SVD Reconstruction Algorithm in 3D SPECT Imaging

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***Abstract*--Singular value decomposition (SVD) method was used for image reconstruction in single photon emission computed tomography (SPECT). The 3D system transition matrix and the projection data were produced by Monte-Carlo simulation based on NCAT human torso phantom. Generalized matrix inverse of system transition matrix was computed. NMSE and Contrast parameters were chosen to evaluate the image quality. The relationship between reserve singular value number and reconstructed image quality is discussed. Reconstructed image in best quality was obtained when the optimized number of preserved singular value was chosen, and compared with routine OS-EM reconstruction methods. Results show that SVD reconstruction algorithm, which can reduce noise influence effectively and improve the reconstruction result greatly, is a valuable image reconstruction algorithm. It can be improved to solve the coded mask SPECT imaging problem.**

I. Introduction

The reconstruction algorithm of tomography in SPECT (single photon emission computed tomography) is always the research hotspot of the area. From early Radon transform based FBP (filtered back projection) method to now widely used OS-EM (ordered subset-expectation maximization) method, we can see the progress of reconstruction algorithm.

The reconstructed image quality is mainly influenced by two aspects. One is the difference between the real physics case and mathematic model we build. We can improve it by describing the imaging physics process more precise. The other important factor is the influence of statistics noise. It is impossible to eliminate the noise completely but we can reduce the influence of noise by developing proper algorithm.

In modern reconstruction theory the process of tomography reconstruction can be equal to the process of solving big linear equation group. Regarding the influence of noise and stabilization of imaging, in our work we use SVD (singular value decompositions) method to reconstruct tomography. The SVD method applied in 2D reconstruction has been discussed in [1] while [2] show us the feasibility of SVD in 3D reconstruction. With the development of computer science, now we can deal with more complicated 3D imaging case with SVD and compare with other reconstruction method.

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II. Theory

A. SVD

The projection image formation process in SPECT may be written in matrix-vector form as:

$$Ay = p\,, \tag{1}$$

where p is the projection data vector and y is the source activity distribution vector. A represents the system transition matrix and can be decomposed as this:

$$A = USV^{\mathrm{T}}, \tag{2}$$

where U and V are orthogonal matrices. S only has values in its diagonal elements called singular values of A. It shapes as this:

$$S = \begin{bmatrix} \Sigma & O \\ O & O \end{bmatrix}, \tag{3}$$

where $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, ..., \sigma_r)$, $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r > 0$. Equation (3) was called SVD of A while $\sigma_1, \sigma_2, ..., \sigma_r$ are singular values. Define $\Sigma^{-1} = \mathrm{diag}(\sigma_1^{-1}, \sigma_2^{-1}, ..., \sigma_r^{-1})$, we can get the reciprocal of S:

$$S^{+} = \begin{bmatrix} \Sigma^{-1} & O \\ O & O \end{bmatrix}. \tag{4}$$

The generalized matrix inverse of A (A^{+}) and the minimum norm least squares solution of y (y^{*}) are given:

$$y^{*} = A^{+}p = VS^{+}U^{\mathrm{T}}p\,. \tag{5}$$

B. Noise restrain and the analysis of stability

We use condition number to evaluate the noise sensitivity to a specific imaging system. It defines as the ratio of maximal and minimal singular value of A:

$$c = \frac{\sigma_{\max}}{\sigma_{\min}} \tag{6}$$

When c comes to a big one, the reconstruction of tomography is very sensitive to noise. The stability is bad that we can't get any available information. Matrix A is called ill-posed.

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In order to reduce the influence of noise, we can retain only the larger singular values in the computation of S^+, which is called truncated SVD.

C. Parameters for evaluate reconstructed images

The following parameters are used in this work to evaluate the quality of reconstruct images:

1. *NMSE*, Normalized mean square error

$$NMSE = \frac{1}{N-1}\sqrt{\sum_{i=1}^{N}\left(\frac{f(i)}{\bar{f}} - \frac{f_0(i)}{\bar{f_0}}\right)^2} \tag{7}$$

where $f_0(i)$ represents source activity distribution, $f(i)$ is the reconstructed image. $\bar{f}$ and $\bar{f_0}$ are the mean value of source image and the reconstructed image respectively. The N is the amount of image pixels counts. Smaller NMSE represents better results.

2. *Contrast*

$$Contrast = \frac{|M-B|}{M+B} \tag{8}$$

where M and B are the mean values of the image object region and background region respectively. Higher Contrast represents better results.

III. Model and Calculation

A. NCAT model and SIMSET simulation program

The NCAT[3] (NURBS-based CArdiac-Torso) model which can truly simulate the complex tissues and structures in human thorax was chosen in our work. The radioactive matter distribution in each organ was defined to produce the radioactive matter activity distribution maps in human body. We chose the heart as the object to simulate and study. The voxel size in the model was defined to $0.5\times0.5\times0.5\text{cm}^3$.

