Separation of Lift-Generated Vortex Wakes Into Two Diverging Parts

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NOMENCLATURE

A = wing planform area, ft^2 (m^2)

AR = b^2/A = wing aspect ratio

b = wingspan, ft (m)

b' = spanwise distance between vortex centers, ft (m)

C_L = $L/q_\infty S$

G_o = $\Gamma_o/b_o U_\infty$

L = $\rho_\infty U_\infty |\Gamma_o|\, b'$, lb (N)

q_∞ = $\rho_\infty U_\infty^2/2$, lb/ft^2 (N/m^2)

t = time, s

T = $t\Gamma_o/b_o^2$

x = distance in flight direction, ft (m)

X = x/b_o

y, z = distance in lateral and vertical directions, ft (m)

u, v, w = velocity components in x, y, and z directions, ft/s (m/s)

U_∞ = velocity of wake-generating aircraft, ft/s (m/s)

Γ = circulation bound in wing, ft^2/sec (m^2/s)

ρ = air density, slugs/ft^3 (kg/m^3)

Subscripts

ell = elliptical spanwise loading

g = wake-generating aircraft

o = reference quantity

pr = vortex pair

RTJ = R. T. Jones weight-optimized span loading

∞ = free-stream condition

SEPARATION OF LIFT-GENERATED VORTEX WAKES INTO TWO DIVERGING PARTS

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I. INTRODUCTION

Efforts are under way to develop reliable prediction methods for the movement and spread of lift-generated vortex wakes during landing and takeoff operations on closely spaced parallel runways at airports (refs. 1–3). Because aircraft fly in close formation (both laterally and along-trail) during these types of operations, the size and spreading rate of the hazardous elements in each of their wakes must be accurately known. Knowledge of the time-dependent outer boundary of the hazardous region behind each aircraft is most important during the minute or so before touchdown because during that time period the multiple aircraft in a landing group are usually near the lift-generated wakes of preceding aircraft in the group. For this reason, any deviations in the dynamics of vortex wakes from conventional rollup and spreading dynamics must be evaluated and, if necessary, incorporated into the avoidance method being used. Separation of vortex wakes into two or more pairs is therefore of interest because the multiple wakes generated may substantially increase the cross-sectional size of the region to be avoided by aircraft during landing and takeoff operations at airports where their flightpaths are most constrained (refs. 4 and 5). A possible advantage that may be brought about by wake division is a more rapid decomposition of the wake (ref. 6).

The study presented uses hypothetical deviations from conventional span loadings to find out which ones are theoretically able to cause a segment of a lift-generated vortex sheet to move either rapidly upward or downward away from the bulk of the vortex wake. It is assumed that the flow-field mechanism that causes departure of wake segments is either a deficiency or an enhancement in the lift carryover across the center part of the wing that is shrouded by the fuselage. The cause for lift deficiency or enhancement across the fuselage is not studied in this paper, but is usually a consequence of the aerodynamic design of the wing-fuselage junction. One cause for wake departures is unwanted flow-separation regions at the sides of the fuselage. The dynamics of the hypothetical span loadings considered were calculated for situations where engine power was both negligible and robust.

II. BACKGROUND ON VORTEX WAKE SEPARATIONS

Both upward and downward wake-departing incidents have been observed in condensation trails, but only an upwardly moving one has been photographed. The photograph (fig. 1) indicates a primary or conventional wake moving downward slowly as expected. Of interest is the fact that a wake segment is also shown as moving upward rapidly. A motion picture of a similar event (ref. 7) shows that the upward-moving wake segment is a small vortex pair moving rapidly upward from the wake just behind the aircraft. It was concluded that the pair of secondary vortices moving upward rapidly were formed as a result of span-load variations on the wing near its junction with and shrouded by the fuselage. Cirrus clouds in the background make the upper trail appear to vary in cross section along its length, but it does not do so. The departing vortex pair appears to be aligned with the centerline of the major part of the wake (ref. 7). It is noted that wake divisions and departures of this kind may be responsible for the vertical enlargement of vortex wakes observed previously in the hazardous parts of wakes of aircraft during the approach of aircraft to airports (refs. 4 and 5). The objective of that program was to determine the vertical distance above lift-generated vortex wakes required for following aircraft if they are to avoid all of the hazardous elements of the wake of a preceding aircraft. It was determined that a consistent safe vertical distance above the flightpath of the wake-generating aircraft must be greater than 262.5 ft (80 m).

