HW 1

Graded

Student

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Total Points

47 / 60 pts

Question 1

Induction Proof

10 / 10 pts

- ✔ **+ 2 pts** Correct Induction Hypothesis and Base Case
- ✔ **+ 7 pts** Correct Induction Step
- ✔ **+ 1 pt** Concluding the inequality
- **− 1 pt** Not properly specifying on what variable induction is applied on
- **+ 0 pts** Incorrect

what was the point of mentioning geometric series ?

Question 2

Bijection between sets

5 / 15 pts

2.1 Even-Odd Subsets

2 / 5 pts

- **+ 0 pts** Incorrect
- **+ 1 pt** Valid Map from set A to B
- **+ 2 pts** Showing it is one to one
- **+ 2 pts** Showing it is Onto
- **+ 2 pts** Point adjustment

1 not properly proven

2 since |A| = |B| ? -- This has to be proven , how could you use it ?

2.2 Digit Sequence

1 / 5 pts

- ✔ **+ 1 pt** Correct bijective function from set A to B
- **+ 2 pts** Showing it is one to one
- **+ 2 pts** Showing it is onto
- **+ 0 pts** Incorrect

5 f(f(x)) is not well defined as domain of f is A.

2.3 Infinite String

2 / 5 pts

- **+ 5 pts** Correct
- **+ 0 pts** Incorrect
- **+ 2 pts** Point adjustment

6 arent you proving opposite thing ?

7 not properly proven

Question 3

Integer or Irrational

10 / 10 pts

- ✔ **+ 10 pts** Correct
- **+ 0 pts** Incorrect

Question 4

Bijections On Infinite Sets

12 / 15 pts

4.1 Countable Set

3.5 / 5 pts

- ✔ **+ 1 pt** Attempted to prove the statement
- **+ 0 pts** Attempted to disprove the statement / Unattempted / Bogus argument
- ✔ **+ 4 pts** Correct argument
- ✔ **− 1.5 pts** Minor flaws / gaps in argument
- **− 3 pts** Major flaws / gaps in argument

You've assumed that A is countable and not considered the uncountable case, although a similar argument can be used.

3 A need not be countable

4.2 Uncountable Set

4.5 / 5 pts

- **+ 0 pts** Attempted to prove the statement / Unattempted
- ✔ **+ 1 pt** Attempted to disprove the statement
- ✔ **+ 2 pts** Gave a correct choice for A, B (eg. any countable A) for which the statement is false
- ✔ **+ 2 pts** Correct proof for the choice using either a diagonalization argument or a known standard separation
- **− 1.5 pts** Minor gaps in argument
- **− 3 pts** Major gaps in argument

− 0.5 pts Point adjustment

4 should say that $A \cup B$ is uncountable, because $A \cup B = \mathbb{R}$

4.3 Quadratic Reals

4 / 5 pts

- ✔ **+ 1 pt** Attempted to prove the statement
- **+ 0 pts** Attempted to disprove the statement / Unattempted
- ✔ **+ 4 pts** Correct argument
- ✔ **− 1 pt** Used a non-trivial fact without proving (which was not shown in class)
- **− 1 pt** Essentially correct, but minor flaws in argument
- **− 3 pts** Major gaps in argument

8 for how many sets are you taking the union (finite/countable/uncountable)?

9 this was not shown in class, so you should at least briefly argue why this is true

Question 5

Predicate Logic

10 / 10 pts

5.1 **Multiple of n** — **3** / 3 pts

- ✔ **+ 0.5 pts** Correct predicate of the original statement
- ✔ **+ 0.5 pts** Correct predicate of negation of the statement
- **+ 0 pts** Attempted to prove the statement / Unattempted
- ✔ **+ 0.5 pts** Attempted to disprove the statement
- ✔ **+ 1 pt** Gave a correct choice of m, n for which the statement is false
- ✔ **+ 0.5 pts** Correctly showed the statement is false for the choice of m, n

