Flight Departure Delay and Rerouting Under Uncertainty In En Route Convective Weather

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Delays caused by uncertainty in weather forecasts can be reduced by improving traffic flow management decisions. This paper presents a methodology for traffic flow management under uncertainty in convective weather forecasts. An algorithm for assigning departure delays and reroutes to aircraft is presented. Departure delay and route assignment are executed at multiple stages, during which, updated weather forecasts and flight schedules are used. At each stage, weather forecasts up to a certain look-ahead time are treated as deterministic and flight scheduling is done to mitigate the impact of weather on four-dimensional flight trajectories. Uncertainty in weather forecasts during departure scheduling results in tactical airborne holding of flights. The amount of airborne holding depends on the accuracy of forecasts as well as the look-ahead time included in the departure scheduling. The weather forecast look-ahead time is varied systematically within the experiments perfomed in this paper to analyze its effect on flight delays. Based on the results, longer look-ahead times cause higher departure delays and additional flying time due to reroutes. However, the amount of airborne holding necessary to prevent weather incursions reduces when the forecast look-ahead times are higher. For the chosen day of traffic and weather, setting the look-ahead time to 90 minutes yields the lowest total delay cost.

NOMENCLATURE

F = set of flights
J = set of sectors
K = set of airports from where flights depart and land
P = planning horizon (in minutes)
N = number of planning stages
E = weather forecast look-ahead time
τ_i = starting time of ith stage
R_f = number of routes available to flight $f \in F$
$l_{f,r}$ = additional flying time of f along route $r \in \{1,..,R_f\}$
d_f = scheduled departure time of f
$\hat{d}_f$ = controlled departure time of f
t_f^r = earliest feasible departure time of f along route r
λ = cost ratio between unit airborne and ground delay of any flight
G = maximum permitted departure delay of any flight

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$S_j(t)$ = time-varying capacity of sector $j \in J$

$Dep_k(t)$ = time-varying departure capacity of an airport $k \in K$

$Arr_k(t)$ = time-varying arrival capacity of an airport $k \in K$.

I. INTRODUCTION

Convective weather is a major cause of disruption of air traffic flow in the National Airspace System. When convective weather occurs, traffic flow management initiatives, such as Ground Delay Programs, Airspace Flow Programs, Playbook Reroutes, and Miles-in-Trail restrictions are implemented.[1] The primary goal of traffic flow management is to mitigate demand-capacity imbalance in the airspace. Assignment of pre-departure delays and reroutes are two commonly implemented traffic flow management initiatives during adverse weather. Such initiatives result in delays and cancellation of flights, which imposes significant cost on passengers and airlines. Uncertainty in weather forecasts present a major challenge to decision makers during the traffic flow management process. It results in aircraft facing longer delays than necessary.

Several previous studies address uncertainty in weather forecast in traffic flow management (see Ref. 2 for a comprehensive review of literature on this topic). Stochastic optimization models have been developed to incorporate uncertainty in weather forecasts within the optimization models for traffic flow management.[3-10] Refs. 3 and 4 proposed two-stage stochastic optimization models for assigning departure delays to aircraft scheduled to arrive at an airport. Ref. 5 developed multi-stage stochastic optimization model that incorporates both stochastic and dynamic aspects of weather. Ref. 6 proposed a model that assigns departure delays to aircraft and dynamically reroutes them when weather impacts arrival-metering fixes at an airport. Refs. 7 and 8 propose models to dynamically reroute aircraft under imprecise information on weather impact along its flight path. A necessary input to the stochastic optimization models is a set of scenarios of weather impact and their probabilities. The lack of such information has led to the development of heuristics that treat weather forecasts as deterministic and applies deterministic optimization models sequentially over time.[9] Ref. 11 proposed a flight scheduling algorithm that is computationally tractable and can take into account airline-specified flight priorities. Computational tractability is necessary when flight scheduling is done at multiple stages to account for uncertainty in traffic demand and weather forecasts. Greedy algorithms such as Ration-by-Distance prioritizes flights based on their flight times while delaying them prior to their departure.[12] During a ground delay program at an airport, certain long-distance flights are exempt from being assigned any delays.[13] The rationale behind such a policy is to prevent long distance flights from facing unnecessary delays if weather forecasts prove wrong and airport capacities turn out to be higher than anticipated. Distance-based exemptions effectively exclude weather forecasts beyond a certain look-ahead time. While such flight exemption policy is applied in practice, a thorough study that correlates the exemption distance with weather forecast accuracy has not been completed until now. In this paper, such a policy is applied towards pre-departure delay and route assignment to flights between multiple airports. The look-ahead time is varied systematically to analyze its effect on overall system-wide delays.

