# A Market Approach to Real-time Departure Runway Scheduling

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## Abstract

Departure runway usage is a major source of delay in the US National Airspace System. Additionally, the current first-come-first-served paradigm for allocating departure runway usage produces strong incentives to airlines to spend all this delay time on the taxiway, even though this can result in wasted fuel. In this paper, a novel dynamic second-price auction method for allocating runway usage is developed. Analytical and simulated results are obtained that suggest the proposed method does not increase total delay, and that almost all delay is spent at the gate (thereby saving fuel). Thousands of simulations of a realistic scenario at John F. Kennedy International Airport indicate that this new mechanism offers two benefits over the current first-come-first-served mechanism. The first benefit suggests this mechanism can reduce airline costs (beyond the savings on fuel). The second benefit suggests the auction mechanism can generate more equitable spread of delay across airlines by an appropriate parametrization over budget constraints.

## Nomenclature

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| $\mathbb{A}$ | The set of airlines. |
| $\mathbb{F}_a$ | The set of flights belonging to airline $a$. |
| $\mathbb{F}$ | The set of all flights (i.e., $\mathbb{F} = \bigcup_a \mathbb{F}_a$). |
| $\mathbb{C}$ | The set of aircraft classes. |
| $c_f$ | The class of flight $f$. |
| $\tau_c$ | The amount of time since the most recent takeoff of aircraft class $c$ occurred. |
| $\alpha_f$ | The minimum amount of time until flight $f$ can be ready to takeoff. |
| $\theta_f$ | A positive real number indicating a flight $f$'s perceived value over time. |
| $G_c(\gamma)$ | A distribution function over priority values $\gamma$ for class $c$ of aircraft. |
| $H_a(\alpha)$ | A distribution function over earliest runway use times $\alpha$ for airline $a$. |
| $b_x^a$ | The bid of airline $a$ in state $x$ |
| $\delta_{c_i,c_j}$ | The minimum required runway separation between leading flight $i$ and trailing flight $j$. |
| $D$ | The separation matrix, with entries $\delta_{c_i,c_j}$. |
| $\mathbf{C}_x$ | A vector of the earliest time that each aircraft class can begin to use the runway when at state $x$ ($\mathbf{C}_x \in \mathbb{R}_+^{|\mathbb{C}|}$). |
| $\mathbf{R}_x$ | The set of remaining flights at state $x$. |
| $P_c^a$ | The probability that airline $a$ believes the next aircraft class to use the runway is $c$. |
| $\$$ | The state space of the auction. $\$ = R_x \times \mathbf{C}_x \times \mathbf{K}_x$ |
| $x$ | A state (i.e., $x \in \$$). |
| $(a)^+$ | $\max\{0, a\}$. |

## Introduction

Airport systems are complicated networks, rich with heterogeneous flight operators with specific financial goals, temporal and spatial uncertainties, and home to one of the largest global transportation economies. Due to increasing demand and fuel costs, air traffic controllers and flight operators are pressured more than ever to use airport resources efficiently. Recently, many have realized that a contributing factor to inefficient operations is due to poor sharing of important information across all stakeholders at airports. Concepts to effectively manage surface traffic through the timely exchange of information are under development both in Europe [1] and the United States (US) [2]. This paper briefly describes different approaches to manage surface traffic, but the primary focus of this paper is to offer an alternate method for allocating runway resources through a novel dynamic second price auction.

Among all resources at US airports, departure runway usage accounts for over 50% of total aircraft delay [3] and is a major network bottleneck [4]. For instance, the authors in [5] show that over 90% of

sequencing decisions at US airport systems are handled on a First-Come-First-Served (FCFS) basis. In the US, FCFS entices airlines to push aircraft back from gates and onto the airport surface as early as possible to get an earlier slot at the runway [1]. This process can cause excessive fuel burn and unnecessary congestion and delays at airports.

European airports, such as Munich [6], have adopted Airport-Collaborative Decision Making (A-CDM) as a standard operational procedure. The purpose of A-CDM is to provide standard departure procedures, enhanced information sharing, and efficient surface scheduling. Coupled with European scheduling tools such as DEparture MANangement (DMAN), A-CDM has shown to reduce holding times at runways by 1.5 minutes per aircraft and significantly increase departure compliance to take-off slots.

To coordinate the pushback process in the US various strategies have been investigated, such as slot allocation procedures [7], aggregate flow measures [8], and individual aircraft control procedures [9]- [11]. While all concepts differ in their execution, they all fundamentally attempt to minimize engine on time by keeping aircraft at their gates for as long as possible while also not degrading runway throughput.

To allow for airlines to communicate priorities and account for constraints at runways, a new dynamic second price auction is introduced. The dynamic second-price auction runs in real-time (dynamic) and issues pushback times to allocate departure runway slots. In this second-price auction, airlines bid for a runway slot by submitting one bid, the highest bidder gets the slot (i.e., issued a pushback time to meet the slot) and only pays the second highest-bid. The mechanism requires only knowledge of each airline's current bid of *virtual* credits to allocate the runway slots, where Estimated-Off-Block-Times (EOBTs) and finances are kept private to each airline. When auction parameters are re-sampled every bidding round, analytical results are derived suggesting that delay for the auction will be approximately efficient. When auction parameters are kept over several rounds, thousands of computer simulations are provided and seeded with data from John F. Kennedy International airport (JFK) to show how the auction competes with a cost optimal solution, delay optimal solution, delay optimal allocation with noise ( [10] analog), and FCFS. The goal is to show that the auction is able to keep total delay [2] approximately the same as other feasible mechanisms, while shifting almost all of it to the gate, thereby greatly reducing fuel waste.

This paper is organized as follows. Problem Statement and Related Work introduces the Departure Runway Scheduling Problem (DRSP) and provides a literature review. The Dynamic Auction Mechanism section describes a simplified variant of the model, where all private information is redrawn every bidding period. Then, a more sophisticated auction is provided, where private information is kept across bidding periods. Finally, section the Simulation section provides simulated results for the auction using a bidding strategy, and compares them with alternative methods of allocating runway usage. Finally a summary of the paper is given in the conclusion section.

# Related Work and Problem Statement

## Literature Review

US associated literature provides evidence of substantial work to find schedules for departure runways and meter gate release times. For example, to achieve less congestion on airport taxiways and at runways, the Collaborative Departure Queue Management [7] model uses a Generalized Ration By Scheduling (GRBS) algorithm [14] that allocates runway slots to airlines in a multi-airline environment. Field evaluations of this model were conducted with two major participants and showed reductions in taxi times (engine-on time). While the model attempts to "equitably" allocate runway capacity, it does so by using updated estimated pushback readiness times from all airlines. The system expects the rate of pushback of each flight operator's aircraft to adhere to the allocation provided

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[1]Airlines are also judged by their on-time performance through published reports, and pilots are motivated to push back since pilots do not start earning their wage until the aircraft has pushed back.

[2]Total delay is the time the aircraft on the airport surface less the minimum time.

during metering. By allowing for manual control over when aircraft can pushback, there is unnecessary uncertainty in actual wheels-off times. Moreover, in a highly competitive airport environment there will be competition for such slots, possibly enticing airliners to submit false estimations of times of readiness to pushback to acquire better takeoff slots.

