Internship
Temporal Matching in Link Streams

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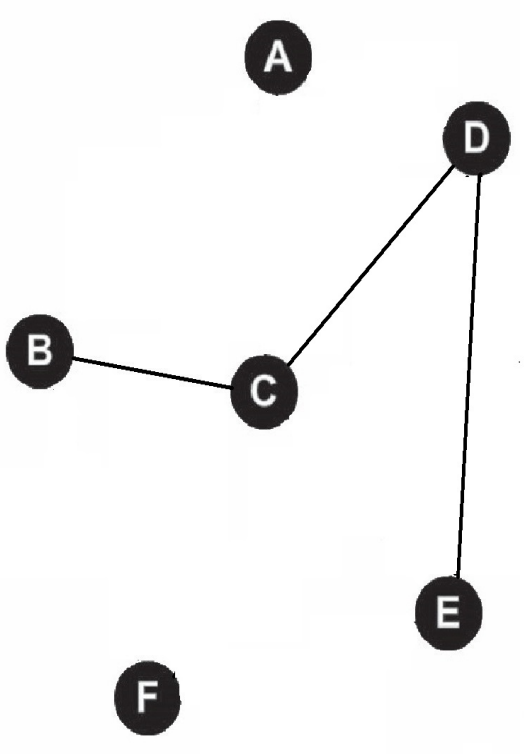
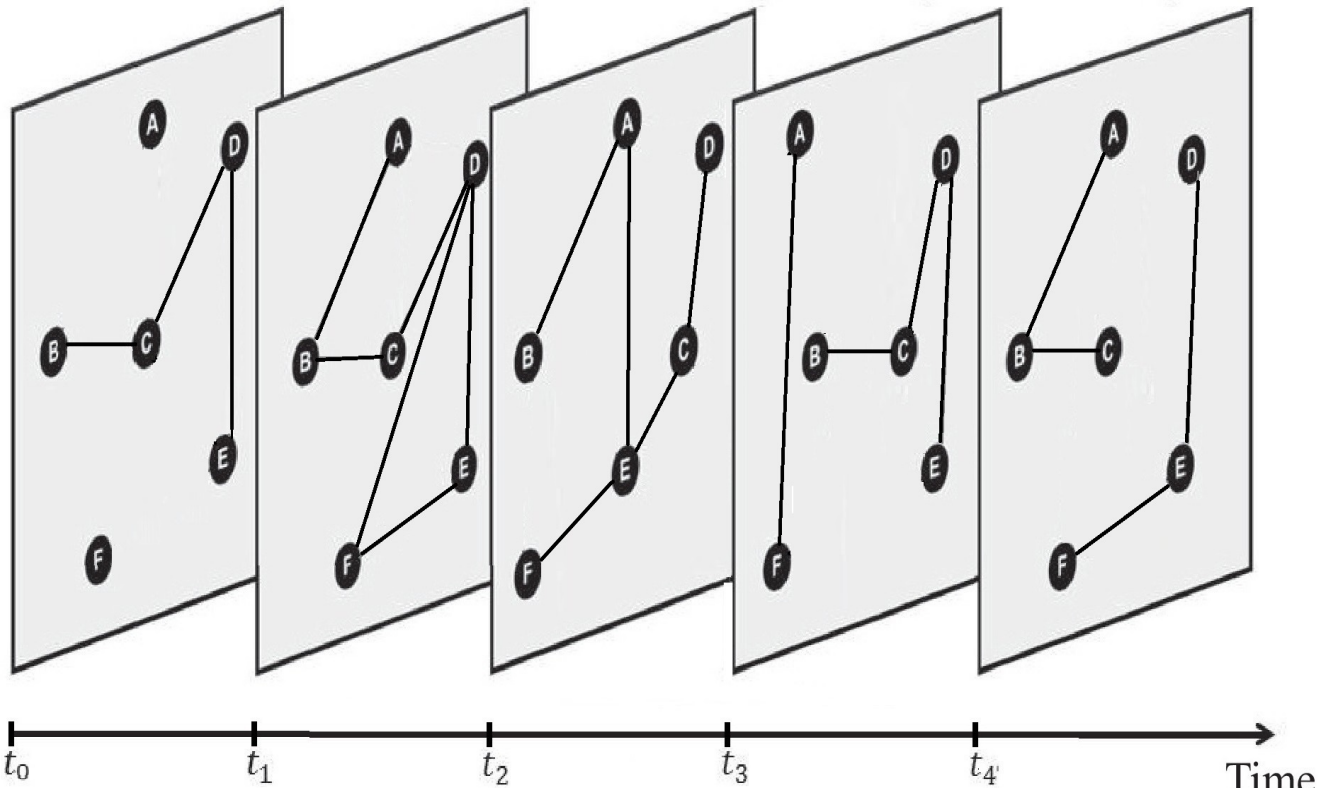
Overview

