

TUTORIAL SHEET 4

1. Consider the following game: the game begins with n match sticks. Two players take turns removing match sticks one, two or three at a time. The player removing the last match stick loses. Use strong induction to prove that if each player plays the best strategy then the first player wins if the remainder of n when divided by 4 is 0, 2, or 3, and the second player wins otherwise.
2. Prove the following by induction: a class of any size of 18 or more can be assembled from student teams of sizes 4 and 7.
3. Suppose that for every pair of cities in a country, there is a direct one way road connecting them in one direction or the other. Use mathematical induction to show that there is a city that can be reached from every other city either directly or via exactly one other city.
4. Use structural induction to show that the number of leaves a full binary tree is 1 more than the number of internal nodes in the tree. A binary tree is said to be a full binary tree if each internal node has exactly two children.
5. A knight on a chessboard can move one space horizontally in either direction and two spaces vertically in either direction, or two spaces horizontally in one direction and one space vertically in either direction. Suppose that we have an infinite chessboard made up of squares (m, n) where m and n are non-negative integers. Show using mathematical induction that a knight starting at $(0, 0)$ can visit every square using a finite number of moves.
6. Show that every natural number has a non-zero multiple which, when written out (base 10), contains only zeroes and ones.
7. Show that for any positive integer n , there is a nonempty configuration of finitely many points on the plane with the property that every point of the configuration is at distance 1 from exactly n of the other points in the configuration.