

TUTORIAL SHEET 6

1. Find a recurrence relation for the number of ternary strings of length n that contain either two consecutive 0s or two consecutive 1s.
2. Find a recurrence relation for the number of ternary strings that do not contain two consecutive 0s or two consecutive 1s.
3. Which of the following statements are correct? Here A, B, C are three events:
 - (i) If A and B are independent, then A and \bar{B} are also independent (here \bar{B} denotes the complement of B).
 - (ii) If A and B are independent, and B and C are independent, then A and C are also independent.
 - (iii) If A and B are independent, and A and C are independent, then A and $B \cup C$ are independent.
 - (iv) If A and B are independent, A and C are independent, and A and $B \cap C$ are independent, then A and $B \cup C$ are independent.
4. There are three fair dice. Let A be the event that the maximum value seen when we throw all three is 6 and B the event that the minimum value seen when we throw all three is 1. Are these two events independent?
5. A family has two children. Each child is equally likely to be a boy or a girl, independently. Let A be the event that the family has two boys, and B be the event that at least one child is a boy. What is $P(A|B)$.
6. There are three doors. Behind one of the doors, there is a prize and behind the other two there is no prize. You pick one door. Monty (who knows where the prize is) opens a different door that has no prize behind it, then offers you a switch to the remaining closed door. Should you switch?
7. A disease affects 1% of the population. A diagnostic test is 95% accurate: it gives the correct result with probability 0.95 (so 5% error rate). A person is randomly selected and tests positive. What is the probability that they actually have the disease?
8. You have three coins in a box. Coin A: a fair coin with $P(H) = 0.5$, Coin B: a double-headed coin with $P(H) = 1$, Coin C: a biased coin with $P(H) = 0.8$. One coin is drawn uniformly at random from the box and the chosen coin is tossed repeatedly until the first **Tail** appears. Suppose the first Tail occurs on toss number 3. What is the probability that you had chosen coin C?