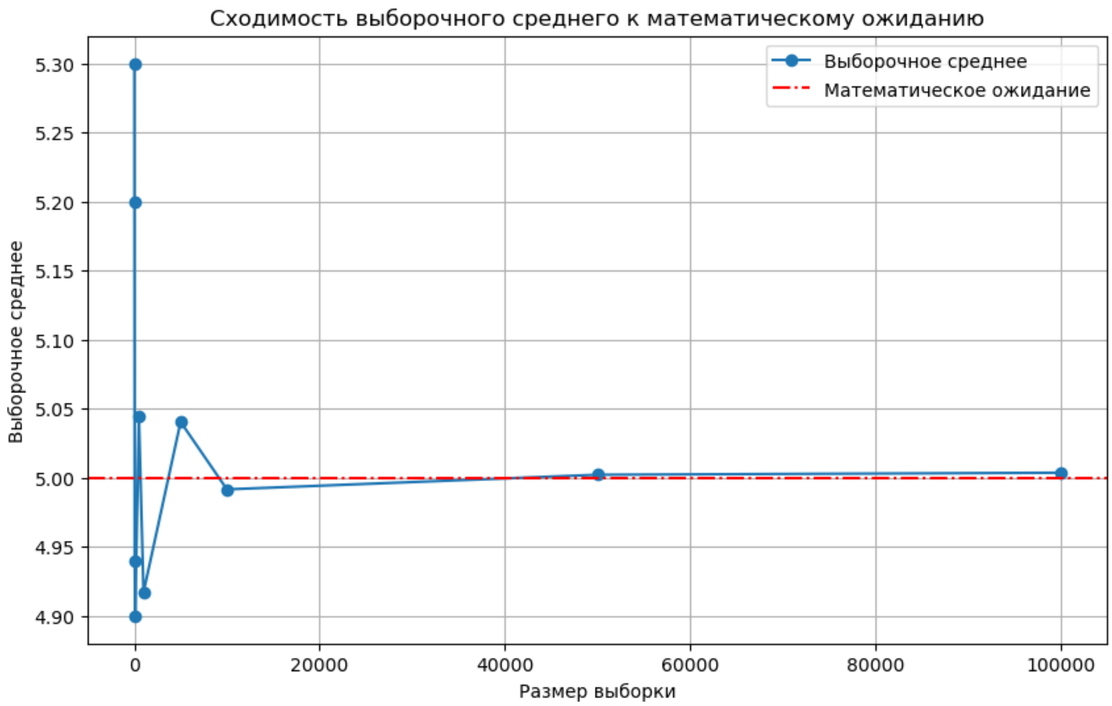
mat_hw_5

December 25, 2024

Распределение Пуассона

[14]:
```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

lambda_expected_value = 5


def sample_mean_convergence():
    sample_sizes = [10, 20, 50, 100, 500, 1000, 5000, 10000, 50000,
 ↪100000]
    means = []

    for size in sample_sizes:
        sample = np.random.poisson(lambda_expected_value, size)
        means.append(np.mean(sample))

    plt.figure(figsize=(10, 6))
    plt.plot(sample_sizes, means, marker='o', label='Выборочное
 ↪среднее')
    plt.axhline(y=lambda_expected_value, color='r', linestyle='-.',
 ↪label='Математическое ожидание')
    plt.title('Сходимость выборочного среднего к математическому
 ↪ожиданию')
    plt.xlabel('Размер выборки')
    plt.ylabel('Выборочное среднее')
    plt.legend()
    plt.grid()
    plt.show()
```

[16]:
```
sample_mean_convergence()
```



[31]:
```
def central_limit_theorem():
    sample_size = 100
    num_samples = 1000
    sample_sums = [np.sum(np.random.poisson(lambda_expected_value,
 ↪sample_size)) for _ in range(num_samples)]
    sample_sums_normalization = (np.array(sample_sums) - sample_size
 ↪* lambda_expected_value) / np.sqrt(sample_size *
 ↪lambda_expected_value)

    plt.figure(figsize=(10, 6))
    sns.histplot(sample_sums_normalization, kde=True, stat="density",
 ↪bins=25, label='Нормализованные суммы выборок')
    x = np.linspace(-4, 4, 1000)
    plt.plot(x, 1 / np.sqrt(2 * np.pi) * np.exp(-x**2 / 2),
 ↪label='Нормальное распределение', color='r')
    plt.title('Центральная предельная теорема')
    plt.xlabel('Нормализованное значение')
    plt.ylabel('Плотность вероятности')
    plt.legend()
    plt.grid()
    plt.show()
```