5.2 **Lower Bound and Least Element** — **3** / 3 pts

- ✔ **+ 0.5 pts** Correct predicate of the original statement
- ✔ **+ 0.5 pts** Correct predicate of negation of the statement
- **− 0.5 pts** Used helper predicates without defining them logically
- **+ 0 pts** Attempted to prove the statement / Unattempted
- ✔ **+ 0.5 pts** Attempted to disprove the statement
- ✔ **+ 1 pt** Gave a correct choice of $S \subseteq \mathbb{R}$ for which the statement is false
- ✔ **+ 0.5 pts** Correctly showed the statement is false for the choice of S

5.3 **Bounded Factors of Composite Numbers** — **4** / 4 pts

- ✔ **+ 1 pt** Correct predicate of the original statement
- ✔ **+ 1 pt** Correct predicate of the negation of the statement
- **− 1 pt** Used helper predicates (other than the given one) without defining them logically
- ✔ **+ 0.5 pts** Attempted to prove the statement
- **+ 0 pts** Attempted to disprove the statement / Unattempted
- ✔ **+ 1 pt** Gave a choice of a non-trivial factor f of n
- ✔ **+ 0.5 pts** Correctly showed that $f \leq \sqrt{n}$

Question assigned to the following page: 1

Q1 Show by induction:

$$\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$$

Intution :

$$\sum_{i=1}^{\infty} \frac{1}{2^{i-1}} = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} \dots \frac{1}{2^k} + \frac{1}{2^{k+1}} \dots \infty$$

Sum of inf GP $= \frac{a}{1-r}$ [a = first term, r = factor of inc/dec

$$= \frac{1}{1-1/2} = 2$$

Hence ~~So~~ $\sum_{i=1}^{\infty} \frac{1}{2^{i-1}} = 2$

So for some constant $k \leq \infty$

$$\sum_{i=1}^{k} \frac{1}{2^{i-1}} \leq 2$$

Our series : $\sum_{i=1}^{n} \frac{1}{i^2} \leq \sum_{i=1}^{n} \frac{1}{2^{i-1}} \leq 2$

Proof by induction :

we want to proove that $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$

we know that $2 - \frac{1}{n} \leq 2$

So ~~by~~ if we prove $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$ then,

by transitivity $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$

Question assigned to the following page: 1

Base case (ε) :

$k=1$

$$S_1 = \sum_{i=1}^{1} \frac{1}{i^2} \leq 2 - \frac{1}{1}$$

$$= \frac{1}{1^2} \leq 2-1$$

$$= 1 \leq 1 \quad \text{(True)}$$

$(k=2)$ $$S_2 = \sum_{i=1}^{2} \frac{1}{i^2} \leq 2 - \frac{1}{2}$$

$$= \frac{1}{1^2} + \frac{1}{2^2} \leq 2 - \frac{1}{2}$$

$$= 1 + \frac{1}{4} \leq \frac{3}{2}$$

$$= \frac{5}{\cancel{4}_2} \leq \frac{3}{\cancel{2}} \Rightarrow 2.5 \leq 3 \quad \text{(True)}$$

Assume its true for some $n=k$

So, $S_k = \sum_{i=1}^{k} \frac{1}{i^2} < 2 - \frac{1}{k}$ is true

Adding $\frac{1}{(k+1)^2}$ both sides

$$S_{k+1} = S_k + \frac{1}{(k+1)^2} \leq \left(2 - \frac{1}{k}\right) + \frac{1}{(k+1)^2}$$