Temporal graph

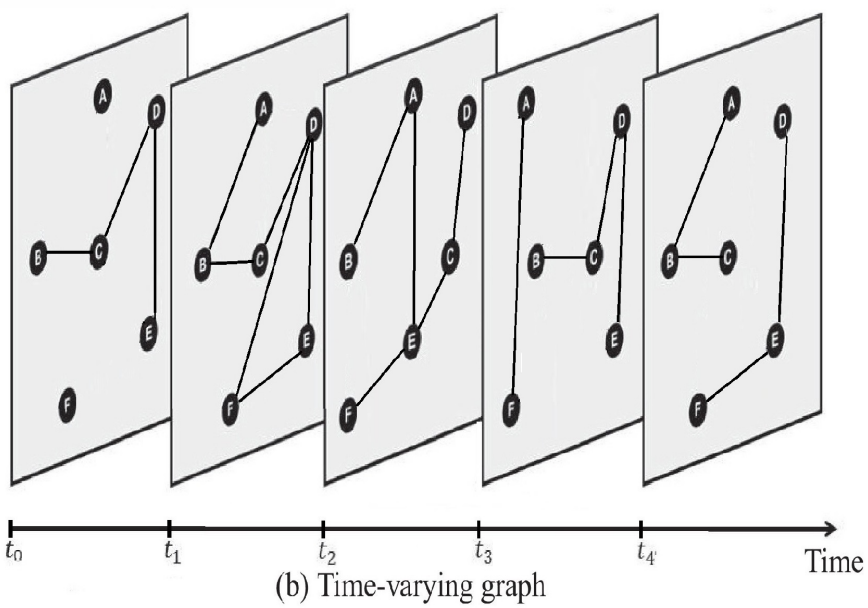


(a) Graph

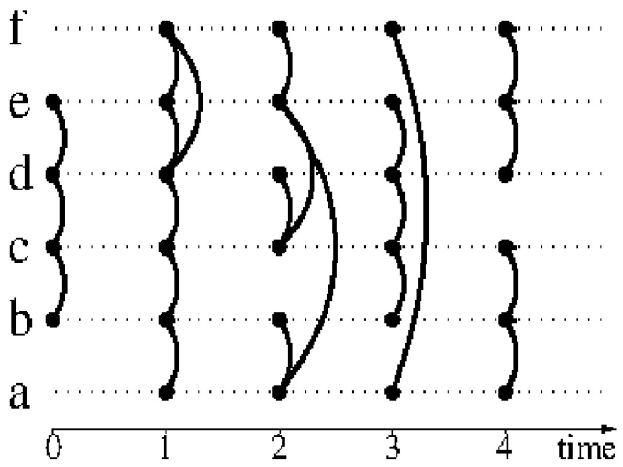


(b) Time-varying graph

Link stream



(b) Time-varying graph



Link stream

Link stream

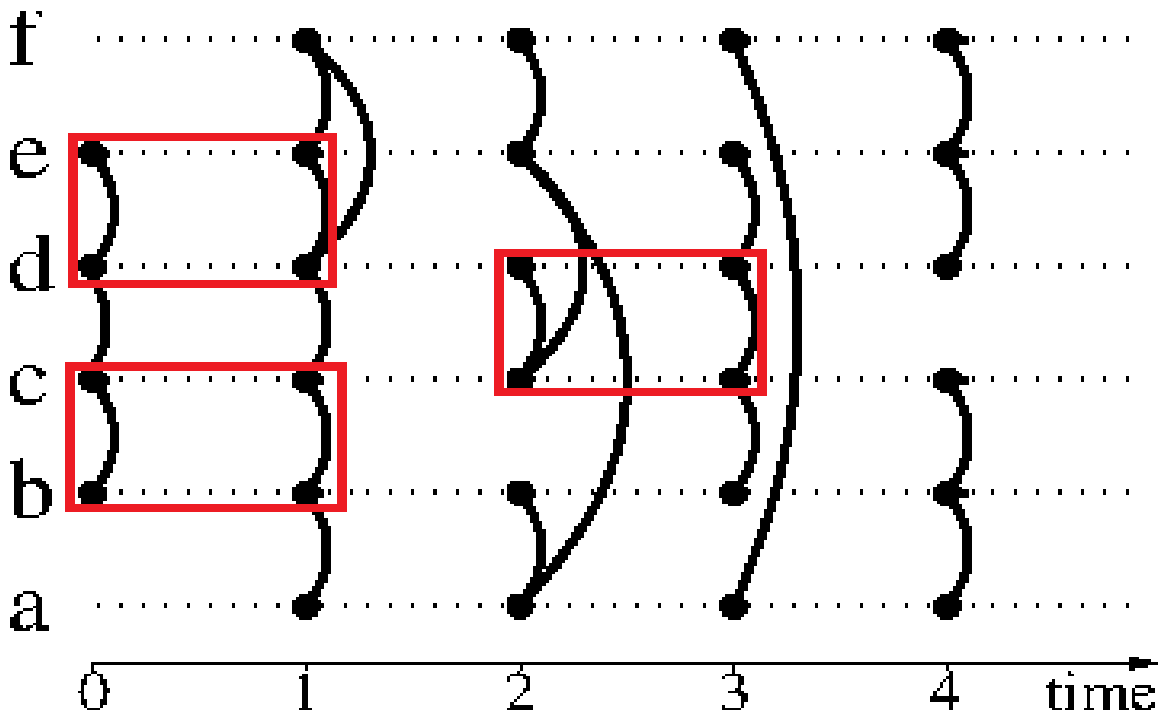
Definition: Let $L = (V,T,E)$:

- V set of vertices in the graph
- T $\subseteq$ N an integer represents a discretized time instant
- E $\subseteq \{(t, uv) : t \in T \wedge u \in V \wedge v \in V \wedge u \neq v\}$

Note :

- A graph G is link stream if $\mid T \mid = 1$
- Link stream is graph if every pair $(t, u, v) \in L$ satisfies t $= t_0$ for same fixed t_0.

Link stream, γ-edges and γ-matching



Temporal matching

Let $L = (V,T,E)$ a link stream and γ an integer
A γ-edges is set of repeated edges between vertices u and v starting at t and ending at t + γ - 1: $\{(t', \{u,v\}) \mid t' \in [t, t+\gamma-1] \}$.