To avoid including airlines in the decision-making process, American authors [10] propose to manage queues at the border between Air Traffic Control (ATC) and airline control (i.e., spots). This concept provides timing advisories to release departures from *spots* that optimize throughput and delays. The concept was tested in a high fidelity human-in-the-loop simulation environment for Dallas/Fort Worth International Airport. Overall, the concept relieves congestion on the taxiways but increases congestion in the ramp areas. A promising result from this research suggests that if aircraft were queued up at their gates fuel use would have decreased by 38%. However, queuing aircraft at gates requires airlines to communicate and collaborate with air traffic control. Moreover, in situations where there are more than one airline, this concept would need to account for various airline preferences.

Authors in [8] use a flow metering strategy called N-Control to control congestion at US airports. This concept was successful at reducing aircraft taxi times, but required that the FAA controlled all airline gate pushbacks. Researchers therefore chose Boston Logan International as their test airport, where most pushbacks are controlled by FAA personnel. At most airports, however, gate pushback is controlled by individual airlines. Even if a third party entity is performing the metering, it is not clear that airlines will use a system where priorities are not communicated. Hence, there is an explicit need for airlines to communicate their priorities over their aircraft.

Like [8], authors in [9] attempt to hold aircraft back at gates at JFK by looking at aggregate statistics on runway throughput. However, the concept is different in that it provides pushback times to each aircraft using the ration-by-schedule algorithm. Airlines get slots proportional to the number of scheduled departures and the capacity of the runway for a given planning period (i.e., 15 minutes). The airline then decides, among their own aircraft, which aircraft uses the owned slots. The system then assigns pushback times to those aircraft. Priorities are included by allowing for inter- and intra-airline departure swaps. To accommodate the many airlines at JFK, a neutral third-party position is created to perform all pushbacks. Using pushback times, the concept has been successful in reducing taxi times and congestion at JFK. With so many airlines, most of the sequencing is done on small subset of the total aircraft that want to pushback. Therefore it is unlikely that the final departure sequence is optimal with respect to inter-aircraft separation. For instance, the simulation section shows a centrally optimal solution can yield 25% lower average delay per aircraft solution than the baseline by re-sequencing.

European airports have approximately 40% less excess taxi out time than US airports [15]. The reason for lower delays may be advances in communication between various stakeholders at European airports and differences in flow control policies. At Zürich and Munich [6] airports, for example, an information sharing system called Airport Collaborative Decision Making (A-CDM) has been implemented to share airport, airline, and ATM data. A-CDM can enable schedulers to optimize departure aircraft by specifying Target Start Up Approval Times (TSATs), from shared EOBT times. Compared to the US, this means that the pushback can be coordinated effectively through a larger degree of information sharing. For example, an evolution based algorithm for optimizing arrivals and departures through a high degree of coordination was presented in [16] for use at Roissy Charles de Gaulle Airport. These times are generated to meet Central Flow Management Unit (CFMU) slots and reduce taxiway congestion.

While meeting CFMU slot times is important, other European literature suggests how to fairly allocate landing slots [18]. Arrival scheduling is similar to departure scheduling, where certain landing sequences can achieve higher throughput and lower delay. Authors in [18] suggest that airlines should share cost information directly with a central planner to achieve a highly efficient outcome. If implemented well, such a model could result in the

highest overall benefits to airlines. Unfortunately, airlines in the US may not be willing to divulge such information and therefore, an indirect mechanism for achieving gate holding may be necessary.

Sealed-bid first-price auctions and combinatorial auctions are suggested by [19] and [20] as general mechanisms to allocating runway resources. In the sealed-bid first price auction, bidders bid for runway slots using real money and the highest bidder wins and pays his bid amount. The term sealed-bid refers to the fact that no other airline sees what any other airline bid. These auctions present a primary and secondary market where runway slots are initially allocated based on a first-price sealed bid auction, then traded on a secondary market.

Authors in [17] provide designs and concepts for mechanisms to allocate runway slots at highly congested airports. They argue that auctions (or market mechanisms) play a fundamental role in the management of air transportation – suggesting that auctions can help manage operational efficiency, political challenges, and safety issues. Moreover, [17] suggests that when designing the auction mechanisms the distribution of weight classes should be considered because the capacity of the runway (or the number of slots to allocate) is dependent on the weight class distribution. The concepts discussed by [17], [19], and [20] work on larger time horizons (i.e., months to days) than the auction considered in this paper. The use of tactical auctions for allocating runway slots still needs to be explored.

In contrast to previous work, this paper finds solutions to a DRSP using a second-price auction model where airline financial models and aircraft pushback times are kept private. This method provides means for airlines to prioritize their aircraft at runways while accounting for inter-aircraft separation at the runway. When all parameters are re-sampled every auction, analytical results show delay will be approximately efficient. Furthermore, a simulation for a more complicated model, based on data from the JFK airport, is carried out. Under mild assumptions, reasonable delay-efficient so-lutions for these simulations are found, indicating that auctions are a promising prospect for efficiently allocating runway usage.

## Departure Runway Scheduling Problem

Here the central planning problem, where all information is known, is described.

Given a set $\mathbb{A}$ of airlines, and each airline $a \in \mathbb{A}$ has a set of flights $\mathbb{F}_a$, all destined to the same runway. Each flight $f \in \mathbb{F}_a (\forall a \in \mathbb{A})$ belongs to a class $c_f \in \mathbb{C}$. Given a separation matrix $D$, with entries $\delta_{c_i, c_j}$ that gives the minimum time after a flight of class $c_i$ uses the runway that a flight of class $c_j$ is able to follow it. Each of these flights $f$ also have a scheduled pushback time $S_f$, an unimpeded taxi time $T_f$, and a minimum time $\alpha_f$ at which they can begin to use the runway unimpededly.[3]

Assuming $t_f$ is the time flight $f$ is assigned to use the runway and $g(t_{f_1}, ..., t_{f_n})$ is a generic cost function, the following Mixed Integer Linear Program (MILP) mathematically defines the problem:

**Minimize** $g(t_{f_1}, ..., t_{f_n})$ (1)
**such that:**

$$z_{f_i f_j}(t_{f_j} - t_{f_i} - \delta_{c_{f_i}, c_{f_j}}) \geq 0 \ \forall f_i \neq f_j \in \bigcup_a F_a \quad (2)$$

$$t_{f_i} \geq \alpha_{f_i} \ \forall f_i \in \bigcup_a F_a \quad (3)$$

$$z_{f_i f_j} \in \{0, 1\} \ \forall f_i \neq f_j \in \bigcup_a F_a \quad (4)$$

$$z_{f_i f_j} + z_{f_j f_i} = 1 \ \forall f_i \neq f_j \in \bigcup_a F_a \quad (5)$$

Remark: $z_{f_i f_j}$ is defined: $z_{f_i f_j} = 1$ when aircraft $f_i$ precedes $f_j$ at the runway, and $z_{f_i f_j} = 0$ indicates that aircraft $i$ does not precede $j$ at the runway. While authors in [12] solve a taxi scheduling problem, many of the constraints for this model are similar.

The above mathematical formulation describes a sequencing problem at the runway, where aircraft are sequenced efficiently with regards to function $g$.