we know that S_{k+1} would be

$$S_{k+1} = \sum_{i=1}^{k+1} \frac{1}{i^2} \leq \left(2 - \frac{1}{k+1}\right)$$

Question assigned to the following page: 1

So, if we prove that

$$\left(2-\frac{1}{k}\right)+\frac{1}{(k+1)^2} \leq 2-\frac{1}{k+1}$$

it will prove $S_{k+1} \leq 2-\frac{1}{k+1}$

$$\cancel{2}-\frac{1}{k}+\frac{1}{(k+1)^2} \leq \cancel{2}-\frac{1}{k+1}$$

$$\frac{1}{(k+1)^2} \leq \frac{1}{k}-\frac{1}{k+1}$$

$$\frac{1}{(k+1)^2} \leq \frac{1}{k(k+1)}$$

We know that $k(k+1) \leq (k+1)^2$

So, $S_{k+1} \leq 2-\frac{1}{k+1}$

therefore $S_k \leq 2-\frac{1}{k} \leq 2$

$$S_k \leq 2 \quad \forall k>1$$

Hence proved

Question assigned to the following page: 2.1

Q2 Show bijection :

(a) X is a finite Set

$A = \{ S \subset X : |S| \text{ is even} \}$ and $B = \{ S \subset X : |S| \text{ is odd} \}$

Assuming prof meant ~~$A \subseteq B$~~ $S \subseteq X$ Question is Done ahead

Given: $A = \{ S \subset X : |S| \text{ is even} \}$
$B = \{ S \subset X : |S| \text{ is odd} \}$

Since its mentioned $S \subset X$ in both cases the Question in itself does not make much sense because -:

1. If $|X|$ is even, let $|X| = n$
   Set A will have $2^{n-1} - 1$ elements
   Set B will have 2^{n-1} elements

2. If $|X|$ is odd, let $|X| = n$
   Set A will have 2^{n-1} elements
   Set B will have $2^{n-1} - 1$ elements

So by observation, here $|A| > |B|$ OR $|A| < |B|$

By definition of bijection, a function needs to be both one-one & onto, So when $|A| \neq |B|$,

if $|A| > |B| \longrightarrow$ Not Injective (onto)

if $|A| < |B| \longrightarrow$ Not Surjective (one-one)

Hence bijection not possible

$\Rightarrow$ Assuming that orignal Question was

$A = \{ S \subseteq X : |S| \text{ is } \cancel{\text{odd}} \text{ even} \}$

$B = \{ S \subseteq X : |S| \text{ is odd} \}$

We know that A contains all even size Subsets, B contains all odd size subsets,

Let total no. of elements in X be n

So Powerset of X will have 2^n elements.

So, A will have 2^{n-1} elements

B will have 2^{n-1} elements

now we take some element i from set X

such that we can do bijection based on presence of i

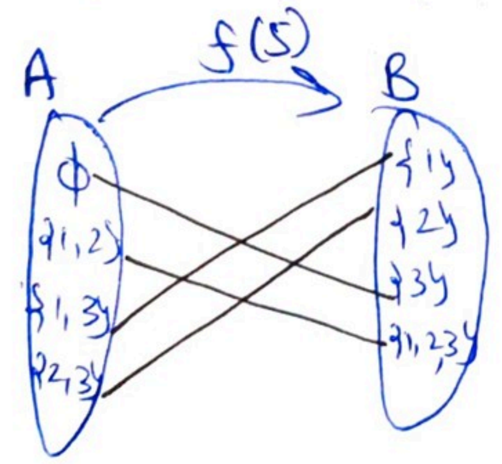
Let the function be $f(x)$ be the mapping ~~bijective~~ function between A and B

$$f(S) = S \Delta \{i\}$$

i.e., if $i \in S$, $f(S)$ maps to some set $S - \{i\}$

if $i \notin S$, $f(S)$ maps to some set $S + \{i\}$

Example $S = \{1, 2, 3\}$



let $i = 3$

Verification | Correctness

$f: A \rightarrow B \quad f(S) = S \Delta \{i\}$

when $|S|$ is even, $|f(S)| = |S| \pm 1$ (Remove or add i)
which is odd

Similarly if $|S|$ is odd $|f(S)| = |S| \pm 1$
which is even

therefore $f(S) \in B$ when $|S|$ is even
and ~~sf~~ preimage of $f(S)$ belongs to A when $|S|$ is odd

proof of ~~onto~~ one-one,
if $f(S_1) = f(S_2)$ (Assuming two elements in B have Same pre image)

then when we ~~toggle~~ remove/add i
it should form $S_1 = S_2$

Thus f must be one-one

for onto,

for every element with even cardinality, then

$\forall_K \; K \subseteq A, \; \underbrace{K \Delta \{i\}}_{\Downarrow} \Rightarrow |f(K)|$ will be odd.

