

Uncertainty Analysis of Integrated Departures and Arrivals: A Los Angeles Case Study

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Integrating departures and arrivals in terminal airspace with shared resources, such as waypoints, fixes and routes, has the potential to provide more efficient operations than segregated operations. However, the benefits of integrated operations may be vulnerable to flight time uncertainty. This paper presents an analysis of the impacts of flight time uncertainty on scheduled integrated operations. Monte Carlo simulations were implemented with perturbations incorporated into flight arrival/departure times. Impacts of the uncertainty on delays and controller interventions were investigated. Two cases in Los Angeles terminal airspace were examined. Results showed the general trend that when uncertainty buffer increased, delay normally increased whereas controller interventions decreased. In addition, results from the Los Angeles cases showed that, even with 60s uncertainty buffer, the schedules of integrated operations generated under deterministic scenarios still had over 90 percent chance of reducing delay over the segregated operations, although the controller interventions increased as a trade-off. On the other hand, in these two cases, the departure time precision showed mixed impact which depended on the intervals in departure sequence, however, the departure time precision showed a stronger relationship with controller interventions than the arrival time precision in both cases.

I. Introduction

In terminal areas, especially in metroplexes where several busy airports are close to each other, hundreds of flights have to fly through a narrow area for departing or arriving in a short time period. In order to solve the challenge in terminal airspace, researchers have addressed arrival scheduling problems^{1–6} and airport surface management problems.^{7–9} Another class of terminal airspace problems arises when different departure and/or arrival flows in a terminal airspace share the same resources such as waypoints, fixes or/and routes. The interactions can happen among departures, arrivals, or between departures and arrivals. Recent studies have shown that integrated arrivals and/or departures in major airports or metroplex areas^{10–12} may have the potential of improving operational efficiency. However, in scheduling problems, results are usually sensitive to flight time uncertainties that are caused by many sources, such as inaccurate wind prediction, error in aircraft dynamics, or human factors. Uncertainty analysis is necessary to evaluate the robustness of the benefits. For arrival scheduling problems, Thipphavong et al¹³ studied the relationship between uncertainty and system performance using Stochastic Terminal Arrival Scheduling Software (STASS).¹⁴ Mulfinger et al¹⁵ also analyzed scheduling benefits of reduced arrival time uncertainty using expanded STASS. Because the interactions between departures and arrivals occur at both merging and diverging points, the impact of the uncertainty may be different from the impact in pure arrival scheduling problems. Therefore, uncertainty analysis is necessary to evaluate the expected benefits for integrated departures and arrivals.

Two different scenarios were investigated. The solutions were first generated for deterministic scenarios, then flight entry times were perturbed in Monte Carlo simulations. Given the deterministic solution, a

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heuristic controller model was used to resolve conflicts caused by perturbations. The impact of the uncertainties on total delays and controller intervention was analyzed based on the outcomes of resolved conflicts. A sensitivity study of the delays and controller interventions with varied precisions of arrivals and departures was then conducted.

In the paper, Section II revisits the problem modelled in previous work. The solutions generated under deterministic cases are presented. Section III presents the method for the uncertainty study, including Monte Carlo simulation set up. Section IV provides the analysis and results.

II. Problem and Model

The interactions between arrivals and departures in Los Angeles terminal airspace were presented in previous work¹² for studying optimal integrated operations in deterministic circumstances. Because the uncertainty investigation in this work is based on the same problem, the model and problem statement from previous work are described in this section.