The second wake-deviation type observed, but not photographed, has been seen by the first author to occur in a few condensation trails, but has not been reported elsewhere. In the cases observed, it appeared that the main part of the exhaust condensation trail appeared as a conventional wake. In addition, one to three other parts of the condensation trail appeared as departing wake segments or vortex pairs moving downward rapidly, rather than upward. The departing pairs were spread along the wake a short distance behind the generating aircraft. Although the departing configurations were not exactly the same as upwardly moving deviation of vortex wake segments, it is assumed here that both forms of wake separation consist of vortex pairs that are initiated by span-load deviations from conventional ones on the wake-generating aircraft. The analysis to follow therefore restricts the span-load variations studied to those in the vicinity of the wing-fuselage junction and over the centerline of the wing that is shrouded by the fuselage, primarily because no other generating mechanism appeared plausible.
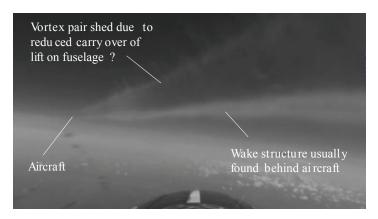

Figure 1. Frame from motion picture taken as side view of condensation wake of aircraft at cruise altitude indicating that one part of the vortex wake is nearly horizontal, as expected, and another part is moving rapidly upward (ref. 7).

III. COMPUTATIONAL METHOD

The same numerical method, point-vortex and point-source singularity structures with soft centers, and dimensionless parameters used previously (ref. 3) are again used here for the present computations. More complex and more representative numerical methods (refs. 8–11) were not used because they require computational times in excess of those allowable for use during aircraft operations on closely spaced parallel runways. The point-vortex and point-source methods used here are not new but have been used extensively in the past to study the dynamics of lift-generated wakes (refs. 12–16). Also, the previous study (ref. 3) considered both elliptic and optimized R. T. Jones span loadings (refs. 17–19). Normalized versions of the two loadings are compared in figure 2. The present paper was restricted to the more efficient R. T. Jones span loading because the two span loadings yield about the same conclusions and because the Jones design most closely approximates the span loadings used on transport aircraft of current design.

The wake structures shed by the unmodified and modified span loadings of Jones used in the study are again represented by 23 point-vortex singularities with soft centers located on each side of the centerline (ref. 3). Because the results did not seem to be very sensitive to the width of the span-load modifications used in the study, the cases presented are restricted to spanwise modifications of 5 vortex spacings (or 22% of the semispan) on each side of the centerline of the wing. Such a restriction did provide an orderly set of results that are believed to be typical of the problem over a range of values from 3 to 7 vortex spacings, and did not appear to eliminate any wake dynamics of interest.

Because all of the computations to be presented assume that the singularities had soft centers, the results have smooth contours. However, if the singular nature of the centers of the point-vortex distribution had not been replaced with soft centers, the smooth and orderly lines of vortex motion illustrated in the figures become quite jagged. In the cases computed with soft cores, the time-dependent rollup of the vortex sheets appears to be quite realistic across the span of the lift distribution being studied, except for the four or five outboard point vortices, which take on an unrealistic kink early in the roll-up process. Calculations made of the same cases but with no soft cores (or with much smaller cores) do not possess this characteristic, but do exhibit erratic motions whenever point vortices approach each other closely. The compromises chosen for the computations therefore carry with
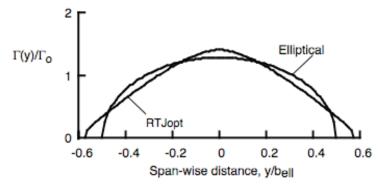
Figure 2. Comparison of span loading used in present study as designed by R. T. Jones with elliptic span loading (refs. 17–19).

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them a distortion in vortex motion most obvious in the wingtip region that appears to be negligible with the remainder of the motion of the singularities. The overall dynamics of the rollup of the wakes studied with soft cores appear to be similar to those calculated by use of no cores, but the smooth variations in the results provided by computations with soft centers or cores are much easier to compare and to interpret.