Equation 2 is a non-linear[4] constraint, and can

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[3]Note, while all $\alpha_f$ are visible in the central planning problem, it is not the case that $\alpha_f = S_f + T_F$. The reason for this is because $\alpha_f$ is highly uncertain. That is, aircraft do not pushback immediately at their scheduled times and taxi according to a nominal taxi time. However, you can imagine all $\alpha_f$ being revealed to the central planner when it needs to formulate a plan.

[4]A linearization can be done simply by writing the equation as: $t_{f_j} - t_{f_i} - \delta_{c_{f_i}, c_{f_j}} \geq -(1 - z_{f_i f_j}) * M$ for some large $M$.

be interpreted as a separation requirement on each aircraft using the runway. That is, if flight $f_i$ precedes flight $f_j$ ($z_{f_i f_j} = 1$), then the time flight $f_j$ uses the runway ($t_{f_j}$) must be after $f_i$ uses the runway ($t_{f_i}$), plus an additional separation ($\delta_{c_{f_i}, c_{f_j}}$). These separation values exist due to operational considerations, such as the departure aircraft wake vortex.

Equation 3 enforces that the time a flight actually takes off ($t_{f_i}$) cannot be earlier than the earliest possible time that the flight could have taken off ($\alpha_{f_i}$).

Equation 4 enforces that $z_{f_i f_j}$ is a binary variable.

Equation 5 enforces that either flight $f_i$ precedes $f_j$, or $f_j$ precedes $f_i$, but not both.

### Objective Functions

For DRSP two objective functions are introduced, $g_1$ for delay and $g_2$ for cost. Solving this linear program for each of these functions will give results for best-case sequencing, which can then be used as a baseline when analyzing alternative methods. Note that since the $\alpha_f$'s are private information to each airline, this optimization is infeasible in practice, and therefore merely provides a lower bound on the best feasible outcome.

The first objective is a *delay* optimal solution where all private information is known exactly:

$$g_1(t_{f_1}, ..., t_{f_n}) = \sum_{i=1}^{n} (t_{f_i} - \alpha_{f_i}) \qquad (6)$$

This equation looks at the amount of time between when a flight was ready to leave, and when it actually leaves, and is used primarily in [12].

To measure airline costs, the following function is used:

$$g_2(t_{f_1}, ..., t_{f_n}) = \sum_{i=1}^{n} \gamma_{f_i}(t_{f_i} - \alpha_{f_i}) + y_{f_i} L \qquad (7)$$

where $\gamma_{f_i}$ represents the importance of flight $f_i \in \mathbb{F}_a$ to airline $a$. In principle, $\gamma_{f_i}$ could be any increasing function (i.e., more delay is always worse than less). This paper assumes a linear function and uses $\gamma_{f_i}$ to represent the slope.

When a flight does not pushback within 15 minutes (900 seconds) of its scheduled gate pushback time, it is recorded as delayed [13]. To capture this effect, $L$ is introduced, the cost to the airline associated to these critical delays. The binary variable $y_{f_i}$ is 1 when flight $f_i$ has exceeded this limit, and 0 otherwise.

When optimizing over this objective function, new constraints and variables are required:

$$0 \leq y_{f_i} + \frac{900 + S_{f_i} - (t_{f_i} - T_{f_i})}{M} \quad \forall f_i \in \bigcup_a F_a \quad (8)$$

$$y_{f_i} \in \{0, 1\} \quad \forall f_i \in \bigcup_a F_a \quad (9)$$

Assuming that all delay occurs at the gate, the actual pushback time of flight $f$ is ($t_f - T_f$). Thus, if $f$ pushes back more than 900 seconds after its scheduled pushback time, the quantity $900 + S_f - (t_f - T_f)$ is negative. For a sufficiently large constant $M$, $-1 \leq \frac{900 + S_f - (t_f - T_f)}{M} < 0$, which forces $y_f = 1$. If $f$ does push back within this interval, then the quantity is positive, which allows $y_f = 0$. Note that since this lowers the objective value, the optimal solution will always select $y_f = 0$, if it is able.

## Dynamic Auction Mechanism

For each auction, every airline $a \in \mathbb{A}$ has two elements of private information for each of their flights: (1) the earliest time a flight $f$ can use the runway ($\alpha_f$) and (2) a weighting factor $\gamma_f$ that corresponds to how much airline $a$ values flight $f$. For private information $\alpha_f$ and $\gamma_f$, it is assumed that each airline knows their own values perfectly, but only has a probability distribution over possible values for their competitors. In addition to private information, each airline $a$ is constrained by a budget $b_a \in \mathbb{R}^+$, allocated for a finite number of auctions.

A state $x$ is an ordered triple containing:

1. $\mathbf{R_x}$, the set of remaining aircraft to use the runway

2. $\mathbf{C_x}$, the vector corresponding to the earliest time that each aircraft class can begin to use the runway

3. $\mathbf{B_x}$, the vector of the remaining budget for each airline

Airlines will bid on slots to use the runway, using virtual credits that can only be spent on these auctions. If an airline wins an auction, they will

be able to send one of their aircraft to the runway next. These auctions will be repeated until the airport is empty, with each winner taking the next slot in the queue. Each auction will have a second-price format, meaning that the highest bidder pays the second highest bid and wins the slot. This auction format was chosen because it is *truthful*. This means that under fairly general assumptions, it is a dominant strategy for each airline to bid exactly its value for a slot.

## Simple Model

A simplified variant of the dynamic auction is discussed. This model makes many assumptions, some of which are not realistic. However, these assumptions do not affect the underlying principles of the situation being modeled. By making these assumptions, a simplified analysis can be made that provides an intuition of the fundamental behavior of the proposed mechanism. Additionally, clean analytical results can be obtained. In the next section, a discussion of ways to extend this model to make it more realistic is provided. Additionally, the next section demonstrates how to simulate a more complicated version of the problem.

At time $t = 0$, every airline independently draws a value $\gamma_c^a$ for every class of aircraft, from the cumulative distribution(s) $G_c(\gamma)$. For now, it is assumed that there is only one class of flights, and then extend the model to multiple classes later. When there is exactly one class $c$ of airplanes, unambiguously use $\delta = \delta_{c,c}$ and $C_x = \mathbf{C}_x(c)$. If it is also assumed that all flights with the same airline/class combination are valued equally, then $\gamma_f = \gamma_c^a$. Since this simplified model only has one class of flights, further simplify $\gamma_c^a$ to $\gamma^a$.

Every flight $f \in \mathbb{F}$ independently draws a value $\alpha_f$ from the distribution(s) $H_a(\alpha)$. For the remainder of this section, ignore the dependency $\alpha$ and write $H_a$. Some factors that might cause these different distributions are that airlines have different nominal taxi speeds, as well as different distances from their terminal to the runway. With each successive time step, a new set of $\gamma^a$ and $\alpha_f$ are drawn, independently from each other and previous values. *Since every round in this simplified model is independent, any strategy that works for one period can*

*also be repeated for arbitrarily many periods.*

If every airline has the same distribution on their values of $\alpha_f$ (with CDF $H$), then the cdf of the minimum over all $\alpha_f$ (i.e. the earliest flight that could possibly take off) is

$$H_{min} = 1 - (1 - H)^{|\mathbb{F}|}. \tag{10}$$

Note that the median of this minimum distribution occurs when $H = 1 - (\frac{1}{2})^{\frac{1}{|\mathbb{F}|}}$. Therefore, even with a moderately large number of aircraft, and the original distribution $H$ is 'reasonable', then every airline can be confident that there will probably be a flight ready to take off very soon.