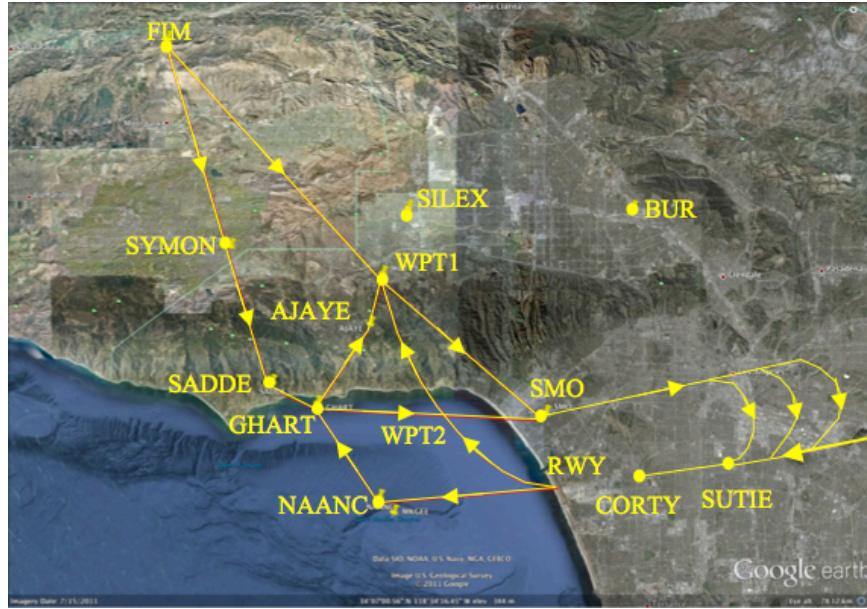


Figure 1. Interactions between SADDE arrivals and CASTA departures

II.A. Problem

Based on the Standard Terminal Arrival Routes (STARs) and the Standard Instrument Departures (SIDs) of Los Angeles terminal airspace, the arrivals from FIM would follow procedure SADDE6 (FIM-SYMON-SADDE-SMO) and the departures to the North need to follow procedure CASTA2 (Runway-NAANC-GHART-SILEX) (see Fig. 1). The arrivals are requested to maintain their flight altitudes above 12,000 feet at Fix GHART and the departures need to remain at or below 9,000 feet at the same fix in order to procedurally avoid potential conflicts between arrivals and departure. If there was no interaction, departures to the north and arrivals from FIM would have flown direct routes. As shown in the Fig. 1, the direct routes would be RWY-WPT2-WPT1 and FIM-WPT1-SMO for departures and arrivals, respectively, where WPT1 and WPT2 are made-up fix names for simplicity. Compared to these direct routes in ideal situations, individual arrival and departure flights following current procedures will approximately fly an extra 60 and 120 seconds, respectively, and would also be flying non-preferred altitudes.

II.B. Modeling

In previous work,¹² three different separation methods were compared. They are spatial, temporal and hybrid separations. Spatial separation uses the same strategy as in the SIDs and STARs to spatially separate interacting flows, which can be treated as current operations. Temporal separation utilizes the direct routes

with conflicts resolved solely with temporal controls. Hybrid separation applies both temporal and spatial separations. Three flows were taken into account in the problem: arrivals from FIM, northbound departures from Runway 24L (shown as “RWY” in Fig. 1), and westbound arrival flights towards SUTIE.

In the formulation of hybrid separation, four design variables were defined for each FIM arrival: d_{1i} is the delay before or at FIM; r_i is the route option, where 0 denotes the direct route and 1 denotes the indirect route; v_i is the aircraft speed between FIM and WPT1 for the direct route or the speed between FIM and WPT2 if the indirect route is chosen; d_{2i} is the delay before or at SUTIE to ensure separation at SUTIE. For a departure flight, three decision variables were defined: d_j is the delay before departure; r_j is the route option, where 0 denotes the direct route and 1 denotes the indirect route. v_j is the speed from departure to WPT1. There is only one decision variable for each arrival flight from the east, d_k , which is the delay time at or before SUTIE. The separation requirements at all fixes are formulated as hard constraints. The objective is to minimize the total delay of all departures and arrivals. More details of the model including route structures and constraints can be found in a previous paper.¹²

III. Deterministic Case Study

To perform the uncertainty study, deterministic solutions are first generated using a non-dominated sorting genetic algorithm (NSGA)¹⁶ as in previous work.¹² To be generalized, two typical cases were chosen from historical traffic data for this study. Case I includes three flows: departures to the North from Runway 24L, arrivals from FIM, and arrivals from the East. Case II contains only two flows without the flow from the East

III.A. Initial condition

Table 1. Initial times in Case I

Order	FIM (sec)	RWY (sec)	SUTIE (sec)
1	0	30	430
2	135	298	671
3	263	540	1070
4	860	1240	1210
5	1230	NA	1376
6	NA	NA	1780

Table 2. Initial times in Case II

Order	FIM (sec)	RWY (sec)
1	39	68
2	446	165
3	728	363
4	1106	529
5	1332	1613
6	1475	1830
7	1613	NA
8	1770	NA

Table 1 shows the initial times for Case I based on traffic data between 10:30AM and 11:00AM (local time) on March 5, 2010. A total of 15 flights were involved including 5 FIM arrivals, 4 departures, and 6 westbound arrivals to SUTIE. Table 2 shows the initial times of 14 flights for Case II based on the traffic data between 9:00AM and 9:30AM (local time) on December 4, 2012. It includes 8 FIM arrivals and 6

departures. The initial times are relative times to simulation start time. The “Order” of each flight is sorted based on initial times.