This paper presents an algorithm for assigning departure delays and reroutes to flights in the presence of en route convective weather. Flight scheduling is conducted at multiple stages and updated weather forecasts are used at each stage. Weather forecasts within a specified look-ahead time are considered, while those beyond this threshold are excluded. When departure times and routes are assigned to aircraft, it is ensured that their 4-dimensional trajectories are free of conflict with forecast weather. Capacities of airports and en route sectors generate additional constraints. Departure delays and route assignments of flights are revised based on the latest forecasts available before its departure. Due to uncertainty in forecasts, flights can be subjected to tactical airborne holding while en route and conflict with weather that was misjudged by the forecasts. Realistic traffic and weather data are used to analyze performance of the scheduling algorithm.

The outline of this paper is as follows. In the next section an algorithm is presented that assigns departure delays and reroutes to flights in the presence of en route convective weather. Flight scheduling is conducted at multiple stages over time in order to incorporate updated forecasts and revised flight schedules. Section III describes the data and experimental setup for a realistic traffic and weather scenario. Section IV presents results, followed by Conclusions (section V) and references (section VI.)

II. DEPARTURE DELAY, REROUTE, AND AIRBORNE DELAY OF AIRCRAFT

This section describes the algorithms used to assign delays and reroutes to aircraft. There are three components of total delay faced by any flight: departure delay, additional flying time due to reroute, and airborne holding. Departure delay and additional flying time of a flight are due to controls assigned prior to its departure. The

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scheduling algorithm for assigning departure delays and pre-departure reroutes is described in Section IIA. Uncertainty in forecast accuracy can necessitate airborne holding to prevent flights from incurring weather delays while en route. Section IIB describes how tactical airborne holding is assigned to aircraft.

A. Dynamic Flight Scheduling (DFS) Algorithm

During the departure scheduling process, flights are ordered according to their scheduled time of departure. Flight scheduling is done at multiple stages so that updated forecasts and revised flight schedules could be considered. At each stage, flights whose scheduled or controlled departure times belong to that stage are assigned delays and reroutes, if necessary. The most recent weather forecast available at the beginning of the stage is used. An important input to the algorithm is the weather forecast look-ahead time. Weather forecasts for times that fall within this specified look-ahead time are considered. For example, if the look-ahead time is set to 60 minutes, weather forecasts beyond one hour are ignored during flight scheduling at any stage. Later, in the experiments, this look-ahead time is varied systematically to study its impact on delays. Another necessary input for each flight is its original flight plan and potential alternative route. The alternative route can be provided by a flight's operating airline, or can be determined using a shortest path finding algorithm.[14] Using these inputs, the algorithm assigns departure delays and reroutes to each flight such that the airspace capacity is not violated. When assigning these controls it is ensured that the 4-dimensional trajectories are free of conflict with forecasted weather. Furthermore, a flight's departure delay is not allowed to exceed G time-units. In the case where it is not possible to find a departure time of a flight such that its 4-dimensional trajectory is free of conflict with weather, the flight takes-off being subject to the maximum permissible delay and faces airborne holding while en route. The algorithm is formally presented below.

Dynamic Flight Scheduling Algorithm

Step 1. Obtain the inputs (see definitions in the Nomenclature): F, J, K, P, N, E, λ, G, and τ_i ($i = 1,..,N$). For $f \in F$ obtain R_f, d_f, and $l_{f,r}$. Obtain capacity data: $S_j(t)$, $Dep_k(t)$, and $Arr_k(t)$.

Step 2. For each flight $f \subseteq F$ set $\hat{d}_f = d_f$. Set the time varying capacities of airports and sectors based on their nominal values reduced by the demand created by exempt flights. Initialize stage $i = 1$.