IV. WAKES STRUCTURES SHED BY HYPOTHETICAL SPAN LOADINGS

A. Conventional Rollup of Wake

Before graphical presentation of the rollup of modified span loadings, the wake dynamics associated with the rollup of the vortex sheet shed by the Jones loading is presented in side view in figure 3. Note that the rollup of the vortex sheet is conventional in that the vortex sheet shed by the wing wraps around the wingtip vortices to form a conventional vortex structure. As time progresses, an orderly vortex structure is formed on each side of the wake centerline. In this case, secondary or departing vortex groups are not formed. If passive (zero strength) sources were placed around the periphery of the engine-exhaust openings (ref. 3), they serve as flow-field markers that indicate the roll-up process of the vortex part of the wake. Also presented in reference 3 is the case when point-source distributions around the peripheries of the engine-exhaust streams are activated to simulate the rollup of the vortex wake as a function of downstream distances in the presence of the jet-exhaust streams of two engines. The results indicate that engine exhaust increases the size of the vortex distribution and makes it appear less organized.
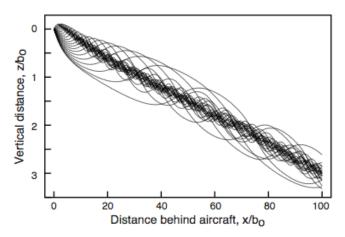

Figure 3. Point-vortex representation of rollup of vortex sheet shed by span loading designed by R. T. Jones for minimum induced drag and wing root bending moment for a given lift and span (ref. 19).

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B. Lift Carryover Reduced by 40%

A 40% lift reduction at the center of the span loading is studied first because it seemed a large and perhaps reasonable amount of lift carryover to lose across the wing-fuselage junction, and their over-lap region. The unmodified and modified span loadings are shown at the top of figure 4a as a verti-cally expanded view to emphasize the difference between the two loadings. The loadings have been normalized in this and in the other cases presented in this paper so that both loadings carry the same amount of lift. As mentioned in the previous section, 23 point vortices (46 total) were used to simu-late the structure of the vortex sheet shed by the Jones span loading. The reduction in lift carryover is spread as an elliptic distribution over 5 vortex spacings, or about $5/23 \approx 22\%$ of the wingspan on each side of the centerline. The presence of one jet-exhaust stream on each side of the wake center-line is simulated by point sources (20 per side, 40 total) that are initially evenly distributed around the periphery of the two engine-exhaust ports as shown in the first parts of figure 4b.

It is at first surprising that departure of a vortex group does not occur in either of the two 40% reduced loading cases shown in figure 4. Both cases first indicate a strong upward motion of the vor-tices near the centerline of the wake. As the computations continue, however, the initial upward motion of the inboard vortices is more than offset by a stronger downward motion brought about by the velocities induced by the outboard vortices. In addition, the possibility for upward departure of a wake segment appears also not to be likely when distributed active sources are added to the flow field below the vortex distribution of the wing (fig. 4b). In that case, the flow field becomes a bit more chaotic but the departure of wake segments does not occur. It is concluded that in both of the cases shown in figure 4, the downward velocity field induced by the outboard vortices is strong enough to overpower the self-induced upward velocities of inboard vortex pairs near the centerline of the wake. It is also observed that, when engine exhaust (represented by open circles for the point-source distributions) is present, the point vortices do not stay tightly grouped, but become spread over a larger cross-sectional area.