III.B. Deterministic solutions

Table 3 and Table 4 show the deterministic solutions from optimizations. Table 3 presents the total delays in Case I with varied uncertainty buffers and separation methods. Table 4 presents the total delays in Case II. Without the arrival flow from the East, Case II showed higher delay savings than Case I. In Case II, the hybrid separation achieved 94%, 83%, and 65% savings over the spatial separation for uncertainty 0s, 30s, and 60s, respectively. In this work, the route options from the deterministic solutions are used and fixed. And if necessary, additional delays are imposed to maintain required separation.

Table 3. Total delay with different separation methods in Case I

Uncertainty buffer	Spatial	Temporal	Hybrid
0 s	1,001s	334 s	275 s
30 s	1,163 s	805 s	778 s
60 s	1,673 s	1,694 s	1,408 s

Table 4. Total delay with different separation methods in Case II

Uncertainty buffer	Spatial	Temporal	Hybrid
0 s	1,185s	61 s	73 s
30 s	1,185 s	336 s	207 s
60 s	1,195 s	781 s	423 s

IV. Monte Carlo Simulation

The stochastic behavior of the flight time errors in arrival and departure flights will result in unexpected separation loss. Therefore, controllers have to spend extra effort to resolve unexpected conflicts, which unavoidably increase total delays and controller interventions. It will essentially reduce benefits claimed under deterministic scenarios. In order to study such behavior, a Monte Carlo simulation was implemented. Heuristic behaviors were modeled to mimic the controller intervention to prevent separation loss and to measure the extra delays.

IV.A. Perturbation in times

As shown in Fig. 2, error sources that follow normal distributions were added in flight arrival times at waypoints FIM and SUTIE and departure times at departure runway, respectively. They will affect FIM arrivals, arrivals from the east, and departures, respectively. By default, the arrival time error has a standard deviation of 30 seconds and a mean of zero second, which has been commonly used as a desired prediction accuracy in many arrival trajectory prediction studies.^{17,18} The departure time error’s standard deviation is 90 seconds and the mean value is 30 second, which is based on the Call For Release (CFR) 3-minute compliance window. The window is often structured to allow departure 2 minutes prior to or 1 minute later than the target coordinated departure time.¹⁹

IV.B. Heuristic controller behavior model

In order to simulate controllers’ behavior of resolving separation loss and to measure the unexpected delay as a result of the intervention, a heuristic model was implemented in the Monte Carlo simulation. The number of times that controllers need to step in to resolve potential conflicts is associated with the increase of controller workload. In this model, the aircraft routes were decided by the solution already generated

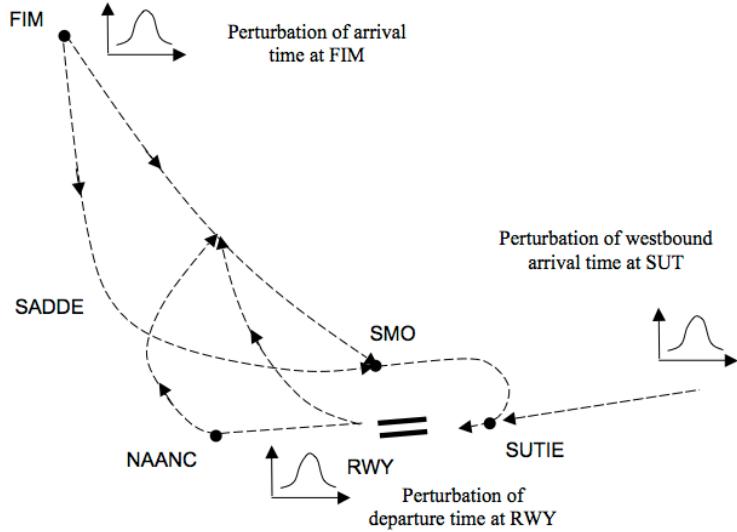


Figure 2. Error sources in Monte Carlo simulation set up

under deterministic scenarios. The route assignments were not changed in Monte Carlo simulations. The simulated controller's intervention behavior was assumed to follow the First-Come-First-Served(FCFS) rule, although in current operations controllers might use improved heuristics other than FCFS. At a waypoint like WPT1, WPT2, and SUTIE, the program will sort the sequence based on the flights' perturbed entry times. Then extra delays are imposed if the separation between any two adjacent aircraft doesn't meet the defined requirement, which is the sum of the separation buffer and the additional buffer. The extra/unexpected delays are propagated to following waypoints if any exist. If there is no perturbation in the entry time as planned in deterministic cases, no extra delay should be imposed and no extra controller intervention should be involved.