Step 3. At stage i, obtain the set of flights $F' \subseteq F$, whose controlled time of departure is within the planning interval $(\tau_i, \tau_{i+1}]$. For each flight $f \in F'$, obtain its controlled departure time $\hat{d}_f$, and a set of available routes R_f. Modify the set R_f depending on new available routes or airline updates. Obtain time-varying weather forecast data available at time τ_i.

Step 4. Order the flights F' based on their increasing $\hat{d}_f$.

Step 5. Obtain the highest ordered flight $f \in F'$. For each available route $r \in R_f$, compute the earliest departure time t_f^r such that the f's trajectory (along route r) is free of conflict with forecast weather, and airspace capacities are not violated if f departs at that time.

Step 6. Select a departure time t'_f and route r'_f for flight f that minimizes the cost function (1). If $t'_f \geq d_f + G$ then reset $\hat{d}_f = d_f + G$. Otherwise, set $\hat{d}_f = t'_f$.

$$\underset{r \in R_f,}{Min} \ (t_f^r - d_f) + \lambda l_{f,r} \tag{1}$$

Step 7. Update sector and airport capacities based on the occupancy times of f. Remove f from F'.

Step 8. If F' is empty set go to step 9. Otherwise, repeat steps 5 – 7.

Step 9. Set $i = i + 1$. If $i > N$ go to step 10. Otherwise, go to step 3.

Step 10. End algorithm.

B. Tactical Airborne Delay

Tactical airborne holding can be achieved using speed control, path stretching, and holding. In this study, airborne holding used as a metric to evaluate the effectiveness of the DFS algorithm applied under uncertain weather conditions rather than a being proposed control strategy. Tactical airborne holding is assigned in a similar way as in Ref. 9. A flight is subject to airborne holding if it is either already in conflict with weather or its current position is

within a very short distance from weather. It faces airborne holding until the weather clears, after which, it resumes flying along its flight plan.

III. DATA DESCRIPTION AND EXPERIMENTAL SETUP

The scheduling algorithm presented above was applied to assign departure delays and reroutes to thousands of flights, whose trajectories were impacted due to weather. Realistic traffic and weather data, described in this section, were obtained to perform the experiments.

A. Weather Data

The Massachusetts Institute of Technology (MIT) Lincoln Lab's Convective Weather Avoidance Model (CWAM) data for June 19, 2007, were used to perform experiments.[15] CWAM data translates raw meteorological data such as precipitation intensity and storm heights into regions of airspace that pilots are likely to avoid. It also provides 0-2 hour weather forecasts during those times. CWAM data were read and translated into two-dimensional polygons using NASA's Future Air Traffic Management Concept Evaluation Tool (FACET).[16] Weather forecasts up to a specified look-ahead time were considered. In this study, CWAM contours representing regions that are likely to be avoided by 60% of flights at an altitude of 33,000 feet were used. This weather data was used for scheduling all flights regardless of their individual cruise altitudes. As an example, Figure 1 shows the CWAM weather regions, drawn as yellow filled polygons, at 11AM Eastern Daylight Time (EDT) as forecasted at 10AM (i.e., 1-hour forecast). Also shown in the figure are the contours, drawn in red, for the weather that actually occurred at 11AM (i.e., Nowcast). The difference between the forecast and actual weather exhibits the level of uncertainty of this particular 1-hour forecast.
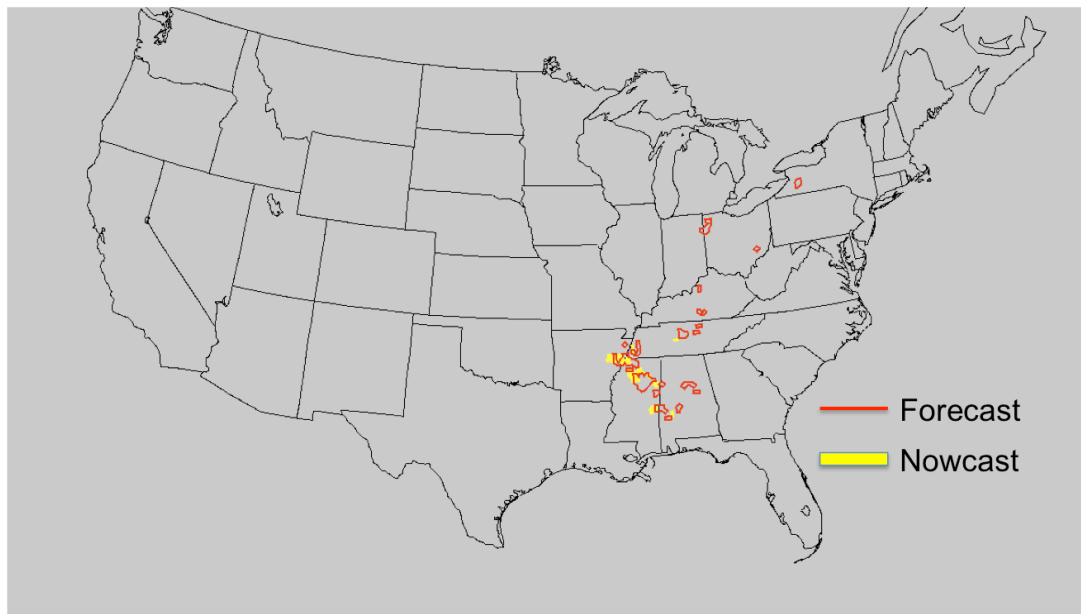