Because the end views presented in figure 4 did not present a coherent picture of the three-dimensional dynamics going on within the wake, a side view of each was prepared, as shown in fig-ure 5. Comparison of the results indicate that across-wake motions include large excursions of groups of vortices as distance behind the wake-generating wing increases, but wake departures do not occur. It was expected that the motion induced on the point-vortex part of the wake by the source distribution (fig. 4b) would assist in the departure process, but it does not appear to do so in the 40% reduction case. Instead, the source distributions move the singularities sideways and then around the periphery of the wake. Because the sources are then on the outside of the wake, they cause an inward motion on the vortex distribution rather than a sustained upward influence. Because the inboard vor-tex pairs produced by the 40% modification are not strong enough to overcome the downwash pro-duced by the wingtip vortex distribution, the vortex-induced velocity field causes the two strongest groups of vortices to orbit about each other, as indicated by the side views shown in figure 5. As a consequence, a significant departing pair of vortices does not occur.
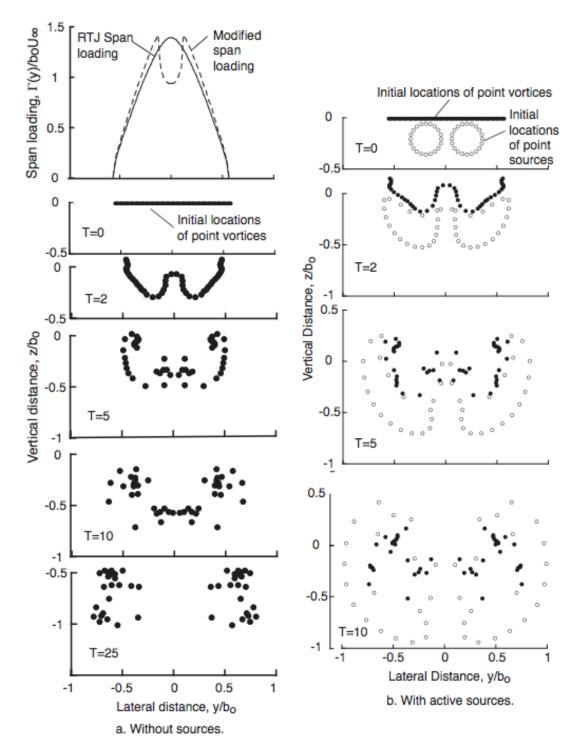
Figure 4. End views of wake dynamics computed for cases when the centerline value of lift carry-over in the fuselage-shrouded region is reduced by 40%. The computations indicate that the loss of lift near the centerline of the wing is not large enough to cause a segment of the vortex sheet to depart from the flow field in an upward direction.
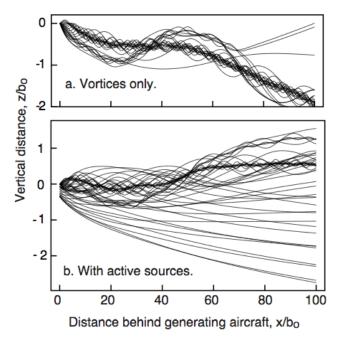
Figure 5. Overview of wake dynamics in side view of the two cases shown in figure 4 (i.e., 40% reduction in centerline lift). Vortex pairs do not depart from main body of wake.

In actual cases, the departing vortices shown in the upper right side of figure 5a at about 50 wingspans behind the generating aircraft are not likely to occur in practice. Such a conclusion is supported by the observation that wake departure usually begins within 10 wingspans behind the aircraft. It is believed that viscous diffusion will have smoothed out motions like those of the two point vortices before the vortex departures shown in figure 5 occur. It is concluded that wake departures will not occur when the modification of centerline lift is 40% or less with or without engine-exhaust streams. For this reason, a larger reduction in span loading near the center of the wake is explored next.

C. Lift Carryover Reduced by 70%

When the centerline lift carryover is reduced by 70% of the nonmodified case, as shown in figure 6, the self-induced up-wash–induced velocities on the inboard vortices are strong enough to overcome the downward-induced velocity field of the outboard vortex groups. As a consequence, an upwardly departing vortex group is predicted by the computational method being used (figs. 6 and 7) for the point-vortex system both without and with the point-source distributions being used to simulate engine-exhaust streams. In these cases, vortices from the centerline region of the wake move upward rapidly during the early stages of rollup. In this case, not all of the upwardly moving point vortices are driven back downward into the main part of the wake. Instead, less energetic vortices near the centerline of the wake move downward and do not depart. However, vortex pairs from near the centerline of the wake are found to be strong enough to produce an upwardly departing vortex group at about $T = 5$ to 10 time units when jet-exhaust streams (sources) are not present, and at about $T = 15$ to 22 time units when exhaust streams are present.
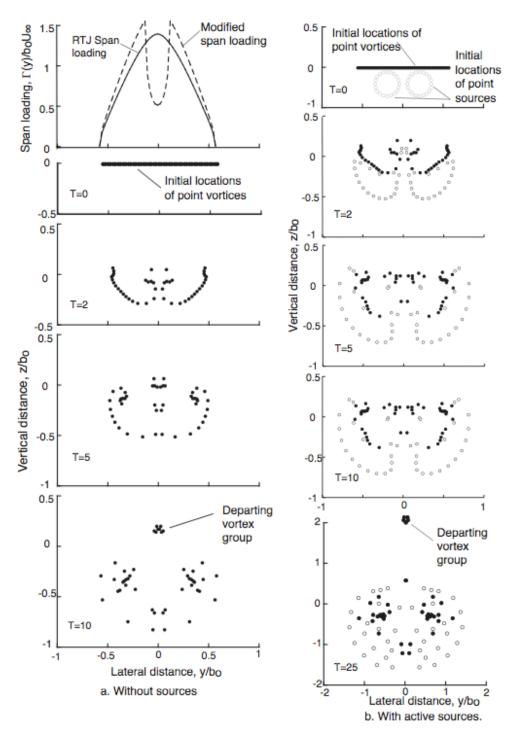
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Figure 6. End views of wake dynamics computed for cases when centerline lift in fuselage-shrouded region is reduced by 70%. These design considerations do cause a vortex pair to depart from the flow field of the bulk of the wake in an upwardly direction both with and without active sources in the flow field.