$|K| \pm 1$, since K was even
$|K| \pm 1$ will be odd.

Since $|A| = |B|$,
Its one-one, onto
hence it will be bijective

Question assigned to the following page: 4.1

4 (a)

A is infinite
B is countable

$A = \{a_1, a_2, a_3 \dots\}$
$B = \{b_1, b_2, b_3 \dots\}$

There will be bijection A to $A \cup B$

~~let set $A \cup B$ be~~

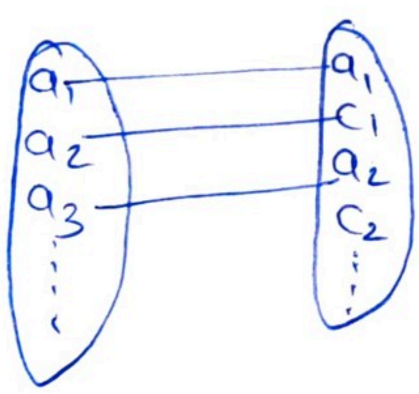
Let $C = B - A$, it will contain all unique elements of B which are not in A

So, $A \cap C = \phi$
$A \cup C = A \cup B$

$$\left.\begin{aligned} x_i &= \cancel{2x_i}\ x_{2i} \text{ if } x_i \in A \\ x_i &= x_{2i+1} \text{ if } x_i \in C \end{aligned}\right\} (A \cup C)$$

~~$f(x) = x.$~~

$$f(x) = \begin{cases} a_{\lceil i/2 \rceil}, & i = \text{odd} \\ c_{i/2} & i = \text{even} \end{cases}$$



This mapping is injective as no two inputs map to same output

~~As~~ $f : (A \cup B) \longrightarrow A$

$A = \{a_1, a_2, a_3 \dots\}$
$B = \{b_1, b_2, b_3 \dots\}$

$f(b_j) = a_{2j+1}$ $f(a_i) = a_{2i}$

Question assigned to the following page: 4.1

Since ?! there is a mapping (injective)
from $A \rightarrow A \cup B$
and
from $A \cup B \rightarrow A$ also injective we can say
that it will bijective

Question assigned to the following page: 5.1

Assignment-1

Q5 (a) If m^2 is Multiple of n, then m is also multiple of n. Here, m and n are +ve integers. You can use N to denote set of positive integers and predicate $P(x,y)$ denote x divides y

$P(m,n) \equiv n \mid m$ (n divides m) or (m is multipe of n)

Given: $m, n \in N$,

Statement: $\forall m \forall n\ (P(m^2,n) \rightarrow P(m,n))$

(i) Its negation will be: $\exists m \exists n\ P(m^2,n) \wedge \sim P(m,n)$

$\sim[\forall m \forall n\ (P(m^2,n) \rightarrow P(m,n))]$

$\exists m \exists n \sim [P(m^2,n) \rightarrow P(m,n)]$

$\exists m \exists n \sim [\sim P(m^2,n) \vee P(m,n)]$

$\exists m \exists n\ \sim(\sim(P(m^2,n))) \wedge \sim P(m,n)$

$\boxed{\exists m \exists n\ P(m^2,n) \wedge \sim P(m,n)}$

(ii) $\forall m \forall n\ (P(m^2,n) \rightarrow P(m,n))$ is False

Idea to disprove: Every natural no. >1 can be broken into products of its Prime Factors