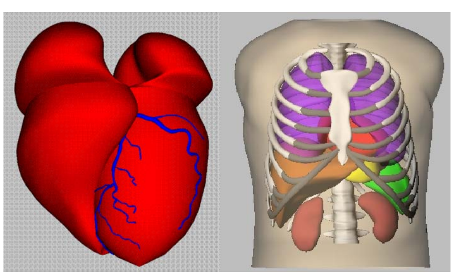

Fig.1 NCAT model[3]

The projection data was produced by Monte-Carlo simulation through the SIMSET[4] (simulation system for emission tomography) program. This program can import the data created by NCAT model then simulate the entire 3D imaging process. During Monte-Carlo simulation, the γ-ray scatter and attenuation in body was fully considered. The parameter of the scatter and attenuation was also given by NCAT model. The projection image pixel size is $0.5\times0.5\text{cm}^2$.

B. The generation of system transition matrix and projection data

The study field of the heart was 34×34×5 voxel cells while the projection images with the size 56×5 were produced in 60 projection angles. The system transition matrix, whose size was 16800×5780, was produced through SIMSET by giving each voxel cell the same emission photons number. The collimators of the system are the LEGP type. 1.0404×10^{10} photons were simulated totally to ensure the precision of the transition matrix.

The projection data based on real radioactive activity map of NCAT was given then. We simulated 3 different noise levels A, B and C for analyzing. The traced photon numbers are 10^7, 3×10^8 and 4×10^9 while total counts are 8.536×10^4、2.02×10^6 和 2.689×10^7 respectively. Form A to C, the noise level was down. A noise-free projection data called 'Ideal' was made by directly multiplying the resource activity distribution and the system transition system.

C. Calculation of SVD and reconstruction

The SVD of system transition matrix was performed using SEGSVD program in LAPACK[5] software kit. On a PC with Pentium4 2.4G CPU, 2GB RAM the SVD calculation costs 8 hours. The matrix U and V generated are 1.13GB and 133MB respectively. Fig.2 shows the singular values spectra of matrix A.

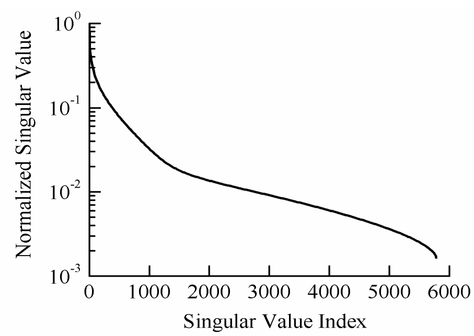

Fig.2 Singular Values Spectra of System Transition Matrix

After SVD calculation, we reconstruct the image with different noise level group A, B and C and different reserved singular value numbers respectively. The reconstruction speed is fast as in former PC it costs only about 30s. To compare with the general reconstruction method, the projection data B was also reconstructed in OS-EM[6] way for which we mark the result as "B(OS-EM)".

IV. Results

NMSE and Contrast parameters were used to evaluate reconstructed images at different noise levels (Ideal, A, B and C). The image qualities changed when reserved singular values numbers were different. In noise free cases, we got best images quality when kept all the singular values. But in other cases (with noise) the best reconstructed result was achieved when we kept proper number of singular values. Fig.3 and Fig.4 show the relationships between NMSE/Contrast and the reserved singular values number. The trends of the results qualities are almost the same with these two parameters.

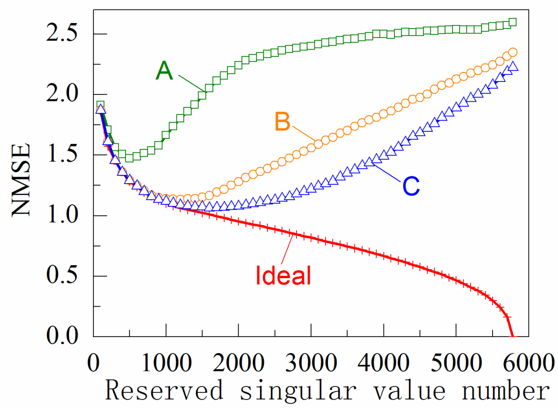

Fig.3 NMSE-Reserved Singular Values Number

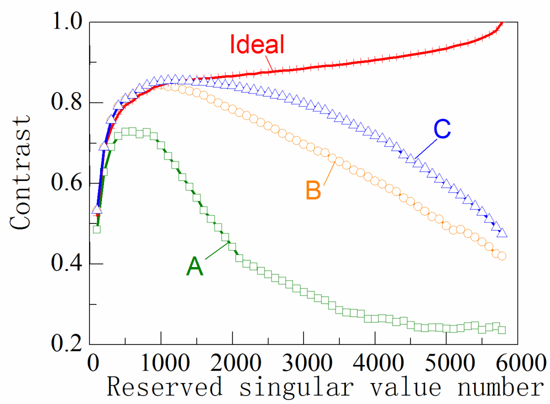

Fig.4 Contrast-Reserved Singular Values Number

The best reserved singular values number was computed according to NMSE value in our work and listed in Table I. Fig.5 lists the comparison of results with different noise level and reconstruction methods. From the top down they are source distribution, SVD reconstructed results of Ideal and C, B, A cases, and finally the OS-EM result of B. Fig.6 show the detail comparison at the section plane. The result of numerical value was list in Table I.