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This configuration and other similar computations indicate that the presence of point sources delays and mixes the motion of wake segments, but does not appear to significantly prevent or encourage upward departures of wake segments. It is also noted that departures of wake segments in the upward direction as induced by large deficiencies in lift carryover consist of point vortices only and, as expected, no point sources accompany or follow the departure. The side views of the motions presented in figure 7 indicate that enough residual circulation from the main wake region is left behind by the departure of a wake segment so that two groups of vortex pairs are formed in the wake. The two groups of vortices then continue to orbit about each other far behind the wake-generating aircraft. The orbiting motion is apparent in both of the cases shown in figures 6 and 7. In all cases observed, the active point sources migrate to the exterior of the point-vortex distribution that forms the main part of the wake. Some of the scatter apparent at earlier times appears to disappear as time increases and as the point sources move to the outside and then to the top of the bulk of the wake. Observations of condensation trails also indicate that engine-exhaust streams and robust vortex regions tend to separate from each other as time or downstream distance increases (ref. 3).

Side views of the two cases studied in figure 6 are presented in side view in figure 7 to indicate the motion of the departing vortices out to downstream distances of about 50 spans.
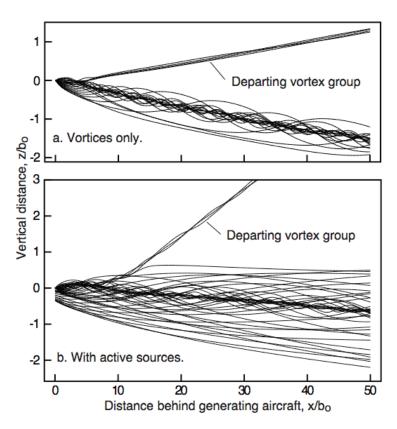

Figure 7. Side views of the two 70% reduced-loading cases presented in figure 6 to show overview of motion of departing vortex groups relative to wake dynamics of primary group of singularities.

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D. Lift Carryover Enhanced by 40%

Because certain configurations of reduced lift carryover brought about upward departure of a segment of the vortex wakes shed by aircraft wings, it seemed logical that enhanced lift carryover might under certain circumstances bring about downward departure of a segment of the vortex sheet shed by a wing. Enhanced lift carryover might be generated by deployment of inboard flaps that extend between the fuselage and the inboard sides of inboard engines. As illustrated in figure 8 by end views and in figure 9 by side views, downwardly moving wake separations do occur if the proper circumstances are present. The most unexpected requirement was that the point-source representations of robust engine-exhaust plumes must be present to produce downward departing wake segments.