So $m = p_1^{a_1} \times p_2^{a_2} \cdots \times p_n^{a_n}$

Similarly $m^2 = p_1^{2a_1} \times p_2^{2a_2} \cdots \times p_n^{2a_n}$

and $n = p_1^{b_1} \times p_1^{b_2} \cdots \times p_n^{b_n}$

now, $n \mid m^2$ means Some $2a_i \geq b_i$

and if $2a_i \geq b_i$ then a_i may or maynot be $\geq b_i$

(counter Example next page)

Question assigned to the following page: 5.1

Counter Example

$n = 9$
$m^2 = 9$

$n | m^2 \rightarrow$ Yes 9 divides 9

$n | m \rightarrow$ No, 9 does not divide 3 yielding result $\in \mathbb{N}$

So Counter Example $\Rightarrow$ $m = 3$, $m^2 = 9$, $n = 9$

Since one counter Example Exists the statement $\forall m \forall n \, (P(m^2, n) \rightarrow P(m, n))$ is False as its not true for all values of (m, n)

4(b) M1

15

B is uncountable set
A is infinite

A to A∪B bijection does not Exist

Proof by Counter example

Let A = set of Natural No.
Let B is set of Real No.

4

A is Countably finite, B is uncountable

~~bute~~ but there is No bijection from
N → R therfore this statement is
false

Question assigned to the following page: 2.2

Q2(b)

A be length n seq. of digits [0, 1, ⋯ 9] whose sum is less than $9n/2$

B be length of n seq. of digits [0, 1, ⋯ 9] whose sum is more than $9n/2$ whose sum is $9n/2$

~~prove n is~~ Here n is positive integer

$A = a_1 a_2 a_3 \cdots a_n$ where $\forall_{a_i}\ a_i \in \{0, 1, \cdots 9\}$

$B = b_1 b_2 b_3 \cdots b_n$ where $\forall_i\ b_i \in \{0, 1, 2 \cdots 9\}$

A = Set of length n seq. whose sum $< 9n/2$
B = Set of length of n seq. whose sum $> 9n/2$

Let f(x) be Mapping function of A to B
such that

$\forall_k\ k \in A,\ f(k) =$ 9's complement of k

Since Sum of digits of $k < \frac{9n}{2}$

9's complement of $k = (\underbrace{999 \cdots 9}_{n \text{ times}}) - k$

$> \frac{9n}{2}$

So, $f(k) =$ Mapping from A to B where we are doing 9's complement of each.

proof of correctness/verification

$Sum(x) = S$

$S < \frac{9n}{2}$

$Sum(f(n)) = (\underbrace{999\ldots9}_{n \text{ times}} - n)$

OR

$9n - S$ which will be $> \frac{9n}{2}$

So $f(n) \in B$

Applying $f(n)$ ~~twiche~~ twice we get n

$f(f(n)) = n$

therefore its both injective & surjective

Question assigned to the following page: 5.2

Q.5 (b)
Every Subset of Real numbers which has a lower bound ~~has a lower bound~~ has a least element.
A subset S has lower bound if there is a number which is smaller than all the numbers in S

Let S be a Subset of R $[S \subseteq R]$
$P(a, S)$: a is lower bound of S
$Q(a, S)$: a is least element of S

Statement : $\forall S,\ S \subseteq R\ [\exists a\ P(a, S) \rightarrow \exists b\ Q(b, S))$

$\Rightarrow \forall S,\ S \subseteq R\ [(\exists a\ \forall x \in S\ (a \leq x)) \rightarrow (\exists b \in S\ \forall x \in S\ (b \leq x))]$

(i) Negation : $\exists S\ [(\exists a \forall x\ (a \leq S)) \wedge (\forall b\ \exists x\ (b > x))]$

$\Rightarrow \sim [\forall S\ [(\exists a\ \forall x\ (a \leq S)) \rightarrow (\exists b\ \forall x\ (b \leq x))]]$

$\Rightarrow \exists S\ \sim(\sim(\exists a\ \forall x\ (a \leq S))) \wedge \sim(\exists b\ \forall x\ (b \leq x))]$

$$\Rightarrow \boxed{\exists S [(\exists a\ \forall x\ (a \leq S)) \wedge (\forall b\ \exists x\ (b > x))]}$$