Two γ-edges $\Gamma_1 = (t_1, u, v)$ and $\Gamma_2 = (t_2, u', v')$ are independent if $t_1 > t_2 + \gamma - 1$. $(t_1 < t_2 + \gamma - 1)$ where $u \neq u'$ and $v \neq v'$.

Temporal matching

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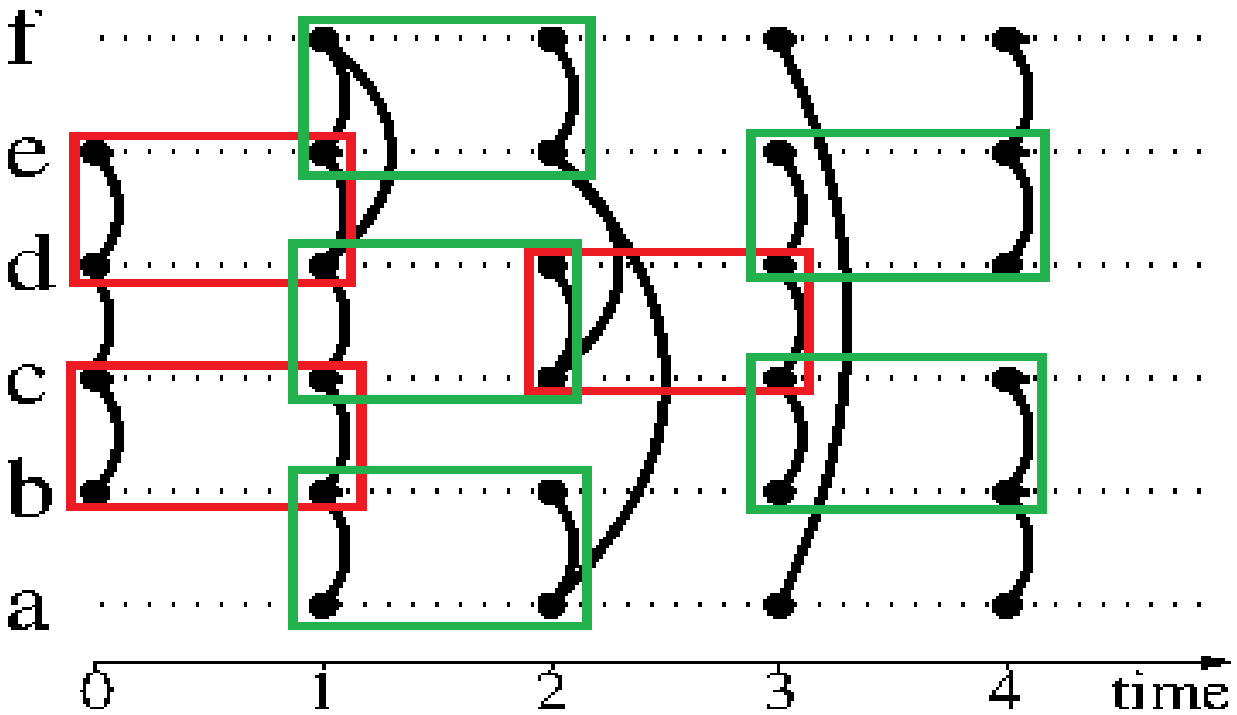
Temporal matching

Definition: A γ-matching is a set of independent γ-edges.

Our question: find maximum γ-matching ?



Our question: find maximum γ-matching ?

Problem γ-matching:

INPUT: link stream L, integer γ
OUTPUT: γ-matching in L of size k

Theorem [Baste et al 2019]

γ-matching is NP-hard for $\gamma > 1$
2 approximation algorithm (BBR19 greedy algorithm)

Our result

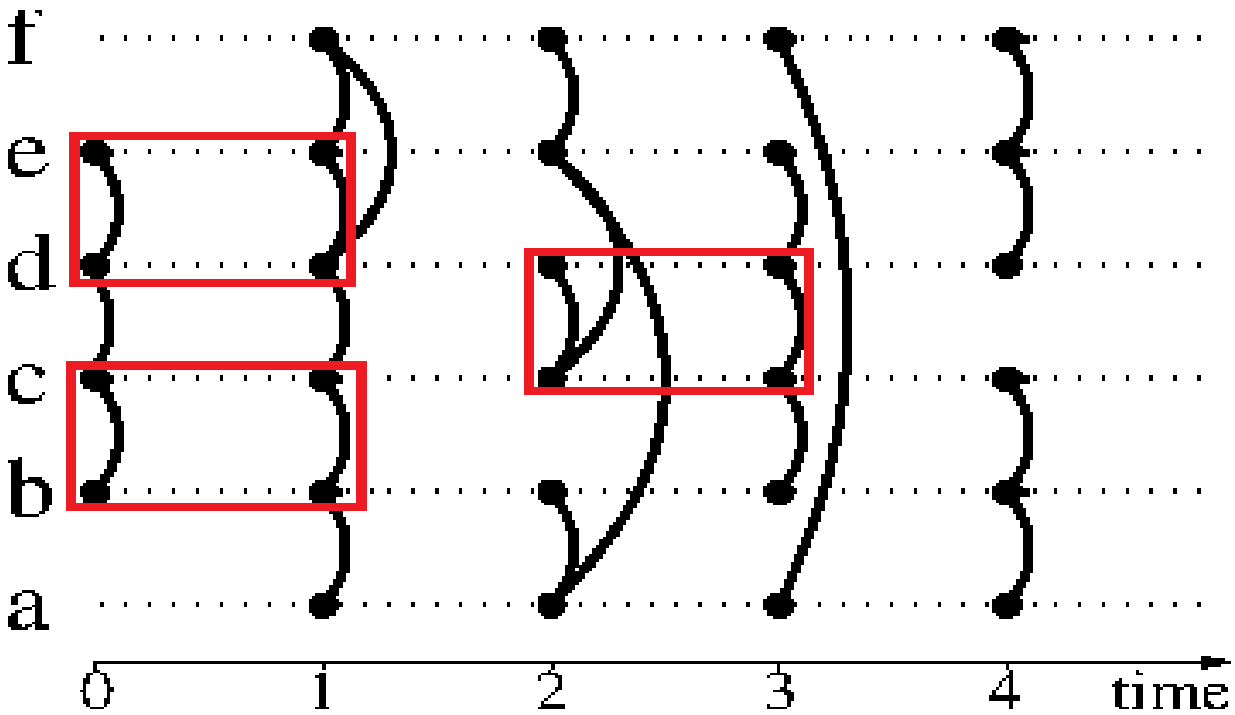
Approximation algorithms:

- Local search heuristic (LS)
- Divide and conquer heuristic (DC)

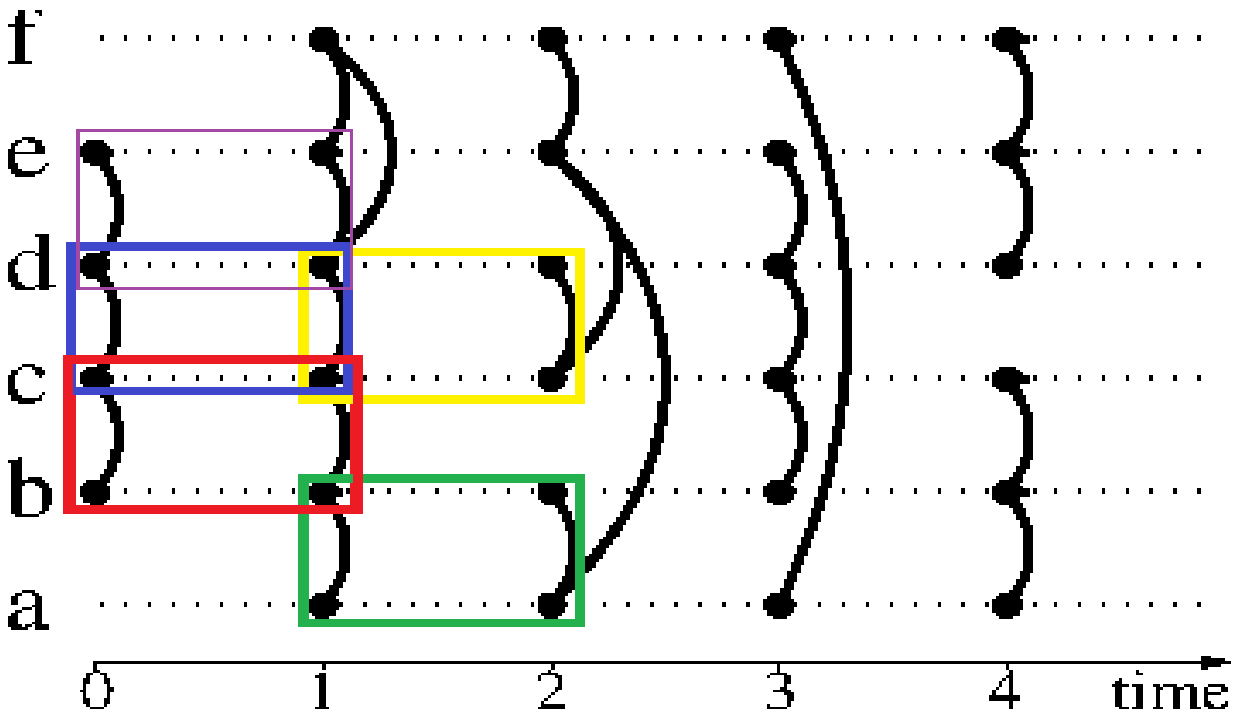
Numerical analysis: approximation

- Generated link stream (3000 tests): moving particles under natural simulation in 2D and 1D euclidean spaces.
- Real world link stream (1000 tests): scenario of Enron dataset.
- Real world linkbstream (1000 tests): scenario of Rollernet (Rollerblading tourning Paris) dataset.

Greedy algorithm (BBR19)



Local search heuristic (LS)