If each airline can have its own distribution over values of $\alpha_f$, then a slightly more complicated formula is used:

$$H_{min} = 1 - \prod_{a \in \mathbb{A}} (1 - H_a)^{|\mathbb{F}_a|}. \tag{11}$$

Even if is not assumed the earliest flight will win the slot, there is a tight upper bound. Within each airline/class combination, the airline will always choose the flight that will be ready first. Since each airline will choose its earliest aircraft of each class, the latest that a winner could possibly use the slot is $\prod_{a \in \mathbb{A}} (1 - (1 - H_a)^{|\mathbb{F}_a|})$. Each term in this product is the distribution over the earliest flight for each airline. Taking the product of all these distributions gives the distribution of the maximum of these minimum values. In other words, it describes a worst-case scenario, where the airline which will be ready last actually wins the auction. For reasonable distributions of $H_a$, this expression will still be quite small, especially if there are significantly more flights than airlines. Thus, even if the earliest aircraft doesn't win the auction, the average amount of delay can't get very large.

When there is not a single dominant airline at the airport, then every airline has a tight distribution predicting that the winning aircraft will be ready to take off very soon after the auction occurs. Thus, as a first order approximation, it is reasonable for airlines to assume that if another airline wins the current slot, they will use the slot immediately. However, this potential additional delay will be incorporated later, when the more complicated model is simulated.

These values, $\alpha_f$ and $\gamma^a$, determine the flight's cost function; this gives the opportunity cost of out-

comes in state $x$. If a flight does not win the current auction, and is therefore delayed by the flight that does win, it enters state $x'$. In this new state, $C_{x'} = \max\{t + \delta, C_x\}$. The first term says that the winning aircraft takes off at time $t$, forcing any other aircraft to wait until time $t + \delta$ before they can follow. The second term says that any other aircraft also needs to wait for the required separation time for any aircraft that took off in even earlier auctions. If the separation distances satisfy the triangle inequality (i.e., $\delta_{i,j} + \delta_{j,k} \geq \delta_{i,k}$ for every triple $(i, j, k)$), then this second term will automatically be satisfied whenever the first one is, reducing the expression to $C_{x'} = t + \delta$. Since the simplified model only has one class of aircraft, the triangle inequality is trivially satisfied, so the reduced formula can be used for the rest of this section.

Additionally, define $\alpha_f^x = \max\{C_x, \alpha_f\}$ as the actual earliest time that flight $f$ can take off in state $x$. This says that first, a flight must wait the necessary separation distances before it can take off, and second, it must wait until it is physically ready to take off (tanks full, luggage/passengers loaded, etc). This allows an expression for the opportunity cost of not getting the next slot in state $x$:

$$\text{Cost}^f = (C_{x'} - \alpha_f^x)^+ \times \gamma^a, \tag{12}$$

In other words, if the flight would not be ready to leave for a long time anyway, then nothing is lost by waiting. However, if the flight that loses could have taken off sooner, then there is an opportunity cost for losing the auction, and this cost is proportional to the amount of delay the losing flight suffers.

As noted earlier, since the separation distances satisfy the triangle inequality, this expression can be simplified:

$$\text{Cost}^f = (t + \delta - \alpha_f^x)^+ \times \gamma^a. \tag{13}$$

If the current state has some flight ready to take off, then it must be the case that $t \geq C_x$, since all flights are of the same class. If $\alpha_f$ is restricted to not being in the past (i.e. $\alpha_f \geq t$), then further simplify:

$$\text{Cost}^f = (t + \delta - \alpha_f)^+ \times \gamma^a. \tag{14}$$

Note that this assumption is only used to simplify notation in this specific case. When actually working with the model, $\text{Cost}^f$ can be substituted with the simplest expression that applies to the case being considered.
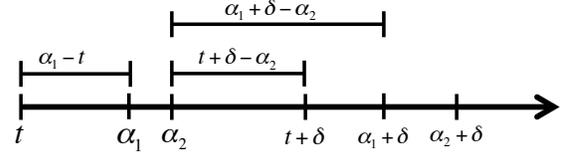


Figure 1: Important values of time.

In Figure 1, important values of time are indicated. Value $t$ corresponds to the earliest instant that any aircraft can use the runway, after the most recent flight has taken off (this most recent takeoff occurred at time $t' = t - \delta$, in the past). Time $\alpha_1$ is the earliest time that airline 1 can use the runway, and $\alpha_2$ is the earliest time for airline 2. Moreover, $t + \delta$ is the earliest time that any airline could use the runway, if the airline who wins the auction is ready to take off immediately. Finally, $\alpha_1 + \delta$ is the earliest time other aircraft can use the runway, if airline 1 wins the auction and begins to use it at $\alpha_1$. Similarly $\alpha_2 + \delta$ is the earliest time other aircraft can use the runway if airline 2 wins.

Suppose that airline 1 wins the auction. In this case, the distance $\alpha_1 - t$ corresponds to the amount of time until the first flight is ready to take off. The distance $t + \delta - \alpha_2$ corresponds to the amount of delay airline 2 would suffer, if it did not win the auction, and the winner was able to takeoff immediately. The longer distance $\alpha_1 + \delta - \alpha_2$ corresponds to the actual amount of time that airline 2 must wait, if airline 1 wins the auction.

Recall the analysis of the distribution $H_{min}$. As long as no airline controls a majority of the flights at the airport, airline 2 will have a tight distribution near $t$ where they expect the earliest flight to be. Therefore, they will rationally believe that the distance $\alpha_1 - t$ will be small, so that the actual delay they suffer by losing the auction $(\alpha_1 + \delta - \alpha_2)$ can be approximated by $t + \delta - \alpha_2$. This delay is then multiplied by $\gamma^a$, the marginal cost of delay.

If a flight wins the current auction, then it will not be delayed by another flight. However, the credits spent on this auction cannot be spent on a later auction, so there is an opportunity cost of $b_t^{a_2}$, the bid of the second highest airline, $a_2$. It is possible to determine an exchange rate between real money and virtual credits, which will allow all costs to be expressed in the same units. Theoretically, this could also be accomplished by charging real money to win

runway slots, though this would cause other, non-technical problems in convincing the airlines to implement this new mechanism.

Since the only difference between the cost functions of flights of the same airline/class combination are their earliest takeoff times, it is a weakly dominant strategy to send out these flights in the order of their $\alpha_f$'s. Therefore, when trying to determine the winner of the auction, one only needs to consider the earliest flight in each group. This will simplify the analysis, since there are now at most only $|\mathbb{A}| \times |\mathbb{C}|$ flights to consider, rather than $|\mathbb{F}|$.

