Capacity of Three-Dimensional Scale Free Wireless Networks

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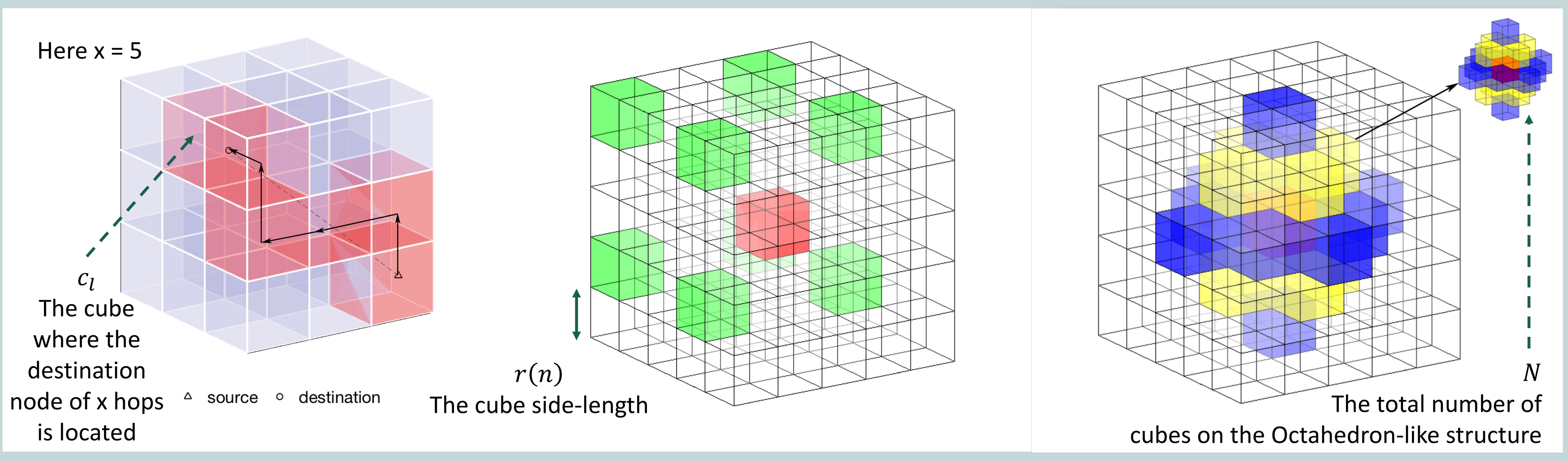
Introduction

- Communications occur increasingly on the wireless mobile terminal in recent years. When the wireless network has **social characteristics**, the probability of communication and the construction of social groups are subject to the social behavior of users, which will affect the original wireless network capacity.
- The **scale free** characteristic of social network will divide nodes into **two classes with obvious different attributes**: one is called "Social-Butterfly" node with large connectivity degree, the other is called normal node correspondingly.
- The social wireless network currently was mostly considered as a 2-D flat network. However, the network will expand from the plane to the **three-dimensional space**, especially in the context of the rapid development of drones in recent years.

3-D System Model

- **Interference Model:**
  Ensure the Network Connectivity
- **Multi-hop Straight-line Routing Strategy**
- **TDMA Transmission Model**
- **Throughput Definition**
- **Multi-hop Representation of 3-D Conditions**



Social Features Parameters

- **γ - Number of Members with Scale-Free Feature:** Assume that each source node S has a social group G and the number of Gs members is a random variable Q. Thus, the probability of group G has $q(q = 1,2,\dots,n-1)$ members is:

$$\Pr\left(Q=q\right)=\frac{q^{-\gamma}}{\sum_{q=1}^{n-1}q^{-\gamma}}=\frac{q^{-\gamma}}{\sigma_1\left(\mathbf{q}\right)},$$

  where γ is the clustering factor related to specific network.

- **α - Membership with Distance-based Power-Law Feature:** S selects any other nodes as its G's member with a Distance-based Power-Law distribution probability. α is the social concentration factor. d_i is the distance between S and node o_i. The probability that G consists of nodes $o_{g_1}, o_{g_2}, \dots,\ o_{g_q}$ is:

$$\Pr\left(G=\left\{o_{g_1},\dots,o_{g_q}\right\}\right) = \frac{d_{g_1}^{-\alpha}\dots d_{g_q}^{-\alpha}}{\sum_{1\le i_1<\dots<i_q\le n}d_{i_1}^{-\alpha}\dots d_{i_q}^{-\alpha}}$$

