

TUTORIAL SHEET 1

1. Let A, B and C be three sets. Show that:

$$A \cup B \cup C = (A - B) \cup (B - C) \cup (C - A) \cup (A \cap B \cap C)$$

2. Prove that for any sets A, B, C, D , if $(A \times B) \cap (C \times D) = \emptyset$, then $A \cap C = \emptyset$ or $B \cap D = \emptyset$.

3. For a set A , let $P(A)$, the power set of A , denote the set $\{C : C \subseteq A\}$. Prove that:

(a) $P(A) \cap P(B) = P(A \cap B)$.

(b) $P(A) \cup P(B) \subseteq P(A \cup B)$ with equality if and only if one of A or B is a subset of the other.

4. Suppose $f : A \rightarrow B$ and $g : B \rightarrow C$ are functions. Show that if $g \circ f$ is a bijection then f is 1-1 and g is onto.

5. Let $f : A \rightarrow B$ be a function.

(a) Let $A' \subseteq A$. Show that $f^{-1}(f(A')) \supseteq A'$ with equality if f is 1-1.

(b) Let $B' \subseteq B$. Show that $f(f^{-1}(B')) \subseteq B'$ with equality if f is onto.

6. Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

7. For each of the statements below, write them in the “if p , then q ” form:

- It is necessary to wash the boss’s car to get promoted.
- Wily gets caught whenever he cheats.
- A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
- It rains whenever the wind blows from the north.
- It is necessary to walk 8 km per day to stay healthy.