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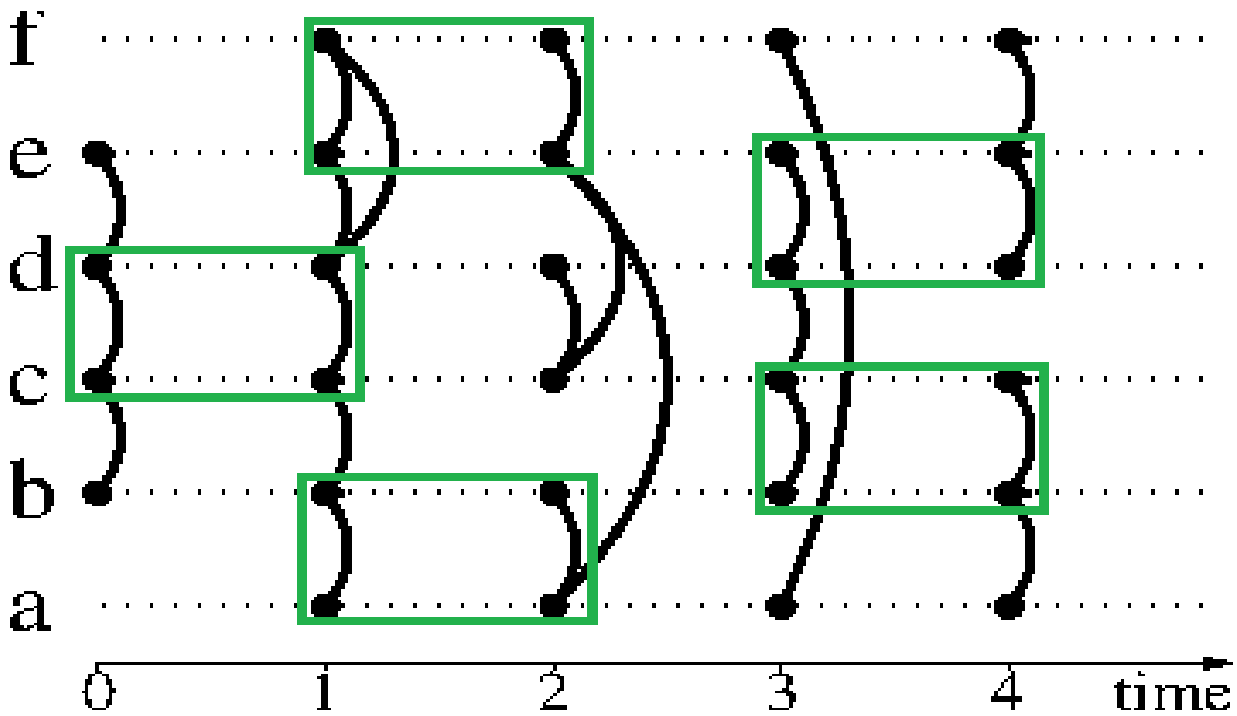
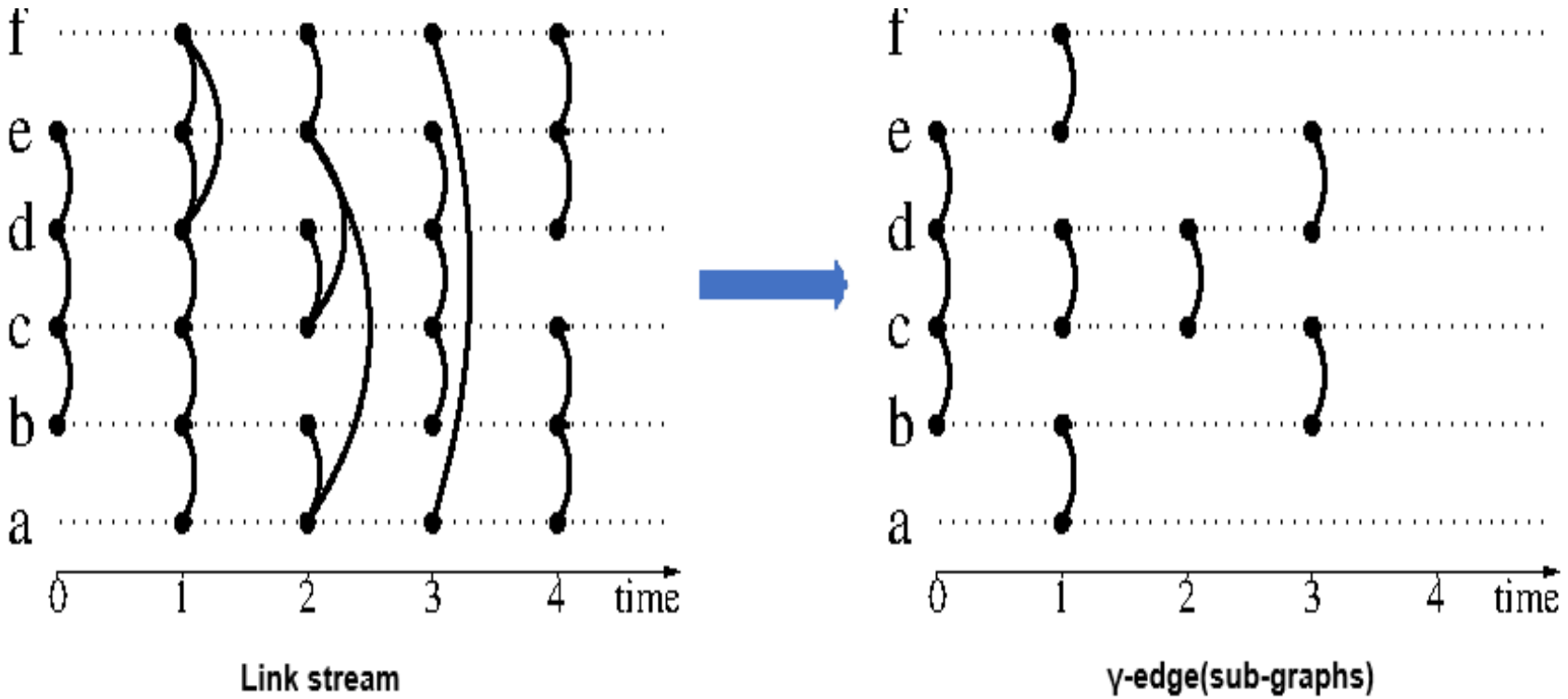