Let $f^* = \arg \min_{f \in \mathbb{F}_a} \alpha_f$. Then the cost if airline $a$ *loses* the auction is

$$\sum_{f \in \mathbb{F}_a} \gamma^a (t + \delta - \alpha_f^x)^+ \tag{15}$$

The cost if it *wins* the auction is

$$b_t^{a2} + \sum_{\substack{f \in \mathbb{F}_a \\ f \neq f^*}} \gamma^a (t + \delta - \alpha_f^x)^+ \tag{16}$$

The difference between these expressions is

$$b_t^{a2} - \gamma^a (t + \delta - \alpha_{f^*}^x)^+ \tag{17}$$

Since the airline wants to minimize their cost, they want to win whenever $b_t^{a2} < \gamma^a (t + \delta - \alpha_{f^*}^x)^+$, and lose whenever the inequality goes the opposite direction. This can be achieved if airline $a$ bids

$$b_t^{a*} = \gamma^a (t + \delta - \alpha_{f^*}^x)^+ \tag{18}$$

These results assume that it is possible for an airline to bid more than its budget, but that higher bids continue to generate additional dis-utility. Again, in terms of real currency, this situation makes more intuitive sense than the virtual credit analog. However, for simulations the hard budget constraint is incorporated.

The first observation from this result is that if there is an aircraft that will be ready to leave before $t + \delta$, then it will beat every aircraft that will not be ready to leave until after $t + \delta$. This is because any aircraft which will not be ready by $t + \delta$ will bid zero, while any aircraft that would be ready would bid a positive amount. This establishes a maximum on the amount of delay possible each round.

Furthermore, define $\varepsilon = \gamma^a - \gamma^{a'}$, and label the airlines such that $\varepsilon \leq 0$. This means that $a'$'s aircraft cost at least as much as $a$'s, or equivalently $\gamma_a \leq \gamma_{a'}$. Also, suppose both airlines have flights

that are able to take off before $t + \delta$. Then

$$b_t^a - b_t^{a'} = \varepsilon(t + \delta - \alpha_{f^*\prime}^x) + \gamma^a(\alpha_{f^*\prime}^x - \alpha_{f^*}^x) \tag{19}$$

If airline $a$ wins the auction, then

$$\alpha_{f^*\prime}^x - \alpha_{f^*}^x \geq (\frac{\gamma^{a'}}{\gamma^a} - 1)(t + \delta - \alpha_{f^*\prime}^x) \geq 0 \tag{20}$$

which means that airline $a$ was indeed ready to take off no later than airline $a'$.

Unfortunately, the converse doesn't quite hold. If $a'$ wins the auction, then

$$\alpha_{f^*\prime}^x - \alpha_{f^*}^x \leq (\frac{\gamma^{a'}}{\gamma^a} - 1)(t + \delta - \alpha_{f^*\prime}^x) \tag{21}$$

However, since this upper bound can be strictly positive, it is not certain that $a'$ was actually ready sooner than $a$. If the two values of $\gamma$ are close to each other, then the maximal value of this error is small. Additionally, if airline $a'$ is causing a large delay by winning this slot ahead of $a$ (i.e., $\alpha_{f^*\prime}^x \to t + \delta$), then the difference between the two airlines' earliest takeoff times is also small. In other words, as the delay gets worse, it means that the maximal difference between the two airlines decreases. This is good, since it rules out the possibility of awarding a slot to a plane that will take a long time to take off when there is another plane ready to take off much sooner.

This difference is important, because $(\alpha_W^x - \alpha_L^x)^+$ is the decrease in throughput caused by airline $W$ winning the auction over airline $L$. If airline $W$ will be ready first, then there is no throughput loss, even though there is a period of time $(\alpha_W^x - t)^+$ when no flight is using the runway. There is no loss, because no matter who won the auction, no one else could have been ready to use the runway either. However, if one of the losing airlines could have been ready sooner, then the time between when they are ready and when the winner actually starts using the runway is wasted, reducing throughput by that amount. Therefore, by showing that this quantity is usually zero, and that when it isn't it is still quite small, it can be seen that this auction mechanism will create only a small loss in throughput.

Note that this small amount of throughput loss is relative to a theoretical ideal arrangement, where all private information is common knowledge to a central planner. This is impossible in practice, due to airline incentives to hide their private information. Therefore, even though the model predicts

some loss in throughput from an ideal level, it is still expected it to perform better than the current FCFS system.

For the simplified model, new values of $\gamma^a$ and $\alpha_f$ are drawn from the same distributions, and each airline is given the same budget (the opportunity cost of bidding was already explicitly incorporated into the cost function in the previous round). Since this new round has no direct connection to the previous round, the same strategy will still apply, and most of the time the flight which will be ready soonest will be selected to leave next. Additionally, under modest assumptions, when errors do occur they will have only a small effect on throughput.

## Multiple Classes

Many of the results found in the previous section can be extended to the case of multiple airlines. First, equation (12) is altered to make $C$ and $\gamma$ depend on the class of the relevant aircraft:
$$\text{Cost}^f = \gamma^a_{c_f}(\mathbf{C}_{x'}(c_f) - \alpha^x_f)^+$$

Each airline can calculate how much (weighted) delay each of their flights would cause to the rest of their fleet if it is selected in the current auction, and choose the flight which will minimize this total:
$$f^* = argmin_{f' \in \mathbb{F}_a} \sum_{\substack{f \in \mathbb{F}_a \\ f \neq f'}} \text{Cost}^f \qquad (22)$$

Then the cost of winning is still given by equation (16). The cost of losing is similar to equation (15), but since this depends on the class of the winner, each airline must take an expected value over the possible winning classes from other airlines. If $P^a_c$ is the probability that airline $a$ believes the next winner will be of class $c$, given that airline $a$ does *not* win the auction, then this expected cost of losing is:
$$\sum_{c \in \mathbb{C}} P^a_c \Big( \sum_{f \in \mathbb{F}_\mathbb{A}} \text{Cost}^f \Big) \qquad (23)$$
Because of this expected value, the terms do not cancel as nicely as in the single-class model. However, a flight will only bid a positive amount if it would actually suffer some additional delay from not winning the current auction. In other words, any flight which will be ready to take off by its maximum separation distance will outbid any flight

which will not. Such a flight $f$ must satisfy:
$$\alpha^x_f < t + \max_{c \in \mathbb{C}} \delta_{c,c_f} \qquad (24)$$
This places a strict upper bound on the amount of delay possible.

# Simulation Model

While closed form analytical results for simple models are given, there are still many more features that need to be incorporate into the model to make it realistic. Since adding these features will make the model significantly more complicated, this more complicated model will be analyzed through computer simulation. This section will explain the additional features, and how they will be incorporated into the simulation algorithm. From this point on, all times are measured in seconds, unless otherwise noted.

One important change is to introduce greater continuity between auctions. Currently, the private information is reset every period. While this makes the problem much more tractable, the ability to model airlines as forward-thinking is lost. To fix this, make $\alpha_f$ and $\gamma_f$ persistent across rounds. Also, a hard budget constraint is implemented, which prevents airlines from bidding more credits than they have remaining.

## Aircraft Readiness

At the beginning of the day, every airline must post a schedule of their flights for that day. However, some flights will be ready to leave before their scheduled times, and some will not be ready until after their scheduled times. These schedules are generally the same every day (with some variation between weekdays/weekends). This means that the actual departure times of these flights can be tracked over time (it would be easy to make this data common knowledge to all the airlines), which can give a probability distribution of the actual departure time for every flight on the schedule.