Examination of the data indicates that lift enhancement causes two vortex pairs of the same sign to be produced. If jet-exhaust gases (sources) are not present, the two rolled-up vortex pairs on each side of the centerline orbit about each other instead of fostering departure of the inboard segments of the vortex wake. Presence of jet-exhaust gases appears to inhibit the orbiting of the two vortices enough to bring about downward departure of a wake segment. If jet-exhaust gases are not present, computations not presented in this paper indicate that departure of a wake segment does not occur for any lift-enhancement cases between 10% and 100%. Also, other computations not presented here indicated that, when the point-source representations of engine exhaust are added to the flow field of the wake, the vortices on each side of the centerline no longer orbit about each other. However, a downward moving wake segment regularly departs the flow field for lift-enhanced wakes by amounts as small as 10% up to 100% enhancement. If point sources are not present in the flow field, the two vortex pairs produced by the span loading orbit about each other (fig. 9a), and departure of a wake segment does not occur. Small enhancement values bring about a low frequency of orbit (long orbit time), and large enhancements bring about high orbit frequencies (short orbit times). Similarly, the velocity of vertical transit of departing vortex pairs varies roughly as the magnitude of the centerline modification; that is, larger modifications result in larger vertical velocities for the departing pair. Modifications larger than 100% were not studied because they seemed unlikely to occur.
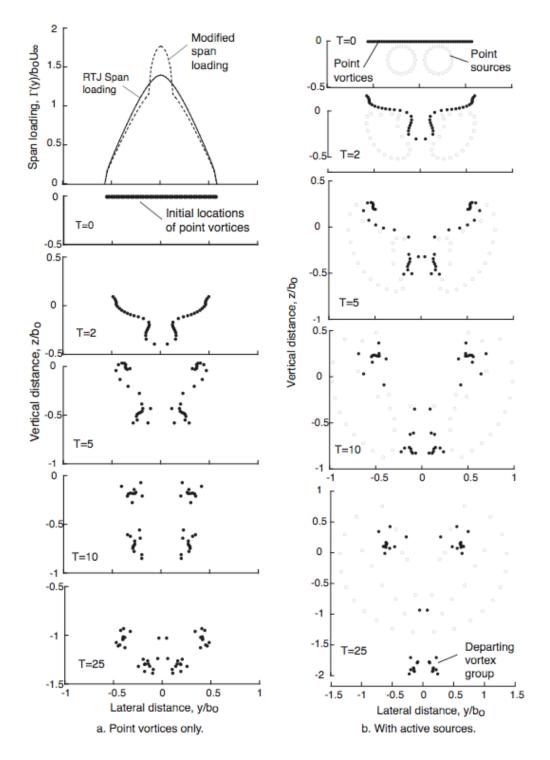
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Figure 8. End views of wake dynamics computed for cases when centerline value of lift has been enhanced by 40%. Vortex pair departs downward only when active sources are present at periphery of engine exhaust streams.
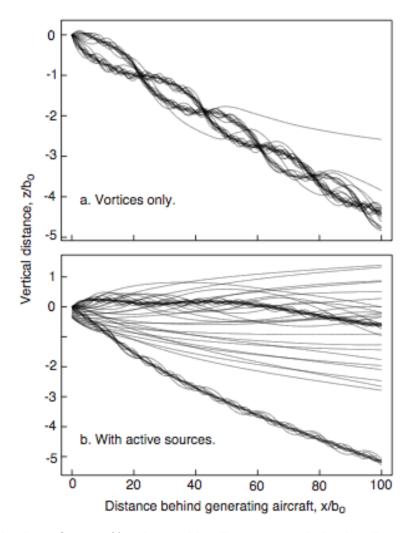
Figure 9. Side view of two 40% enhanced-loading cases to help visualize wake behavior illustrated in figure 8.

V. CONCLUSIONS

As part of an ongoing study of the spreading rate of lift-generated vortex wakes, the present investigation considers possible reasons as to why segments of lift-generated wakes sometimes depart from the main part of the wake to move rapidly in either an upward or downward direction. It is assumed that deficiencies or enhancements of the lift carryover across the fuselage-shrouded wing are the driving mechanism for departures of wake segments. The computations presented first indicate that upwardly departing wake segments that were observed and photographed could have been produced by a deficiency in lift carryover across the fuselage-shrouded part of the wing. Computations made of idealized vortex wakes indicate that upward departure of a wake segment requires a centerline reduction in the span loading of 70% or more, whether the engines are at idle or robust thrust.

Similarly, it was found that downward departure of wake segments is produced when the lift over the center part of the wing is enhanced. However, it was also found that downward departures do not occur without the presence of robust engine-exhaust streams (i.e., engines must not be at idle). In those cases, downward departures of a wake segment occur when the centerline value of the loading is enhanced by any amount between about 10% and 100%.

Observations of condensation trails indicate that downward departure of wake segments is rare. Upward departures of wake segments appear to be more common but still rare. A study to determine the part of the aircraft that causes wake departures has not been carried out. However, even though departures of wake segments rarely occur, some aircraft do regularly shed these wake structures. If aircraft safety is to be assured to a high degree of reliability, and a solution for eliminating them is not implemented, existing guidelines for the avoidance of vortex wakes (refs. 1 and 3) may need to be broadened to include possible increases in wake sizes caused by vertical departures of wake segments. Further study may indicate that it is not possible to modify existing aircraft enough to prevent wake departures. Wake-avoidance guidelines must then be adjusted to provide the desired degree of safety. It appears that steps to avoid upwardly moving wake segments have already been incorporated into the avoidance procedures used for aircraft on approach to runways at the Frankfurt Airport (refs. 6 and 7). The uncertainty in the prospects for compromises in flight safety caused by rapidly upwardly or downwardly moving wake segments suggests that aircraft should not be permitted to fly above or below each other during operations in the airport vicinity where aircraft are likely to be closely spaced (ref. 20).

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