Divide and conquer heuristic (DC)



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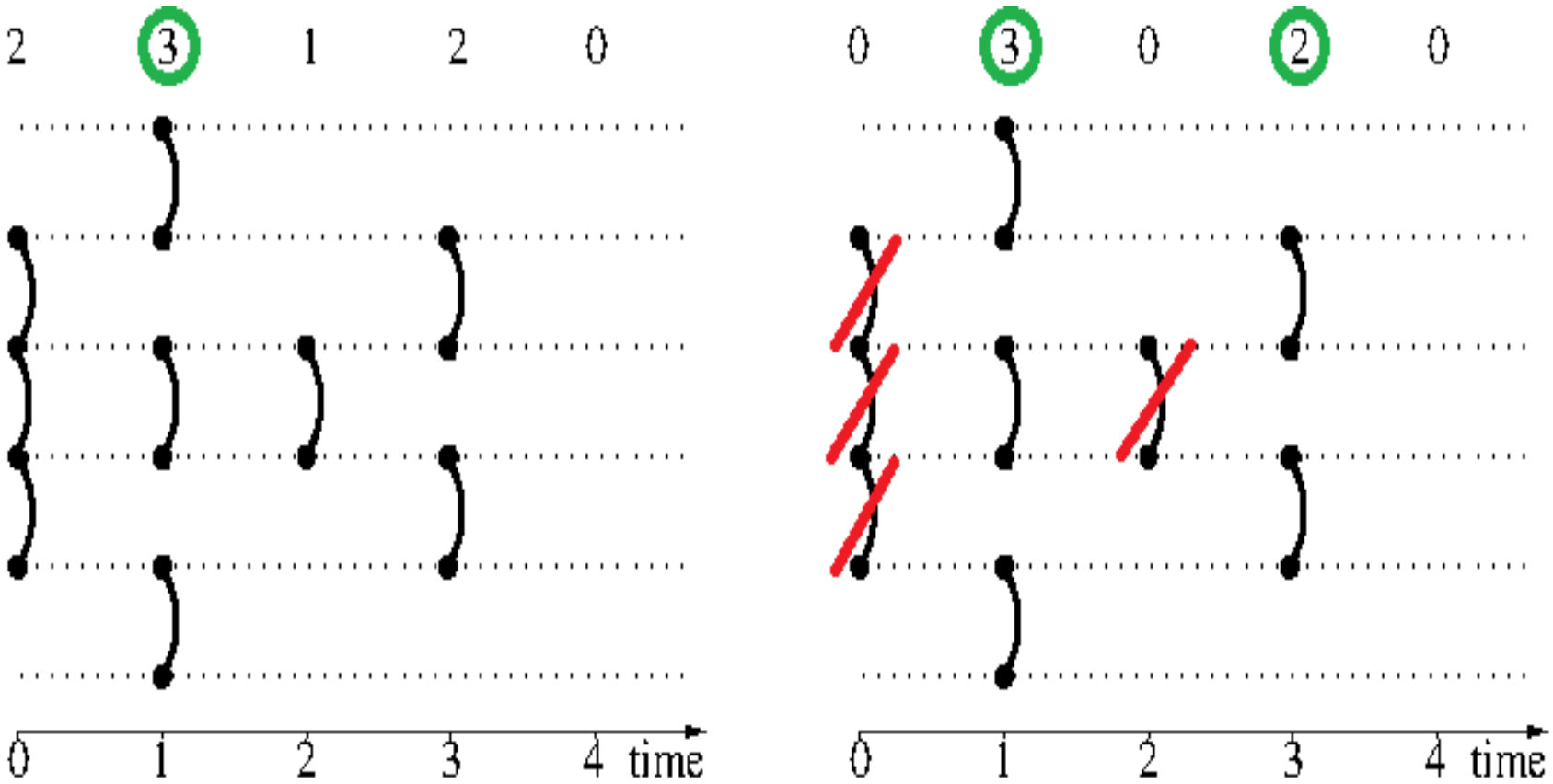
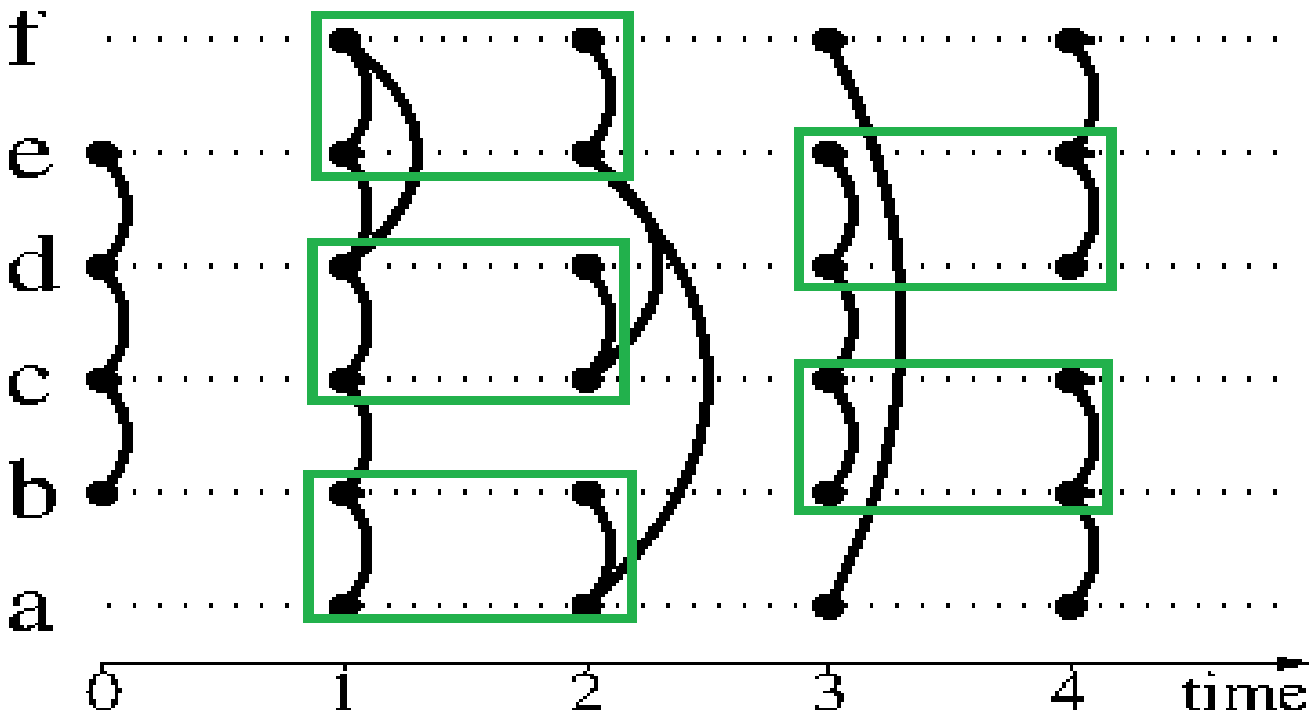