  The probability of an arbitrary node o_k being a member of G is:

$$\Pr\left(o_k\in G\right)=\frac{d_k^{-\alpha}\sigma_{q-1}\left(\mathbf{d}_\mathbf{n}^{\bar{\mathbf{k}}}\right)}{\sigma_q\left(\mathbf{d}_\mathbf{n}\right)}$$

- **β - Nodes Communications:** The probability of S chooses which of its group members to communicate based on distance and renders a Power-law distribution. β reflects the communication activity level. So the probability of o_k is the destination node D in G is:

$$\Pr\left(D=o_k\left|o_k\in G\right.\right)=\frac{d_k^{-\beta}}{\sum_{i=1}^{q}d_i^{-\beta}}=\frac{d_k^{-\beta}}{\sigma_1\left(\mathbf{d}_\mathbf{q}\right)}$$

Main References

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Main Results

- **Capacity of "Social-Butterfly" Nodes:**

$$\lambda_{\max 1}=\begin{cases}\Theta\left(\dfrac{n^{\frac{\beta-1}{3}}}{\log^{\frac{\beta+2}{3}}(n)}\right) & 0\le\beta\le 1\\ \Theta\left(\dfrac{1}{\log(n)}\right) & \beta>1\end{cases}$$

- **Capacity of Normal Nodes:** Notice that all scale free networks are sparse in reality, namely, $\gamma > 2$.

α / $\alpha+\beta$	$0\le\alpha<3$	$\alpha=3$	$\alpha>3$
$\gamma>1$			
$0\le\alpha+\beta<2$	$\Theta\left(r^{\beta-1}(n)\right)$	-	-
$\alpha+\beta=2$	$\Theta\left(r^{3-\alpha}(n)\ln\left(r^{-1}(n)\right)\right)$	-	-
$2<\alpha+\beta<4$	$\Theta\left(r^{\beta-1}(n)\right)$	$\Theta\left(r^{\beta-1}(n)\ln^{-1}\left(r^{-1}(n)\right)\right)$	$\Theta\left(r^{\alpha+\beta-4}(n)\right)$
$\alpha+\beta=4$	$\Theta\left(r^{3-\alpha}(n)\ln\left(r^{-1}(n)\right)\right)$	$\Theta(1)$	$\Theta\left(\ln\left(r^{-1}(n)\right)\right)$
$\alpha+\beta>4$	$\Theta\left(r^{3-\alpha}(n)\right)$	$\Theta\left(\ln^{-1}\left(r^{-1}(n)\right)\right)$	$\Theta(1)$

- **Result of Threshold q_0:** Here β is fixed and different α is taken into consider. q_0 is the threshold to separate nodes into two parts, which is discovered **66.7%** of the maximum q for all nodes. We find that q_0 increases with n and decreases with α.

n / q_0	100	200	500	1000	2000	5000	10000
$\alpha=1$	20	50	100	270	480	1000	2000
$\alpha=2$	12	30	60	110	200	380	500
$\alpha=3$	8	12	25	40	60	90	130

Idea of the Proof of Capacity

- The upper bound of the capacity λ_{max}:

$$\lambda_{\max}=\Theta\left(\frac{W}{nE\left[X\right]\left(Mc_1r\left(n\right)\right)^3}\right)=\Theta\left(\frac{1}{E\left[X\right]\log n}\right)$$

- X is the number of hops. $E[X] = \sum x\,P(X = x)$. A large number q_0 divides hops into two categories: $E_1[X]$ ($q_0 \ge q$), $E_2[X]$ ($q_0 < q$). Take $E_1[X]$ as an example:

$$E_1\left[X\right]\equiv\sum_{x=1}^{\frac{1}{r(n)}}x\sum_{l=1}^{N}\sum_{o_k\in c_l}\sum_{q=q_0}^{n}\frac{q^{-r}d_k^{-\alpha-\beta}\sigma_{q-1}\left(\mathbf{d}_\mathbf{n}^{\bar{\mathbf{k}}}\right)}{\sigma_1\left(\mathbf{q}\right)\sigma_1\left(\mathbf{d}_\mathbf{q}\right)\sigma_q\left(\mathbf{d}_\mathbf{n}\right)}$$

$$E_1\left[X\right]\equiv\left(r\left(n\right)\right)^3\int_1^{\left\lceil\frac{1}{r(n)}\right\rceil+1}\left(v^{3-\beta}+v^{1-\beta}\right)dv$$

  Then we can calculate the theorem of capacity λ_{max1}. With the same method $E_2[X]$ and λ_{max2} can be derived.
- Math Method：Strong law of LLN; Riemann-Stieltjes integral