There are several reasons that the actual departure times can differ from the posted times. For example, if a flight takes off for a trip $A \to B$ an hour late, the flight using that same plane traveling $B \to C$ will probably be delayed about an hour as

a result. This delay can be foreseen while the first flight is sitting delayed at airport $A$, and is common knowledge to all airlines. Therefore, they would not rationally expect the flight to depart for $C$ until much later than scheduled, and would take this into account when bidding in the auction. There is some randomness in the actual takeoff time of the flight $A \rightarrow B$, which will be resolved at the time of takeoff.

There is a smaller amount of randomness in the flight time between $A$ and $B$. This can depend on factors like the weather. Given historical data for the actual transit times between airports, airlines will have a distribution over when a given aircraft will arrive at its destination, given the type of aircraft and the time it departs.

After a flight arrives at $B$, it must perform all preflight procedures. The delays that result from preflight procedures are private information to the airline. They occur on a much shorter timescale, and are more difficult for other airlines to observe. Therefore, only the airline will know how long these delays are likely to take, while other airlines can only guess, based on a wider distribution of historical delay.

With this in mind, a description of the way that the $\alpha_f$'s will work in this simulation is presented.

First, every airline publishes a schedule for the day of expected takeoff times for all their flights. Until the plane arrives at the departure airport, this distribution is so wide (relative to the separation distances between aircraft classes), that it does not affect the bidding policy of any airlines. After the plane arrives at its departure gate, the actual readiness time $\alpha_f$ will be revealed to the airline that owns the flight, and every other airline will have a tighter distribution over this value $\alpha_f$. This distribution the other airlines know will be the same as the distribution from which the $\alpha_f$'s are drawn.

Only flights whose planes are currently at the airport will be allowed to bid for slots. For the purpose of simulation, this will be approximated by saying that only flights with $\alpha_f - t < 30$ minutes are eligible to enter an auction at time $t$.

## Weighting Factors

Instead of every airline/class combination having a single weight $\gamma_c^a$, every flight will have its own $\gamma_f$. However, flights from the same airline/class combination will be drawn from the same distribution, while different combinations can potentially be drawn from different distributions.

## Timing

The timing of the auctions in the simplified model was driven by the separation distances required between aircraft types, which are on the order of 1-2 minutes. However, between when a flight wins an auction and when it takes off, it also needs to taxi from its gate to the runway, which takes more like 10-15 minutes. There is currently quite a bit of variation in taxi out times, mostly resulting from long queues at the runway. Since this system will remove the FCFS incentives which generate these long queues, most flights should be able to move at nominal taxi speed, reducing both the average and the variance in taxi times.

In the model, it is assumed that the uncertainty in the delay occurs entirely at the gate. Given the time that a flight pushes back, and the sequence of aircraft that have preceded it (and their pushback times), it should be possible to calculate a flight's takeoff time with certainty. If all airlines had the same taxi times, the last winner pushes back at time $t$, and the minimal separation distance to follow that winner is $\delta_{min}$, then run the next auction can be run at any time between $t$ and $t + \delta_{min}$ without changing the outcome. Any flight that would be ready to push back during that interval would wait until $t + \delta \geq t + \delta_{min}$ to actually push back, and any flight that won without being ready would still push back at their earliest pushback time. Whenever the flight leaves, it will head straight to the runway and takeoff, without any additional delay. By having flights wait at their gates, fuel waste will be reduced.

Unfortunately, since taxi times are not the same for all airlines, using this method may leave the runway unused for significant periods of time. To reduce this decrease in throughput, a buffer is created by running several auctions at once. At the beginning of the day, $k$ auctions are run to assign the

first $k$ slots. Each of these $k$ flights will be given a requested pushback time, which would give it a just-in-time departure if all previous flights make their requested pushback times. When the first flight in the buffer actually pushes back, the $k + 1^{st}$ auction occurs. This process continues through the day, keeping a buffer of flights to maintain runway usage. Later the affect the buffer size $k$ has on taxi times and airline costs can be observed.

## Objective Function

The airlines do not place any inherent value in the virtual credits; the credits are only useful to the extent that they can be exchanged for slots to use the runway. What airlines care about is reducing delay, and they will attempt to use their credits to compete with other airlines for delay reduction. Let $\tau_f$ be the actual time that flight $f$ departs the airport. Then the objective function that each airline cares about is:

$$\text{Total Cost}_a = \sum_{f \in \mathbb{F}_a} \gamma_f (\tau_f - \alpha_f) + y_f L \qquad (25)$$

Minimizing this function minimizes the total cost to each airline for the delay of their aircraft over the course of the day.

Note that the model does not take into account which other airlines win which slots. The only extent to which an airline will care about the actions of its competitors is in how those actions affect delay for its aircraft. Since this is already included in the actual takeoff times of all the flights, there is no need to include any explicit information about other airlines' takeoff times. This is assuming that all airlines care primarily about their own delay, and will not delay their own aircraft to cause additional delay for their competitors.

## Simulation Values

In order to test the auction model, it was important to use an airport that was frequently busy/over capacity (such that some sort of optimization would have noticeable effects), and that did not have a single dominant airline (since a system like [10] would work better than an auction in such an environ-

ment). An excellent candidate would be JFK International Airport. Unfortunately, the data source [21] is missing all international and cargo flights that makes up a significant portion of flight operations at JFK. Therefore, the available data from JFK is used to simulate an airport with an appropriate number of flight operations.

In the Simulation section results are presented comparing an auction approach to delay optimal and cost optimal solutions under various levels uncertainties. Approximately 1000 unique scenario instances were run for each mode[5].

## Penalty

To discourage airlines from bidding for slots they aren't ready to use immediately, a penalty function is implemented. If $d$ is defined as the difference between the requested and actual takeoff times for the flight, then the winner would actually pay $b_t^{a_2} * 1.03^d$. Since the airlines would know both of these times with certainty during the auction, they could shade their bids by $1.03^{-d}$.

## Strategy

To determine which of their flights to send out, each airline performs exactly the same calculations using equation (22). Unfortunately, determining the probability of each possible winner (both flight and actual readiness time) proved too difficult. Instead, a heuristic strategy was used. Each airline begins with the average amount of credits they have remaining per flight, and multiplies that by a factor that represents how valuable the current auction is, compared to what they expect the remaining auctions to be.

The first factor in this multiplier is $\gamma_f$, since more valuable flights should command higher bids. The next factor is inversely proportional to the amount of time until the flight could be ready to push back, since flights that will not be ready for a long time should have lower bids. The third factor relates to long delays. If an airline could get a flight out before the long delay would occur, then the factor is inversely proportional to the amount of time

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[5]For details about the values used to generate these scenarios please contact the authors. Most of the data was generated using sources from [21].

until this penalty point occurs; that is, if a flight is just before the time when it could avoid the long delay, it will have a very high bid. If a flight is already past the point of receiving a long delay, then the factor is a constant, which is small enough to prevent these delayed flights from beating out on-time flights. The final factor is a constant, designed to make the average bid approximately equal to the average number of credits per flight.