Figure 1: CWAM one hour forecast and actual occurrence of convective weather at 11AM EDT on 06/19/2007

B. Traffic Data

The Enhanced Traffic Management System[17] (ETMS) data for August 24, 2005 was used to generate traffic information required by the DFS algorithm. The weather on this day was relatively mild. This allowed for obtaining information on schedule, trajectories and unimpeded travel times of flights, and traffic loads at various sectors under nominal conditions. The traffic data included take-off times and trajectories of flights that originated from, or were destined to, a U.S. airport. The data also included the original and modified flight plans, which consisted of waypoints along a flight's scheduled trajectory, cruise altitude, and ground speed. The ETMS data were read by

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FACET. A Java-based program using the FACET Application Programming Interface was developed to generate flight-specific information required as input to the DFS algorithm.

C. Alternative Route Generation

Alternative, weather-free, routes for flights were provided as input to the DFS algorithm, so that pre-departure rerouting could be implemented. In practice, alternative routes can be provided by airlines. However, for the experiments performed in this paper, Dijkstra's algorithm[18] was implemented to find the shortest-distance weather-free route between two airports during the time-horizon defined by the weather forecast look-ahead time. The algorithm proposed in Ref. 18 is a variant of the Dijkstra's algorithm[14], which finds the shortest path between two nodes in a network where link costs vary over time.