Divide and conquer heuristic (DC)



Discussion

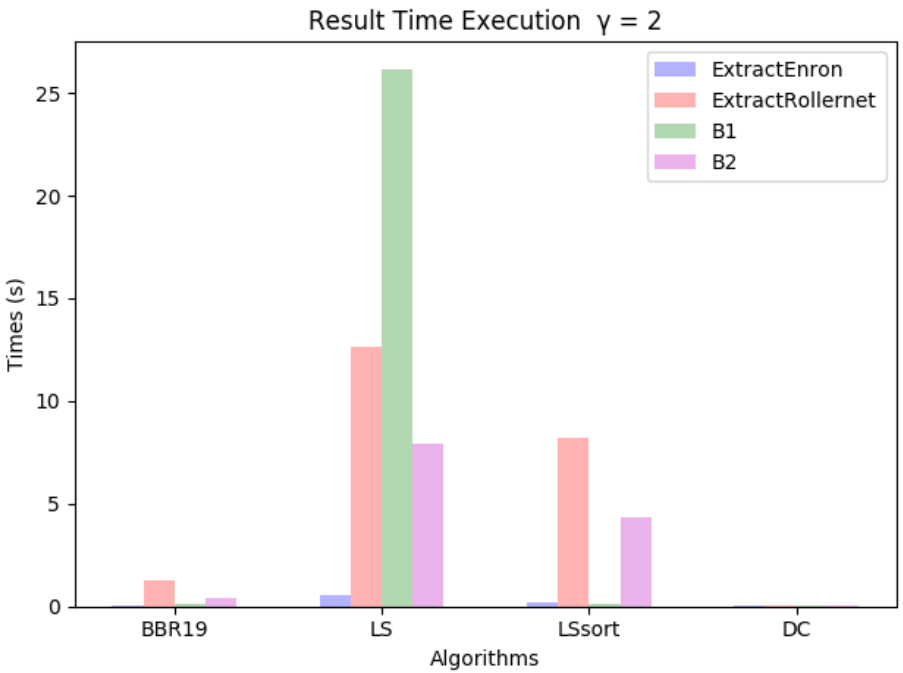
Algorithms

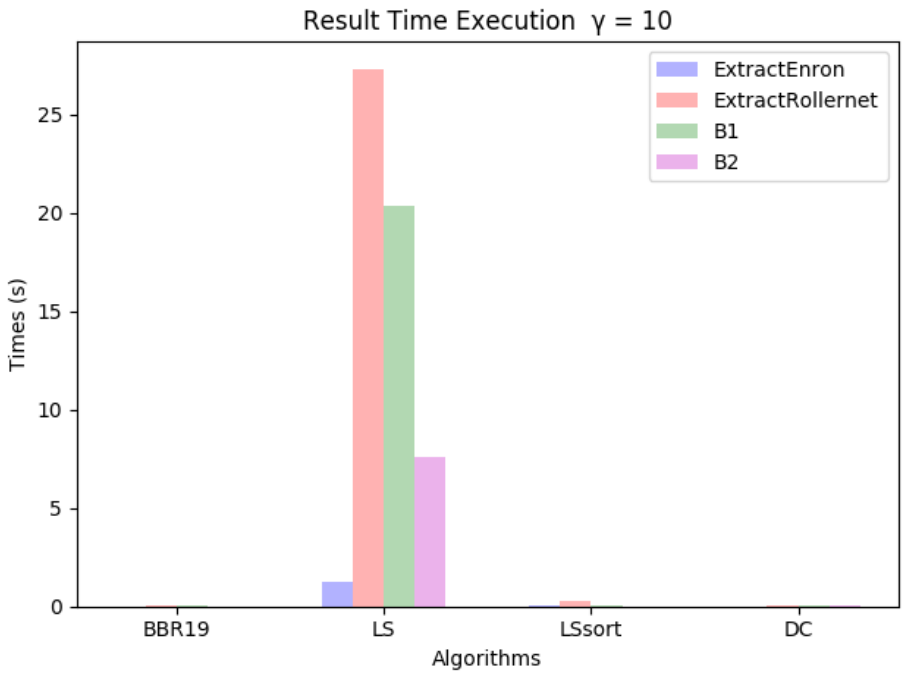
- BBR19 (J. Baste et al) in time $O(n\tau + m)$, where $\tau = |T|$, n $= |V|$ and m $= |E|$.
- LS (Local search heuristic) is very long.
- DC (Divide and conquer heuristic) on $O(m\ log\ m + (\text{m-}\tau')^2)$, where $1 < \tau' < \tau$