## Simulations

One of the first tasks of the simulation was to determine how large the buffer $k$ should be for the auction. If it is too small, then the runway goes unused. If it is too large, then delay shifts from the gate to the airport surface. The following plots in Fig. 2 show relevant statistics for $1 \leq k \leq 7$:
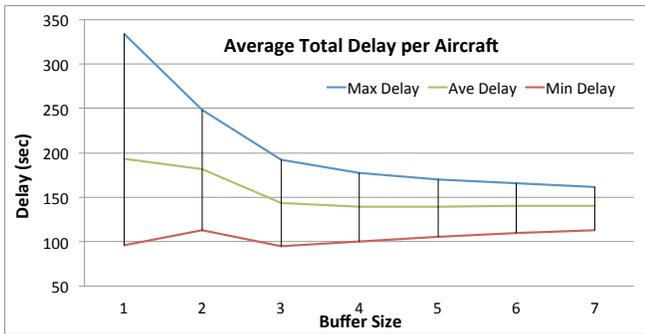


Figure 2: Average total delay per aircraft as a function of buffer size (k).
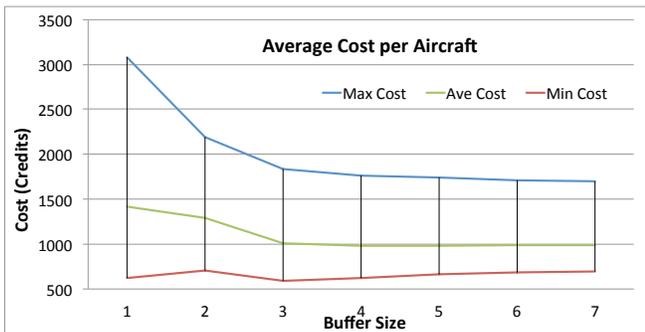


Figure 3: Average total cost per aircraft as a function of buffer size (k).

In Figures. 2 and 3, that total delay (gate plus taxi delay) and total cost per aircraft both gradually decrease with buffer $k = 1$ to $k = 3$, then

stabilize for $k \geq 4$. The average delay completely stabilizes, while the max and min delay appear to be converging to the average. This phenomena is likely a function of the airport taxi-out times, where larger differences in taxi-out times will lead to runway down-time if a large enough buffer is not provided. This suggests that a good buffer will be large to avoid the runway down-time and force a smaller spread of delay among aircraft. However, as evident in the next plots, the buffer has a direct affect on the trade-off between taxi delay and gate delay.
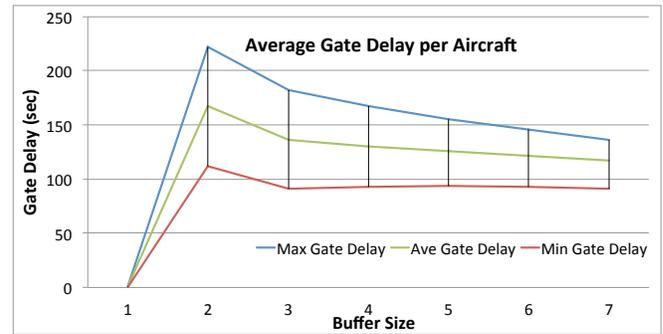


Figure 4: Average gate delay per aircraft as a function of buffer size (k).
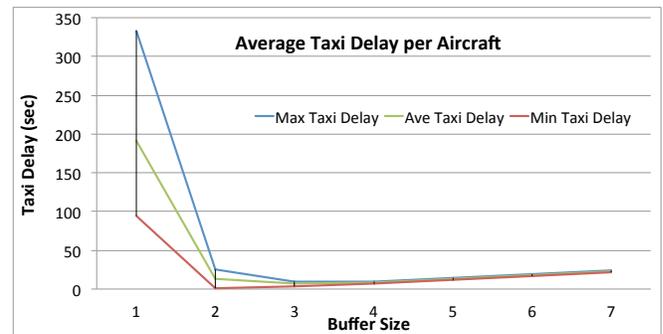


Figure 5: Average taxi delay per aircraft as a function of buffer size (k).

Now consider Figure 4 and Figure 5 which show the average gate and taxi delay as functions of the buffer size, respectively. First note that the gate delay rapidly increases from $k = 1$ to $k = 2$, since at $k = 1$ no aircraft are held at the gate. A close visual inspection of Figure 5 indicates that taxi delay is minimized at $k = 4$ and then begins increasing slightly. Since the goal is to reduce engine on time (i.e., taxi time), while keeping the total delay and

cost as small as possible, a good $k$ value will prevent large taxi-out delay and reduces the total delay and cost.

Summarizing this analysis from all the plots considered so far, $k = 4$ is a good buffer value. This results in about approximately 100 seconds of delay per aircraft at the gate, and less than about 20 seconds of delay per aircraft on the taxiway.

The next set of charts (Figs. 6-9) compares several different methods of allocating runway use. Each chart is constructed in a similar manner, so before discussing their results their structure is discussed. For each chart, the first two bars correspond to results from the MILP introduced in the Departure Runway Scheduling Problem section, optimizing for cost and delay. These results both assume a central planner with access to information that is not available in practice, but they do provide a lower-bound on the best outcome possible. Since ATC will not have perfect information regarding aircraft pushback, the $\alpha$ values are perturbed, by adding uniform random noise from an interval $\pm120$s or $\pm240$s around the true value. The next two bars correspond to MILP solutions for minimizing delay, but with these noise terms, as an approx-imation of mechanisms that try to optimize runway schedules without receiving information from airlines regarding pushback. These methods are called "Noise: 120s" and "Noise: 240s" for 120s and 240s of noise, respectively. The fifth bar corresponds to FCFS, the method actually used at most airports currently. The final three bars correspond to different implementations of the auction mechanism. The first allocates every airline the same number of credits per flight for their initial budgets. The second varies this credit assignment to try to equal-ize delay across the airlines, and the third does the same to equalize avoidable costs.

Moreover, 4 new metrics are used in the fol-lowings figures: (1) avoidable cost, (2) unavoidable (cost), (3) avoidable long delay, and (4) unavoid-able long delay. Avoidable cost is the cost that air-lines can control through bidding (i.e., optimizing equation (25)). Unavoidable cost is the cost that arises due to the randomization of $\alpha_f$ which could be larger than $900 + S_f$. Similarly, avoidable long delay is a long delay that could be avoided through winning auctions. Finally, an unavoidable long de-lay is a long delay that arises due the randomization of $\alpha_f$.
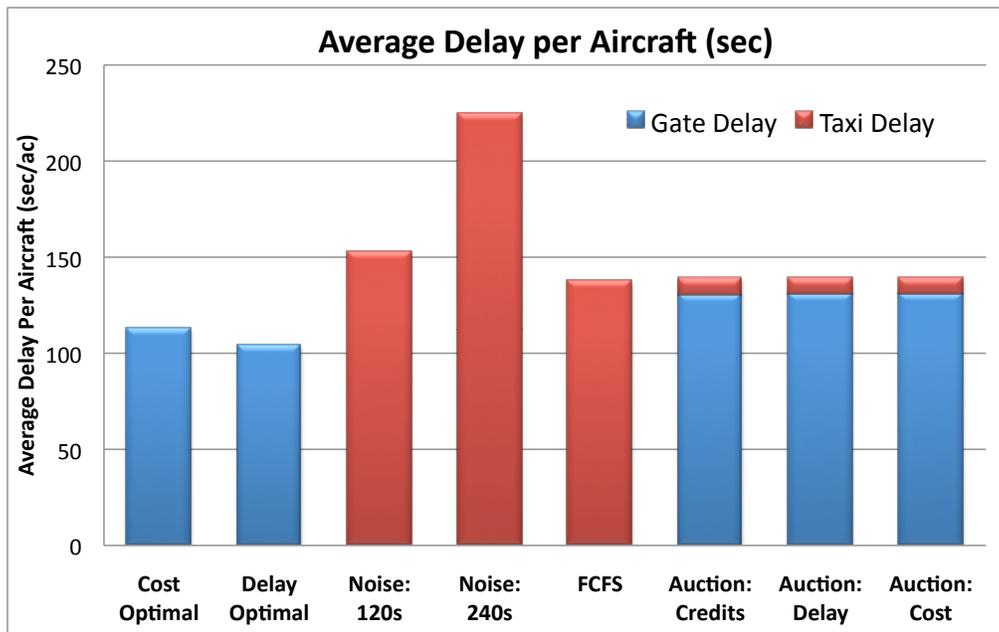