To implement the Dijkstra's algorithm to find the shortest weather-free route between two airports, a grid was drawn across the weather impacted region of the National Airspace System. The distance between two adjacent nodes in the grid was set to approximately 20 nautical miles. The size and location of the grid could be varied to test the sensitivity of the grid geometry on the solutions obtained from the shortest path algorithm. However, this was left as an exercise to be conducted in the future. The time-varying link costs were generated using the weather contours and the geometry of the grid. If weather blocked a link during at a particular time-interval, the associated link traversal during that time was set to a large value. As time progressed the weather moved, and hence, the shortest weather-free route between two airports depends on the scheduled departure time. Figure 2 shows the weather-free, shortest routes between Boston Logan International Airport (BOS) and Chicago O'Hare International Airport (ORD) for three different departure times.
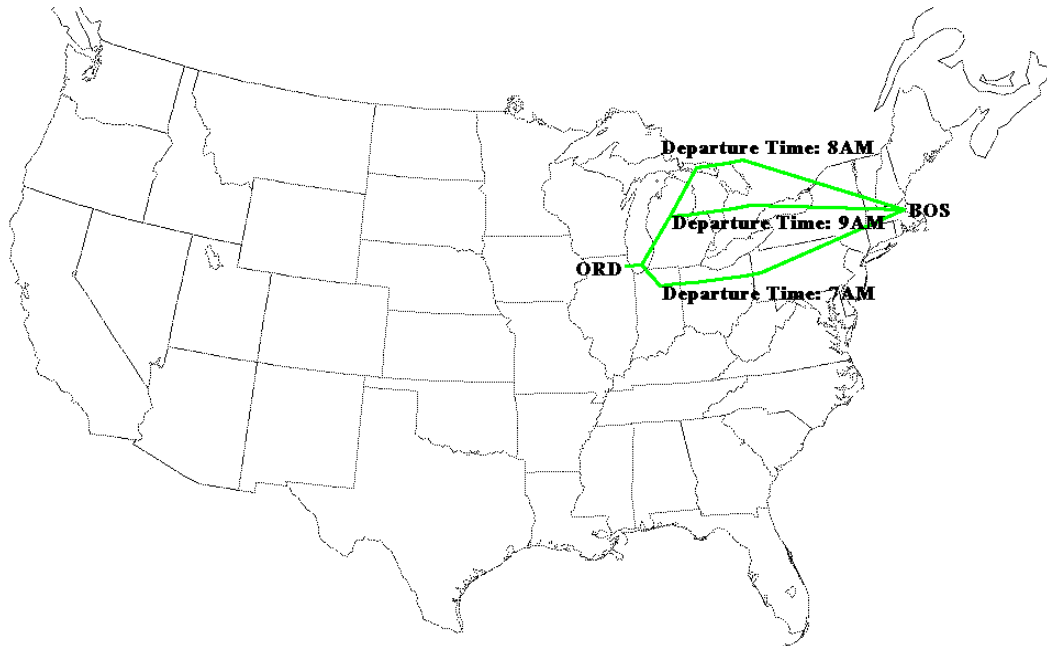

Figure 2: Alternative, weather-free routes between Boston and Chicago for different scheduled times (in EDT) of departure

D. Experimental Setup

The TFM planning horizon P was chosen to be from 6:00 to 12:00 EST. Flights that departed between 6:00 – 9:00 EST were assigned controls, whereas those already airborne at the start of planning horizon and the international flights were treated as exempt and were not assigned any delays. Previous studies have applied similar methods to reduce the computation load.[19] In total, there were 8600 flights. The planning horizon P was divided into 24 planning stages (i.e., $N = 24$), each of which was of 15-minute duration. The start time of each planning stage i, τ_i, was computed accordingly. At each planning stage i, the CWAM weather forecast was obtained for every 15-

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minutes starting at τ_i and continuing upto $\tau_i + E$. In the experiments, the look-ahead time E was varied between 0 to 120 minutes.

For each flight in the system, FACET was used to generate the set of sectors along the flight's path and the corresponding sector occupancy times. The time-varying demands in a sector that is caused by the exempt flights were subtracted from the sector's Monitor Alert Parameter (MAP) values to generate residual capacities, which were used in Step 5 of the scheduling algorthim.

In the experiments performed, each flight had at most two potential routes. The primary route of all flights was assumed to be the one based on their original flight plan. As mentioned above, Dijkstra's algorithm was applied to obtain weather-free alternative routes for flights. The algorithm requires as input a feasible departure time in order to generate a reroute. For a flight f, an "on-time" departure window that was within 15-minutes of its controlled time of departure, $\hat{d}_f$, was used as input to the route generation algorithm. A weather-free alternative route, if found within this time-interval, was used as input to the DFS algorithm. In certain cases the algorithm was unable to find alternative route of a flight primarily due to presence of weather in the vicinity of its origin or destination airport. In such cases, flights operating between the weather impacted airports faced departure delays without any option to reroute.

A set of feasible departure times was generated for the primary route of each flight. For a potential time of departure of a flight, which started as the flight's scheduled departure time, if the flight could depart on a given route without intersecting any of the forecast weather cells, then that time is counted as a feasible departure time for that route. A maximum of five hours departure delay was allowed for any flight (i.e., $G = 300$). For any flight, the cost of additional flying time due to reroute was assumed to be twice more expensive than holding the flight prior to its departure (i.e, the cost ratio λ was set to 2 during scheduling process). The cost of tactical airborne holding was assumed to be higher, and therefore, λ for airborne holding was set to 4.

