Matrix Methods and ML Assignment 5

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July 23, 2023

A)

$$X = \begin{bmatrix} 1+\gamma & 1-\gamma \\ 1-\gamma & 1+\gamma \\ 1-\gamma & 1+\gamma \\ 1+\gamma & 1-\gamma \end{bmatrix} \frac{1}{2\sqrt{2}}$$

$$w_0 = \begin{bmatrix} \gamma+1 \\ \gamma-1 \end{bmatrix} \frac{1}{\gamma\sqrt{2}} \quad \Rightarrow \|\omega_0^2|| = \sqrt{\gamma^2+1}\left(\frac{1}{\gamma\sqrt{2}}\right) \tag{1}$$

$$\tag{2}$$

$$\gamma = .1, \quad X = \begin{bmatrix} 1.1 & .9 \\ .9 & 1.1 \\ .9 & 1.1 \\ 1.1 & .9 \end{bmatrix} \tag{3}$$

$$\tag{4}$$

$$\text{condition } \# = \frac{1.1}{.9} \simeq 1.2222 \tag{5}$$

$$\left\|\omega_1^2\right\|^2 = \sqrt{2\left(.1^2+1\right)}\left(\frac{1}{.1\sqrt{2}}\right) = 10.05 \tag{6}$$

$$\tag{7}$$

$$\gamma = 10^{-8}, \quad X = \begin{bmatrix} 1+10^{-8} & 1-10^{-8} \\ 1-10^{8} & 1+10^{-8} \\ 1-10^{-1} & 1+10^{-8} \\ 1+10^{-8} & 1-10^{-8} \end{bmatrix} \tag{8}$$

$$\text{condition } \# = \frac{1+10^{-8}}{1-10^{-8}} = 1.0000002 \tag{9}$$

$$\left\|w_0^2\right\|^2 \sqrt{2\left(10^{-16}+1\right)}\left(\frac{1}{10^{-8}\sqrt{2}}\right) = 1 \times 10^8 \tag{10}$$

$$\tag{11}$$

B)

$w = w_0 + w_e = \begin{bmatrix} \gamma+1 \\ \gamma-1 \end{bmatrix} \frac{1}{\gamma\sqrt{2}} + \begin{bmatrix} \frac{\epsilon}{2}(\gamma-1) \\ \frac{\epsilon}{2}(\gamma+1) \end{bmatrix} \frac{1}{\gamma\sqrt{2}}$
the norm of $||w_e||_2^2$ is equal to $\frac{\epsilon}{2}||w_e||_2^2$
$\|w_\epsilon\|_2^2$ when $\epsilon = .01$ and $\gamma = .1 \Rightarrow 5.025 \times 10^{-2}$ $\|w_\epsilon|_2^2$ when $\epsilon = .01$ and $\gamma = 10^{-8} \Rightarrow 5 \times 10^5$

C)

$w = VS^{-1}U^Ty$
Low ranth upprox of w

$$\begin{bmatrix} \frac{\epsilon+2}{\sqrt{2}} \\ \frac{\epsilon+2}{\sqrt{2}} \end{bmatrix}$$

Standard w

$$\begin{bmatrix} (\gamma+1)\left(1+\frac{\epsilon}{2}\right) \\ (\gamma-)\left(1+\frac{\epsilon}{2}\right) \end{bmatrix} \frac{1}{\gamma\sqrt{2}}$$

the low rank approx is not dependent on γ so it's the same for all γ