(ii) Disprove : L.H.S.
Let S be in Range (α, β)
$\forall a,\ a \in R\ \exists b\ b = (a-1)$ such that b is lower bound of S
So, $\exists a\ P(a, S)$ is always true
RHS $\exists b\ Q(b, S)$,
$\exists b\ \forall x\ x \in S,\ b \leq x$ $\quad \cancel{\alpha \leq b \leq \beta}$ $\quad \frac{\alpha+b}{2} < b,\ \frac{\alpha+b}{2} \in R$
Since S is in (α, β) $\quad \alpha < b < \beta,\ \forall \frac{\alpha+b}{2}$

Hence $\frac{\alpha+b}{2}$ is always $< b$ and greater than α

So, $\exists_b Q(b,S)$ is false

and

$\forall_S \left(\exists_a P(a,S) \rightarrow \exists_b Q(b,S)\right)$ becomes false

$(T \rightarrow F \Rightarrow F)$

<u>Proof by Counter Example :</u>

Let $S \in (0,1)$

$\exists_a P(a,S) \Leftrightarrow \exists_{(0\text{-}1)} P(-1,S)$ True

$\Rightarrow -1 \leq x, \forall_x\ x \in S$ ~~$-1 \leq x \forall_x\ x \in S$~~

$\exists_b Q(b,S) \Leftrightarrow \exists_{\frac{\alpha+b}{2}}, \frac{\alpha+b}{2} \in R$ and

$\frac{\alpha+b}{2} < b$

Let $\alpha = 0,\ \beta = 1$

$b = \frac{0+1}{2} = 0.5,\ b \in R$

$b = \frac{0.5+0}{2} = 0.25,\ b \in R$

$b = \frac{0.25+0}{2} = 0.125,\ b \in R$

$\vdots$

So on, $\forall_b, \exists_c\ .\ c = \frac{\alpha+b}{2}, \alpha < c < b$

Q 2(c) 8

A - Contains inf length string over $\{1, 2, 3\}$

B - Contains inf length string over $\{4, 5\}$

Let the function be defined as prefix replacement of encoding of strings

~~Let, 1 denote to prefix 45~~
~~2 denote to prefix 54~~
~~3 denote to prefix 44 or 55~~ (will fail) 333 ....

Let 1 denote 45
2 denote 44
3 denote 5

now our $f(x)$ can read first symbol and replace prefix with $\{1, 2, 3\}$

$$f(x) \overset{\text{Read next}}{=} \begin{cases} 4, & \text{Read next} \begin{cases} 4, \rightarrow 44 \text{ replace by } 2 \\ 5, \rightarrow 45 \text{ replace by } 1 \end{cases} \\ 5, & \text{replace by } 3 \end{cases}$$

So for every encoding over $\{1, 2, 3\}$ will have exactly one mapping over set B of encodings over $\{4, 5\}$

So $f : A \rightarrow B$ is injective

Now, for $f : B \rightarrow A$

Let 4 denote $\rightarrow$ 1
54 denote $\rightarrow$ 2
55 denote $\rightarrow$ 3

Question assigned to the following page: 2.3

So, we can map all encodings over $\{4,5\}$ to set A of encodings over $\{1,2,3\}$

$$f(n) \begin{cases} 1 & \text{, replace by } 4 \\ 2 & \text{, replace by } 54 \\ 3 & \text{, replace by } 55 \end{cases}$$

6

this way, every encoding over $\{4,5\}$ will have exactly one mapping over A of encoding over $\{1,2,3\}$

So, $f: B \rightarrow A$ is also injective

Since $f: A \rightarrow B$ is injective & $f: B \rightarrow A$ is also injective, we can say that $f: A \rightarrow B$ is bijective function.