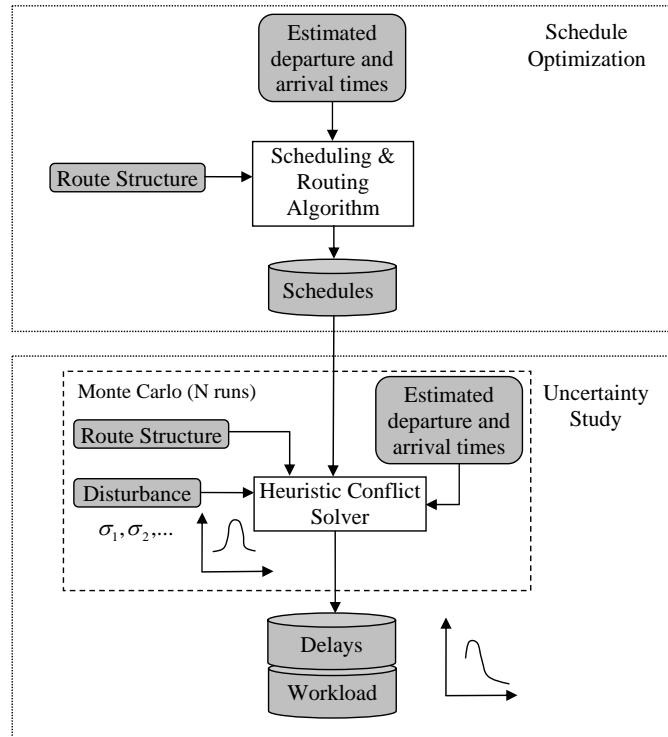


Figure 3. Flow Chart

Figure 3 describes the entire work flow. The first box contains the schedule optimization that was developed and described in previous work.¹² The second box shows the uncertainty study. The “Heuristic Conflict Solver” is the function that mimic a controller’s intervention. The Monte Carlo simulation in this work includes 5,000 simulations and it can be run in less than one second on a MacOS platform with 2x2.66GHz 6-Core Intel Xeon and 8GB RAM. By feeding in live traffic, the first part can find deterministic solutions and the second part can be used as a quick reference to see if uncertainty can reduce or eliminate the benefit. The decision maker can thus decide if suggested operations should be executed.

V. Results

This section presents the impact of flight time uncertainty on delay reduction and controller intervention. The robustness of these performance measures to the departure and arrival time precision is also investigated.

V.A. Delay distribution

As in previous work,¹² three separation strategies were investigated. Besides the minimum separation requirement, uncertainty buffers of 0, 30, or 60 seconds were included when generating the schedule solution in deterministic scenarios. Figures 4(a) and 4(b) present the comparison among different strategies with varied buffers for Case I and II, respectively. In the horizontal axis, letters “S”, “H”, and “T”, represent spatial, hybrid, and temporal separations, respectively. The numbers that follow these letters denote the values of extra buffers in seconds. For instance, “H30” means hybrid separation with 30 seconds buffer. The results in the figures are shown in Box-and-Whisker plots. In each case, the top and bottom horizontal lines are the maximum and minimum delays. The top and bottom boundaries of the narrow long box represent 90th and 10th percentiles, respectively. And the top and bottom boundaries of the wide short box are 75th and 25th percentiles. The horizontal line in the wide box is the mean value of the Monte Carlo simulation.

