon. In other words, the buying or selling done by a trader affects the decisions of his neighbouring traders, and this local interaction is one of the causes for the deviation from normal distribution (fat-tails).