The total departure delay and additional flying time due to longer reroutes for various weather forecast look-ahead times, E, were obtained by applying the DFS algorithm. Tactical airborne delays were obtained as a result of weather impacting routes of aircraft after they takeoff. The total cost of delay for each case was obtained by weighting the delay types by their cost-ratio mentioned earlier. Delay costs under various forecast look-ahead times were compared to that obtained under perfect information. Delays under perfect information on weather were obtained by applying the DFS algorithm with actual weather (i.e., Nowcast) and a large value of E. In this case, the DFS algorithm generates the same solutions as the Priority-Based Scheduling algorithm[11] when scheduled departure times are used in ranking flights. The following section presents the results.

IV. RESULTS

Figure 3 shows the distribution of delays under various look-ahead times. As expected, total departure delay generally increases with look-ahead time. This is because the further the look-ahead time, more constraints are imposed during the departure scheduling process. Under longer look-ahead times, some flights are rerouted to prevent them from facing long departure delays. The total departure delay in the case of $E = 120$ minutes is slightly lower than that in $E = 90$ minutes. However, more flights are rerouted in the case of $E = 120$ minutes, which results in about 35% higher additional flying time. Departure delay and additional flying time under $E = 60$ minutes are lower compared to those under $E = 90$, and 120 minutes.

Airborne delays generally decrease with increasing look-ahead time. The exception is in the case of $E = 120$ minutes, where both airborne holding and additional flying times are higher than in case of $E = 90$ minutes. This is primarily because when $E = 120$ minutes, flights are released with lower departure delays either along their primary route or their alternative route. However, due to inaccuracy of weather forecasts the amount of airborne holding in this case is significant. Since all flights are exempt in the case of $E = 0$ minutes, the total delay is absorbed through tactical airborne holding.

The total delays increase with increasing values of E primarily because the increase in departure and rerouting delays offset the reduction in tactical airbone delays. Delays costs, however, vary differently. Figure 4 shows the variation of total cost of delay with look-ahead times. As mentioned before, the cost of tactical airborne holding was assumed to be higher than the cost of additional flying time due to reroute. As evident from Fig. 4, the total cost of delay is lowest at $E = 90$ minutes. The total cost of delay is highest in the case of $E = 0$ minutes due to severe airborne holding faced by flights.

Figure 5 compares the total delay costs under various look-ahead times with those under perfect information. Under perfect information, the total departure delay and additional flying time obtained were 10,190 minutes and 4,284 minutes. There is no airborne holding under perfect information. This is intuitive, as any necessary airborne

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holding would be translated back to ground delays in this case. As seen from Fig. 5, the total delay cost under imperfect weather forecast varies between 260% to nearly 310% of that under perfect information. Thus the excess delay faced due to imprecise forecast varied between 160 – 210%. It is the lowest for a look-ahead time $E = 90$ minutes. Figure 5 suggests significant loss occurring due to uncertainty in weather forecasts. It must be noted that these results are for a specific instance of weather and traffic.
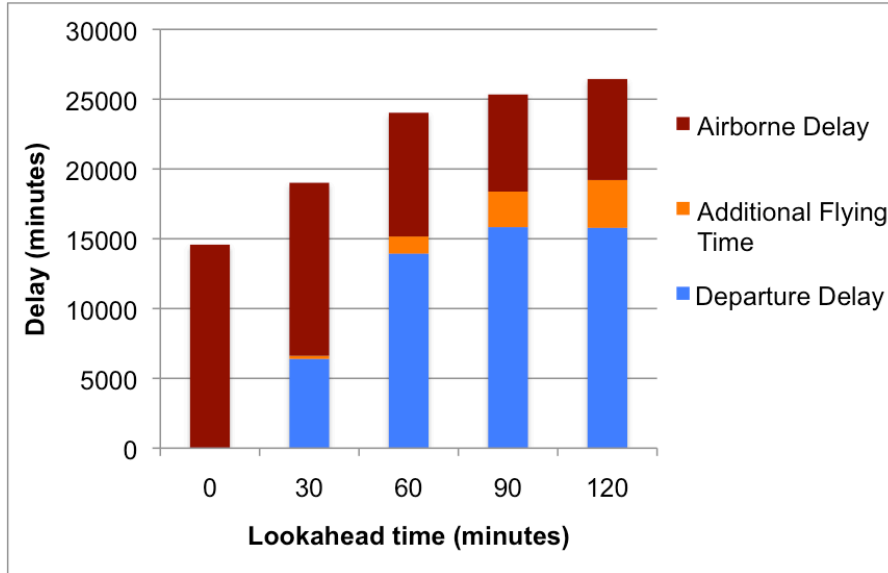