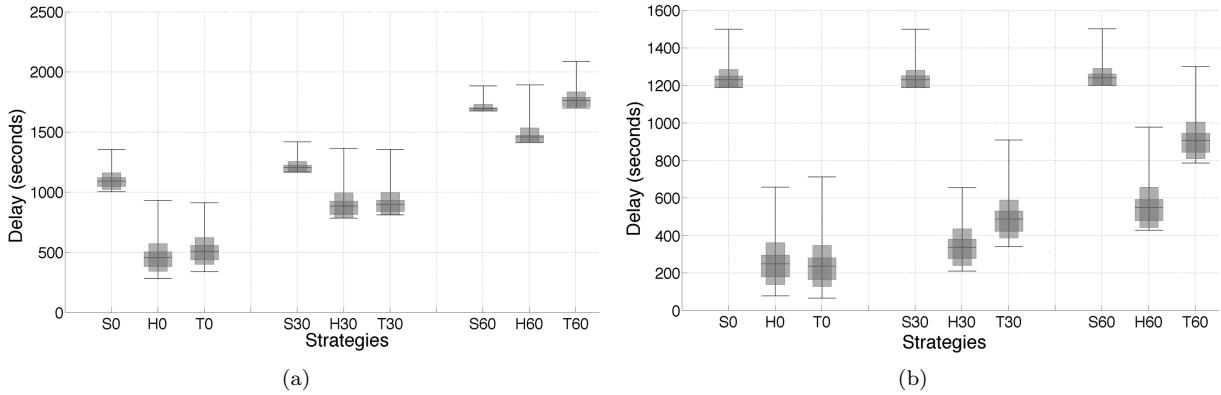


Figure 4. Delay distribution under uncertainty ($\sigma_{dep} = 90s$, $\mu_{dep} = 30s$; $\sigma_{arr} = 30s$, $\mu_{arr} = 0s$) (a) Case I (b) Case II.

From the figures, it is noted that the hybrid separation showed greater delay reduction compared with spatial separation. In the cases of 60 seconds buffer, Case II showed 55% saving at 90th percentiles. There was still 20% delay savings even when comparing the worst case of hybrid solutions with the best case of spatial solutions according to the 5,000 Monte Carlo simulations. Although in Case I, the delays from the hybrid and spatial separations are similar in the worst case, the hybrid separation still has at least 9% delay savings over the spatial separation 90% of the time even with 60 second buffer. It is also noticed that generally as the buffer increases, the uncertainty of the delay reduction decreases, which means the benefit is more robust to flight time uncertainty if a larger extra buffer is applied in the deterministic solutions. As a trade-off, the differences of mean values between “spatial” and “hybrid” decreases, as do the benefits from hybrid separation over spatial separation. According to these uncertainty analysis charts, in both cases the integrated arrivals and departures showed advantages in delay saving, but the levels of savings are quite different in different scenarios.

V.B. Controller intervention distribution

By counting the times that controllers have to intervene to resolve any separation loss, a controller intervention distribution can be generated under stochastic scenarios. Figure 5(a) and 5(b) show the comparison among different strategies and uncertainty buffers for Case I and Case II, respectively. By increasing uncertainty buffers in Case I, the average intervention drops. However, the uncertainty characteristics showed great connection to the schedules. For instance, comparing “S0” and “S60” in Fig. 5(a), the controller intervention decreases from 3 to 1. Whereas Case II’s trend wasn’t as strong, “S0”, “S30”, and “S60” showed similar mean controller intervention. On the other hand, a trade-off exists between delay reduction and intervention. A significant benefit was gained in delay reduction as discussed in the previous section, whereas controller intervention increases as shown in Fig. 5(a) and 5(b). For example, comparing “S0” and “H0” in Case I, the controller intervention changed from 3 to 5 with about 60% increase. A similar situation happened in Case II. Based on the comparison, a buffer of 30 or 60 seconds might be recommended to balance the trade-offs.