Note

The algorithms BBR19, LS and DC are greedy algorithms therefore 2-approximation algorithms)

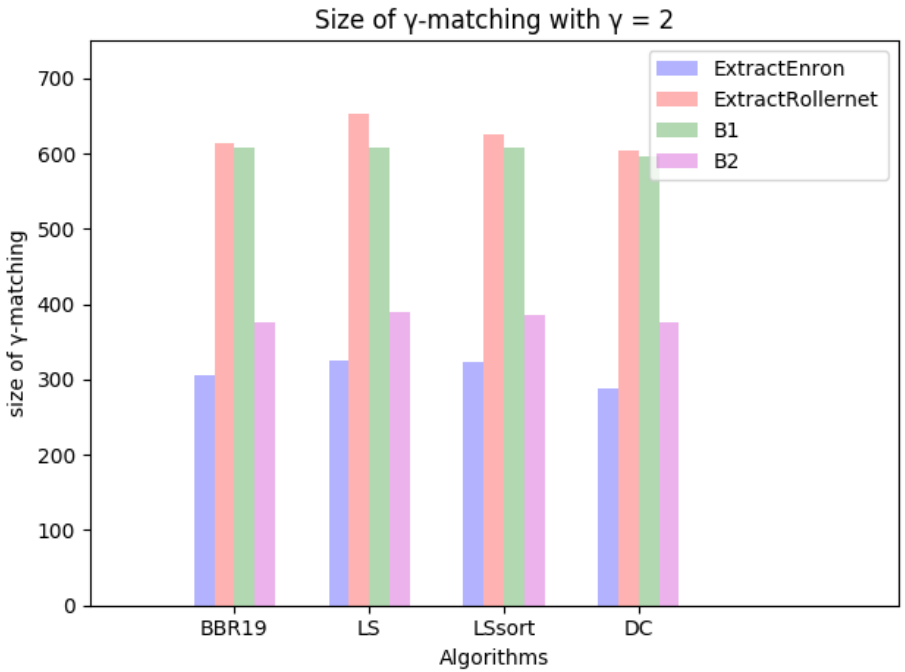
Discussion

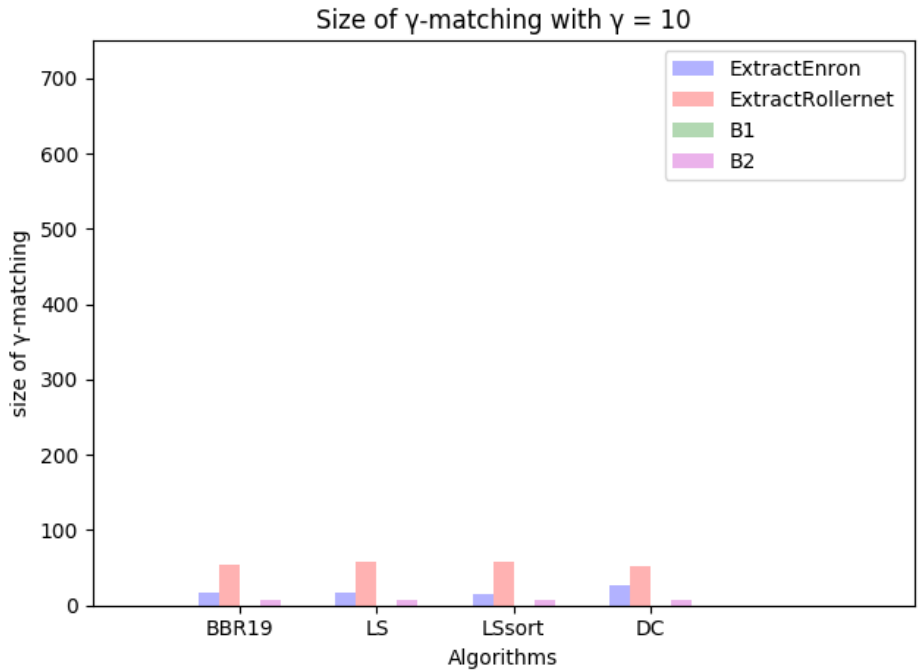




- BBR19 depends mainly on $|E|$ and $|\gamma\text{-edges}|$.
- LS depends on $|E|$ and $|\gamma\text{-edges}|$ but mainly on the density of the sub-graphs for each instance.
- DC depends on $|T|$ and the position of the maximum matching for each iteration.

Discussion





- The order of the edges in the link stream has an impact on the size of γ-matching
- The more one increases the value of γ the smaller the size of γ-matching
- For γ=10 the size of γ-matching=0.

Conclusion

Our result:

- Implementation of two heuristic (LS and DC).
- Generation of 3000 tests.
- # Lines of code $\approx$ 2400 (Python, bash).

Analysis:

- The order of the edges in the link stream has an impact on the results.
- The execution time does not always depend on the size of γ-edges.

`https://github.com/KatiaAMICHI/Temporal-Matching-in-Link-Streams`

Thank you,Questions?