Figure 6: Comparison of Runway Allocation Mechanisms: Average Delay Per Aircraft

Both FCFS and the Noise methods create strong incentives to push back as early as possible. Figure 6 demonstrates the primary advantage of the auction mechanism; delay can be shifted from the airport surface to the gate, greatly reducing fuel waste. Total delay is similar to FCFS, and slightly less than the Noise methods, so overall airport efficiency will not significantly decrease.
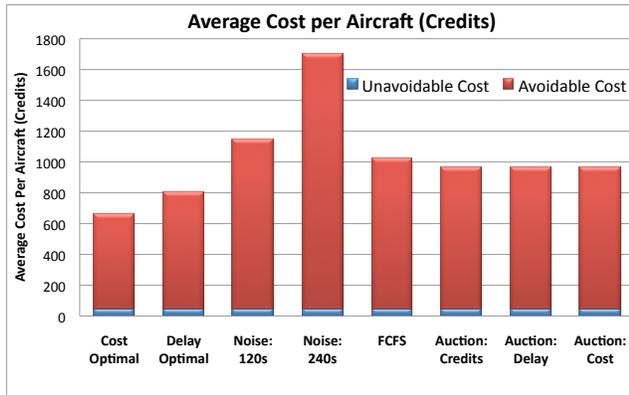


**Figure 7: Comparison of Runway Allocation Mechanisms: Average Cost**

Figure 7 shows the average cost per flight. The blue portion corresponds to the cost of long delays that were unavoidable, because the flights' internal delays were over 15 minutes, while the red portion corresponds to those costs which were theoretically avoidable. The auction achieves better cost results than FCFS and the Noise methods.
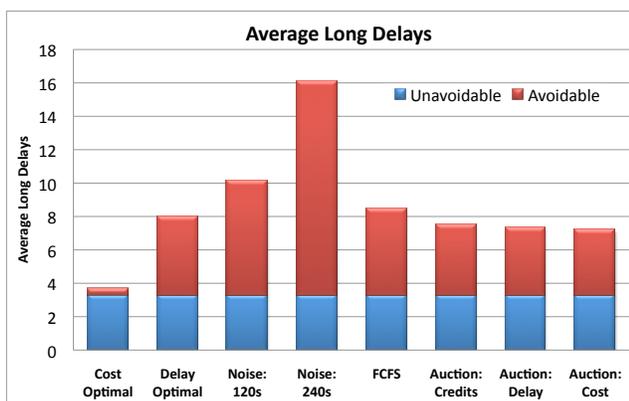


**Figure 8: Comparison of Runway Allocation Mechanisms: Average Long Delays**

Figure 8 shows the number of delays greater than 15 minutes: blue are unavoidable, and red

are avoidable. It is observed that cost optimization nearly eliminates avoidable long delays, since they are so detrimental to airline cost functions. Additionally, the auction results in fewer long delays than the Noise methods or FCFS, demonstrating that the auction has successfully incorporated airline preferences to some extent.
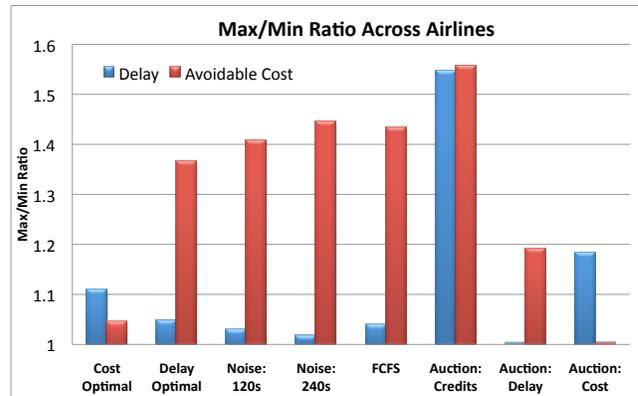


**Figure 9: Comparison of Runway Allocation Mechanisms: Max Delay to Min Delay Ratio**

Finally in Figure 9, blue represents the ratio between the highest/lowest average delay airlines, and red represents the ratio between highest/lowest average cost airlines. The closer these ratios are to 1, the more equal the distribution of delay (cost) between airlines. First, notice that in general delay is easier to balance than cost. Second, when credits are distributed to airlines purely based on the number of flights, the result is less equitable than any alternative method. However, the results are highly sensitive to the credit distribution; by altering the initial budgets, the central planner can achieve arbitrarily close distributions of either delay or cost. Additionally, balancing one will result in significantly more balance in the other, compared to an even credit distribution. Attempting to balance delay results in distributions that are significantly more equal for both delay and cost, when compared to any of the feasible alternative methods.

# Conclusion and Next Steps

In this paper, an innovative method for allocating runway usage is described, based on a dynamic second-price auction. Clean analytical results for a simplified version of the mechanism show that usually the auction will succeed at creating an efficient flight ordering at the runway in real-time.

Furthermore, a more complicated version of the auction is simulated and compared with other methods that could be used to solve the same problem. The auction mechanism greatly decreases taxi delay, which would lead to significant fuel savings. This savings comes without any noticeable increase in total delay. It also results in slightly lower costs and number of long delays, compared to feasible alternatives. Finally, through strategic budget allocation, the FAA is able to equalize average delay across airlines, generating a more equitable solution than the other mechanisms.

The auction mechanism allows airlines to keep their financial information private and still communicate priorities via bidding. Since all currency for the auction is in the form of virtual credits, the mechanism accomplishes its goals without having to severely change the political and financial atmosphere at airports. In addition, since bids are the only data processed, the mechanism might alleviate costs in developing central information exchange, airline operational standards, and/or political costs.

Fundamentally, there are still many hurdles to tackle to make use of the proposed approach. While the mechanism focuses on providing departure aircraft pushback times, arrival aircraft need to be accounted for during this process. The reason is that arrivals and departure aircraft share gates, taxiways, and other airport resources. Hence, it is important to extend the mechanism further to either work around arrival aircraft, or include arrivals in decision making process. Equally as important, a more detailed study on the feasible strategies of the second price auction needs to be carried out. Currently, the strategies developed for simulating airline behavior are based on intuition. Since strategies that are efficient and constitute equilibrium can be good predictors of bidding behavior, additional analysis of new strategies should be analytically or numerically justified. At a minimum, high fidelity human-in-the-loop studies should be carried out to determine bidding behavior norms, human machine interfaces, and provide guidance on the operational concept.

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