Figure 3: Delay distribution under various look-ahead times
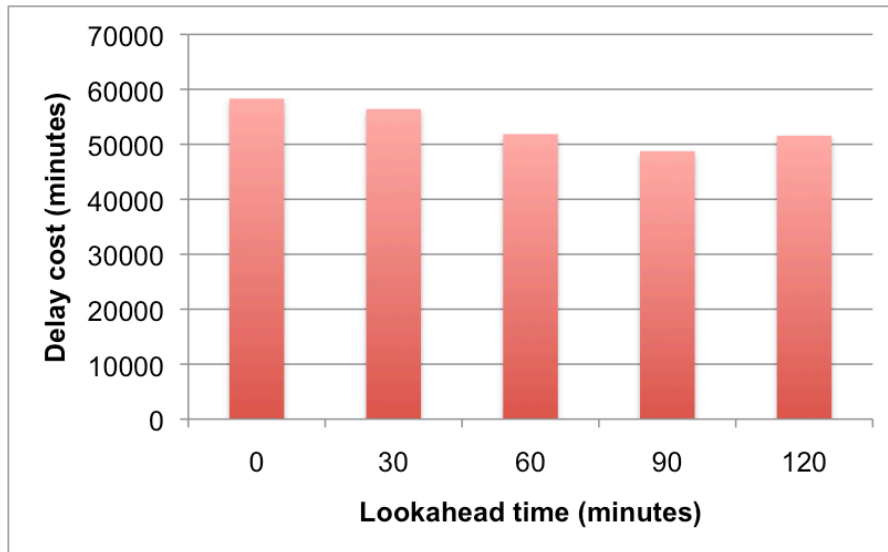

Figure 4: Total delay cost under various look-ahead times

Figure 6 shows the departure delay and additional flying time under various look-ahead times as a percentage of respective delays under perfect information. In general, the departure delays under imprecise weather information are higher than those under perfect information. The difference between the delay occurring under perfect and imperfect information on weather can be considered as "unnecessary". Departure delay is highest in the case of $E = 90$ minutes. Whereas, in the case of E = 30 minutes, the departure delay is lower than what is needed under perfect information on weather. When the look-ahead time E is 0 minutes, there is no departure delay or rerouting. In these

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cases, however, severe airborne holding significantly increases the total cost of delay. Based on the results in Fig. 6, under all look-ahead times, fewer flights are rerouted than necessary under perfect information of weather.
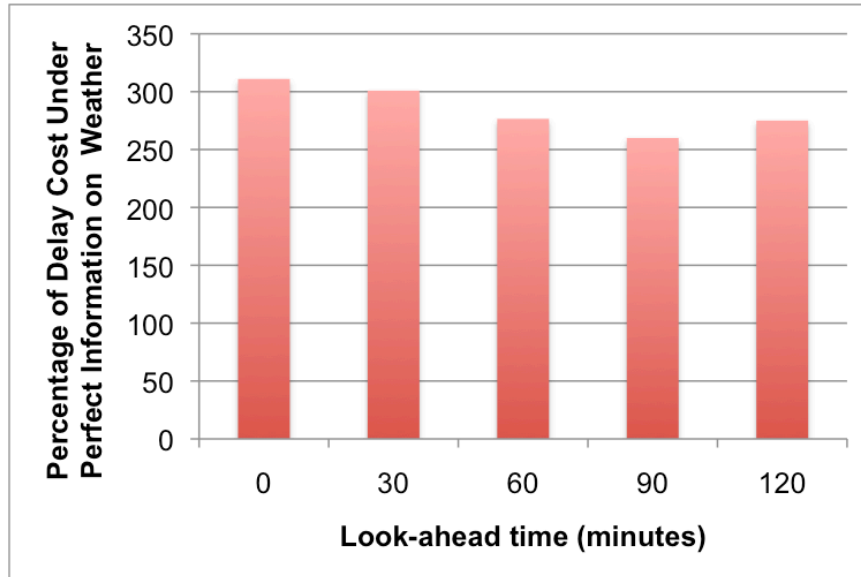