[32]:
```
central_limit_theorem()
```

```
c:\Users\Katie\miniconda3\envs\mltest\Lib\site-
packages\seaborn\_oldcore.py:1119: FutureWarning: use_inf_as_na option
  ↪is
deprecated and will be removed in a future version. Convert inf values
  ↪to NaN
before operating instead.
  with pd.option_context('mode.use_inf_as_na', True):
```

Доверительные интервалы

[34]:
```
import pandas as pd
from scipy.stats import t
```

[35]:
```
def generate_samples():
    small_sample = np.random.poisson(lambda_expected_value, 20)
    medium_sample = np.random.poisson(lambda_expected_value, 500)
    large_sample = np.random.poisson(lambda_expected_value, 10000)
    return small_sample, medium_sample, large_sample
```

[93]:
```
def asymptotic_cpt(sample, confidence=0.95):
    n = len(sample)
    mean = np.mean(sample)
    std_error = np.std(sample, ddof=1) / np.sqrt(n)
    z = t.ppf((1 + confidence) / 2, df=n-1)
```

```
    return round(mean - z * std_error,3), round(mean + z * std_error,
↪3)
```

[94]:
```
def exact_cpt(sample, confidence=0.95):
    return asymptotic_cpt(sample, confidence)
```

[95]:
```
def bootstrap_cpt(sample, stat_function, confidence=0.95,
↪n_bootstrap=1000):
    bootstrap_samples = np.random.choice(sample, (n_bootstrap,
↪len(sample)), replace=True)
    stat_values = np.apply_along_axis(stat_function, 1,
↪bootstrap_samples)
    lower = np.percentile(stat_values, (1-confidence)/2 * 100)
    upper = np.percentile(stat_values, (1+confidence)/2 * 100)
    return round(lower, 3), round(upper, 3)
```

[96]:
```
def summarize_intervals(sample, sample_name):
    mean_asymptotic = asymptotic_cpt(sample)
    mean_exact = exact_cpt(sample)
    mean_bootstrap = bootstrap_cpt(sample, np.mean)
    median_bootstrap = bootstrap_cpt(sample, np.median)
    mode_bootstrap = bootstrap_cpt(sample, lambda x: np.bincount(x).
↪argmax())
    variance_bootstrap = bootstrap_cpt(sample, np.var)

    return {
        "Sample": sample_name,
        "Asymptotic CPT (Mean)": mean_asymptotic,
        "Exact CPT (Mean)": mean_exact,
        "Bootstrap CPT (Mean)": mean_bootstrap,
        "Bootstrap CPT (Median)": median_bootstrap,
        "Bootstrap CPT (Mode)": mode_bootstrap,
        "Bootstrap CPT (Variance)": variance_bootstrap
    }
```

[97]:
```
small_sample, medium_sample, large_sample = generate_samples()
```

[98]:
```
intervals = [
    summarize_intervals(small_sample, "Small"),
    summarize_intervals(medium_sample, "Medium"),
    summarize_intervals(large_sample, "Large")
]
```

[99]:
```
intervals_df = pd.DataFrame(intervals)
intervals_df
```

```
[99]:    Sample Asymptotic CPT (Mean) Exact CPT (Mean) Bootstrap CPT (Mean)
 ↪ \
0   Small       (3.086, 5.614)   (3.086, 5.614)       (3.15, 5.551)
1  Medium       (4.698, 5.086)   (4.698, 5.086)      (4.706, 5.082)
2   Large       (4.975, 5.062)   (4.975, 5.062)      (4.972, 5.061)

  Bootstrap CPT (Median) Bootstrap CPT (Mode) Bootstrap CPT (Variance)
0            (2.5, 6.0)           (1.0, 8.0)             (3.74, 9.347)
1            (4.0, 5.0)           (4.0, 5.0)             (4.33, 5.426)
2            (5.0, 5.0)           (4.0, 5.0)            (4.844, 5.105)
```

1. Ширина доверительных интервалов уменьшается с ростом объема выборки.
2. Эфронов метод позволяет оценить параметры, которые трудно вычислить аналитически.