Q 4(c)

A real number is called Quadratic when it is a root of a degree two polynomial with integer coefficients, the number of Quadratic reals is countable

Let the Quadratic equation be $ax^2+bx+c=0$

$a, b, c \in Z$ aur $a \neq 0$

To prove : No. of Quadratic reals are countable

~~No~~ with 3 variables each equations solutions can be mapped from (a,b,c) to (S_1, S_2)

where S_1, S_2 are possible solutions.

for any equation $ax^2+bx+c=0$,

we can have -:

→ 2 solutions
→ 1 solution (both same)
→ No solution

So $(a,b,c) \in Z^3$

Z^3 is also countable

and each (a,b,c) pair can have finitely many sol's. (≤ 2)

So Union of ~~countable~~ countably finite sets is also countable

∴ hence No. of Real Quardratics are countable

Question assigned to the following page: 3

01)

Q3

(1)

Prove $\log_7 n$ is either integer or an irrational number when n is Positive integer

Proof by Contradition
Lets Assume

$\log_7 n$ is Rational but not integer

$$\log_7 n = \frac{a}{b} \quad , \quad \frac{a}{b} \in \mathbb{Q} \overset{\text{and}}{\wedge} \frac{a}{b} \notin \mathbb{Z}^+$$

taking 7 as base both side, doing exbonent

$$7^{\log_7 n} = 7^{a/b}$$

by prop of log $a^{\log_b n} = \cancel{b}\ \cancel{n^{\log_a b}}\ n^{\log_b a}$

$$n^{\log_7 7} = 7^{a/b}$$

$$n = 7^{a/b}$$

$$n^b = 7^a$$

Since RHS is 7 and LHS can be brokeninto prime factors multiple

$$n = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n}.$$

but LHS has only 7 so, n can be written as 7^k for some $k > 0$, $k \in \mathbb{Z}^+$

$$n^b = (7^k)^b$$

$$\Rightarrow (7^k)^b = 7^{(a)}$$

Question assigned to the following page: 3

Removing power both side

$kb = a$

from ~~from~~ $\log_7 n = \frac{a}{b}$, substitute $a = kb$

$\log_7 n = \frac{kb}{b}$

$\log_7 n = k$ — Contradiction found,

~~So, k is some integer for $n = 7^k$~~

therefore k must be some integer when in $\log_7 n$

n is some power of 7 otherwise must be irrational.

Question assigned to the following page: 5.3

Question 5 (c)

If a positive no. n is composite then, it has a factor (Other than 1) which is at Most $\sqrt{n}$. Here you can use N to denote the set of positive numbers and the predicate $P(x,y)$ indicating x divides y.

Statement: $\forall n,\ n \in N\ [(\exists a\ \exists b\ (a>1 \wedge b>1 \wedge n/a=b) \Rightarrow$ ($a, b \in N$)

$(\exists a\ a \in N,\ (a>1 \wedge P(a,n) \wedge a \leq \sqrt{n})]$

$P(a,n)$: a divides n

Negation: $\exists n\ [\exists a\ \exists b\ a,b \in N (a>1 \wedge b>1 \wedge n/a=b)$

$\wedge \sim [\exists a\ a \in N,\ a>1 \wedge P(a,n) \wedge a \leq \sqrt{n})]$

$\Rightarrow \boxed{\exists n\ [\exists a\ \exists b\ a,b \in N\ ((a>1) \wedge (b>1) \wedge (n/a=b)) \wedge \forall a\ (a<1 \vee \sim P(a,n) \vee a>\sqrt{n})]}$

Proof by Contradiction

Assume $\exists a\ \exists b\ a,b \in N,\ a>\sqrt{n},\ b>\sqrt{n},\ \frac{n}{a}=b$

So $a>\sqrt{n},\ b>\sqrt{n}$

$\frac{n}{a}=b \Leftrightarrow a \cdot b = n$

$a>\sqrt{n},\ b>\sqrt{n}$

So $a \cdot b > \sqrt{n} \cdot \sqrt{n}$

$a \cdot b > n$

which is contradiction of $a \cdot b = n$ or $\frac{n}{a} = b$