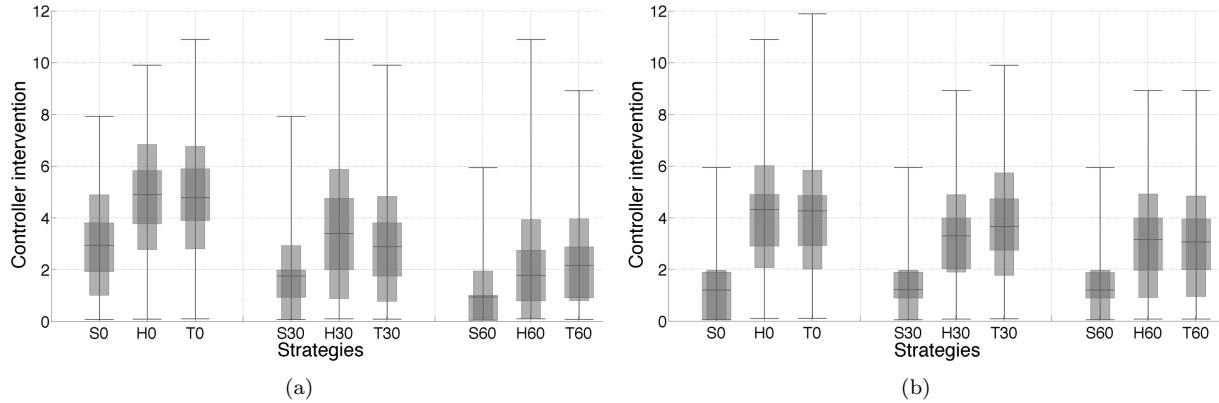


Figure 5. Controller intervention distribution under uncertainty ($\sigma_{dep} = 90s$, $\mu_{dep} = 30s$; $\sigma_{arr} = 30s$, $\mu_{arr} = 0s$) (a) Case I (b) Case II.

V.C. Sensitivity to departure and arrival time precisions

The above studies assumed that the standard deviation of departure times is 90 seconds with a mean of 30 seconds and the standard deviation of arrival times is 30 seconds with a zero mean. Questions may arise regarding the impact of departure and arrival time precisions. How would departure and arrival time precisions affect delay reduction and controller intervention? In this section, a constant buffer of 30 seconds was used but arrival and departure time standard deviations were varied. To be conservative, the difference between the 75th percentile of the “H30” distribution and the 25th percentile of the “S30” was used to calculate delay robustness.

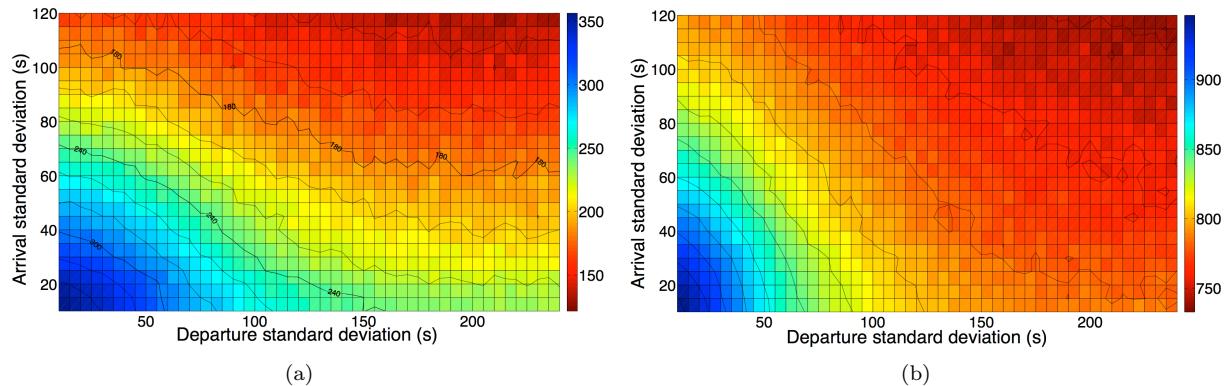


Figure 6. Delay reduction under varied uncertainties (extra separation buffer = 30 s) (a) Case I (b) Case II.

As shown in Figs. 6(a) and 6(b), cold color represents high delay savings and warm color denotes low delay savings. The saving ranges anywhere from 140 seconds to 340 seconds in Case I and from 740 seconds to 940 seconds in Case II. In Fig. 6(a), the delay saving would be 5 minutes for an arrival deviation of 30 seconds and a departure deviation of 60 seconds. In Fig. 6(b), the same precision yielded 14 minutes savings. It is noted that in Case I, the delay reduction is less sensitive to the departure time precision than arrival time precision when the departure deviation exceeds 80 seconds. For example, with an arrival time precision of 30 seconds, the difference of delay reductions at departure time precisions of 2 minutes and above are similar. In Case II, similar phenomenon showed up when the departure deviation exceeded 120 seconds. One explanation for this phenomenon could be the relatively bigger gaps among departures than gaps in arrivals. For instance, the departure and arrival intervals in Case II are 352 s and 247 s respectively. The big gaps in departure flows provided the flexibility in absorbing delays caused by flight time errors. Another explanation would be the relatively low number of departure flights. Although these hypotheses need to be further verified, this kind of information is helpful to traffic managers when they are making decisions.