Figure 5: Total cost of delay under various look-ahead times expressed as percentage of that under perfect information on weather
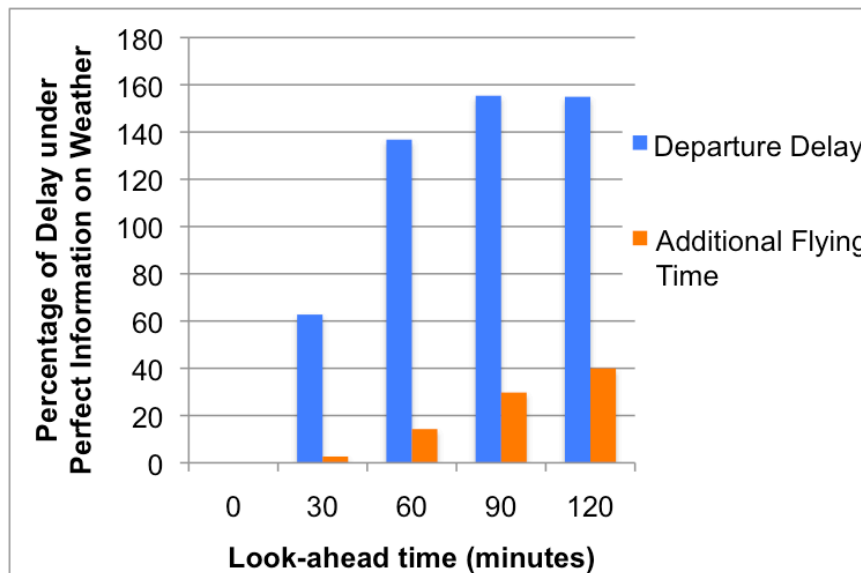

Figure 6: Departure delay and additional flying time expressed as percentage of delays under perfect information on weather

V. CONCLUSIONS

In this paper, an algorithm that assigns departure delays and reroutes to flights in presence of en route convective weather was presented. The algorithm assigns controlled departure times and reroutes to flights at multiple stages using updated weather forecasts and flight schedules available at the beginning of each stage. During each planning interval, weather forecasts up to a certain look-ahead time are considered, while those beyond this threshold are

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excluded. Using the forecast weather and flight plans, departure delays and reroutes are assigned to flights ensuring that their 4-dimensional trajectories are free from conflict with predicted weather. It is also ensured that departure delays are sufficient to prevent violations of en route sector capacities. Departure delays and route assignments of flights are revised based on the latest forecasts available before its departure. Due to uncertainty in forecasts, some flights are subject to tactical airborne holding while en route.

The weather forecast look-ahead time is varied systematically to study its impact on overall delays. Realistic traffic and weather data were used to analyze performance of the scheduling algorithm. In general, departure delays and rerouting increase with longer look-ahead time. However, due to presence of uncertainty in the weather forecasts, airborne holding becomes necessary to prevent aircraft from flying through severe weather. In some cases, tactical airborne holding is severe. Airborne holding is highest in the case when all flights are exempted from facing departure delays or reroutes. Based on the experimental results, a look-ahead time of 90 minutes produced lowest total delay cost. The delay cost obtained for various look-ahead times varied between 260 – 310% of delays obtained under perfect information on weather. Such results indicate that for the chosen experimental scenario, the quality of forecasts was not very good. It also indicates the value of accurate weather forecasts. Total delays and delay costs vary significantly with changes in look-ahead time. The best look-ahead time depends on the accuracy of forecasts. Future research will focus on scoring weather forecasts and correlating forecast accuracy with best look-ahead time.

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