TABLE I
NUMERICAL RESULT COMPARISON

Object	Best Reserved Singular Number	NMSE	Contrast
Ideal	5780	0.0005	0.9999
A	500	1.4708	0.7282
B	1200	1.1331	0.8391
C	1600	1.1062	0.8518
B(OS-EM)	-	1.2355	0.8231

It shows that the SVD result of noise free case has the best imaging quality which has no difference with the source image. SVD results of C, B and A verified their noise level which also represent in their different best reserved singular numbers. The different reconstruction algorithm results of B show that SVD result is better than OS-EM result.

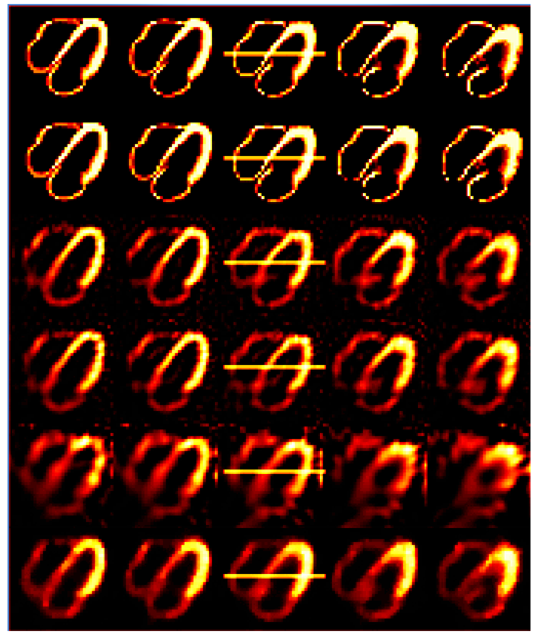

Fig.5 Source data and Reconstructed Results. From the top down there are source activity distribution, noise-free, C, B and A (SVD) reconstructed result and B (OS-EM) reconstructed result.

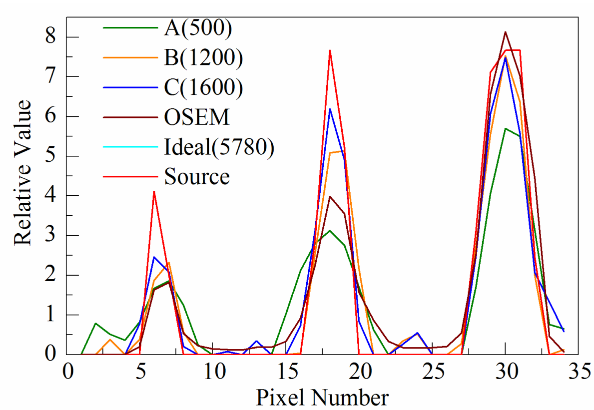

Fig.6 Comparison of Section Plane. SVD reconstructed result of noise-free case is completely the same as The source activity distribution. For B case, the SVD reconstructed result is better than OS-EM result.

V. CONCLUSION

The SVD reconstruction algorithm, which has good imaging quality, is a hopeful reconstruction algorithm with the development of computer technology. Direct 3D imaging can also use SVD to get a better result. The best reserved singular number which represents the noise level of the imaging can be confirmed with NMSE and Contrast standard.

Another important application of SVD algorithm is to evaluate the mathematic model of the system. Because of its good imaging quality, SVD can reduce the influence of algorithm itself to reconstruction process.

The coded mask SPECT imaging which now has no effective algorithm is similar to this direct 3D imaging. To use SVD algorithm on this will be a good effort. It is also where our future work lies.

VI. REFERENCES

[1] Smith M F, Floyd Jr E, Jaszczak R J, et al. "Reconstruction of SPECT images using generalized matrix inverses". *IEEE Trans Med Imag*, 1992, 11: 165-175.

[2] Smith M. F., "Out-of-plane photon compensation for 3-D SPECT image reconstruction with generalized matrix inverses". *IEEE trans Nucl Sci*, 1994, 41: 2820-2830.

[3] Segars W. P. "Development and Application of the New Dynamic NURBS-based Cardiac-Torso (NCAT) Phantom". The University of North Carolina at Chapel Hill, 2001.

[4] Harrison R L, Vannoy S D, Haynor D R, et al. "Preliminary experience with the photon history generator module of a public-domain simulation systems for emission topography". *IEEE Nucl Sci Symp Med Imag Conf*: Norfolk, Va, IEEE, 1994.

[5] Anderson E Z, Bai C, Bischof S B, et al. *LAPACK user's guide*. http://www.netlib.org/lapack/lug/ lapack _lug. html, third edition, 1999.

[6] Ma T, Jin Y. "Efficient analytical scatter modeling in fully 3-D iterative single photon emission computed tomography reconstruction". *IEEE Med Imag Conf*: Portland, IEEE, 2003.