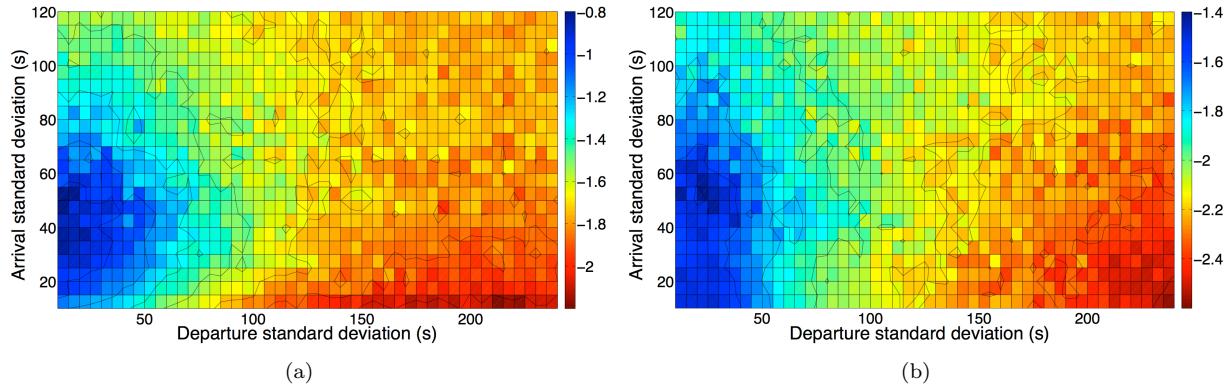


Figure 7. Controller intervention increase under varied uncertainties (extra separation buffer = 30 s) (a) Case I (b) Case II.

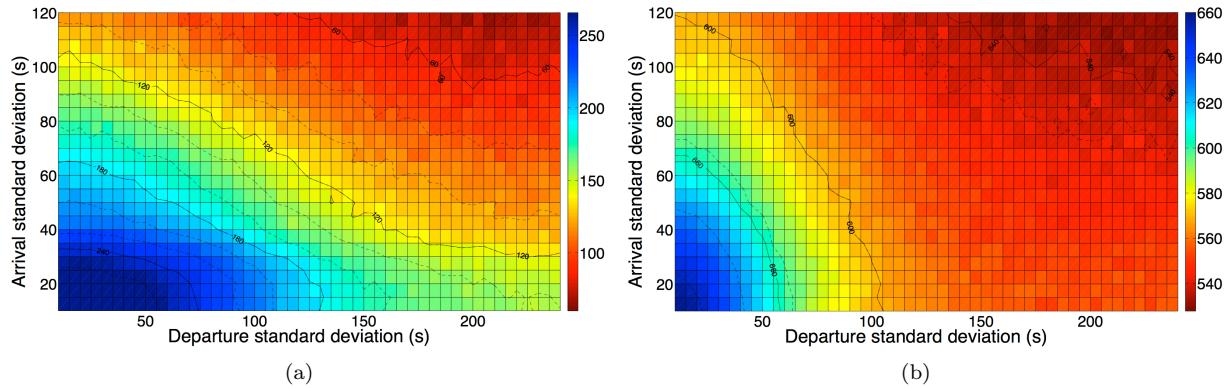


Figure 8. Delay reduction under varied uncertainties (extra separation buffer = 60 s) (a) Case I (b) Case II.

Figures 7(a) and 7(b) present the controller intervention with varied departure and arrival time precisions. Cold colors represent low controller intervention increase when comparing hybrid with spatial, and warm colors denote high intervention increase. The controller intervention showed a different pattern from the delay savings. The patterns in both figures implied that departure time precision dominated impacts on controller interventions, whereas arrival time precision showed much less effect. For instance, in Case I, with the arrival time deviation of 30 seconds, the controller intervention increase over “spatial” is around 1.3 (shown as negative 1.3 in the figure) when the departure time deviation is 60 seconds. The controller intervention increase changes to 1.6 when the departure time deviation is 90 seconds. Whereas, the controller intervention increase stayed the same when increasing arrival time deviation to 60 seconds and keeping departure time deviation at 60 seconds. One explanation could be that, although big gaps in departure flows provided the

flexibility in absorbing delays caused by flight time errors, controller interventions were still unavoidable.

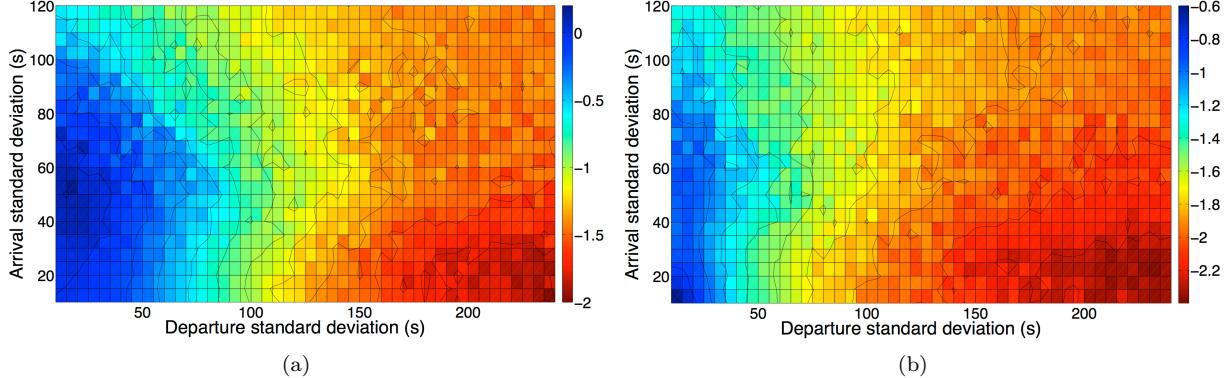


Figure 9. Controller intervention increase under varied uncertainties (extra separation buffer = 60 s) (a) Case I (b) Case II.

Figures 8(a), 8(b), 9(a) and 9(b) show the delay reduction and controller intervention increase for 60 seconds buffer in both cases. Although the delay savings decreased and the controller intervention increased, similar trends still persist with 60 seconds buffer. However, the impact of departure precision on delay reduction has slightly increased compared with the results under 30 second buffer. This change may result from the decreased gaps between departure flights due to the increased buffers. Therefore, the ability to absorb delays has been weakened. It can be expected that the delay will be more sensitive to departure precision when the departure flights get closer or the buffer increases.

V.D. Discussion

This post analysis of the uncertainty presents a clear picture of the stochastic characteristics of the solution generated under deterministic scenarios. This stochastic analysis is supplemental and necessary to the deterministic optimization. For example, in Case I, with a 60 second buffer, a great delay savings with more than 90% chance should be expected. A second example is: given an arrival deviation of 30 seconds, if the departure precision varies from 2 minutes (-1.5 minutes/+2.5 minutes) to 3 minutes (-2.5 minutes/+3.5 minutes), the expected delay savings in Case I should not change too much according to the uncertainty analysis. Compared to Case I, Case II still has significant delay savings even in the worst case, which makes Case II a good candidate for integrated operations.

The study of these two scenarios demonstrated that analysis results are highly correlated to the scenario setups. These case dependent results suggest the necessity of an advisory tool which has the capability of real-time uncertainty analysis. If the uncertainty analysis can be combined with the schedule optimization in an advisory tool, according to the results provided by such a tool, decision makers would be able to issue the "H30" solutions in both cases with great confidence in expecting good delay savings and a manageable controller intervention increase.

VI. Conclusions

Integrated operations between arrivals and/or departures provides a way to improve operation efficiency in terminal airspace. Previous studies¹² showed great benefits in deterministic circumstances. Because benefits from schedulers could be sensitive to flight time uncertainty, the robustness of the gained benefits must be investigated under uncertainty scenarios. This paper presents a method and analysis for the uncertainty study of integrated operations using two representative Los Angeles cases. Perturbations were incorporated into flight entry times in Monte Carlo simulations. Solutions generated in deterministic scenarios were used as references. In each simulation, the routes were fixed but extra delays were imposed if necessary to avoid separation loss caused by the perturbations in flight entry times. Impacts of the uncertainties on total delays and controller interventions were then presented and analyzed. A sensitivity study of the delays and interventions with varied precisions of arrivals and departures was also carried out.

The study showed that in the two sample cases, deterministic solutions using hybrid separation still had

substantial delay reduction over the spatial separation solutions 90% of the time when a 60 second buffer was included. As a trade-off the controller intervention increased. When the buffer decreased, the intervention increase was high. For instance, in Case I, when the buffer was reduced to zero, the intervention increased from 3 in spatial separation to 5 in hybrid separation – a 60% increase. But in terms of absolute values, the intervention should still be manageable in these two sample cases. The sensitivity analysis of departure and arrival time precisions showed that the departure time precision had less impact on delay reduction than the arrival time precision, which might be caused by a sparse departure queue. But the departure time precision presented strong correlation with the controller intervention possibly due to the unavoidable need for conflict resolution. These analysis results are dependent on scenario setups, and they are supplemental and necessary components to the deterministic optimization and could be helpful to decision makers.

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